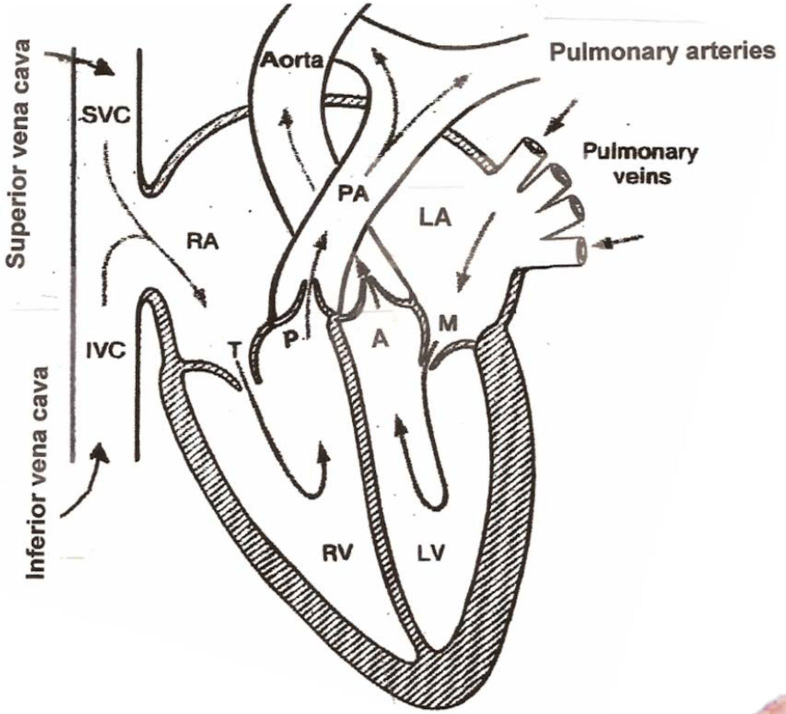
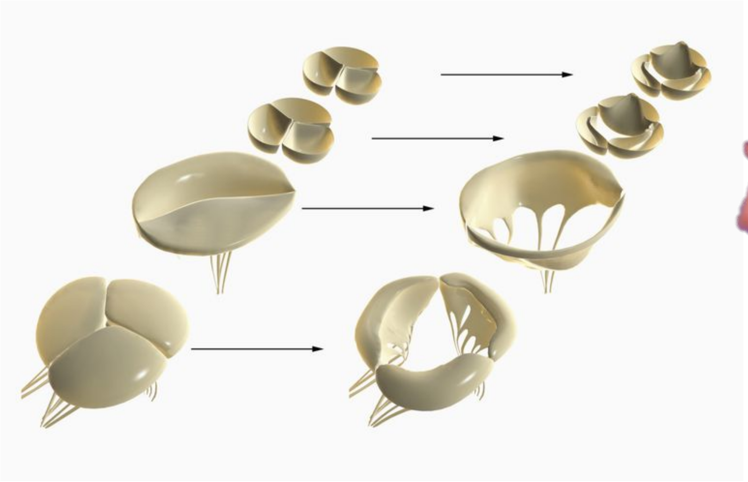
Hearth Models

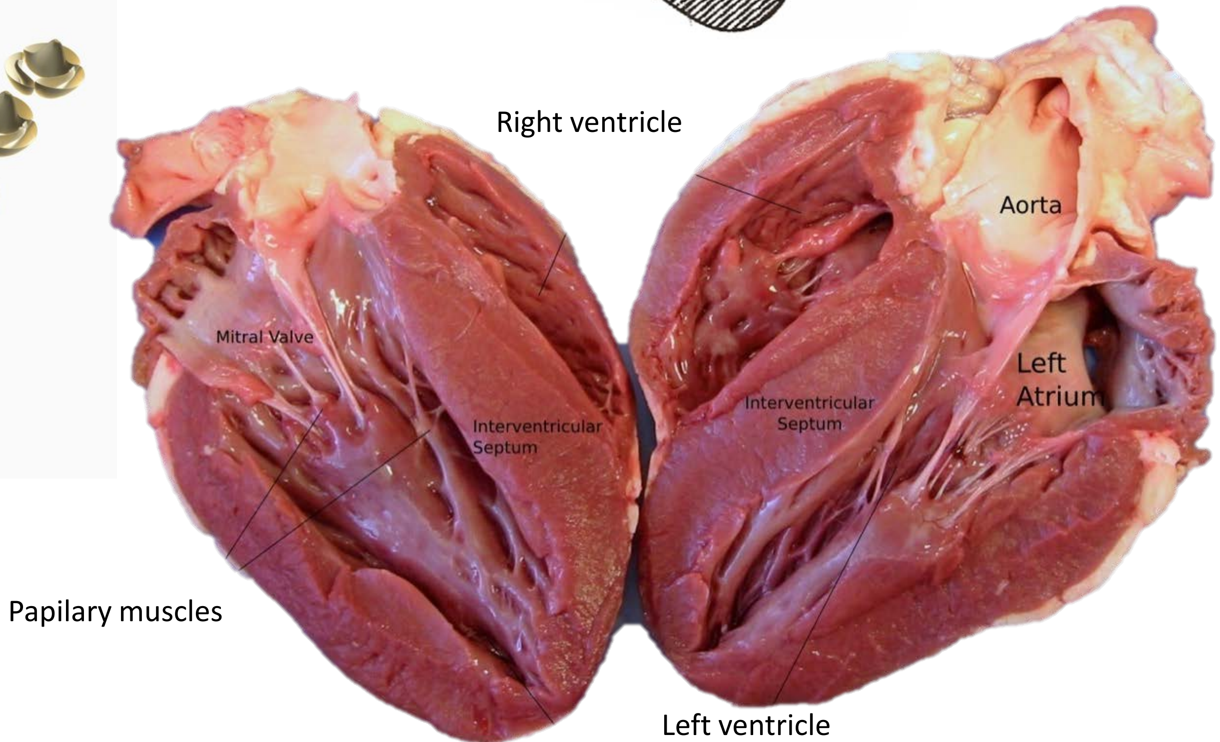
Biomechanics lecture

Anatomy of the human heart

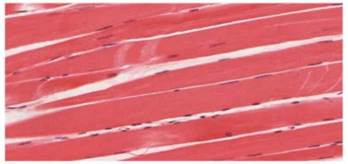
- **Hearth** chambers
  - Right atrium
  - Left atrium
  - Right ventricle
  - Left ventricle
- Valves
  - aortic valve
  - pulmonary valve
  - mitral valve
  - tricuspid valve
- Main cycles
  - Diastole (filling)
  - Systole (ejection)



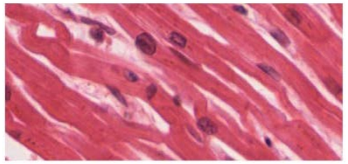




Anatomy of the human heart

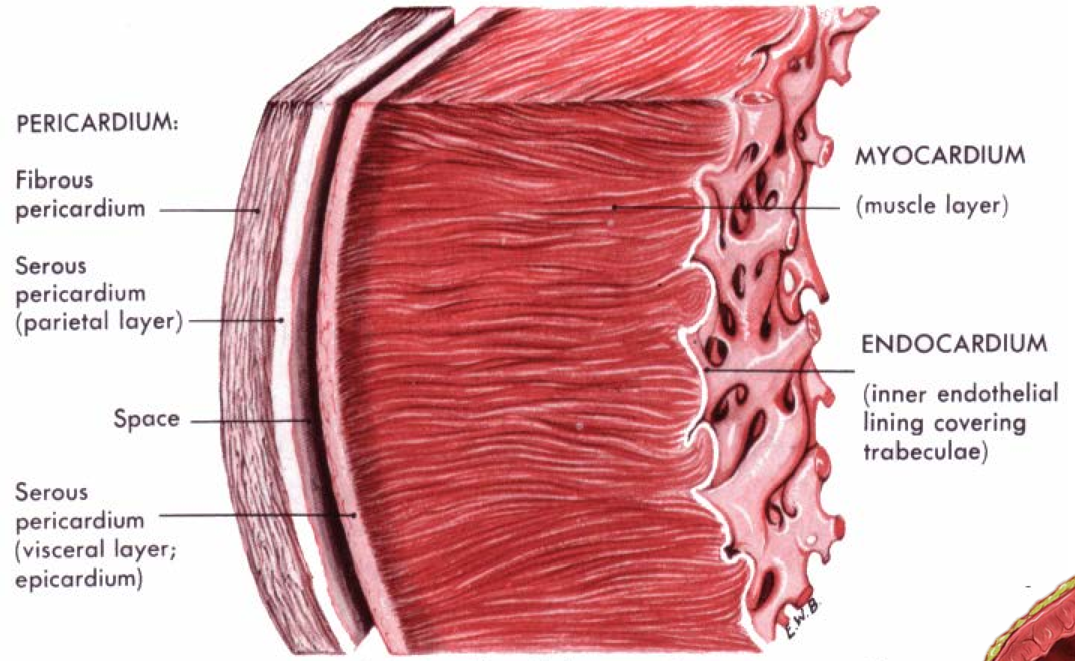


Skeletal muscle



Myocardium

- Thee layers
  - Pericardium
  - Myocardium
  - Endocardium

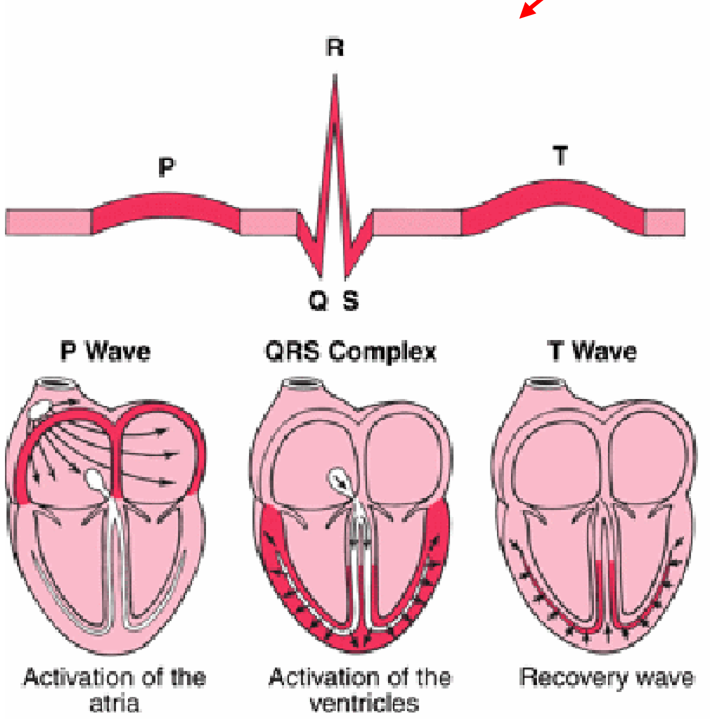


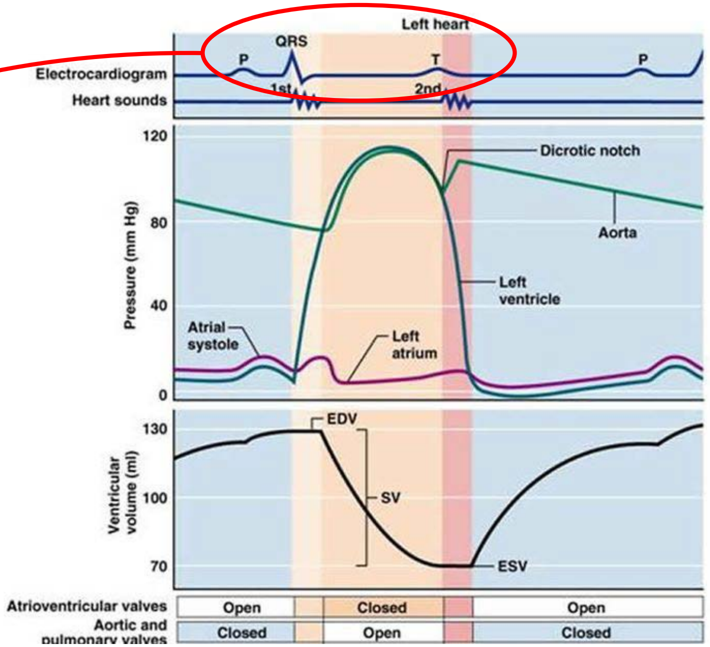
Section of the heart wall showing the components of the outer pericardium (heart sac), muscle layer (myocardium), and inner lining (endocardium).

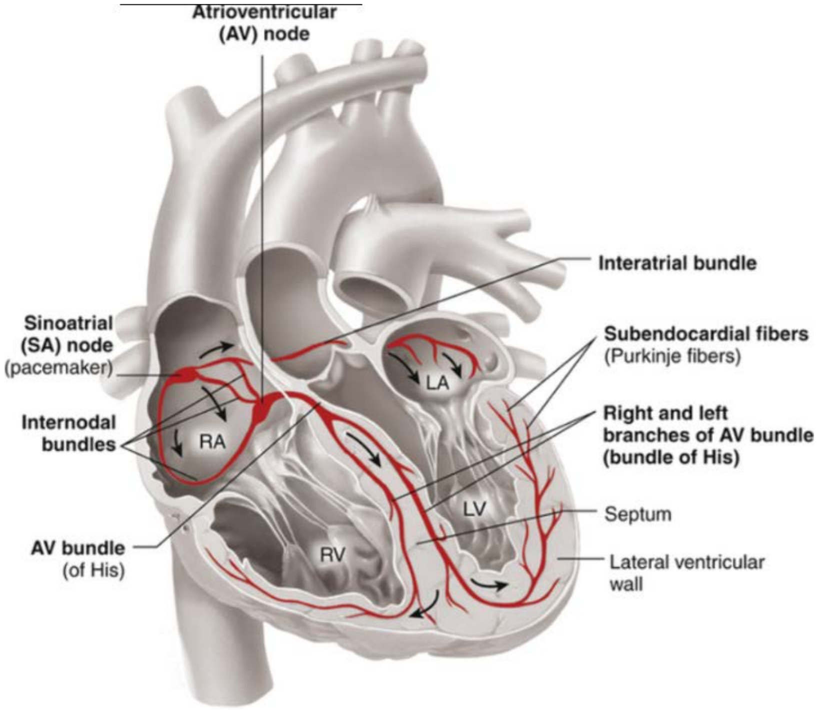
- Hearth muscles
  - ➢ Myocardium

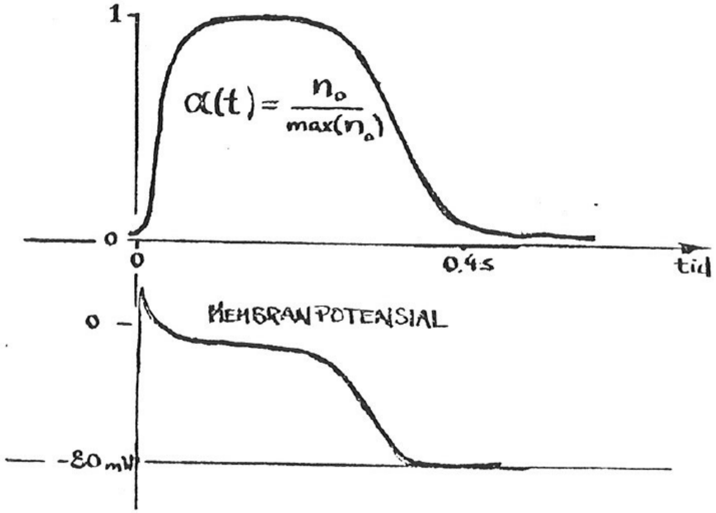
Anatomy of the human heart

- Electrical activity of the heart



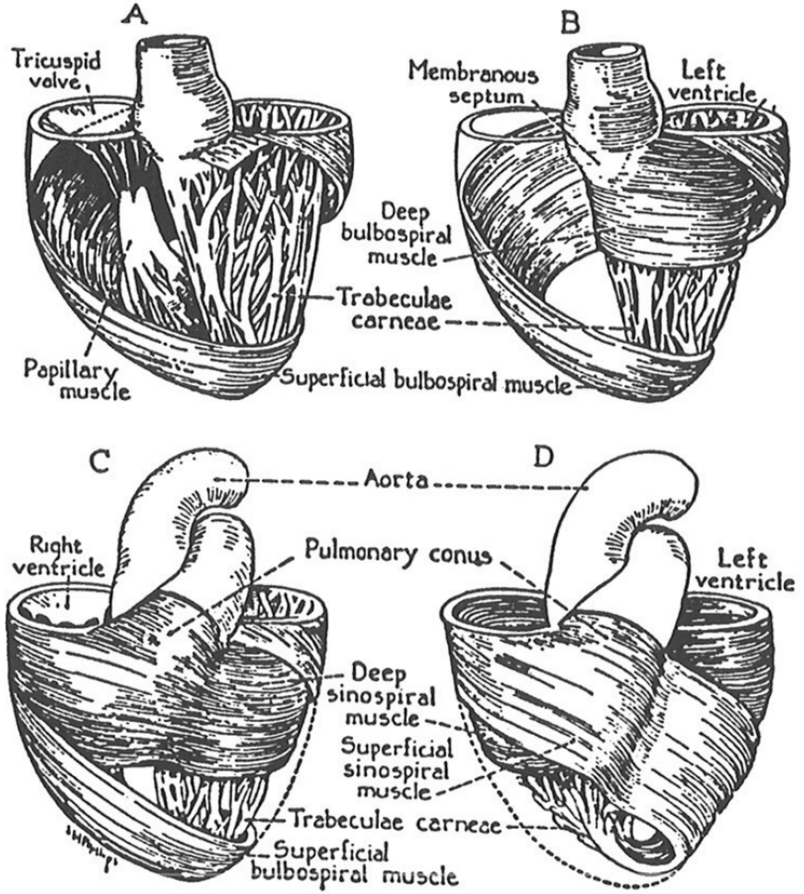


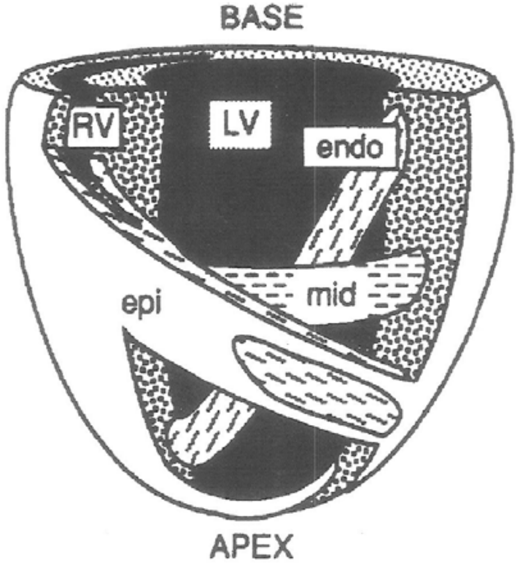




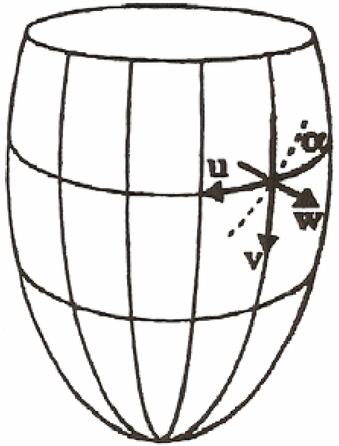
Anatomy of the human heart

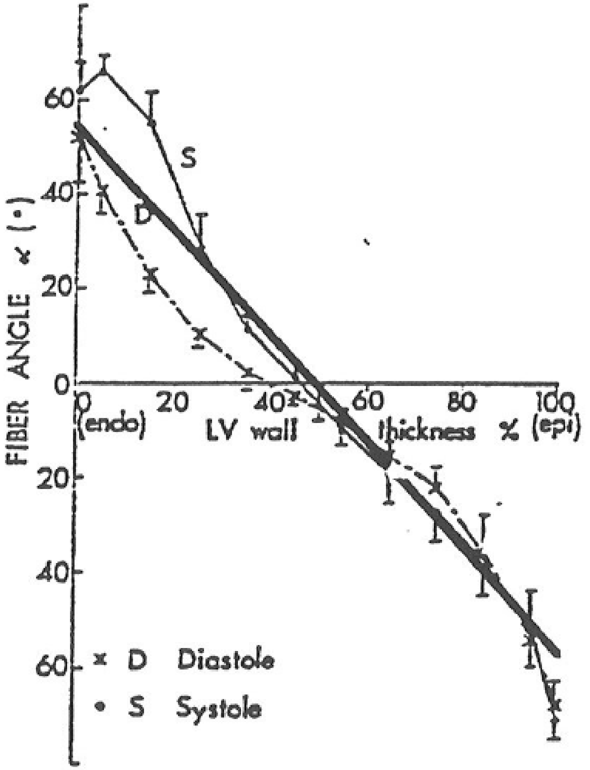
- Myocardium fiber directions





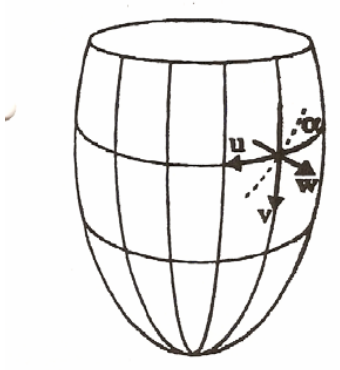
Arts et al, 1979.

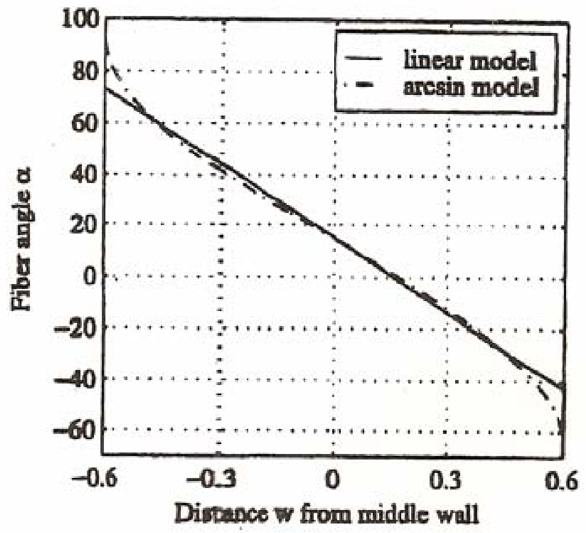


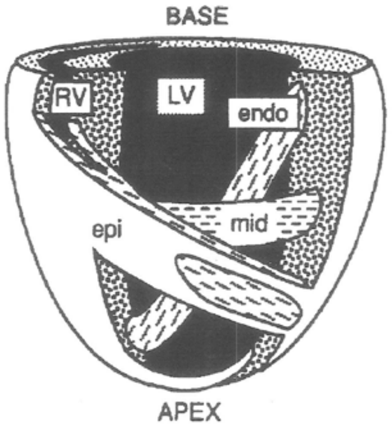


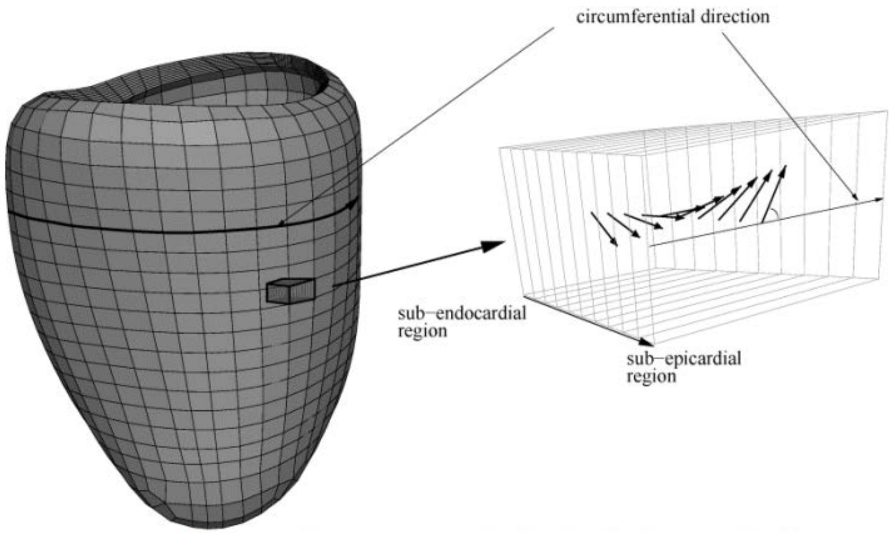
Models of the left ventricle

- **The early, simple models** of the left ventricle assume a thin-walled chamber obeying Laplace's law
- **In a newer generation of models**, the left ventricle is considered to be a thick-walled spherical or ellipsoidal shell having isotropic material properties
- **A more accurate description** of the geometry of the LV requires the use of finite element methods, applied to an ellipsoidal or more realistic geometry with isotropic or anisotropic material properties





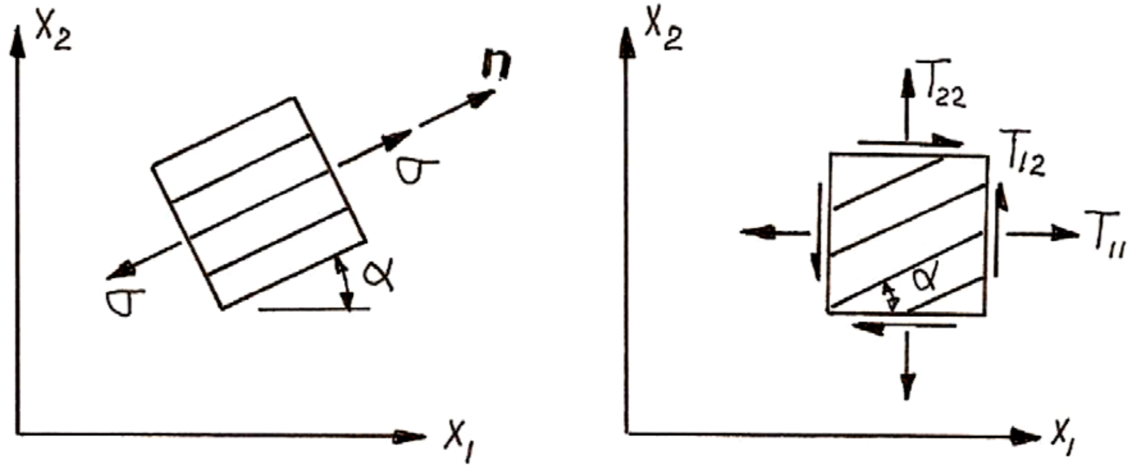




Finite element model of a left ventricle

The fluid-fiber continuum

- Chadwick et al, 1982, proposed a mechanical model of the LV in which he regarded the myocardium as a fluid-fiber continuum.

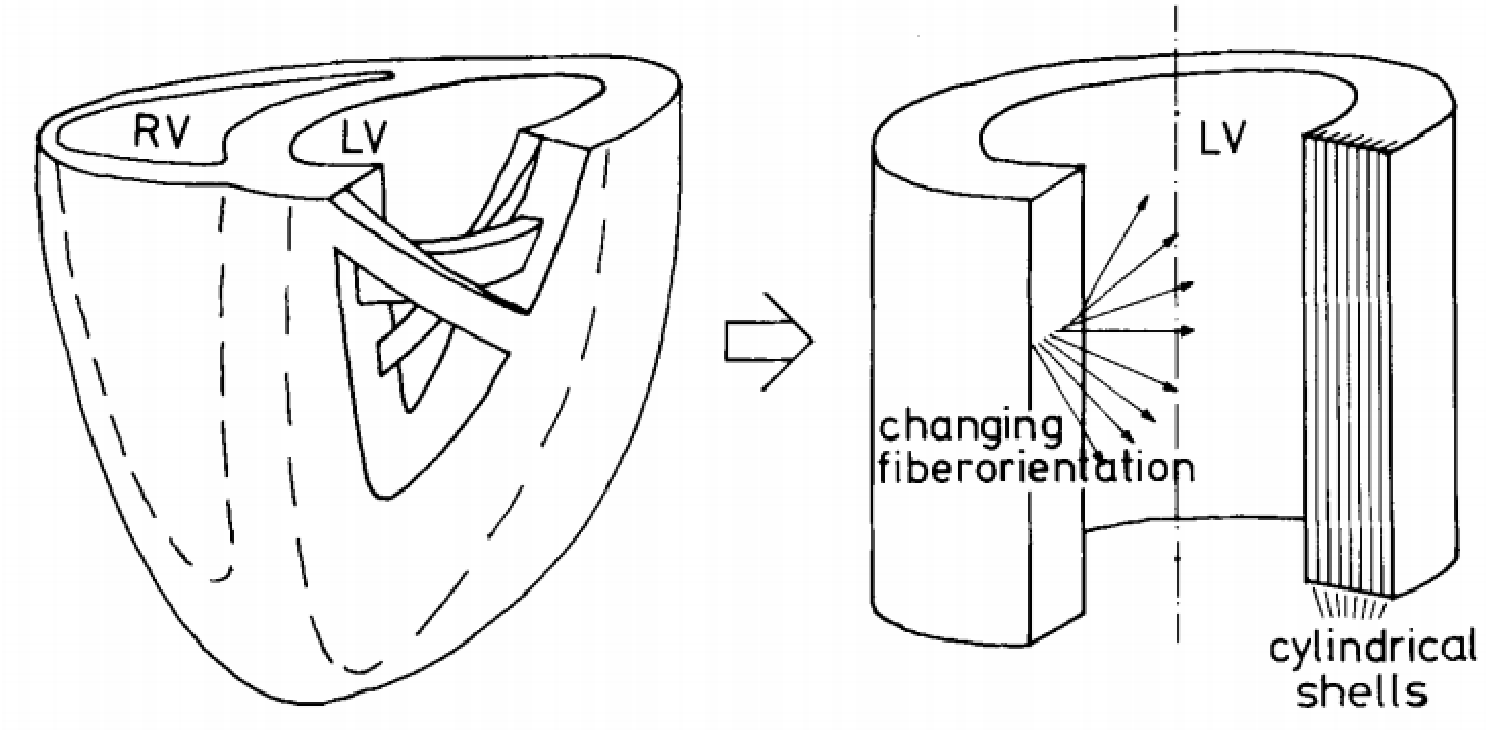


$$T_{ij} = -p\,\delta_{ij} + \sigma n_i n_j$$

$$T_{11} = -p + \sigma\cos^2\alpha \;,\quad T_{22} = -p + \sigma\sin^2\alpha \;,\quad T_{33} = -p \;,\quad T_{12} = \sigma\sin\alpha\cos\alpha$$

The Arts model of the left ventricle

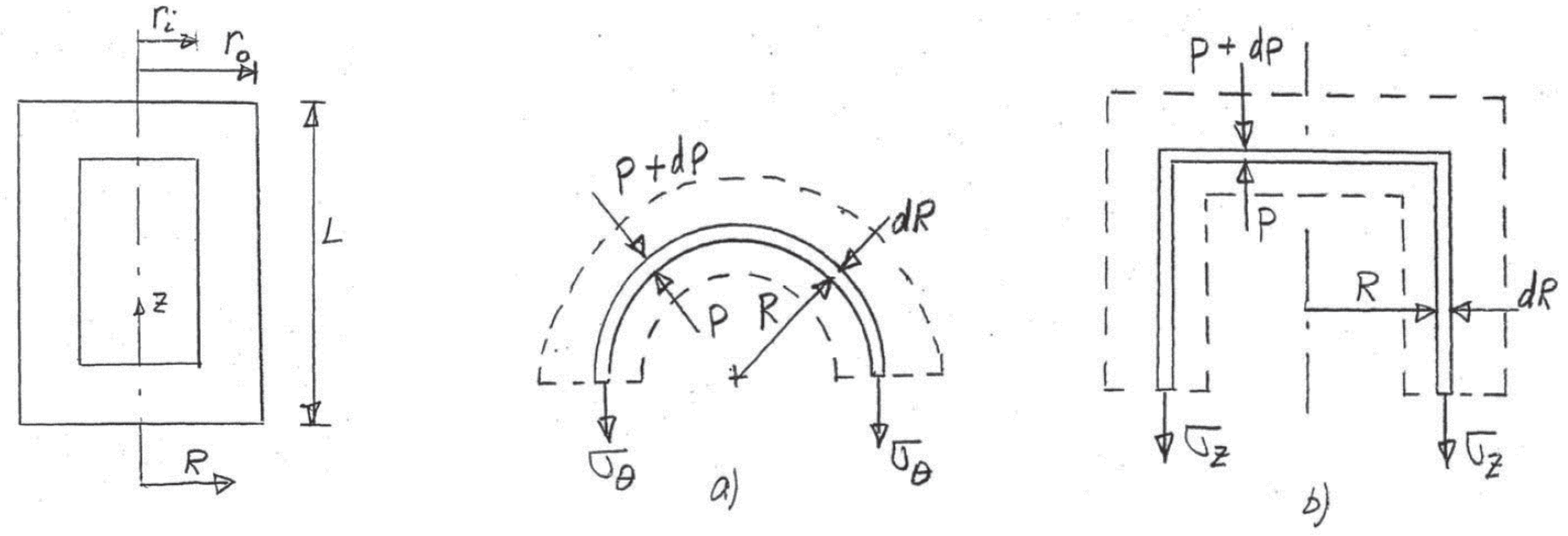
- Arts et al, 1979, the left ventricle is a modeled as a thick-walled cylinder composed of 8 concentric cylindrical shells



The Arts model of the left ventricle

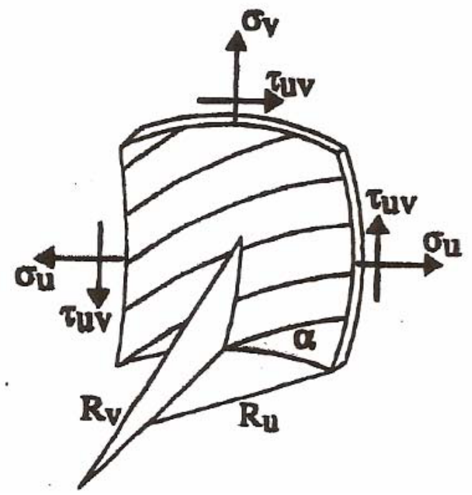
- Arts et al, 1979, the left ventricle is a modeled as a thick-walled cylinder composed of many concentric cylindrical shells

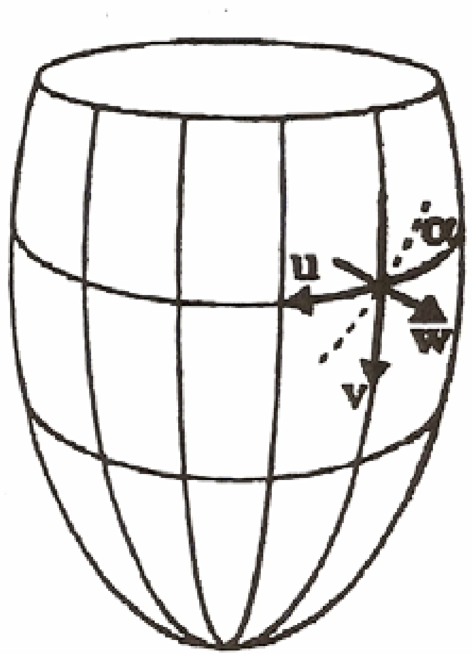
$$\frac{p_{lv}}{\sigma} = \frac{1}{3}\ln\left[1 + \frac{V_w}{V_{lv}}\right]$$



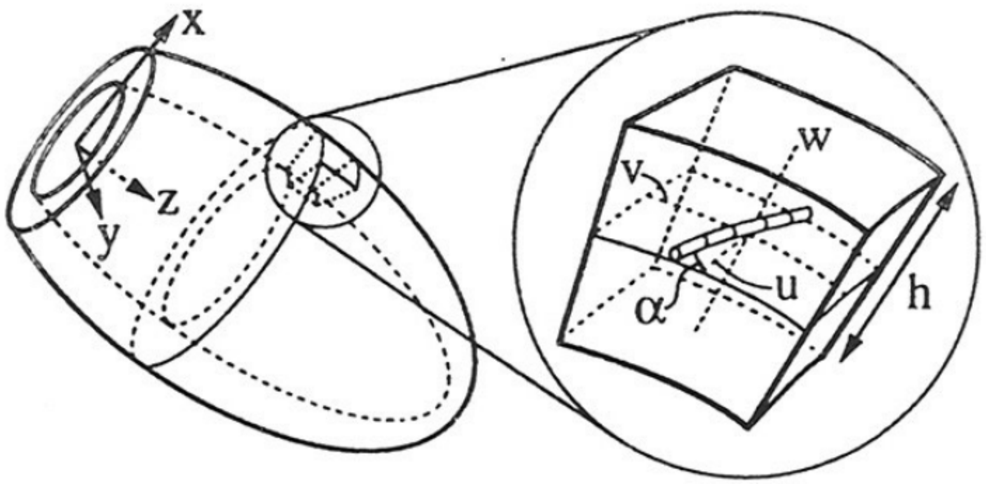
The ellipsoidal model

- Rabben et al, proposed an ellipsoidal model of where the left ventricle is modeled as a thick-walled shell. The middle surface of the thick-walled shell is an axisymmetric ellipsoid





1. The surfaces w = constant are isobars, i.e. surfaces of constant intra-myocardial pressure.
2. The u-lines and v-lines are along the principal curvature directions. The radii of curvature are R_u with respect to the u-lines and R_v with respect to the v-lines.
3. The w-coordinate is normal to the isobars.
4. The muscle fibers are parallel to the isobars.



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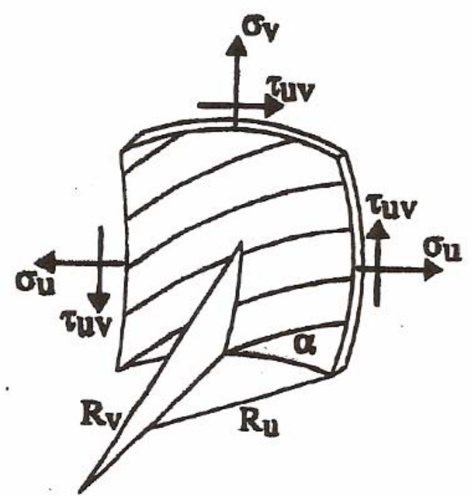
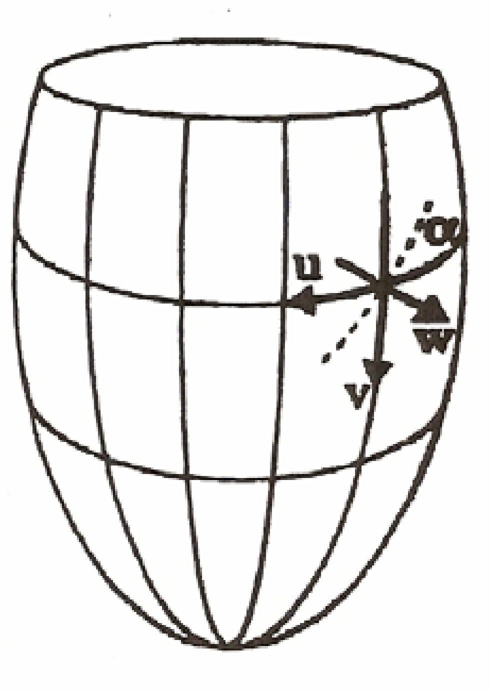
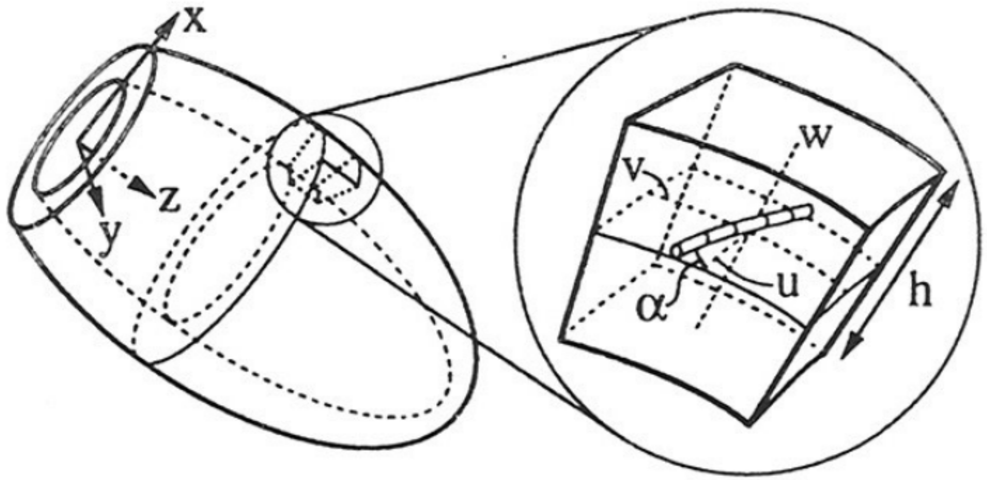




$$T_{11} = -p + \sigma\cos^2\alpha\ ,\quad T_{22} = -p + \sigma\sin^2\alpha\ ,\quad T_{33} = -p\ ,\quad T_{12} = \sigma\sin\alpha\cos\alpha$$

$$\sigma_u = \sigma\cos^2\alpha\ ,\quad \sigma_v = \sigma\sin^2\alpha\ ,\quad \sigma_w = 0\,,\quad \tau_{uv} = \sigma\sin\alpha\cos\alpha$$

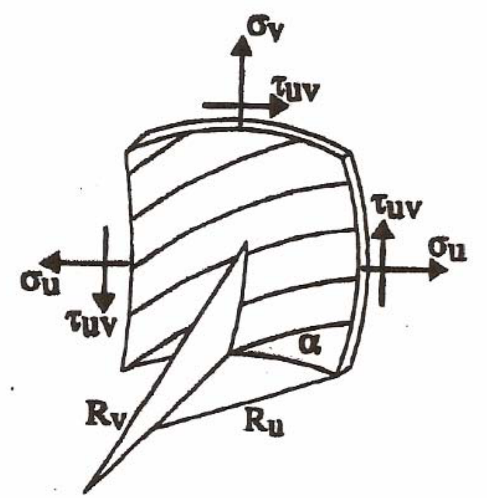


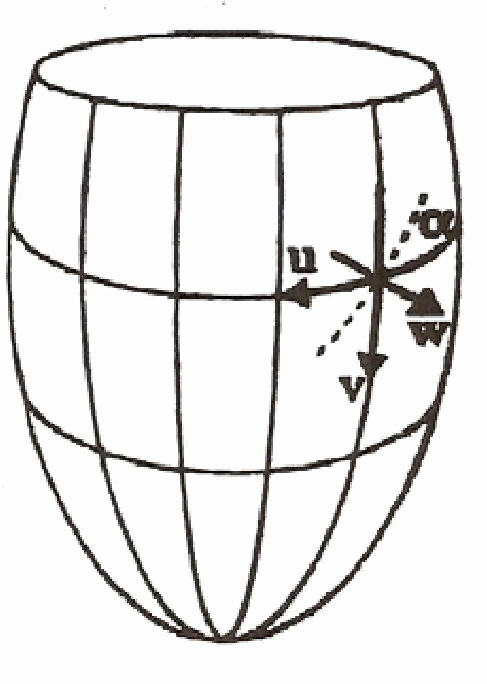
The ellipsoidal model

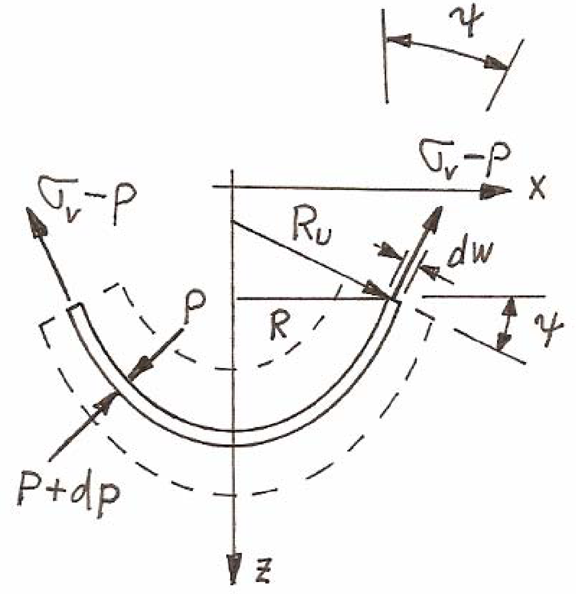
- Equilibrium

$$-\frac{dp}{dw}=\frac{2\sigma}{R_u\left(3-R_u/R_v\right)}$$

$$\left\{\begin{array}{l}\sigma_u=\sigma\cos^2\alpha\ ,\quad \sigma_v=\sigma\sin^2\alpha \\ \sigma_u+\sigma_v=\sigma \\ -\dfrac{dp}{dw}=\dfrac{\sigma_u}{R_u}+\dfrac{\sigma_v}{R_v}\qquad \text{(Laplace law)} \\ \text{Equilibrium in z-direction}\end{array}\right.$$

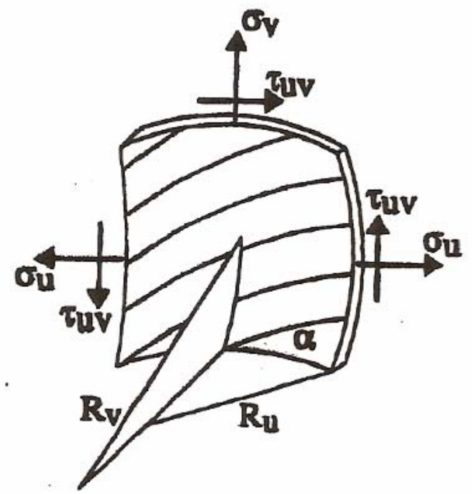


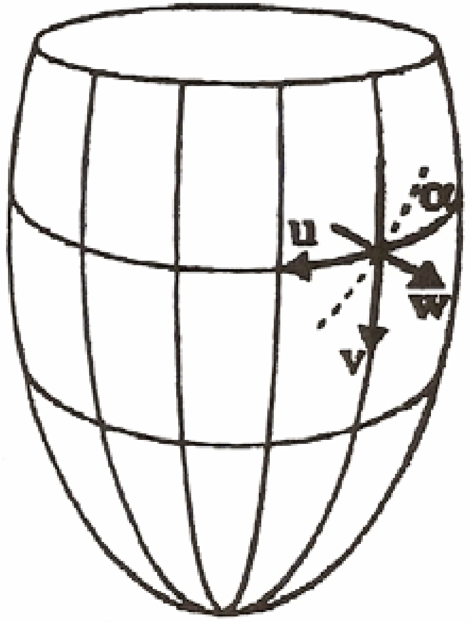




The ellipsoidal model

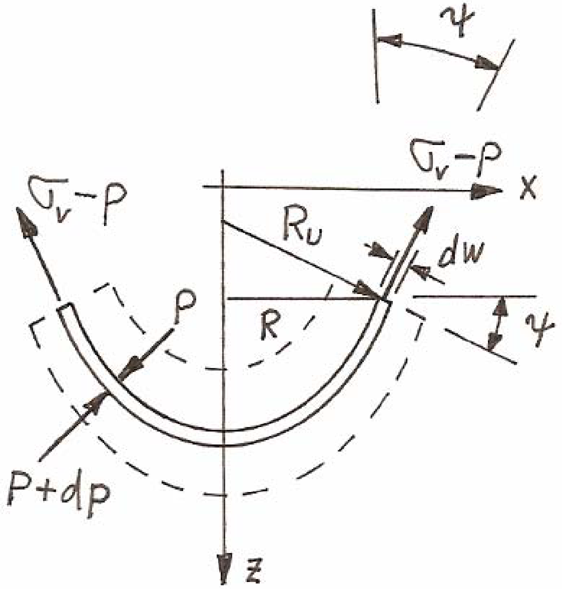
- Equilibrium





$$-\frac{dp}{dw}=\frac{2\sigma}{R_u\left(3-R_u/R_v\right)}$$

$$\left\{\begin{array}{l} \sigma_u=\sigma\cos^2\alpha\ ,\quad \sigma_v=\sigma\sin^2\alpha \\ \sigma_u+\sigma_v=\sigma \\ -\dfrac{dp}{dw}=\dfrac{\sigma_u}{R_u}+\dfrac{\sigma_v}{R_v} \qquad \text{(Laplace law)} \\ \text{Equilibrium in z-direction} \end{array}\right.$$

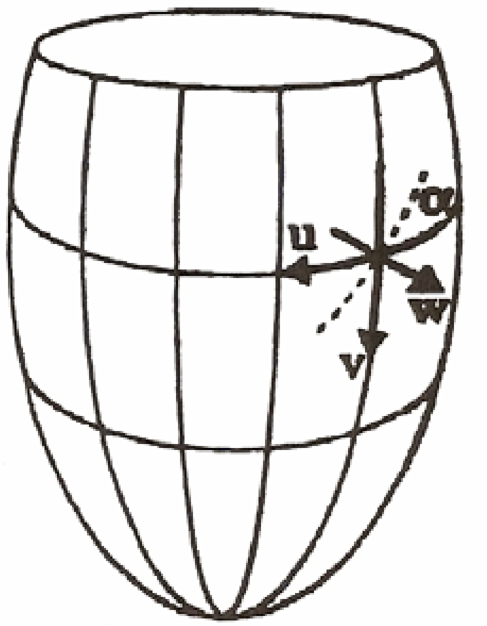


Integration through the thickness of the wall then gives:

$$\frac{\sigma}{p_{lv}}=\left[\frac{2}{3}\ln\left(\frac{R_{mu}+h/2}{R_{mu}-h/2}\right)+\frac{1}{3}\ln\left(\frac{3R_{mu}-R_{mv}+h}{3R_{mu}-R_{mv}-h}\right)\right]^{-1}$$

$$R_u=R_{mu}+w\quad ,\quad R_v=R_{mv}+w\quad \text{where}\quad -\frac{h}{2}\le w\le\frac{h}{2}$$

The ellipsoidal model



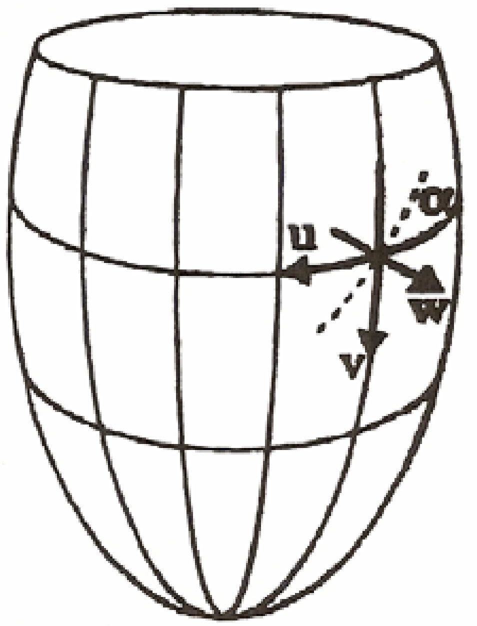
- Fiber directions

Balance equations

$$-\frac{dp}{dw} = \frac{\sigma\cos^2\alpha}{R_{mu}+w} + \frac{\sigma\sin^2\alpha}{R_{mv}+w}$$

$$\frac{\sigma}{p_{lv}} = \left[\frac{2}{3}\ln\left(\frac{R_{mu}+h/2}{R_{mu}-h/2}\right) + \frac{1}{3}\ln\left(\frac{3R_{mu}-R_{mv}+h}{3R_{mu}-R_{mv}-h}\right)\right]^{-1}$$

The ellipsoidal model



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Constraints

$$\int_{-h/2}^{h/2}\left[\frac{\cos^2\alpha}{R_{mu}+w}+\frac{\sin^2\alpha}{R_{mv}+w}\right]dw=\frac{2}{3}\ln\left(\frac{R_{mu}+h/2}{R_{mu}-h/2}\right)+\frac{1}{3}\ln\left(\frac{3R_{mu}-R_{mv}+h}{3R_{mu}-R_{mv}-h}\right)$$

$$T=\int_{-h/2}^{h/2}(\tau_{uv}\cdot R)\cdot 2\pi R\,dw=\sigma\pi\cos^2\psi_m\int_{-h/2}^{h/2}\sin 2\alpha\,(R_{mu}+w)^2\,dw=0$$

The ellipsoidal model

- Fiber directions

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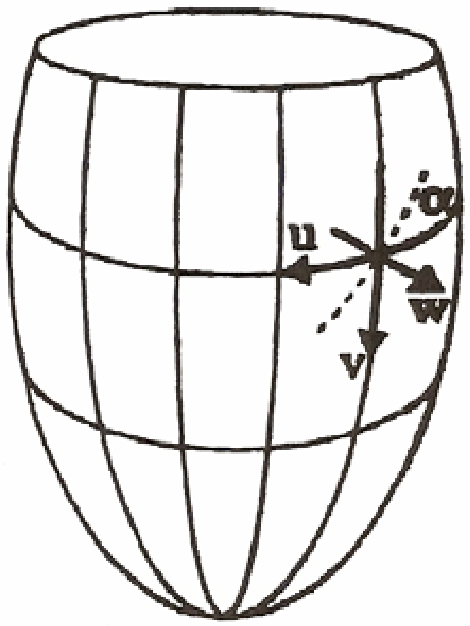
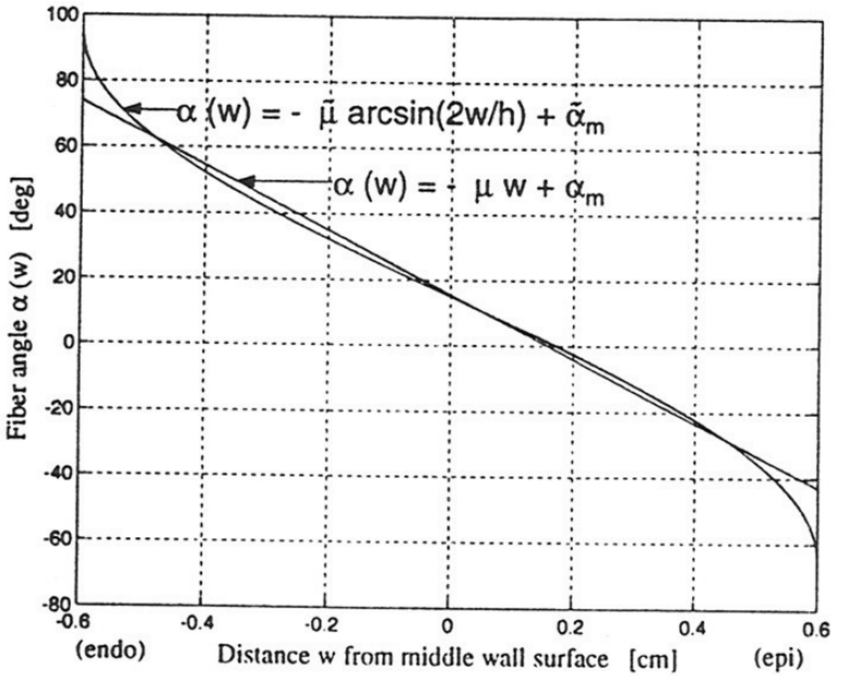
Fiber directions models

$\alpha=-\mu w+\alpha_m$ linear model

$\alpha=-\hat{\mu}\arcsin\left(\frac{2w}{h}\right)+\hat{\alpha}_m$ arcsin model





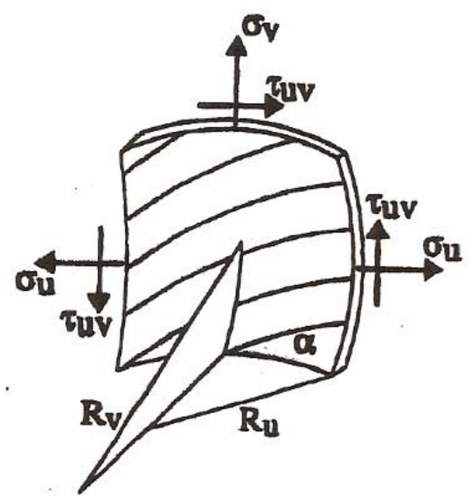
The ellipsoidal model

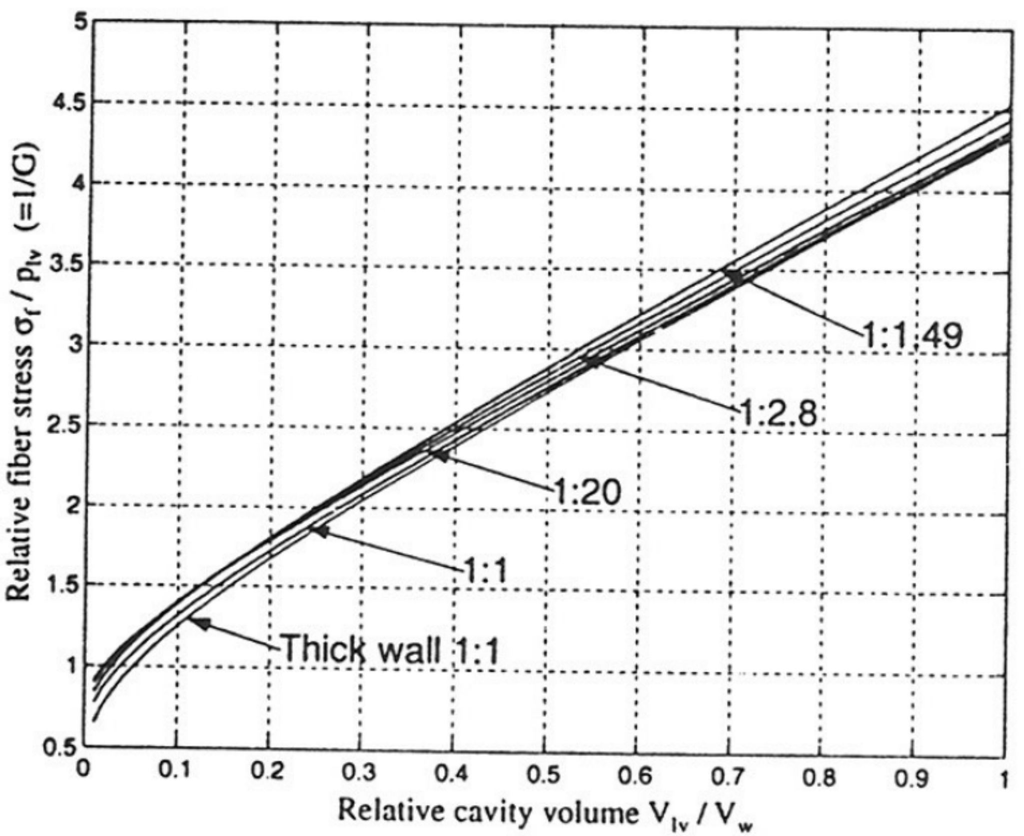
- Fiber stress

$$\frac{\sigma}{p_{lv}} = \left[\frac{2}{3}\ln\left(\frac{R_{mu}+h/2}{R_{mu}-h/2}\right)+\frac{1}{3}\ln\left(\frac{3R_{mu}-R_{mv}+h}{3R_{mu}-R_{mv}-h}\right)\right]^{-1}$$

$$\frac{\sigma}{p_{lv}} = \frac{2h}{R_{mu}\left(3-\frac{R_{mu}}{R_{mv}}\right)}$$

thin wall ventricle
$R_u \approx R_{mu}$





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$$\frac{\sigma}{p_{lv}} = \frac{2h}{R_{mu}\left(3-\frac{R_{mu}}{R_{mv}}\right)}$$

thin wall ventricle
$R_u \approx R_{mu}$

$$\frac{p_{lv}}{\sigma} = \frac{1}{3}\ln\left[1+\frac{V_w}{V_{lv}}\right]$$

Arts model