PROBLEM SET 1

TKT4150 Biomechanics

Main topics: Eulerian and Lagrangian coordinate systems. Extensive and intensive properties. Reynolds' transport theorem. Cauchy's theorem. Cauchy equilibrium equations.

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1 Eulerian vs. Lagrangian coordinate systems

In continuum mechanics, a coordinate system has to be defined. Two main alternatives exist: Eulerian and Lagrangian.

a) Explain the difference between the two coordinate systems, and provide a simple drawing to support it.

b) Based on the different coordinate systems, we get different kinds of derivatives. Show how you can use the chain rule to establish the *material derivative*.

2 Reynolds' transport theorem

The fundamental principles in mechanics; mass, momentum and energy conservation, are all valid for particle mechanics. In cases where it is inconvenient or impossible to track particles in a system, a different approach is sought. The equations representing these principles can be transformed to be valid for a predefined bounded volume; a control volume. This is done using the Reynolds' transport theorem.

a) Explain the differences between an *extensive* and an *intensive* property.

b) Derive Reynolds' transport theorem.

c) Use Reynolds' transport theorem to establish the control volume version of the mass conservation equation.

d) Consider Figure 1. Apply Reynolds' transport theorem applied to momentum to determine the pressure drop from A_1 to A_2. Show your control volume. You may neglect viscous effects along the wall of the pipe.

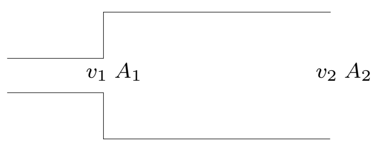

Figure 1: *Fluid flows through a narrow pipe with area A_1 at velocity v_1 into a larger pipe with area A_2 with a velocity v_2 shortly after the junction.*

3 Velocity field

A velocity field is given by

$$v_1 = \frac{-\alpha x_1}{t_0 - t}, \quad v_2 = \frac{\alpha x_2}{t_0 - t}, \quad v_3 = 0 \tag{1}$$

where α and t_0 are constants.

a) Show that the flow is volume preserving, or *isochoric*.

b) Determine the local acceleration, the convective acceleration and the particle acceleration $\dot{\mathbf{v}}$.