Divide-and-Conquer 2

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MIN, MAX problem

$T(N) = T\left(\frac{N}{2}\right) + T\left(\frac{N}{2}\right) + 2, n > 2$

$1, n = 2$

$0, n = 1$

$T(N) = 2\left[2T\left(\frac{N}{4}\right) + 2\right] + 2 = 4T\left(\frac{N}{4}\right) + 4 + 2$

$T(N) = 4\left[2T\left(\frac{N}{8}\right) + 2\right] + 4 + 2 = 8T\left(\frac{N}{8}\right) + 8 + 4 + 2$

$T(N) = 2^k * T\left(\frac{n}{2^k}\right) + 2^k + \cdots + 2^1$

Relationship between n and k

$\frac{n}{2^k} = 2, n = 2^{k+1}$

$T(N) = 2^k * \frac{2}{2} * T\left(\frac{n}{2^k}\right) + 2 + 4 + 8 + \cdots$

$= \frac{2^{k+1}}{2} T(2) + 2 + 4 + 8 + \cdots$

$= \frac{n}{2} * 1 + 2 + 4 + 8 + \cdots$

(geometric series)

$\frac{a(1 - r^n)}{1 - r} = \frac{2(1 - 2^k)}{1 - 2} = -2 + 2^{k+1}$

$T(n) = \frac{n}{2} - 2 + \ 2^{k+1} = \frac{n}{2} - 2 + n$

$= 1.5n - 2 = O(n)$

Brute force:

$T(n) = 2n - 2 = O(n)$

Quick sort:

A number selected at random

After one iteration, the number is placed in the correct position with the numbers smaller than it on the left and numbers bigger than it on the right

$$T(N) = 2T\left(\frac{N}{2}\right) + n$$

Worst case:

$$T(N) = T(n-1) + n = O(n^2)$$

Best case:

$$T(N) = O(nlogn)$$

Average case:

$$T(N) = O(n)$$