# Wave propagation in blood vessels

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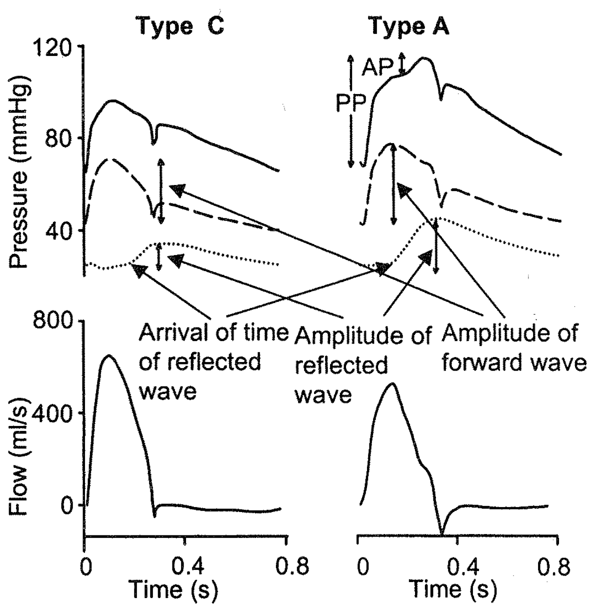
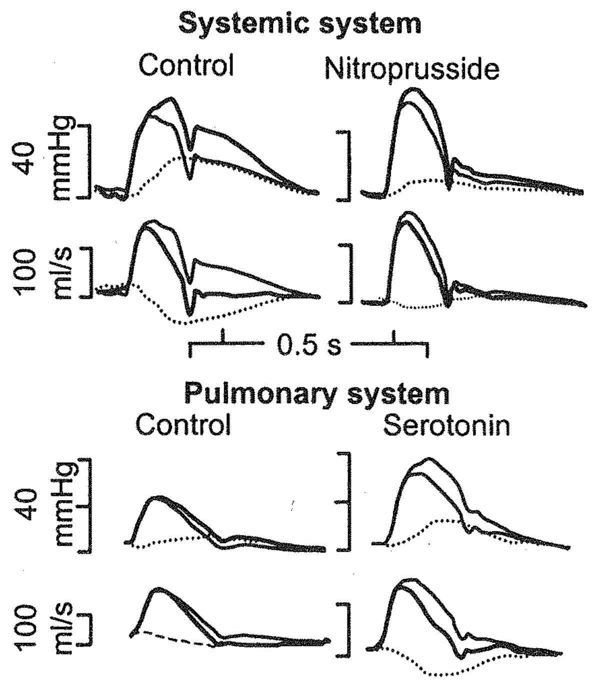
## Wave separation: physiological and clinical relevance




- Useful for quantification of of wave reflection
- Young healthy adult (C)
  - Small amplitude of reflected wave
  - Late arrival
  - No significant increase in late systole
- Old subject (A)
  - Higher amplitude of reflected wave
  - Arrives earlier
  - Significant late systolic peak

# Impact of vascular bed on reflections




- Reflections less significant in pulmonary circulation than in the systemic arterial tree
- Nitroprusside
  - Dilates *systemic* bed
  - Reflections decrease
- Serotonin
  - Constricts *pulmonary* bed
  - Reflections increase
- Pressure and flow waves are similar when reflections are small

# Validity of wave equations

- Caution
  - Wave results valid for straight, cylindrical, elastic tube with a liquid not flowing?
  - Flow requires pressure gradient
  - Variable pressure yields variable wall stresses and tube will taper
  - Variable incremental E-modulus as the vessel wall is nonlinear
- Valid for
  - Small taper
  - Small variations in E-modulus

## Progressive waves superimposed on steady flow

- Black board derivation

## Laminar flow in straight pipe

Rigid pipe, stationary, incompressible Newtonian fluid
Look for: $v_x = u(r)$, $v_r = v_\theta = 0$
Cauchy-equation

$$\frac{\partial p}{\partial x} = \frac{1}{r}\,\frac{\partial}{\partial r}\left(r\,\tau_{xr}\right), \quad \tau_{xr} = \mu\frac{\partial u}{\partial r}, \quad \frac{\partial p}{\partial x} = konst.$$

By integration

$$u(r) = -\frac{1}{4\mu}\,(a^2 - r^2)\,\frac{dp}{dx} \tag{1}$$

Flow:

$$Q = 2\pi \int_0^a u\,r\,dr, \quad R = -\frac{\pi a^4}{8\mu}$$

$$\boxed{Q = R\,\frac{dp}{dx}} \tag{2}$$

# Simplified solution for flow in compliant vessel

- Navier-Stokes equations for blood
- Navier equations for vessel wall
- Must be solved simultaneously

Simplification

- Poiseuille flow for the fluid
- p(A) relation for vessel wall

# Poiseuille

$$\frac{dp}{dx} = -\frac{8\mu}{\pi a^4}\,Q = -\frac{8\pi\mu}{A^2}\,Q$$

Compliance $\qquad C = \frac{\partial A}{\partial p}$

$$\frac{dp}{dx} = \frac{\partial p}{\partial A}\frac{dA}{dx} = \frac{1}{C}\frac{dA}{dx} = -\frac{8\pi\mu}{A^2}\,Q$$

$$A^2\frac{dA}{dx} = \frac{1}{3}\frac{d}{dx}\left(A^3\right) = -8\pi\mu CQ$$

Integration

$$A(x)^3 = A(0)^3 - 24\pi\mu CQx$$

Constitutive model: $A(p) = A_0 + C\,(p - p_0)$

# Pressure and flow for stationary flow in compliant vessel

$$Q(x) = \frac{A(0)^3 - A(x)^3}{24\pi\mu C x}, \quad p(x) = p_0 + \frac{A(x) - A(0)}{C}$$