1.一番低い次数の文字を見つけて因数分解

(1)

$$
\begin{aligned}
&9b^2+3ab-2a-4\\
&=9b^2+(3b-2)-4\\
&=\left(9b^2-4\right)+(3b-2)a\\
&=(3b+2)(3b-2)+(3b-2)a\\
&=(3b+a+2)(3b-2)
\end{aligned}
$$

(2)

$$
\begin{aligned}
&x^3+x^2y-xz^2+yz^2\\
&=x\left(x^2-z^2\right)-\left(x^2-z^2\right)y\\
&=(x-y)(x+z)(x-z)
\end{aligned}
$$

(3)

$$
\begin{aligned}
&a^3b+16-4ab-4a^2\\
&=\left(a^3b-4ab\right)-\left(4a^2-16\right)\\
&=ab\left(a^2-4\right)-4\left(a^2-4\right)\\
&=(ab-4)(a+2)(a-2)
\end{aligned}
$$

(4)

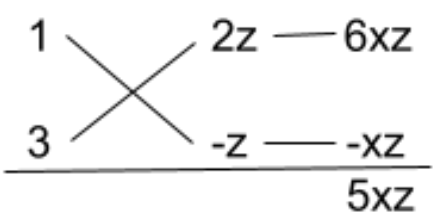
$$
\begin{aligned}
&x^3y+x^2-xyz^2-z^2\\
&=y\left(x^3-xz^2\right)+\left(x^2-z^2\right)\\
&=xy\left(x^2-z^2\right)+\left(x^2-z^2\right)\\
&=(xy+1)(x+z)(x-z)
\end{aligned}
$$

(5)

$$
\begin{aligned}
&6x^2-yz+2xz-3xy\\
&=\left(6x^2+2xz\right)-y(z+3x)\\
&=2x(3x+z)-y(3x+z)\\
&=(2x-y)(3x+z)
\end{aligned}
$$

(6)

$$
\begin{aligned}
&3x^2-2z^2+4yz+2xy+5xz\\
&=\left(3x^2+5xz-2z^2\right)+2y(2z+x)\\
&=(x+2z)(3x-z)+2y(x+2z)\\
&=(x+2z)(3x+2y-z)
\end{aligned}
$$



2.一つの文字について見て、たすき掛けを2回する

(1)

$$
\begin{aligned}
&x^2-xy-2y^2-x-7y-6\\
&=x^2-(y+1)x-\left(2y^2+7y+6\right)\\
&=x^2-(y+1)x-(y+2)(2y+3)\\
&=(x+y+2)(x-2y-3)
\end{aligned}
$$

(2)

$$
\begin{aligned}
&3x^2+7xy+2y^2-5x-5y+2\\
&=3x^2+(7y-5)x+\left(2y^2-5y+2\right)\\
&=3x^2+(7y-5)x+(y-2)(2y-1)\\
&=(x+2y-1)(3x+y-2)
\end{aligned}
$$

(3)

$$
\begin{aligned}
&x^2-2xy-3y^2+6x-10y+8\\
&=x^2-(2y-6)x-\left(3y^2+10y-8\right)\\
&=x^2-(2y-6)x-(y+4)(3y-2)\\
&=(x+y+4)(x-3y+2)
\end{aligned}
$$

(4)

$$
\begin{aligned}
&2x^2-5xy-3y^2+7x+7y-4\\
&=2x^2-(5y-7)x-\left(3y^2-7y+4\right)\\
&=2x^2-(5y-7)x-(y-1)(3y-4)\\
&=(x-3y+4)(2x+y-1)
\end{aligned}
$$

(5)

$$
\begin{aligned}
&6x^2+5xy+y^2+2x-y-20\\
&=6x^2+(5y+2)x+\left(y^2-y-20\right)\\
&=6x^2+(5y+2)x+(y-5)(y+4)\\
&=(2x+y+4)(3x+y-5)
\end{aligned}
$$

3.一つの文字を見て、同じような係数をたくさん作る

(1)

$$
\begin{aligned}
&a^2b+ab^2+b^2c+bc^2+c^2a+ca^2+2abc\\
&=(b+c)a^2+\left(b^2+2bc+c^2\right)a+bc(b+c)\\
&=(b+c)a^2+(b+c)^2a+bc(b+c)\\
&=(b+c)\left(a^2+ab+ac+bc\right)\\
&=(b+c)\left(a^2+(b+c)a+bc\right)\\
&=(b+c)(a+b)(a+c)
\end{aligned}
$$

(2)

$$
\begin{aligned}
&a^2(b-c)+b^2(c-a)+c^2(a-b)\\
&=(b-c)a^2-b^2a+c^2a+b^2c-bc^2\\
&=(b-c)a^2-\left(b^2-c^2\right)a+bc(b-c)\\
&=(b-c)a^2-(b+c)(b-c)a+bc(b-c)\\
&=(b-c)\left(a^2+(b+c)a+bc\right)\\
&=(b-c)(a-b)(a-c)
\end{aligned}
$$

(3)

$$
\begin{aligned}
&abc+ab+bc+ca+a+b+c+1\\
&=a(bc+b+c+1)+(bc+b+c+1)\\
&=(a+1)(b(c+1)+(c+1))\\
&=(a+1)(b+1)(c+1)
\end{aligned}
$$

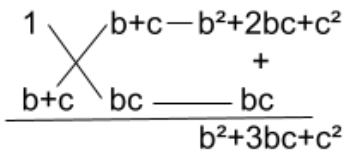
(4)

$$
\begin{aligned}
&a^2b+ab^2+a+b-ab-1\\
&=(b-1)+\left(ab^2-ab\right)+\left(a^2b+a\right)\\
&=(b-1)+ab(b-1)+a(ab+1)\\
&=(ab+1)(b-1)+a(ab+1)\\
&=(ab+1)(a+b-1)
\end{aligned}
$$

(5)

$$
\begin{aligned}
&a^2(b+c)+b^2(c+a)+c^2(a+b)+3abc\\
&=a^2(b+c)+b^2c+b^2a+c^2a+c^2b+3abc\\
&=a^2(b+c)+a\left(b^2+c^2+3bc\right)+bc(b+c)\\
&=(a+b+c)(ab+bc+ca)
\end{aligned}
$$

※aについてのたすき掛けを行う



(6)

$$
\begin{aligned}
&a^3(b-c)+b^3(c-a)+c^3(a-b)\\
&=a^3(b-c)+b^3c-b^3a+c^3a-c^3b\\
&=a^3(b-c)-\left(b^3-c^3\right)a+bc\left(b^2-c^2\right)\\
&=a^3(b-c)-(b-c)\left(b^2+bc+c^2\right)a+(b-c)(b+c)bc\\
&=(b-c)\left(a^3-\left(b^2+bc+c^2\right)a+bc(b+c)\right)\\
&=(b-c)\left(a^3-ab^2-abc-ac^2+b^2c+bc^2\right)\\
&=(b-c)\left(b^2(c-a)+b\left(c^2-ac\right)-\left(ac^2-a^3\right)\right)\\
&=(b-c)\left(b^2(c-a)+bc(c-a)-a\left(c^2-a^2\right)\right)\\
&=(b-c)\left(b^2(c-a)+bc(c-a)-a(c+a)(c-a)\right)\\
&=(b-c)(c-a)\left(b^2+bc-a(c+a)\right)\\
&=(b-c)(c-a)\left(b^2+bc-ac-a^2\right)\\
&=(b-c)(c-a)\left(c(b-a)+\left(b^2-a^2\right)\right)\\
&=(b-c)(c-a)(c(b-a)+(b+a)(b-a))\\
&=(b-c)(c-a)(b-a)(c+b+a)\\
&=-(a-b)(b-c)(c-a)(a+b+c)
\end{aligned}
$$

(7)

$$
\begin{aligned}
&ab(a+b)+bc(b+c)+ca(c+a)+3abc\\
&=a^2b+ab^2+b^2c+bc^2+c^2a+ca^2+3abc\\
&=(b+c)a^2+\left(b^2+3bc+c^2\right)a+bc(b+c)\\
&=(a+b+c)(ab+bc+ca)
\end{aligned}
$$

※(5)の問題と過程が同じになる

(8)

$$
\begin{aligned}
&a(b+c)^3+b(c-a)^3+c(a-b)^3\\
&=ab^3-ac^3+bc^3-a^3b+a^3c-b^3c\\
&=a^3(c-b)-a\left(c^3-b^3\right)+bc\left(c^2-b^2\right)\\
&=a^3(c-b)-a(c-b)\left(c^2+cb+b^2\right)+bc(c-b)(c+b)\\
&=(c-b)\left(a^3-a\left(c^2+cb+b^2\right)+bc(c+b)\right)\\
&=(c-b)\left(a^3-ac^2-abc-b^2a+bc^2+b^2c\right)\\
&=(c-b)\left(c^2(b-a)+c\left(b^2-ab\right)+\left(a^3-b^2a\right)\right)\\
&=(c-b)\left(c^2(b-a)+bc(b-a)-a\left(b^2-a^2\right)\right)\\
&=(c-b)\left(c^2(b-a)+bc(b-a)-a(b+a)(b-a)\right)\\
&=(c-b)(b-a)\left(c^2+bc-a(b+a)\right)\\
&=(c-b)(b-a)\left(c^2+bc-ab-a^2\right)\\
&=(c-b)(b-a)\left((bc-ab)+\left(c^2-a^2\right)\right)\\
&=(c-b)(b-a)((c-a)b+(c+a)(c-a))\\
&=(c-b)(b-a)(c-a)(b+c+a)\\
&=(a-b)(b-c)(c-a)(a+b+c)
\end{aligned}
$$

4.平方完成をしてから2乗-2乗の因数分解をする

(1)

$$
\begin{aligned}
&x^4+4x^2+16\\
&=\left(x^2+4\right)^2-4x^2\\
&=\left(x^2+4\right)^2-(2x)^2\\
&=\left(x^2+2x+4\right)\left(x^2-2x+4\right)
\end{aligned}
$$

(2)

$$
\begin{aligned}
&x^4-7x^2y^2+y^4\\
&=\left(x^2+y^2\right)^2-9x^2y^2\\
&=\left(x^2+y^2\right)^2-(3xy)^2\\
&=\left(x^2+3xy+y^2\right)\left(x^2-3xy+y^2\right)
\end{aligned}
$$

(3)

$$
\begin{aligned}
&4x^4+1\\
&=\left(2x^2+1\right)^2-4x^2\\
&=\left(2x^2+2x+1\right)\left(2x^2-2x+1\right)
\end{aligned}
$$

(4)

$$
\begin{aligned}
&x^4+3x^2+4\\
&=\left(x^2+2\right)^2-x^2\\
&=\left(x^2+x+2\right)\left(x^2-x+2\right)
\end{aligned}
$$

(5)

$$
\begin{aligned}
&x^4+11x^2y^2+y^4\\
&=\left(x^2-y^2\right)^2-9x^2y^2\\
&=\left(x^2-y^2\right)^2-(3xy)^2\\
&=\left(x^2+3xy-y^2\right)\left(x^2-3xy-y^2\right)
\end{aligned}
$$

(6)

$$
\begin{aligned}
&x^4-9x^2y^2+16y^4\\
&=\left(x^2-4y^2\right)^2-x^2y^2\\
&=\left(x^2+xy-4y^2\right)\left(x^2-xy-4y^2\right)
\end{aligned}
$$

(7)

$$
\begin{aligned}
&4x^4+11x^2y^2+9y^4\\
&=\left(2x^2+3y^2\right)^2-x^2y^2\\
&=\left(2x^2+3y^2\right)^2-(xy)^2\\
&=\left(2x^2+xy+3y^2\right)\left(2x^2-xy+3y^2\right)
\end{aligned}
$$

5.まとめ

(1) 次数の小さいものをくくる

$$
\begin{aligned}
&a^3+a^2b-a\left(c^2+b^2\right)+bc^2-b^3\\
&=a^3+a^2b-ac^2-ab^2+bc^2-b^3\\
&=-c^2(a-b)+\left(a^3-b^3\right)+ab(a-b)\\
&=(a-b)\left(-c^2+\left(a^2+ab+b^2\right)+ab\right)\\
&=(a-b)\left(-c^2+a^2+2ab+b^2\right)\\
&=(a-b)\left(\left(a^2+2ab+b^2\right)-c^2\right)\\
&=(a-b)\left((a+b)^2-c^2\right)\\
&=(a-b)(a+b+c)(a+b-c)
\end{aligned}
$$

(2) 展開して一つの文字について見る

$$
\begin{aligned}
&a(b+c)^2+b(c+a)^2+c(a+b)^2-4abc\\
&=ab^2+ac^2+bc^2+ba^2+ca^2+cb^2+2abc\\
&=a^2(b+c)+a\left(b^2+2bc+c^2\right)+bc(b+c)\\
&=a^2(b+c)+a(b+c)^2+bc(b+c)\\
&=(b+c)\left(a^2+a(b+c)+bc\right)\\
&=(b+c)(a+c)(a+b)\\
&=(a+b)(b+c)(c+a)
\end{aligned}
$$

(3) 一つの文字について見る

$$
\begin{aligned}
&a^2b-ab^2-b^2c+bc^2-c^2a-ca^2+2abc\\
&=a^2(b-c)-a\left(b^2-2bc+c^2\right)-bc(b-c)\\
&=a^2(b-c)-a(b-c)^2-bc(b-c)\\
&=(b-c)\left(a^2-a(b-c)-bc\right)\\
&=(b-c)(a-b)(a+c)\\
&=(a-b)(b-c)(c+a)
\end{aligned}
$$

6.まとめ

(1) 展開して一つの文字について見る

$$
\begin{aligned}
&(x+y)(y+z)(z+x)+xyz \\
&=xz^2+y^2z+yz^2+x^2y+x^2z+xy^2+3xyz \\
&=x^2(y+z)+x\left(z^2+3yz+y^2\right)+yz(y+z) \\
&=(x+y+z)(x(y+z)+yz) \\
&=(x+y+z)(xy+yz+zx)
\end{aligned}
$$

(2) 一番次数の低いものをくくる

$$
\begin{aligned}
&6a^2b-5abc-6a^2c+5ac^2-4bc^2+4c^3 \\
&=b\left(6a^2-5ac-4c^2\right)-\left(6a^2c-5ac^2-4c^3\right) \\
&=b\left(6a^2-5ac-4c^2\right)-c\left(6a^2-5ac-4c^2\right) \\
&=(b-c)\left(6a^2-5ac-4c^2\right) \\
&=(b-c)(2a+c)(3a-4c)
\end{aligned}
$$

(3) 同じような係数を多くつくる(-4ab=-2ab+(-2ab))

$$
\begin{aligned}
&\left(a^2-1\right)\left(b^2-1\right)-4ab \\
&=a^2b^2-a^2-b^2+1-4ab \\
&=\left((ab)^2-2ab+1\right)-\left(a^2+2ab+b^2\right) \\
&=(ab-1)^2-(a+b)^2 \\
&=(ab-1+a+b)(ab-1-a-b) \\
&=(ab+a+b-1)(ab-a-b-1)
\end{aligned}
$$