Arterial stenosis

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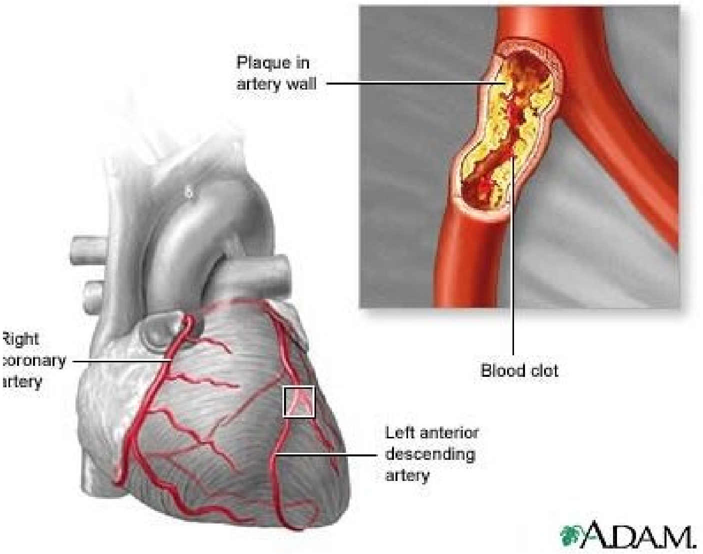
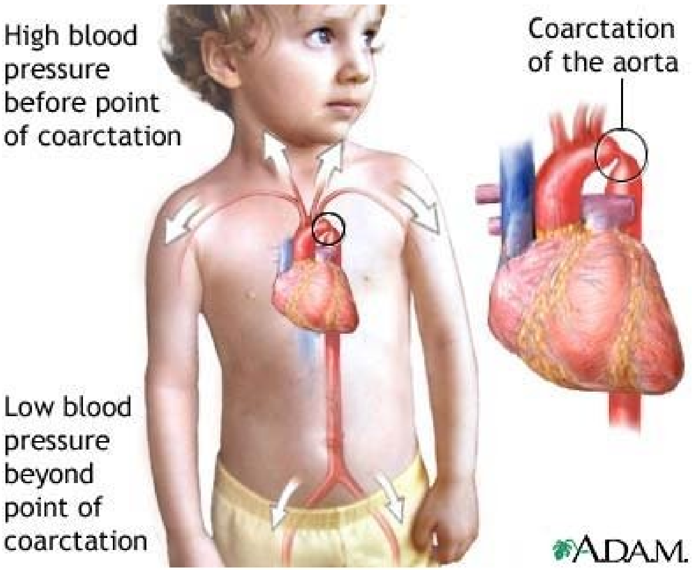
Outline

Atherosclerosis



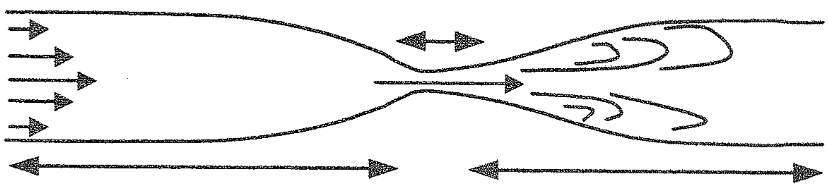
- Fatty material is deposited in the vessel wall
- Impairment of blood flow
- Symptoms: Chest pain
- No symptom until complication occurs

Coarctation of the aorta



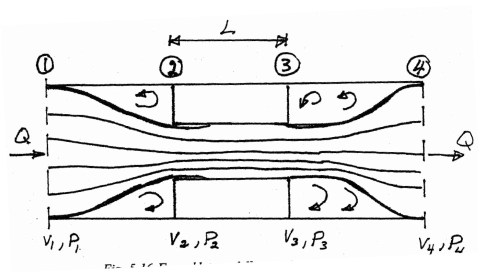
- Narrowing of the aorta
- Birth defect
- Normally
  - High blood pressure in upper body and arms
  - Low blood pressure in lower body and legs
- Symptoms
  - Localized hypertension
  - Cold feet/legs
  - Decreased exercise performance
  - Heart failure

Geometrical description of a stenosis/coarctation



- Converging section. Bernoulli applies
- Narrow long section. Poiseuille's law applies.
- Diverging section. Turbulent with significant losses.

Stenosis pressure loss derivation



- Conservation of mass

$$
\begin{aligned}
v_1 = v_4 \quad &= \frac{Q}{A_0} \\
v_2 = v_3 \quad &= \frac{Q}{A_s}
\end{aligned}
$$

- 1 to 2 Bernoulli

$$p_1 - p_2 = \frac{\rho Q^2}{2A^2}\left(\left(\frac{A}{A_s}\right)^2 - 1\right)$$

- 2 to 3 Poiseuille

$$p_2 - p_3 = \frac{8\pi\mu l_s}{A_s^2} Q$$

- 3 to 4 Euler's first axiom

$$
\begin{aligned}
p_3 A - p_4 A &= \rho v^2 A - \rho v_S^2 A_s \\
&= \rho \frac{Q^2}{A}\left(1 - \frac{A}{A_s}\right)
\end{aligned}
$$

The pressure loss in a stenosis

$$\Delta p = \underbrace{\frac{8\pi\mu l_s}{A_s^2}\, Q}_{Poiseuille} + \underbrace{\frac{K_t\,\rho}{2A_0^2}\left(\frac{A_0}{A_s}-1\right)^2 Q^2}_{Borda-Carnot} = a_1\, Q + a_2\, Q^2$$

- $K_t \approx 1.5$ empirical coefficient

Physiological and clinical relevance of arterial stenosis

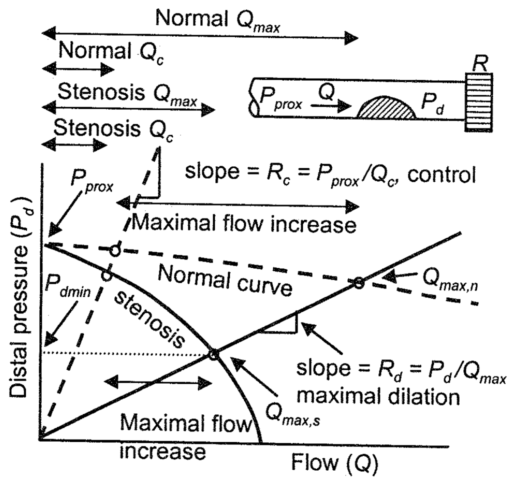
- $\Delta p \propto A_0^2$ and Q^2 for stenosis
- $\Delta p \propto Q$ for laminar straight pipe flow

- Example I
  - Patient with mild coarctation
  - $\Delta p \approx 10\ mmHg$ at rest
  - Walking $\Rightarrow$ peripheral bed dilates to increase pressure drop and perfusion
  - Increase to $3Q$ $\Rightarrow \Delta p \approx 90\ mmHg$ ...impossible
  - Dilatation does not increase flow sufficiently

- Example II
  - $\Delta p \propto \left(\frac{A_0}{A_s}-1\right)^2$
  - 80% area stenosis $\left(\frac{A_0}{A_s}-1\right)^2 = 16$
  - 90% area stenosis $\left(\frac{A_0}{A_s}-1\right)^2 = 81$
  - 90% is approximately 5 times more severe than 80%

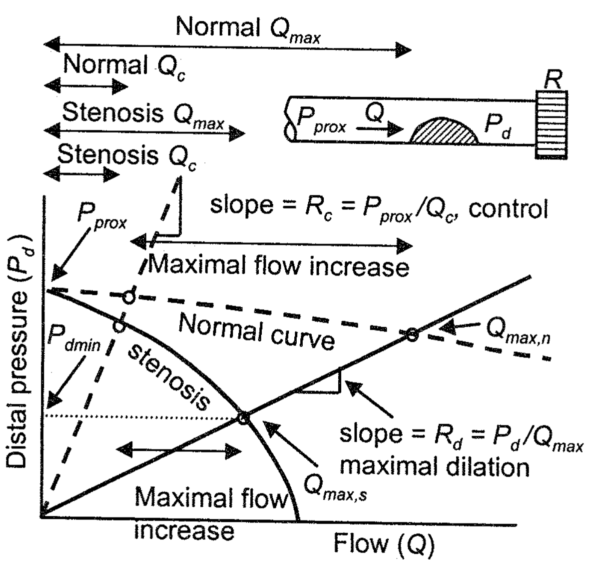
Flow reserve (FR)

- Motivation
  - Angiography gives inaccurate functional information
  - Methods for functional description needed



- Definition
  - $FR = Q_{\max}/Q_c$
  - $Q_{\max}$ = max dilatation flow
  - Q_c = control flow
- Illustration
  - P_d distal (downstream) to stenosis
  - Dilatation $\Rightarrow R_c \downarrow R_d$
  - Stenosis $\Rightarrow P_d \downarrow$ and $Q_{\max,s} \downarrow$
- Stenosis impact
  - Limits flow for max dilatation
  - $\Rightarrow FR \downarrow$
  - $FR(stenosis, R_{c,d})$
  - Negligible in control
  - Severe for max dilatation

Fractional Flow Reserve (FFR)



$$FFR = \frac{Q_{\max,s}}{Q_{\max,n}} = \frac{(p_d - p_v)/R_s}{(p_{prox} - p_v)/R_n} \approx \frac{p_d}{p_{prox}}$$

- Assumptions
  - Microvascular resistance equal for normal and stenosed conditions
  - Venous pressure is small with respect to p_d, $p_v \leq p_d$
- FFR < 0.74 is pathological

Summary

- Stenosis (coarctation)
  - Narrow section of a blood vessel
- $\Delta p \propto \left(\frac{A_0}{A_s}\right)^2$
  - 90% occlusion much worse than 80%
- $\Delta p \propto Q^2$
  - Small influence at rest
  - Severe influence for max dilatation
- Flow reserve $FR = Q_{\max}/Q_c$
- Fractional Flow Reserve $FFR = \frac{Q_{\max,s}}{Q_{\max,n}} \approx \frac{p_d}{p_{prox}}$