

## 5. Tutorial 4

Monday, March 1, 2021 10:20 AM

① Series:  $a, a+c, a+c^2, \dots, a+c^m$   
 $a > 0, c > 0, m > 0$

$a=1$   
 $c=2$

1	3	5	9	17	33	65	127	257
0	1	2	3	4	5	6	7	

in less than linear time:

1)  $(A, l, h)$   $(A, 0, 7)$   $\text{mid} = \frac{0+7}{2} = 3$

2) check  $a[3] = a+c^3 = 1+2^3 = 9$   
 error in right half

$(A, 4, 7)$   $\text{mid} = \frac{4+7}{3} = 6$

check  $a[6] = a+c^6 = 1+2^6 = 67 \neq 127$   
 error in left half

$O(\log n)$

② Sorting DOB

1. 05 04 1986  
 2. 06 03 2011  
 3. 03 02 1925  
 4. 22 04 1986  
 ...

$O(k \cdot n) \rightarrow O(8 \cdot n)$

number of times  
you sort

$k \neq n$

$O(n^2)$  is never  
reached.

1. Sort by year, then  
month and day

2. Use bucket/radix Sort

3.  $O(n \log n)$  for sort  
buckets (less than  
 $n \log n$  because many numbers  
repeat)

4. 3 set of buckets: bit by bit

Ⓐ for year

Ⓑ for month within year bucket

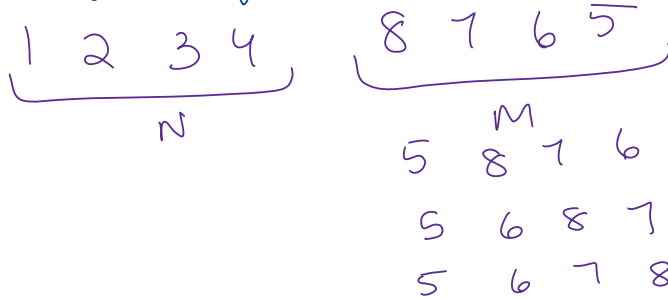
Ⓒ for day within month bucket

③ Array A of  $M+N$  elements

first N sorted

first  $N$  sorted  
last  $M$  unsorted

1. fully sorting using insertion sort



1	3	5	7	2	8	6	4
1	2	3	5	7	8	6	4
1	2	3	4	5	7	8	6
1	2	3	5	5	6	7	8

Shifting happens only  $n$  times,  $n+m$  elements shifted

Complexity =  $O(m(m*n))$

If  $m = O(1)$ :

Using insertion sort,  $O(n)$

If  $m = O(\log n)$

Using insertion sort,  $O(n \log n)$

If  $m = O(n)$

Using insertion sort,  $O(n^2)$

4.  $T(n) = 4T(n/2) + n$

Let  $a \geq 1$  and  $b > 1$  be constants, let  $f(n)$  be a function, and let  $T(n)$  be defined on the nonnegative integers by the recurrence

$$T(n) = aT(n/b) + f(n),$$

where we interpret  $n/b$  to mean either  $\lfloor n/b \rfloor$  or  $\lceil n/b \rceil$ . Then  $T(n)$  has the following asymptotic bounds:

1. If  $f(n) = O(n^{\log_b a - \epsilon})$  for some constant  $\epsilon > 0$ , then  $T(n) = \Theta(n^{\log_b a})$ .
2. If  $f(n) = \Theta(n^{\log_b a})$ , then  $T(n) = \Theta(n^{\log_b a} \lg n)$ .
3. If  $f(n) = \Omega(n^{\log_b a + \epsilon})$  for some constant  $\epsilon > 0$ , and if  $af(n/b) \leq cf(n)$  for some constant  $c < 1$  and all sufficiently large  $n$ , then  $T(n) = \Theta(f(n))$ . ■

$T(N) = aT(N/b) + f(n)$   
 $T(N) = 4T(N/2) + n$   
 $a=4, b=2, f(n)$   
 $f(n) \leq n^{\log_2 4}$   
 $n \leq n^2$ , true  $\Rightarrow f(n) \in n^{\log_2 4} \Rightarrow f(n) \in \Theta(n^2)$   
 $T(n) \in \Theta(n^2)$

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a)  $T(n) = 5T(n/3) + n \lg n$   
 $n \lg n \leq n^{\log_3 5} \approx n^{1.5}$  } deduced from graph  
 $T(n) = \Theta(n^{1.5})$

b) c) cannot be solved (unequal division)

d)  $T(n) = 16T(n/4) + n^2 \Rightarrow \Theta(n^2 \lg n)$

e) cannot be solved (not dividing)