PROBLEM SET 5

TKT4150 Biomechanics

Main topics: Hyper-elasticity.

$\boxed{1}$ **Mechanics of hyper-elastic rabbit skin**

The skin of a rabbit is modelled using a hyper-elastic model, where $S_{ij} = \rho_0 \frac{\partial \phi}{\partial E_{ij}}$.
Assume the strain energy density (per unit mass) is given by:

$$\phi = \frac{1}{2\rho_0}[\alpha_1 E_{11}^2 + \alpha_1 E_{22}^2 + \alpha_3 E_{12}^2 + \alpha_3 E_{21}^2 + 2\alpha_4 E_{11}E_{22}+ \tag{1}$$

$$c \cdot \exp(a_1 E_{11}^2 + a_2 E_{22}^2 + a_3 E_{12}^2 + a_3 E_{21}^2 + 2a_4 E_{11}E_{22})] \tag{2}$$

where

$$\alpha_1 = 1020Pa, \quad \alpha_3 = 500Pa, \quad \alpha_4 = 254Pa, \quad c = 0.779Pa \tag{3}$$

$$a_1 = 3.79, \quad a_2 = 12.7, \quad a_3 = 1.25, \quad a_4 = 0.587 \tag{4}$$

and ρ_0 is the mass density of the skin. It is further assumed that the rabbit skin is incompressible.
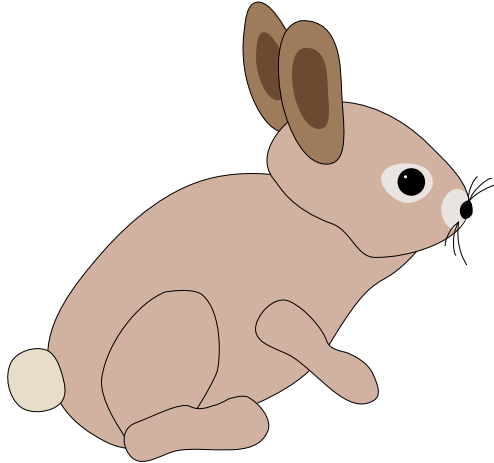

***Figure 1:** Rabbit.*

a) Establish the expressions for the Piola-Kirchoff stress components S_{ij} as expressions of the strain components E_{ij}.

b) Choose $E_{12} = E_{22} = 0$. Determine the Cauchy stresses T_{11} and T_{22}, as functions of the stretch λ_1 and the strain component E_{11}.

c) Assume a state of pure shear:

$$x_1 = X_1 + \gamma X_2, \quad x_2 = X_2, \quad x_3 = X_3 \tag{5}$$

Determine the Cauchy stresses T_{ij} in terms of the Piola-Kirchoff stresses S_{ij} for this deformation state. Which components of the Green strain tensor affect the resulting Piola-Kirchoff and Cauchy stresses? What happens to the Cauchy stresses when we assume small deformations?

d) Establish an expression for the shear strain γ_{12} (expressed in terms of γ), given the same deformation state.

e) Choose $E_{12} = E_{22} = 0$. Plot the function $T_{11}(\lambda_1)$ in the interval $1 < \lambda_1 < 2$.