Vessel wall mechanics

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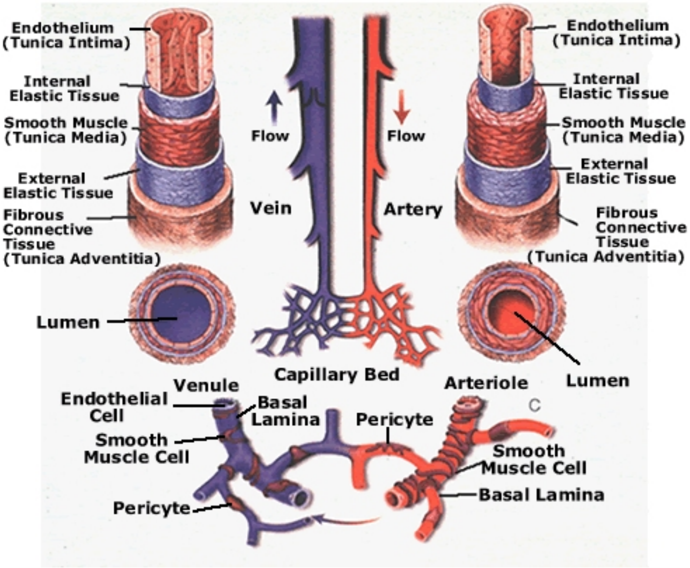
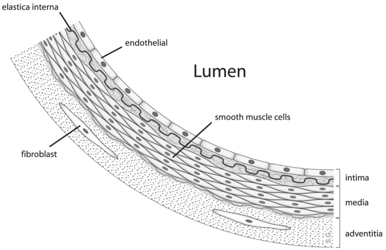
Outline

- Morphology of blood vessels
- Mechanical properties of constituents
  (from chap5. Mechanics of the vessel wall. Cardiovascular fluid dynamics. Fvd Vosse)
- Refer to previous generalized models
- Derive displacement for simplified model

Morphology of arteries and veins



The intimal layer (tunica intima)



- Innermost layer of all blood vessels

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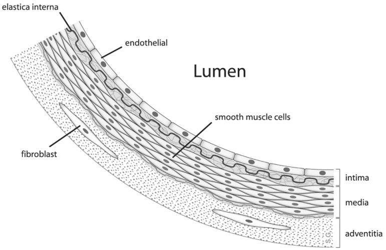

- Innermost layer of all blood vessels
- Two structures separated by thin basal lamina
  - Single layer of endothelial cells
  - Subendothelial layer of a few collagen bundles and elastic fibrils

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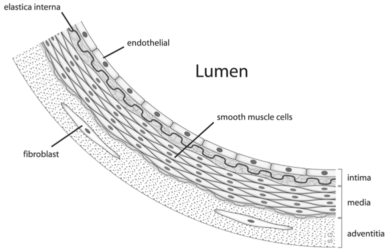

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  - Long axis || to the blood vessel

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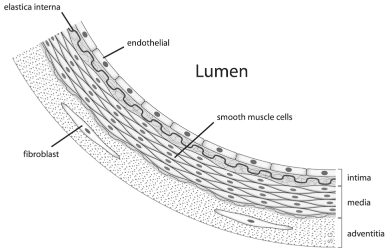

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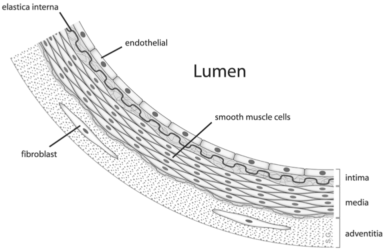

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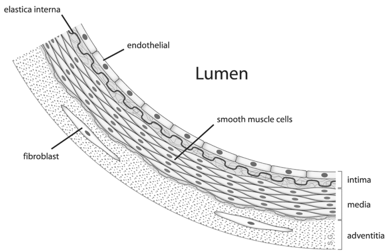

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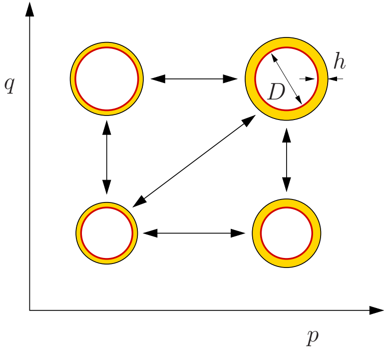
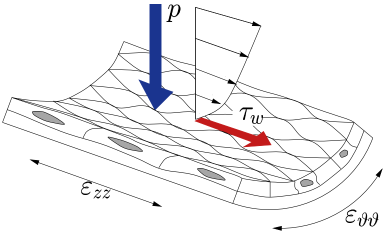
- No contribution to overall vessel mechanics
  - Small thickness
  - Small stiffness

Important parameters for vascular hemodynamics[1]





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[1] vd Vosse 2008

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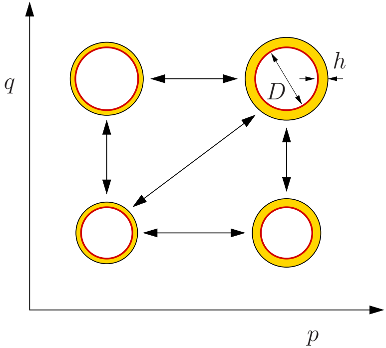

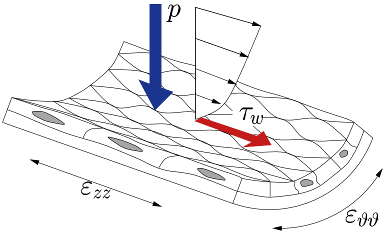

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- flow $p \uparrow$ $\Rightarrow$ wall shear stress $\tau_w \uparrow$ $\Rightarrow$ diameter $D \uparrow$

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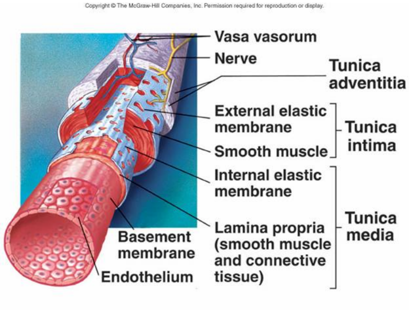
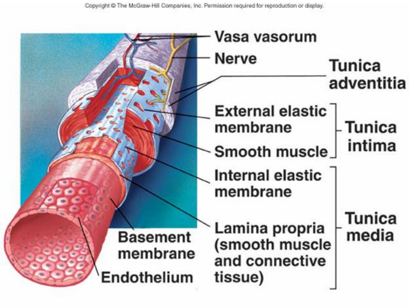
Tunica media



- Media, between intima and adventitia
- Large variations in different regions

Tunica media



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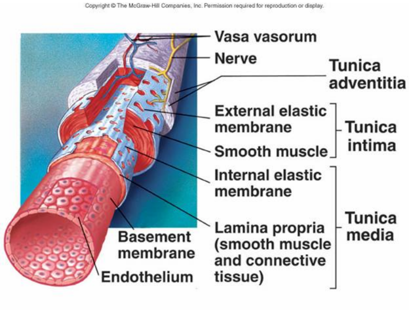

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  - 40-60 EL
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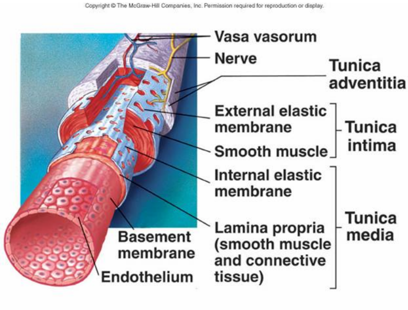

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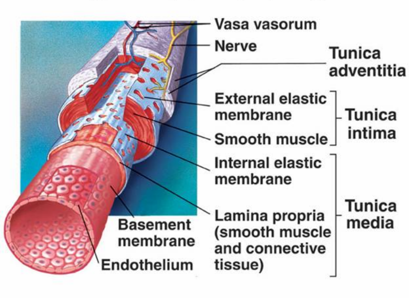
  - SMC $\uparrow$ and EL $\downarrow$ toward the periphery
  - El (3 μm average), concentric, equidistant
  - EL interconnected by elastic fibrils

Tunica media



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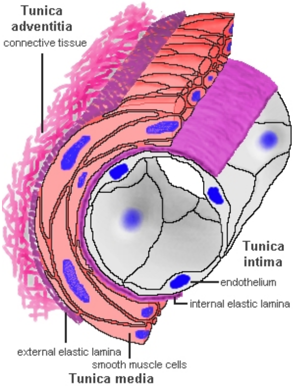
- Media has great strength and elasticity

Tunica adventitia



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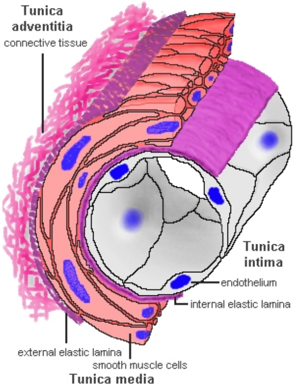
- May be as thick as media

Tunica adventitia



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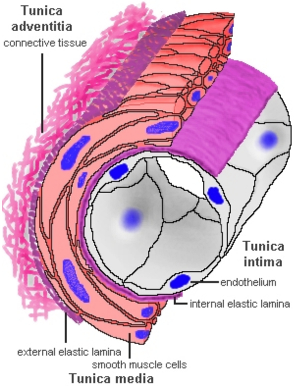

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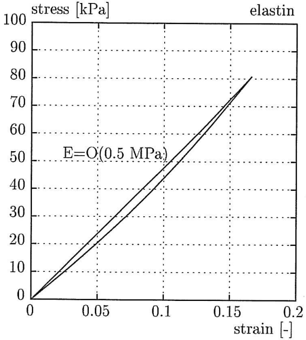
- Purpose
  - Connect blood vessel to surrounding tissue
  - Vaso vasorum (nutrient vessels) in large arteries

Main constituents of vessel wall

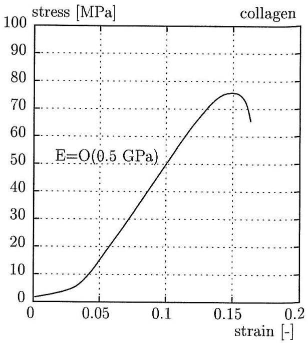
- Elastin fibers
- Collagen fibers
- Smooth muscle cells

Elastin: mechanical properties



- $\approx$ linear stress-strain relation
- $\eta \approx$ 0.5 MPa
- Remains elastic up to stretch ratios $\approx$ 1.6
- Hardly hysteresis in stress-strain curves

Collagen: mechanical properties



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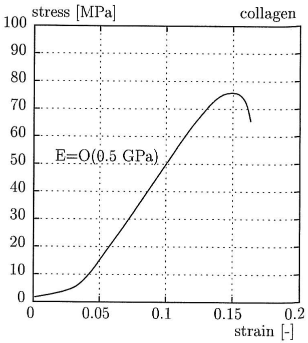

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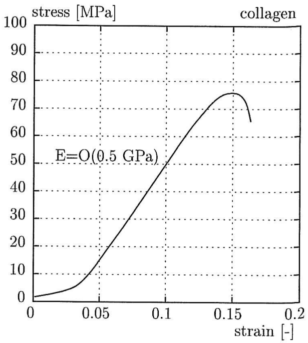

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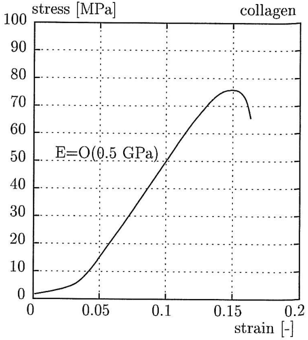

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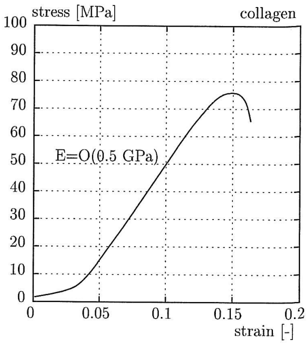

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  - Helices form subfibrils and fibrils ($d \approx 20 - 40\ nm$)
  - Fibrils form fibers ($d \approx 0.2 - 12\ \mu m$)

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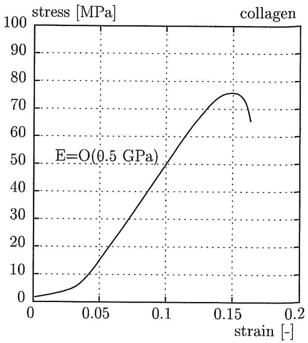

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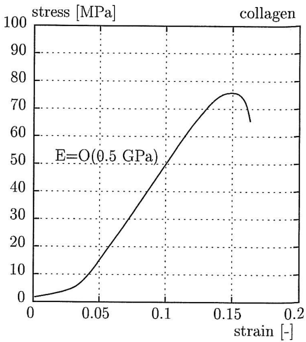

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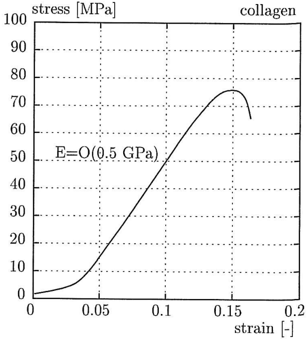

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  - $\eta \approx 0.5\ GPa$

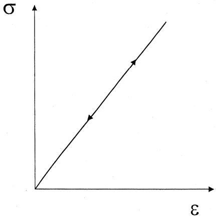
Smooth muscle cells: mechanical properties

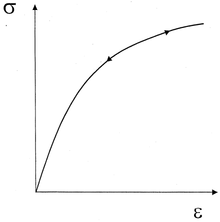
- $\eta \approx 0.5$ *MPa* $\approx$ as for elastin
- $\eta \approx 0.1$ *MPa* when relaxed
- $\eta \approx 2$ *MPa* at max activation

Fundamental properties of elastic materials

- Reversibility
  Identical loading/unloading stress-strain curves
- Path and rate independence
  The stress depends only on the level of strain - not strain history or rate
- Non-dissipative
  The deformation energy may be recovered upon unloading

Uniaxial behavior





Incompressible isotropic hyperelastic materials

- Modified strain energy function:

$$\bar{\phi} = \phi(\mathbf{C}) + p(J - 1)$$

  - p undetermined Lagrange multiplier
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- Second PKS

$$\mathbf{S} = 2\,\frac{\partial \bar{\phi}}{\partial \mathbf{C}} = 2\frac{\partial \phi(\mathbf{C})}{\partial \mathbf{C}} + pJ\mathbf{C}^{-1}$$

Examples of isotropic incompressible nonlinear hyperelastic materials

- Mooney-Rivlin
  - Used for rubber
  - $\phi = c_1(I_1 - 3) + c_2(I_2 - 3)$
- Neo-Hookean
  - $\phi = c_1(I_1 - 3)$

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  - $\phi = c_1(I_1 - 3)$
- Transversely isotropic membrane shells
  - Application to mitral valve mechanics
  - Fiber directions $\mathbf{a}_0$ must be accounted for
  - Additional invariants introduced
  - $I_4 = \mathbf{a}_0\mathbf{C}\mathbf{a}_0$
  - $I_5 = \mathbf{a}_0\mathbf{C}^2\mathbf{a}_0$

Thick walled cylinder with internal and external pressures

$$
\begin{aligned}
u(R) = \frac{1}{2\eta} \frac{a}{1-(a/b)^2} \\
\left[\left(\frac{a}{R} + (1-2\nu) \left(\frac{a}{b}\right)^2 \frac{R}{a} \right) p \right]
\end{aligned}
$$

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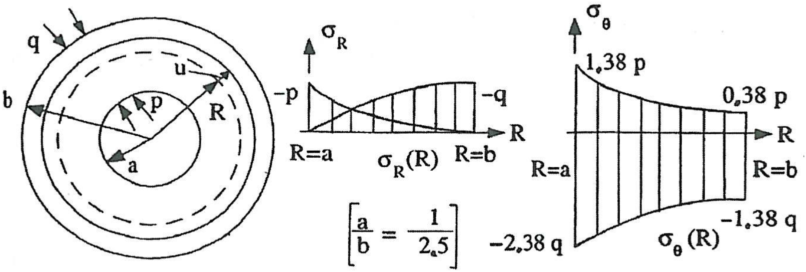
$$\sigma_\theta(R) = \frac{2\mu}{1-\nu}\left(\frac{u}{R} + \nu\frac{du}{dR}\right)$$

Solution thick walled cylinder with internal and external pressures



Summary

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