PROBLEM SET 12

TKT4150 Biomechanics

Main topics: Arterial stenosis. Dimensional analysis with the Buckingham Π-theorem.

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(1) Arterial stenosis

A patient is suffering from a narrowing of the aorta, due to a calcium build-up. The dimensions and modelling assumptions of the stenosis are shown in Figure 1.

a) Using the assumptions indicated in the figure, show that the pressure drop over the section is:

$$\Delta p = \frac{8\pi\mu L_s}{A_s^2} Q + \frac{\rho}{2A_0^2}\left(\frac{A_0}{A_s} - 1\right)^2 Q^2 \tag{1}$$

b) During systole, the patient has an average blood flow of 54 ml/s. Blood is assumed to have density and viscosity $\rho = 1060 kg/m^3$ and $\mu = 3.5 \cdot 10^{-3} Pa \cdot s$. Further, the dimensions characterising the stenosis are given in Table 1. The flow during systole is assumed steady (a fairly crude assumption). What pressure increase is needed, for the flow rate to be maintained?

c) What consequences do stenoses imply for affected patients? How does the area of a stenosis affect the pressure drop?

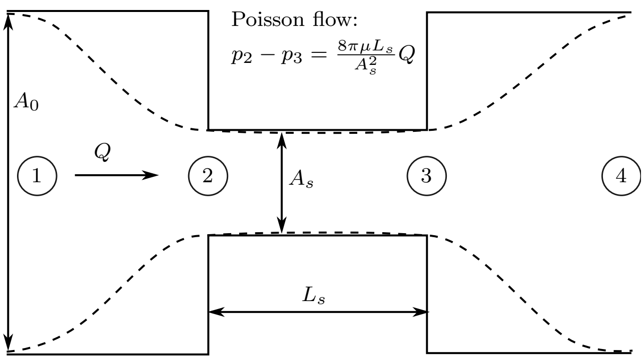

Figure 1: *Modelling of an arterial stenosis. Assumptions made in different regions are: $1 \rightarrow 2$: Bernoulli, $2 \rightarrow 3$: Poisson flow and $3 \rightarrow 4$: Conservation of momentum.*

Table 1: *Dimensions characterizing the stenosis.*

A_s	A_0	L_s
$0.75cm^2$	$3.5cm^2$	$1cm$

2 Dimensional analysis

A pipe has length L, diameter D and wall roughness ϵ, and is filled with a fluid with density ρ and viscosity μ. The flow has the average velocity V. The pressure drop can be assumed a function like this:

$$\Delta p = f(\rho, \mu, V, D, L, \epsilon) \tag{2}$$

a) Use the Buckingham Π-theorem to determine three non-dimensional terms which constitute the variables of a non-dimensional version of Equation 2.