Seifert van Kampen Lecture

MAT327

Let $(G, \cdot)$ be an arbitrary group. Denote its identity element as 1, and for arbitrary $x \in G$, denote its inverse as x^{-1}, and $x^0 = 1$. Let $x^n = \underbrace{x \cdot x \cdots x}_{n}$.

Definition 1. *Suppose $\{G_\alpha\}$ is a collection of subgroups of G. These subgroups* **generate** *G if for any $x \in G$ we have*

$$x = x_1 \cdot x_2 \cdots x_n$$

where $x_i \in G_{\alpha_i}$. The above product is a **word** *representing x. A* **reduced word** *is one where no two adjacent elements in the product are in the same subgroup.*

Definition 2. *Let G have a collection $\{G_\alpha\}$ as subgroups. Suppose $G_{\alpha_1} \cap G_{\alpha_2} = \{1\}$ when $\alpha_1 \neq \alpha_2$. Then G is the* **free product** *of G_α is for any $x \in G$, there is a unique reduced word representing x:*

$$G = \dot{\prod_{\alpha \in J}} G_\alpha$$