On Satisficing in Quantitative Games

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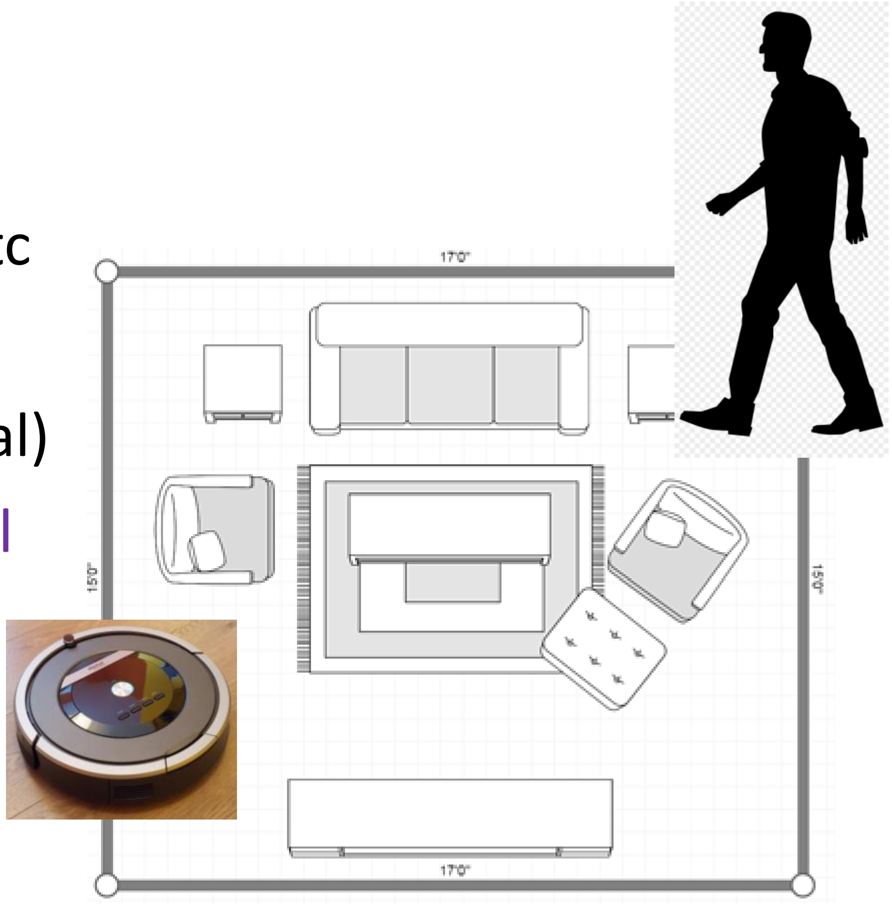
Synthesis from Quantitative Constraints

Quantitative constraints

- Resource consumption, quality measures, rewards, etc

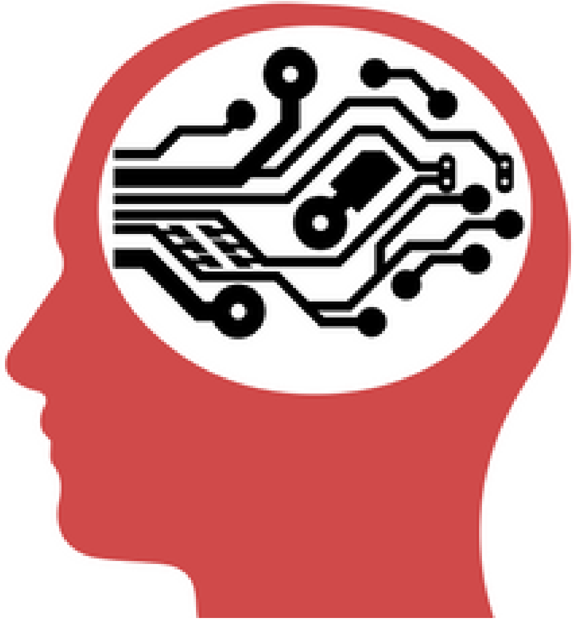
- Controller with minimal battery consumption (Optimal)
- Controller ensures battery never goes below a critical threshold (good-enough)



Satisficing: To search for “good enough” solutions

- Herb A. Simons

Benefits of satisficing



Computationally Easier

Quantitative

+

Temporal

Amenable to combination of constraints

In this paper:

Formulate the Satisficing Problem in Quantitative Games

Approach 1: Via Optimization

- Inherits deficiencies of optimization ☹

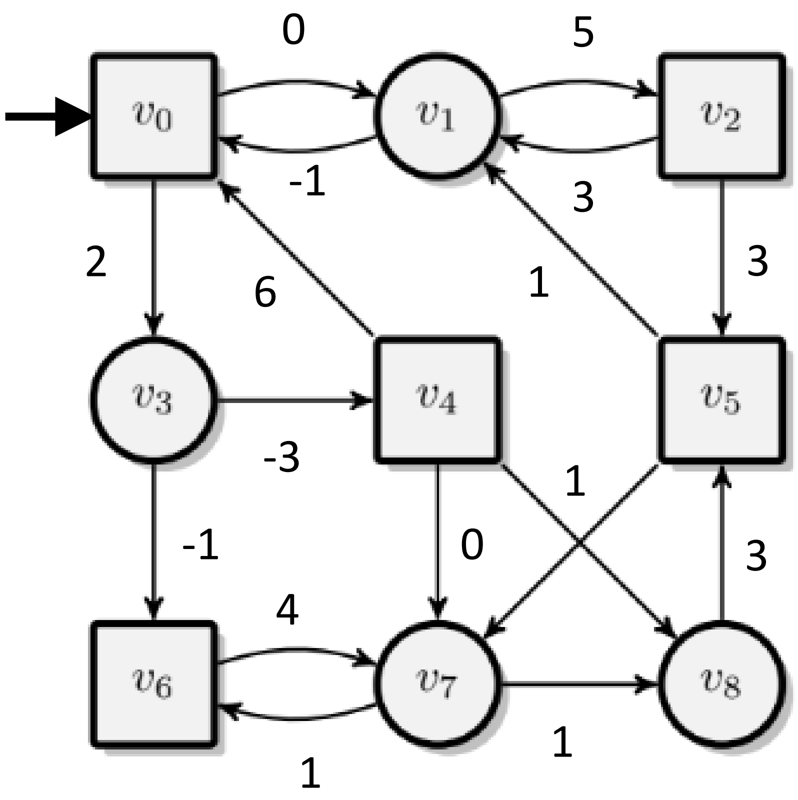
Approach 2: Via Automata-based methods

- Demonstrates theoretical and empirical benefits

Quantitative Game

- States partitioned into two types
  - Each player owns one type of states
- Play
  - Begins in initial state
  - From each state, its player choses the next state
- Cost of a play:
  - Discounted-sum
  - For weight sequence A and discount factor $d > 1$,

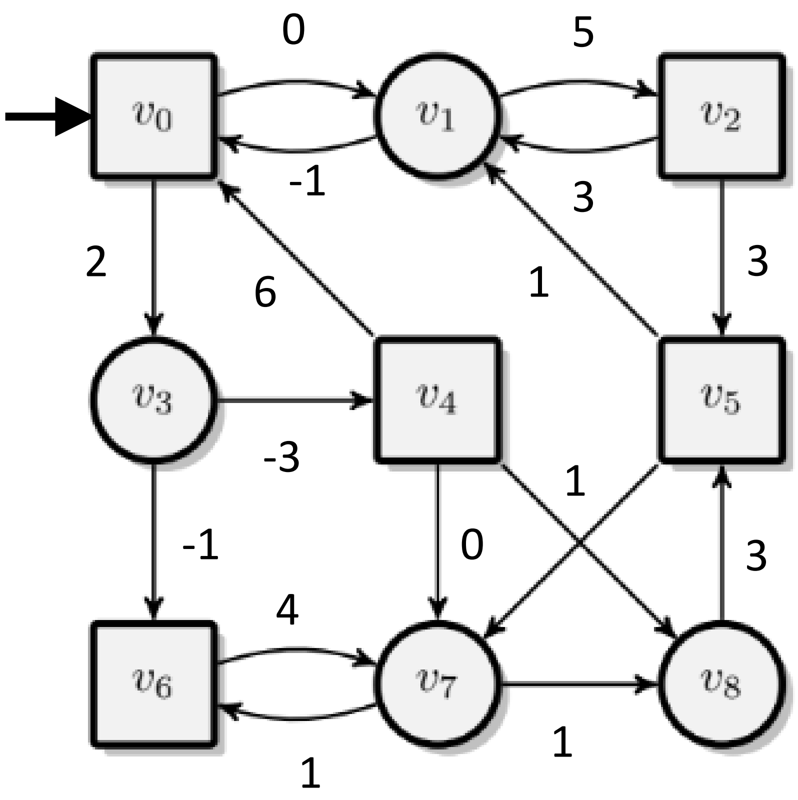
$$DS(A, d) = A[0] + \frac{A[1]}{d} + \frac{A[2]}{d^2} + \cdots$$



Quantitative Game (cont…)

- Adversarial players
  - Max-player: To maximize cost of plays
  - Min-player: To minimize cost of plays

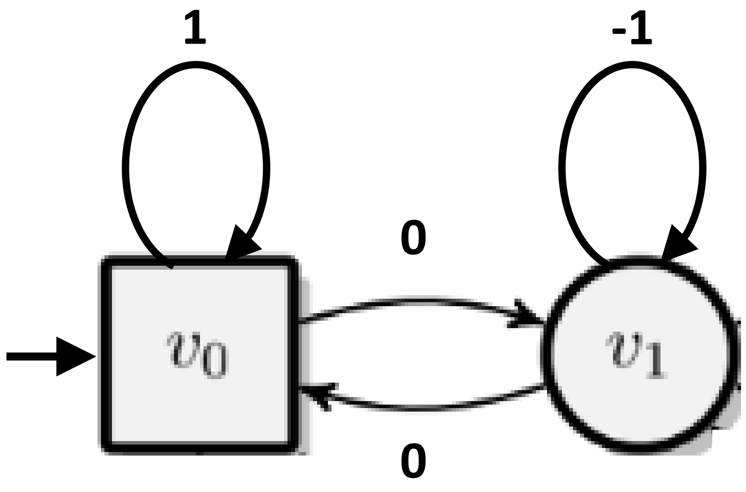
- Strategy for a player
  - Determines which state to go next based on the history of a play?



Satisficing Problem

- Strategies that obey a threshold
- Threshold could be a physical constraint

Satisficing Problem
Given a threshold value v,
Does the Max-Player have a strategy such that cost of all plays exceeds v?



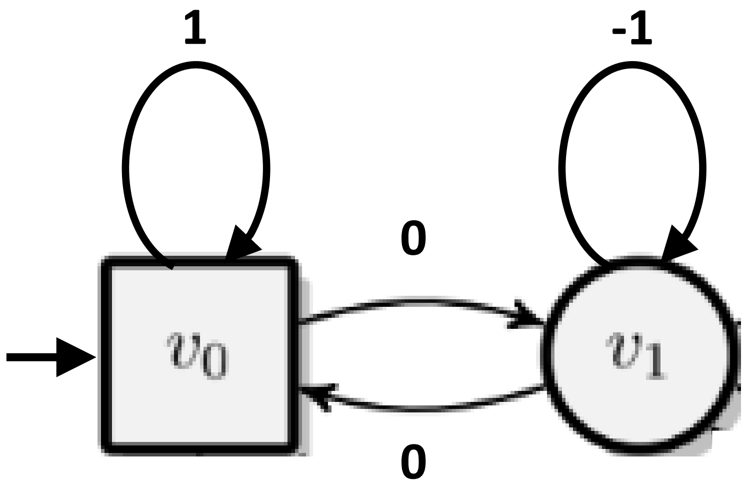
Example: Threshold value 0.5

"Max-player wins"

Satisficing via Optimization

Optimization

- To compute the optimal cost
  - Optimal cost is when both players play optimally
  - Pseudo-polynomial time [Shapely 1950]

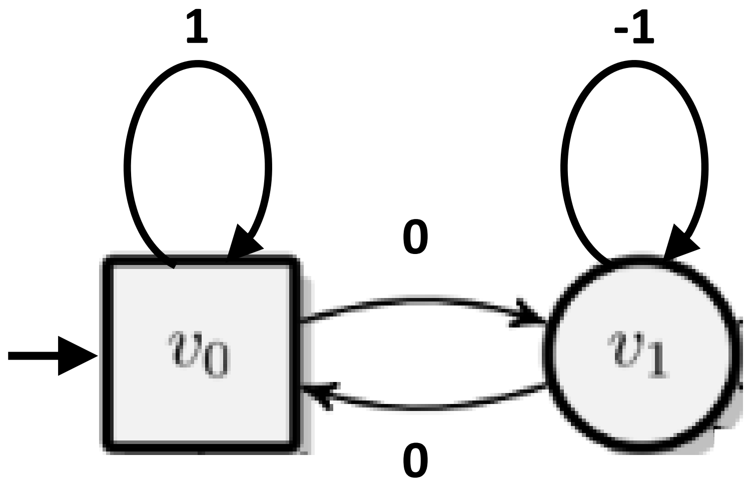


Satisficing via Optimization

- Max-player wins iff Threshold ≤ Optimal Cost

Satisficing via Optimization: Drawbacks

- Same worst-case complexity as optimization
  - No improvement in complexity
- Unsound for Satisficing + Temporal Goals
  - Satisficing threshold: 0.5
  - Temporal goal: Visit v_1



So far on Satisficing

- Satisficing via Optimization ☹
  - No complexity improvement over optimization
  - Unsound with temporally extended goals
- Quantitative + Qualitative goals is challenging [Kwiatkowska, FSE 2006]
  - Inherent incompatibility of techniques
    - Quantitative uses numerical methods
    - Qualitative uses automata-based methods
- Satisficing via an automata-based method?
  - Eliminate the incompatibility!

Comparison is the fundamental operation

Max-player wins

If cost of plays exceeds v

$\equiv$ $DS(A, d) > v$, where A is weight-sequence of a play

How to perform comparison?

Comparator automata

[Bansal, Chaudhuri, and Vardi. FoSSaCS 2018; Bansal et. al. CAV 2018; Bansal and Vardi, CAV 2019]

- Weight sequences are infinite-length words
  - Finite alphabet $\Sigma = \{-\mu, \dots, \mu\}$ for integer $\mu > 0$

Given, discount factor $d > 1$

equality or inequality relation $R \in \{\leq, <, \geq, >, \neq, =\}$

integer $\mu > 0$

Comparator automata (comparator) accepts $A \in \Sigma^\omega$ iff $DS(A, d)\ \boldsymbol{R}\ 0$

Comparator is ω-regular iff the discount factor is an integer
For integer discount factors, Comparator is a deterministic safety/co-safety automata with $O(\mu)$ states

Satisficing via Comparators

Max-player wins

If cost of plays exceeds 0

$\equiv$ $DS(A, d) > 0$, where A is weight-sequence of a play

When the discount factor is an integer

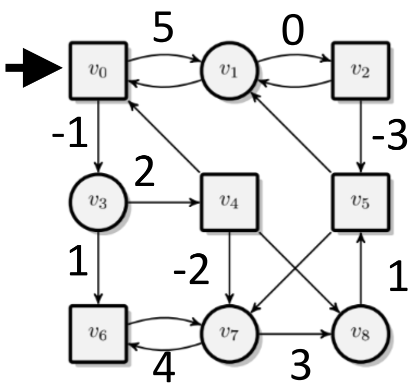
$\equiv$ A is accepted by a comparator for $>$

$\equiv$ Comparator for $>$ captures winning condition

$\equiv$ Safety/co-safety automata captures winning condition

Satisficing via Comparators: Reduction

Threshold value 0



Integer $d > 1$
Maximum of absolute value of weights μ

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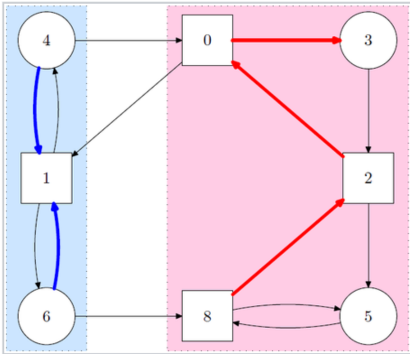
Deterministic safety/co-safety automata for Comparator for d, μ, R

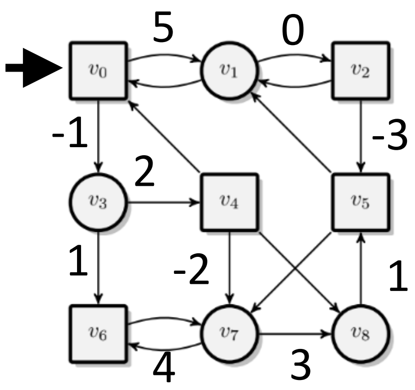
→



Safety or reachability game

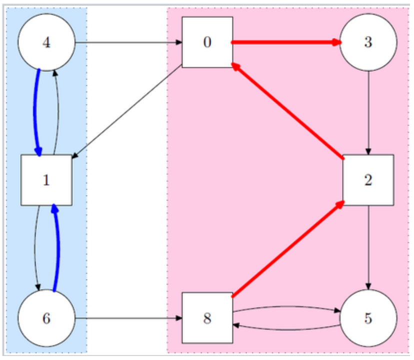
Satisficing via Comparators: Reduction

Threshold value 0



Integer $d > 1$
Maximum of absolute value of weights μ

Deterministic safety/co-safety automata for Comparator for d, μ, R



Safety or reachability game

Comparator automata with arbitrary thresholds

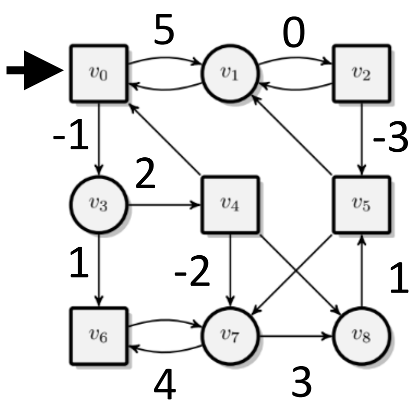
- Comparator automata (comparator) for d, R, μ, v
  - Rational value v
  - Accepts weight sequence $A \in \Sigma^\omega$ iff $DS(A, d)\ \boldsymbol{R}\ v$

Comparator is ω-regular iff the discount factor is an integer

Comparator for an integer discount factor is a deterministic safety/co-safety automata with $O(\mu \cdot n)$ states where n is a parameter in v

- Rational number has a lasso-word representation
- Product of comparator with lasso-word for threshold doesn't work
  - Transition on a × Transition on b → Transition on a+b ?
  - Distorts the alphabet from $\{-\mu, \dots, \mu\}$ to $\{-\mu + k\ \ \dots, \mu + k\}$

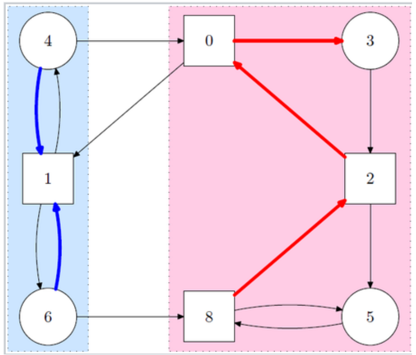
Satisficing via Comparators: Algorithm



- Integer $d > 1$
- Threshold v
- Maximum of absolute value of weights μ

Synchronized product with

Deterministic safety or co-safety automata for comparator for d, μ, R, v



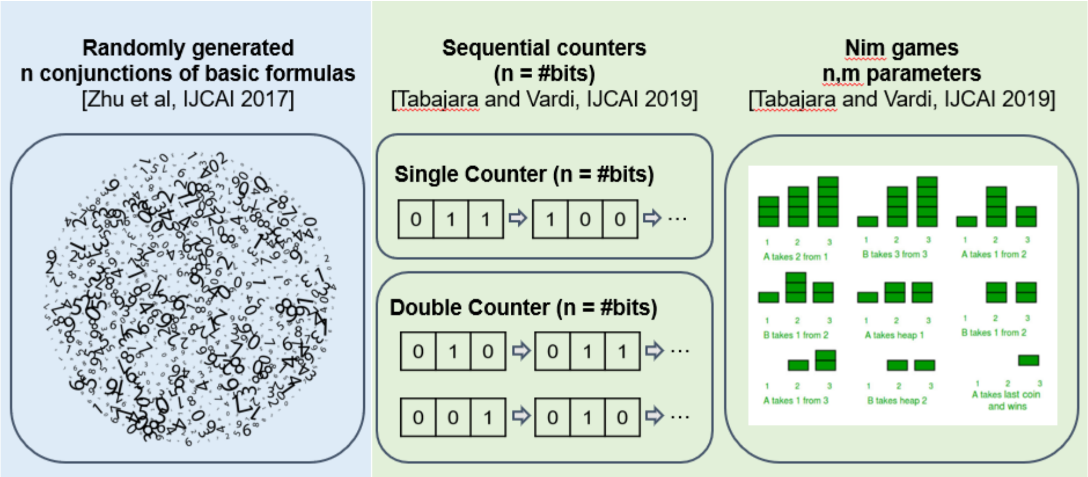
Safety or reachability game

For integer discount factors $d > 1$,

1. Satisficing reduces to solving a safety/reachability game
2. Satisficing takes $O\big((|V| + |E|) \cdot \mu \cdot n\big)$
   where threshold v has an n-bit representation

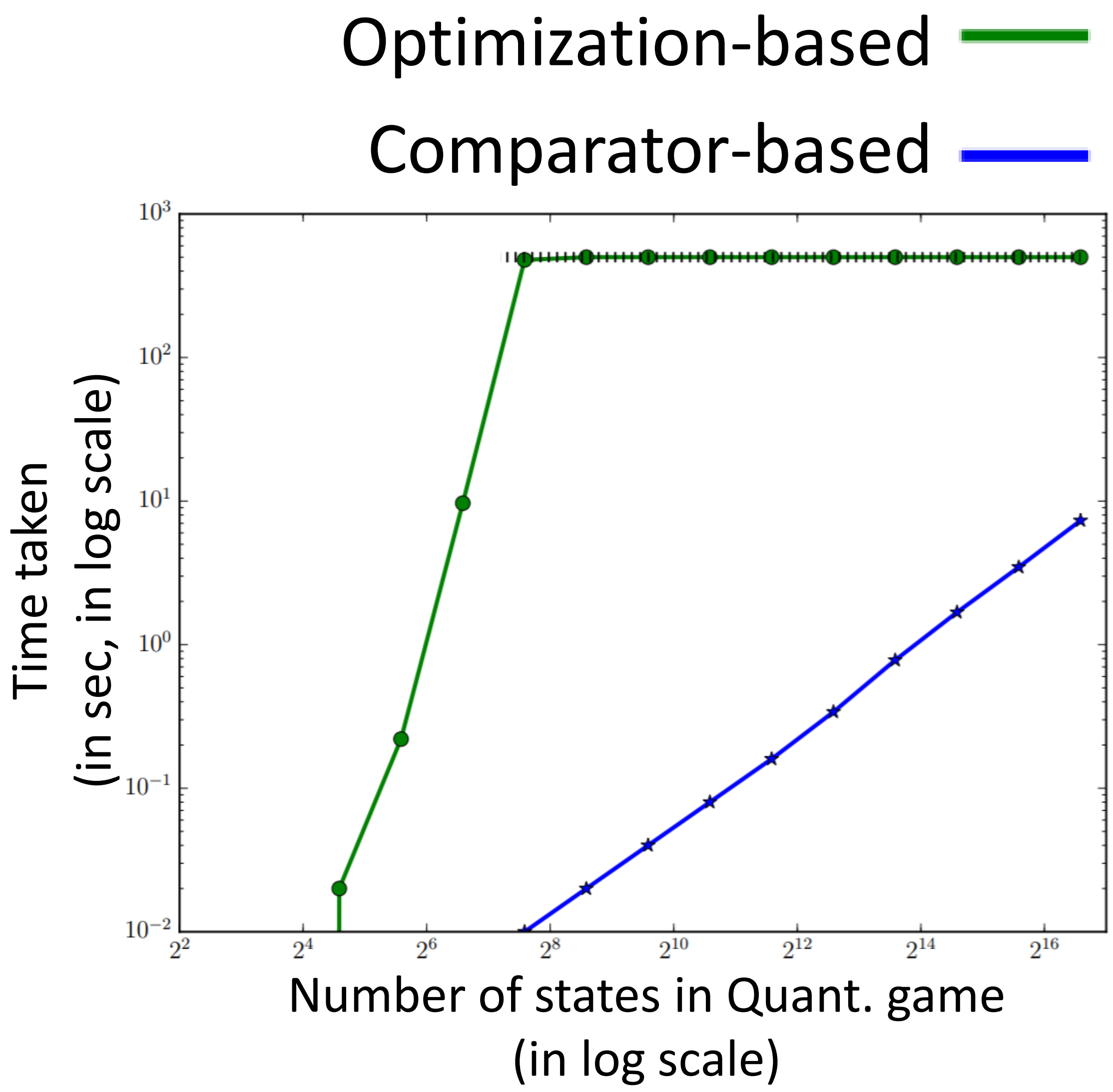
Optimization vs Comparators - I

300 benchmarks created from suite of LTLf benchmarks

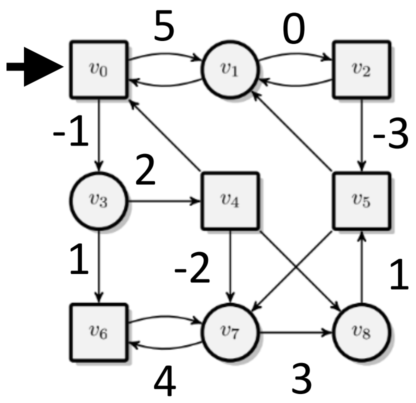


Optimization: ~140/300

Comparator: ~ 285/300



Optimization vs Comparators - II

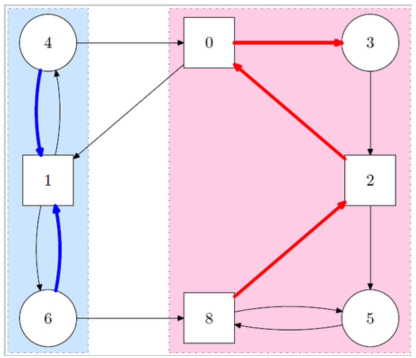


Quantitative Game,
Integer $d > 1$,
Maximum of absolute value of weights μ
+
LTL formula φ

Synchronized product with

Deterministic safety or co-safety automata for DS Comparator for $d, \mu, >, v$

Deterministic Parity Automata for LTL formula φ



Parity Game

Winning in Parity Game iff
Exists satisficing strategy that satisfies the LTL formula

In a nutshell

On Satisficing in Quantitative Games

- Satisficing is an effective alternative to optimization
- Comparator-based approach is scalable, efficient, has broad applicability

- Fractional discount factors [Under Submission]
  - Comparators for (additive) approximations of DS
  - Applications in planning under adversarial domains

- Algorithmic improvements: Leverage progress in qualitative reasoning
- Satisficing in other domains: MDPs, w/o partial information

Back-up slides

Optimization vs Comparators - III

Optimization-based

- Value Iteration
- Early-termination Heuristic

Comparator-based exhibits robust performance