Tutorial 2

Monday, February 8, 2021 10:15 AM

1. Big-O time complexity

Expression	Dominant terms	O()
$5 + 0.001n^3 + 0.025n$	$0.001n^3$	n^3
$500n + 100n^{1.5} + 50nlog(10)n$	$100n^{1.5}$	$n^{1.5}$
Type equation here.	$2.5n^{1.75}$	$n^{1.75}$
	n^2logn	n^2logn
	$3\log(8)\, n$	$\log(8)\, n$
	$0.01\mathrm{n}^2$	$100\mathrm{n}^2$
	$100\mathrm{n}^2$	
	$0.5n^{1.25}$	
	$0.01nlogn$	
	n^3	
	$0.003\log(4)\, n$	

$(logn)^2 < logn$
$\log(logn) < logn$

2.
$Ta(n) = 5 * n * \log(10)\, n$
$Tb(n) = 25 * n$

Assume that nlogn is the upper bound of n
25n = O(5nlogn)

f(n)<= c*g(n)
25n <= c* 5nlogn
5 <= c * logn
Let c=5
5<=5*5 (n >10^5)
-->assumption is true

3. Image

$T(n) = 2T(n/2) + cn^2$

$T(n) = O(n\log_2 n)$ ← closed form

$T(n) = k \cdot T(n/k) + c k \cdot n$ $[\ O(c,k,n)\ ]$

$T(1) = 0$

assume, $n = k^m$

$T(k^m) = k\,T\left(\frac{k^m}{k}\right) + c \cdot k \cdot k^m$

$= k\,T(k^{m-1}) + ck^{m+1}$

$\frac{T(k^m)}{k^{m+1}} = \cancel{k}\,\frac{T(k^{m-1})}{k^{m+1}} + c$

$\frac{T(k^m)}{k^{m+1}} = \frac{T(k^{m-1})}{k^m} + c$

reduces till $T(1) = 0$

$\frac{T(k^{m-1})}{k^m} = \frac{T(k^{m-2})}{k^{m-1}} + c$ $m-2=0 \Rightarrow k^{m-2} = 1$, $m = 2$

$\cancel{ke}\ \frac{T(k^1)}{k^2} = \frac{T(k^0)}{k} + c$

$\cancel{ke}\ T(k) = 0 + ck^2$

$T(k^m) = k^{m+1} \cdot c$ [back substitution]

$T(n) = k^m \cdot k\ c$

$T(n) = n \cdot k \cdot c$

$= n \cdot \log n \cdot c$

1. Loop 1: logn
   Loop 2: logn
   Loop 3: n/2

   Complexity: $n(logn)^2$