# PROBLEM SET 11

TKT4150 Biomechanics

**Main topics:** Arts heart model.

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## 1 Arts model (exam 2011)

*We consider a simplified model of left ventricular dynamics: the Arts model. In this model, the left ventricle is assumed to be a thick-walled cylinder of thickness consisting of many thin-walled cylinders of thickness dr. Let's assume that the stresses in a cylindrical surface of radius r can be expressed in the cylindrical coordinate system ($\mathbf{e}_r$, $\mathbf{e}_\theta$, $\mathbf{e}_z$), where the z-direction is parallel to the axis of symmetry of the left ventricle, as:*

$$\sigma_r = -p \tag{1}$$
$$\sigma_\theta = -p + \sigma cos^2\alpha \tag{2}$$
$$\sigma_z = -p + \sigma sin^2\alpha \tag{3}$$
$$\tag{4}$$

*where $\sigma$ is the stress in the myocardial muscle fiber oriented in the direction $\mathbf{n} = cos\alpha\mathbf{e}_\theta + sin\alpha\mathbf{e}_z$. The model is shown in Figure 1. Assume that we consider one such thin-walled cylinder, with thickness dr. Two equilibrium sketches are given in Figure 2, corresponding to a thin-walled container. Hint: when $d\theta$ is small, $d\theta = sind\theta$.*

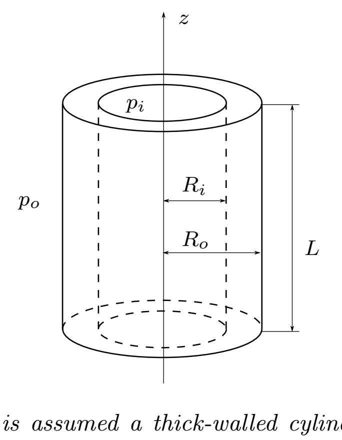


***Figure 1:*** *The left ventricle is assumed a thick-walled cylinder in Arts model, consisting of several thin-walled cylinders. The outer and inner radii $R_o$ and $R_i$; the outer and inner pressures $p_o$ and $p_i$; and the length of the cylinder $L$, are denoted on the figure.*

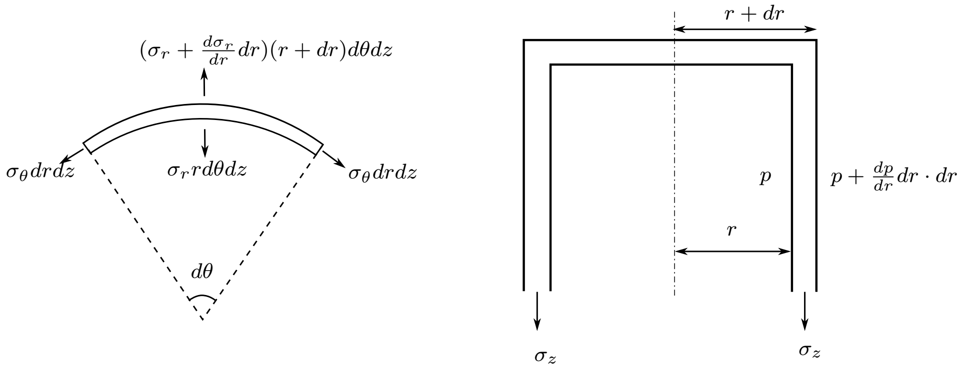


***Figure 2:*** *Equilibrium sketches of thinwalled piece.*

a) Write the equilibrium in the r-direction and show that:

$$\frac{dp}{dr} = \frac{-\sigma cos^2\alpha}{r} \tag{5}$$

b) Write the equilibrium in the z-direction and show that:

$$\frac{dp}{dr} = \frac{-2\sigma sin^2\alpha}{r} \tag{6}$$

c) Express $\frac{dp}{dr}$ with respect to $\sigma$ and $r$ only.

d) Let's introduce the following relations:

| | | |
|---|---|---|
| Left ventricular pressure: | $p_{LV} = p_o - p_i$ | (7) |
| Volume of left ventricular wall: | $V_W = \pi L(R_o^2 - R_i^2)$ | (8) |
| Volume of left ventricular cavity: | $V_{LV} = \pi L R_i^2$ | (9) |

Integrate the result found in c) from $R_i$ to $R_o$ and express a relationship between $p_{LV}$, $\sigma$, $V_W$ and $V_{LV}$.