TKT4150 Biomechanics Problem set 2

Exercise 1: Cauchy's stress theorem used on the foot of a running human

a) Rewrite the Cauchy stress theorem in index notation.

b) The foot of a running human is exposed to a force from the ground, as is seen in Figure 1.

Let's say the stress matrix corresponding to a point on the foot where the foot hits the ground is assumed to be

$$\mathbf{T} = \begin{bmatrix} 0 & 20 & 0 \\ 20 & -40 & 0 \\ 0 & 0 & 0 \end{bmatrix} MPa \tag{1}$$

Find an estimate of the angle, α, on which the force from the ground acts on the foot.

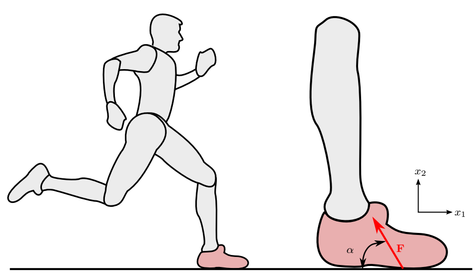

Figure 1: Running human.

Exercise 2: Cauchy equations

a) Use the Cauchy equations (1.89) and the fact that a fluid at rest has an isotropic state of stress (1.86), to derive the formula for the pressure in a still body of fluid as a function of the vertical distance z from a free surface (assume a pressure p_0 at the surface).

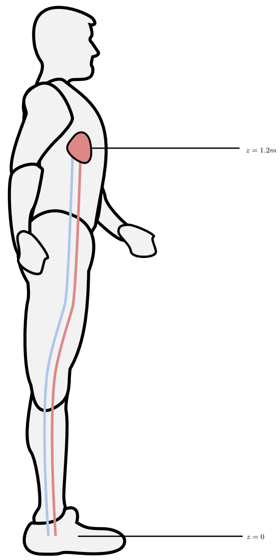

Figure 2: Sketch of arteries and veins between heart and foot.

b) The average static pressure $p - p_1$, where p_1 is the atmospheric pressure, may in veins (p_v) and arteries (p_a) near the heart region be set equal to

$$p_{vh} = 1kPa, \quad p_{ah} = 10kPa \tag{2}$$

Use this to make an estimate of p_v and p_a in the foot of a standing human, when the elevation between the foot and the heart is equal to $1.2m$, as indicated in Figure 2. The density of blood is assumed $\rho = 1000kg/m^3$.

Exercise 3: Computational Prinicpal Stress Analisys

For the following stress matrix:

$$\mathbf{T} = \begin{bmatrix} 90 & -30 & 0 \\ -30 & 120 & -30 \\ 0 & -30 & 90 \end{bmatrix} MPa \tag{3}$$

you will calculate the principal stresses, and directions, as well as use `Python`, `Matlab` or another computational tool of your choice to calculate these quantities as well. The file `exercise_6.ipynb` is an IPython Notebook to help you get started if you would like. You can upload this to https://tmpnb.org and get started if you do not have access to `Python` on your local computer. Remember that to save it you will need to download the notebook, as https://tmpnb.org does not provide permanent storage.

a) Establish the principal stresses σ_1, σ_2 and σ_3, and the corresponding directions $\mathbf{n}_1$, $\mathbf{n}_2$ and $\mathbf{n}_3$ with hand-calculations.

Hint. Remember that $\|\mathbf{T}\|$ is the **Frobenius Norm**. It may be easier to solve the cubic polynomial $\det(\sigma\mathbf{1} - \mathbf{T}) = 0$ using cofactor expansion to find the determinant as opposed to the formula in terms of invariants.

b) Find the numerical values for the three stress invariants I, II and III, using `Python` or `Matlab`.

Hint. Remember that $\|\mathbf{T}\|$ is the **Frobenius Norm**.

c) Use the invariants to determine the characteristic polynomial from the eigenvalue problem. Furthermore, plot the polynomial and graphically determine the principal stresses.

d) Use `Python` or `Matlab` to find the solutions of the characteristic polynomial.

e) Use the linear algebra library of `Python` or `Matlab` to directly solve for the principal stresses and corresponding directions.

f) Plot the principal direction vectors $\mathbf{n}_i$ (i=1,2,3) using `Python` or `Matlab`. (Make sure it is possible to see how they are oriented relative to each other. It may be necessary to have two figures from different angles.)

g) The maximum shear stress is found in an orientation between the directions of the smallest and largest principal stresses, σ_1 and σ_3, and in the plane normal to the direction of σ_2. Determine the orientation where the stress element gets the largest shear stress, and the numerical value of this stress. Plot the found normal vector together with the vectors in the previous problem.