Chapter 5

Mechanics of the vessel wall

5.1 Introduction

In the arterial system, the amplitude of the pressure pulse is that large that the arteries significantly deform during the cardiac cycle. This deformation is determined by the mechanical properties of the arterial wall. In this section an outline of the mechanical properties of the main constituents of the vessel wall and the wall as a whole is given taking the morphology as a point of departure. A simple linear elastic model for wall displacement due to change in transmural pressure will be derived.

5.2 Morphology

The vessel wall consists of three layers: the intima, the media and the adventitia. The proportions and composition of the different layers vary in different type of blood vessels. In figure 5.1 these layers and their composition are depicted schematically showing transversal sections through different kind of blood vessels.

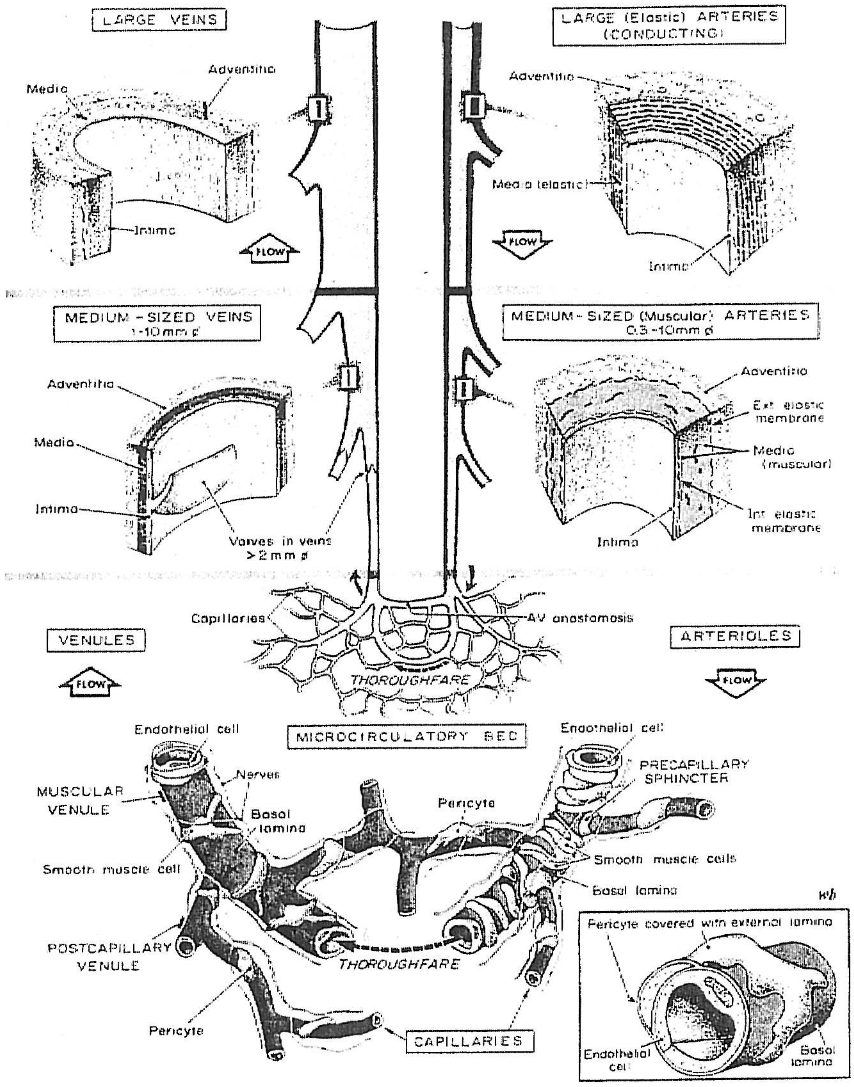


Figure 5.1: Morphology of principle segments of blood vessels in mammals (from Rhodin, 1980) .

The intimal layer. The intimal layer is the innermost layer of all blood vessels. This layer is composed of two structures, a single layer of endothelial cells and a thin subendothelial layer, separated by a thin basal lamina. The **endothelial cells** are flat and elongated with their long axis parallel to that of the blood vessel. They have a thickness of $\pm$ 0.2 -0.5 μm, except in the area of the nucleus, which protrudes slightly into the vessel. The endothelium covers all surfaces that come into direct contact with blood. It is important in regeneration and growth of the artery which is controlled by variations of wall shear stress and strain distributions induced by the blood flow and the wall deformation respectively. The **subendothelium layer** consists of a few collagenous bundles and elastic fibrils. Due to its relatively small thickness and low stiffness the intimal layer does not contribute to the overall mechanical properties of the vessel wall. An exception to this is found in the microcirculation, where the intimal layer is relatively large. Here, however, mechanical properties are mainly determined by the surrounding tissue of the vessel.

The tunica media. The media is the thickest layer in the wall and shows large variation in contents in different regions of the circulation. It consists of elastic lamina and smooth muscle cells. In the human aorta and in large arteries, 40-60 of these lamina exist and almost no smooth muscle cells are found. These arteries therefor are often referred to as elastic arteries. Toward the periphery the number of elastic lamina decreases gradually and a larger amount of smooth muscle cells are found (muscular arteries). The elastic lamina (average thickness 3 μm) are concentric and equidistantly spaced. They are interconnected by a network of elastic fibrils. Thus structured, the media has great strength and elasticity. The smooth muscle cells are placed within the network of elastic fibrils and have an elongated, but irregular shape.

The tunica adventitia. The tunica adventitia of elastic arteries generally comprises only 10 % of the vascular wall. The thickness, however, varies considerably in different arteries and may be as thick as the media. The adventitia is composed of a loose connective tissue of elastin and collagen fibres in mainly longitudinal direction. The adventitia serves to connect the blood vessels to its surrounding tissue and in large arteries it harbors the nutrient vessels (arterioles, capillaries, venules and lymphatic vessels) referred to as vaso vasorum.

5.3 Mechanical properties

As described in the previous section, the main constituents of vascular tissue are elastin, collagen fibers and smooth muscle cells.

Elastin is a biological material with an almost linear stress-strain relationship (fig. 5.2). It has a Young 's modulus of $\approx$ 0.5 MPa and remains elastic up to stretch ratios of $\approx$ 1.6 (Fung, 1993a). As can be seen from the stress-strain curves the material shows hardly any hysteresis.

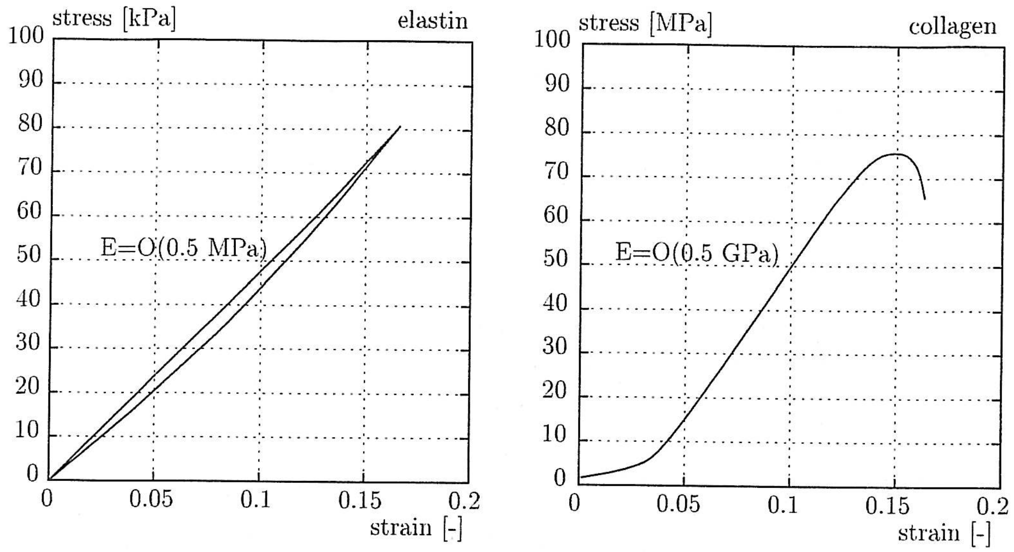


Figure 5.2: Left: Stress-strain relationships of elastin from the ligamentum nuchae of cattle. Right: Typical stress-strain relationship of collagen from the rabbit limb tendon. (Both after Fung, 1993a).

Collagen is a basic structural protein in animals. It gives strength and stability and appears in almost all parts of the body. The collagen molecule consists of three helically wound chains of amino-acids. These helices are collected together in micro-fibrils, which in their turn form subfibrils and fibrils. The fibrils have a diameter of 20-40 nm, depending on species and tissue. Bundles of fibrils form fibers, with diameters ranging from 0.2 to 12 μm. The fibers are normally arranged in a wavy form, with typical "wavelengths" of 200 μm (Fung, 1993a). Due to this waviness the stress-strain relationship shows a very low stiffness at small stretch ratios (fig. 5.2). The stiffness increases fast once the fibers are deformed to straight lines, the Young 's modulus of the material then reaches $\approx$ 0.5 GPa. At further stretching, the material finally fails at 50-100 MPa longitudinal stress.

Smooth muscle cells appear in the inner part of the tunica media and are oriented longitudinally, circumferentially or helically. The Young's modulus is in the order of magnitude of the one of elastine ($\approx$ 0.5 MPa). When relaxed it is about 0.1 MPa and when activated it can increase to 2 MPa. Especially in the smaller arteries and arterioles they strongly determine the mechanical properties of the arterial wall and are responsible for the ability to regulate local blood flow.

Elastic and Viscoelastic behavior

Due to the properties of its constituents, its specific morphology and its geometry, the arterial wall exhibits a non-isotropic nonlinear response to cyclic pressure loads. An important geometrical parameter is the ratio between diameter and thickness of the arterial wall. This ratio depends on the type of artery but is O(0.1) in many cases.

Moreover the vessels are thetered mainly longitudinal by the surrounding tissue. Due to these complex properties it formally is not possible to define a Young's modulus as can be done in linear elasticity theory. Still in order to obtain a global idea of the elastic behavior it is possible to lump all properties together as if the vessel wall was homogeneous. This can be done by measuring the stress strain relationship and use the result to define an *effective* Young's modulus. In figure 5.3 a typical stress strain relationship of a large artery is shown. The stiffness in longitudinal direction is higher then in circumferential direction especially at larger stretch ratios yielding a different effective incremental Young's modulus. Still the relation is non-linear due to the wavy form of the collagen fibres that consequently contribute to the stiffness only at higher stretch ratios. It will be clear that linear elasticity can not be applied straight forward. Linearization, however, about an equilibrium state (for instance the mean or diastolic pressure) yields a *linearized* or *incremantal* effective Young's modulus that in many cases is approriate the use in linear elasticity analysis.

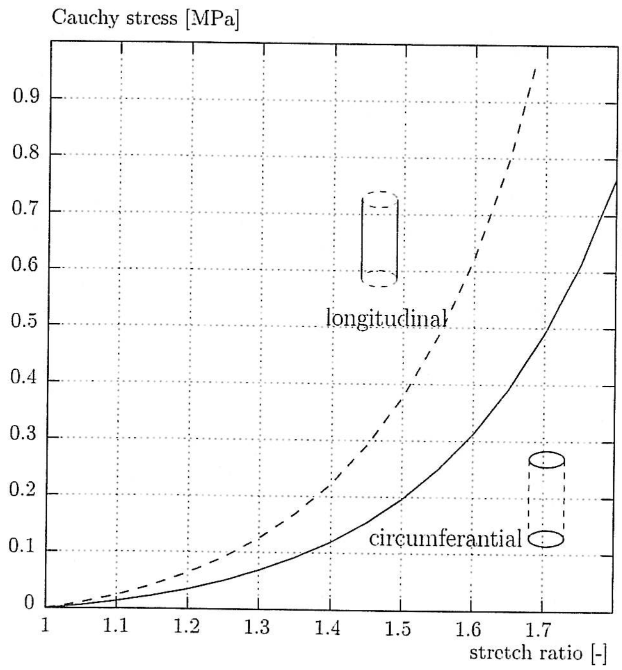


Figure 5.3: Typical stress-strain relationship (broken line = longitudinal, solid line = circumferential) of aortic wall material (after Kasyanov and Knet-s, 1974) .

Vascular tissue normally is viscoelastic. When a cyclic load is applied to it in an experiment, the load-displacement curve for loading differs from the unloading curve: hysteresis due to viscoelasticity is found (see figure 5.2). Moreover, the curves change after several repetitions of the same loading/unloading cycle. After a certain number of repetitions, the loading-unloading curve doesn't change anymore, and the loading/unloading curves almost coincide. The state of the specimen then is called preconditioned (Fung *et al.*, 1979; Fung, 1993a). How well this state resembles the in-vivo situation is not reported.

5.4 Incompressible elastic deformation

In section 2.2.4 it has been shown that in general elastic solids can be described by the constitutive equation $\boldsymbol{\sigma} = -p\boldsymbol{I} + \boldsymbol{\tau}(\boldsymbol{B})$. The most simple version of such a relation will be the one in which the extra stress $\boldsymbol{\tau}$ linearly depends on the Finger tensor. Materials that obey such a relation are referred to as linear elastic or neo-Hookean solids. Most rubber like materials but also to some extend biological tissues like the arterial wall are reasonably described by such a neo-Hookean model.

5.4.1 Deformation of incompressible linear elastic solids

The equations that describe the deformation of incompressible linear elastic solids are given by the continuity equation (2.38) and the momentum equation (2.39) together with the constitutive equation:

$$\boldsymbol{\sigma} = -p\boldsymbol{I} + G(\boldsymbol{B} - \boldsymbol{I}) \tag{5.1}$$

with p the pressure and G the shear modulus. Note that the linear relation between the extra stress and the strain is taken such that if there is no deformation ($\boldsymbol{B} = \boldsymbol{I}$) the strain measure $\boldsymbol{B} - \boldsymbol{I} = \boldsymbol{0}$. The momentum and continuity equations then read:

$$\begin{cases} \rho\dfrac{\partial \boldsymbol{v}}{\partial t} + \rho(\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = \rho\mathrm{f} - \nabla p + G\nabla\cdot(\boldsymbol{B} - \boldsymbol{I}). \\ \nabla\cdot\boldsymbol{v} = 0 \end{cases} \tag{5.2}$$

After introduction of the non-dimensional variables $\boldsymbol{x}^* = \boldsymbol{x}/L$, $t^* = t/\theta$ and characteristic strain γ, the characteristic displacement is $U = \gamma L$ and the characteristic velocity is $V = U/\theta = \gamma L/\theta$. Using a characteristic pressure $p^* = p/p_0$, the dimensionless equations for elastic deformation then become (after dropping the superscript *):

$$\begin{cases} \dfrac{\rho\gamma L}{\theta^2}\dfrac{\partial \boldsymbol{v}}{\partial t} + \dfrac{\rho\gamma^2 L}{\theta^2}(\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = \rho g\mathrm{f} - \dfrac{p_0}{L}\nabla p + \dfrac{\gamma G}{L}\nabla\cdot(\boldsymbol{B} - \boldsymbol{I}) \\ \nabla\cdot\boldsymbol{v} = 0 \end{cases} \tag{5.3}$$

Unfortunately in the solid mechanics community it is not common to use dimensionless parameters and we have to introduce them ourselves or use an example to show which of the terms are of importance in this equation. If we assume that we deform a solid with density $\rho = O(10^3)[kg/m^3]$, shear modulus $G = O(10^5)[Pa]$ and characteristic length $L = O(10^{-2})[m]$ with a typical strain $\gamma = O(10^{-1})$ in a characteristic time $\theta = O(1)[s]$, the terms at the left hand side have an order of magnitude of $O(1)[Pa/m]$ and $O(10^{-2})[Pa/m]$. The deformation forces at the right hand side, however, have an order of magnitude $O(10^4)[Pa/m]$. As a consequence the terms at the left hand side can be neglected. Gravity forces can not be neglected automatically but are often not taken into account because they work on the body as a whole and do not induce extra deformation unless large hydrostatic pressure gradients occur. The momentum equation then reads:

$$\begin{cases} \nabla \cdot \boldsymbol{\sigma} = -\nabla p + G\nabla \cdot (\boldsymbol{B} - \boldsymbol{I}) = \boldsymbol{0} \\ \\ det(\boldsymbol{F}) = 1 \end{cases} \tag{5.4}$$

Note that the continuity equation is expressed in terms of the deformation tensor $\boldsymbol{F}$ (see below). Equations (5.4) can be solved after applying boundary conditions of the form (see (2.43)):

$$\begin{array}{lll} \text{in normal direction:} & & \\ \quad \text{prescribed displacement:} & & \\ & (\boldsymbol{u} \cdot \boldsymbol{n}) = u_{n_\Gamma} & \\ \quad \text{or prescribed stress:} & & \\ & (\boldsymbol{\sigma} \cdot \boldsymbol{n}) \cdot \boldsymbol{n} = (\boldsymbol{s} \cdot \boldsymbol{n}) & \\ \text{in tangential directions:} & & \\ \quad \text{prescribed displacemnet:} & & \\ & (\boldsymbol{u} \cdot \boldsymbol{t}_i) = u_{t_{i_\Gamma}} & i = 1, 2 \\ \quad \text{or prescribed stress:} & & \\ & (\boldsymbol{\sigma} \cdot \boldsymbol{n}) \cdot \boldsymbol{t}_i = (\boldsymbol{s} \cdot \boldsymbol{t}_i) & \end{array} \tag{5.5}$$

In linear elasticity the incompressibility constraint ($det(\boldsymbol{F}) = 1$) is oftenly circumvented by assuming the material to be (slightly) compressible. In that case it is convenient to decompose the deformation in a volumetric part and an isochoric part according to:

$$\boldsymbol{F} = J^{1/3}\tilde{\boldsymbol{F}} \tag{5.6}$$

with J the volume ratio defined by:

$$J = det(\boldsymbol{F}) = \frac{dV}{dV_0} \tag{5.7}$$

Consequently $det(\tilde{\boldsymbol{F}}) = det(J^{-1/3}\boldsymbol{I}) \cdot det(\boldsymbol{F}) = 1$ and an equivalent neo-Hookean model can be taken according to:

$$\boldsymbol{\sigma} = \kappa(J - 1)\boldsymbol{I} + G(\tilde{\boldsymbol{B}} - \boldsymbol{I}) \tag{5.8}$$

with κ the compression modulus and $\tilde{\boldsymbol{B}} = \tilde{\boldsymbol{F}} \cdot \tilde{\boldsymbol{F}}^c = J^{-2/3}\boldsymbol{F} \cdot \boldsymbol{F}^c$.

5.4.2 Approximation for small strains

For small strains γ (i.e. $||\nabla \boldsymbol{u}|| \approx ||\nabla^0 \boldsymbol{u}|| << 1$) the deformation tensor $\boldsymbol{F}$ can be written as:

$$\boldsymbol{F} = (\nabla^0 \boldsymbol{x})^c = (\nabla^0(\boldsymbol{x}^0 + \boldsymbol{u}))^c = \boldsymbol{I} + (\nabla^0 \boldsymbol{u})^c \approx \boldsymbol{I} + (\nabla \boldsymbol{u})^c \tag{5.9}$$

The volume ratio for small deformations then yields:

$$J = det(\tilde{\boldsymbol{F}}) \approx det(\boldsymbol{I} + (\nabla \boldsymbol{u})^c) \approx 1 + tr(\nabla \boldsymbol{u}) \tag{5.10}$$

The isochoric part of the deformation then is given by:

$$\begin{aligned}\tilde{\boldsymbol{F}} &= J^{-1/3}\boldsymbol{F} \\ &\approx (1 + tr(\nabla\boldsymbol{u}))^{-1/3}(\boldsymbol{I} + (\nabla\boldsymbol{u})^c) \\ &\approx (1 - \frac{1}{3}tr(\nabla\boldsymbol{u}))(\boldsymbol{I} + (\nabla\boldsymbol{u})^c)\end{aligned} \tag{5.11}$$

This finally yields the isochoric Finger tensor:

$$\begin{aligned}\tilde{\boldsymbol{B}} &= J^{-2/3}\boldsymbol{F}\cdot\boldsymbol{F}^c \\ &\approx (1 - \frac{2}{3}tr(\nabla\boldsymbol{u})(\boldsymbol{I} + (\nabla\boldsymbol{u})^c)(\boldsymbol{I} + (\nabla\boldsymbol{u})^c)^c \\ &\approx \boldsymbol{I} + (\nabla\boldsymbol{u})^c + \nabla\boldsymbol{u} - \frac{2}{3}tr(\nabla\boldsymbol{u})\boldsymbol{I}\end{aligned} \tag{5.12}$$

The constitutive relation for compressible elastic deformation for small strains then reads:

$$\begin{aligned}\boldsymbol{\sigma} &= \kappa tr(\nabla\boldsymbol{u})\boldsymbol{I} + G((\nabla\boldsymbol{u})^c + \nabla\boldsymbol{u} - \tfrac{2}{3}tr(\nabla\boldsymbol{u})\boldsymbol{I}) \\ &= (\kappa - \tfrac{2}{3}G)tr(\nabla\boldsymbol{u})\boldsymbol{I} + G(\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^c)\end{aligned} \tag{5.13}$$

Together with the definition of infinitesimal strain:

$$\boldsymbol{\epsilon} = \frac{1}{2}(\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^c) \tag{5.14}$$

we finally obtain:

$$\boldsymbol{\sigma} = (\kappa - \frac{2}{3}G)tr(\boldsymbol{\epsilon})\boldsymbol{I} + 2G\boldsymbol{\epsilon} \tag{5.15}$$

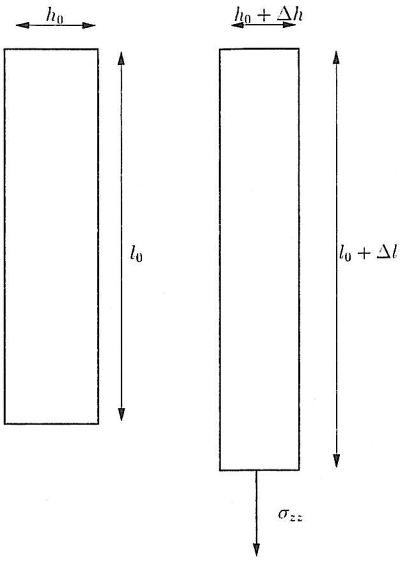
It can readily be verified that consequently

$$\boldsymbol{\epsilon} = \frac{(1+\mu)}{E}\boldsymbol{\sigma} - \frac{\mu}{E}tr(\boldsymbol{\sigma})\boldsymbol{I} \tag{5.16}$$

with :

$$\begin{aligned}&\kappa = \frac{E}{3(1-2\mu)} \qquad G = \frac{E}{2(1+\mu)} \\ &\text{or} \\ &\mu = \frac{3\kappa - 2G}{6\kappa + 2G} \qquad E = \frac{9\kappa G}{3\kappa + G}\end{aligned} \tag{5.17}$$

Note that the Young's modulus E and Poisson ratio μ can be determined by a tensile test. Using the expression for the strain given by (5.16) it follows that:



$$\epsilon_{zz} = \frac{\Delta l}{l_0} = \frac{1}{E}\sigma_{zz}$$

$$\epsilon_{xx} = \epsilon_{yy} = \frac{\Delta h}{h_0} = -\frac{\mu}{E}\sigma_{zz}$$

and consequently:

$$E = \frac{\sigma_{zz}}{\epsilon_{zz}}$$

$$\mu = -\frac{\epsilon_{xx}}{\epsilon_{zz}}$$

5.5 Wall motion

Consider a linear elastic thin walled tube with constant wall thickness h , density ρ_w, Young's modulus E and Poisson's ratio μ (see figure 5.4).

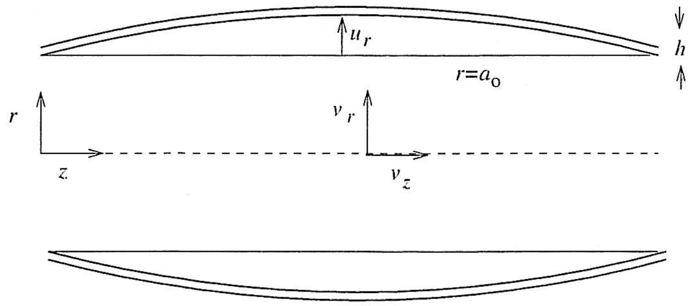


Figure 5.4: Distensible tube with radius $a(z,t)$, wall thickness h

If the thickness of the wall is assumed to be that small that $\sigma_{rr} = 0$, the momentum equation in z-direction is given by:

$$\varepsilon_{zz} = \frac{\partial u_z}{\partial z} = \frac{1}{E}(\sigma_{zz} - \mu\sigma_{\phi\phi}) \tag{5.18}$$

with u_z the wall displacement in axial direction. Assuming again axial restraint ($u_z = 0$) then $\varepsilon_{zz} = 0$ and thus:

$$\sigma_{zz} = \mu\sigma_{\phi\phi} \tag{5.19}$$

In circumferential direction the strain $\varepsilon_{\phi\phi}$ is given by:

$$\begin{aligned}\varepsilon_{\phi\phi} &= \frac{(a_0+u_r)d\phi - a_0 d\phi}{r d\phi} \approx \frac{u_r}{a_0} \\ &= \frac{1}{E}(\sigma_{\phi\phi} - \mu\sigma_{zz}) = \frac{\sigma_{\phi\phi}}{E}(1-\mu^2)\end{aligned} \tag{5.20}$$

with u_r the wall displacement in radial direction. This implies the following expression for the circumferential stress:

$$\sigma_{\phi\phi} = \frac{E}{1-\mu^2}\frac{u_r}{a_0} \tag{5.21}$$

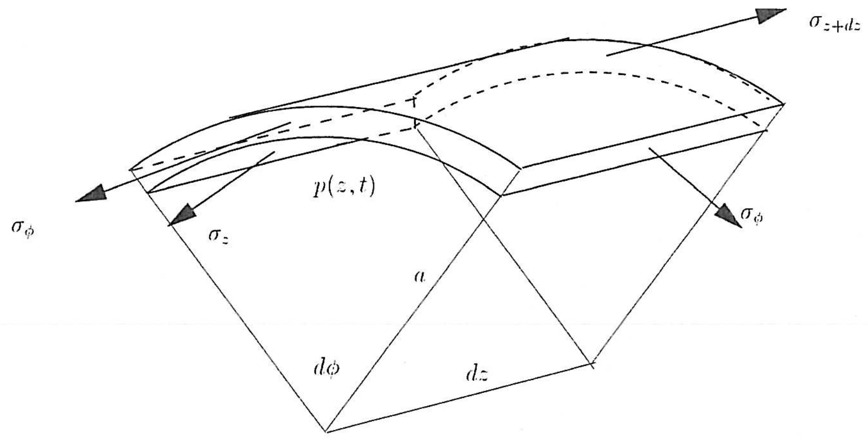


Figure 5.5: Stresses in tangential and circumferential direction.

If the tube is loaded with an internal (transmural) pressure $p(z,t)$ the momentum equation in radial direction reads (see figure 5.5):

$$\rho_w a_0 d\phi h dz \frac{\partial^2 u_r}{\partial t^2} = p(z,t) a_0 d\phi dz - 2\sigma_{\phi\phi}\sin(\tfrac{1}{2}d\phi) h dz \tag{5.22}$$

Since $\sin(d\phi) \approx d\phi$ this yields together with (5.21):

$$\rho_w h \frac{\partial^2 u_r}{\partial t^2} = p - \frac{hE}{(1-\mu^2)}\frac{u_r}{a_0^2} \tag{5.23}$$

If we neglect inertia forces we obtain

$$u_r = \frac{(1-\mu^2)a_0^2}{hE}p \tag{5.24}$$

The cross-sectional area of the tube is given by:

$$A = \pi(a_0+u_r)^2 = \pi a_0^2 + 2\pi a_0 u_r + \pi u_r^2 \approx \pi a_0^2 + 2\pi a_0 u_r \tag{5.25}$$

This yields the compliance of the tube to be:

$$C = \frac{\partial A}{\partial p} = \frac{2\pi a_0^3}{h}\frac{(1-\mu^2)}{E} \tag{5.26}$$

Oftenly instead of the compliance C the distensibility

$$D = \frac{1}{A_0} C = \frac{2a_0}{h} \frac{(1 - \mu^2)}{E} \tag{5.27}$$

is used. In the next chapter D will be used to derive expressions for the propagation of pressure waves in distensible tubes.

5.6 Summary

A short introduction to vessel wall mechanics based on morphology and material properties of the main constitutents (elastine and collagen) is given. Although the constitutive behavior of vessel wall is anisotropic and visco-elastic simple linear elastic models based on thin walled tubes can be valuable. To this end, simple expressions for the compliance and distensibility of thin walled tubes are derived and related to parameters that can be derived from tensile tests.