

4. TVS + Job Scheduling

Sunday, February 21, 2021 8:32 AM

Greedy

1. Knapsack
2. Tree vertex splitting
3. Job scheduling with deadlines
4. Minimum spanning tree

Greedy

TVS

1. For every node assign a value
2. For node u , $d(v) + \text{weight}(u, w) \geq \text{delta}$, then split the node $w \rightarrow \text{parent}$
Value of leaf node is always zero
3. All values of $d(v)$ initialized to zero (initially)
4. $d(T) = \max(d(T), d(v) + w(T, v))$ [for non-leaf nodes]

Algorithm TVS(T, δ)
// Determine and output the nodes to be split.
 $w()$ is the weighting function for the edges.
if ($T \neq \emptyset$) then
 $d(T) := 0$
 for each child v of T do
 {
 TVS(v, δ);
 $d(T) := \max(d(T), d(v) + w(T, v))$;
 }
 if ((T is not the root) and
 ($d(T) + w(\text{parent}(T), T) > \delta$)) then
 {
 write (T); $d(T) := 0$;
 }

Job scheduling with deadlines:

- Profit for each job
- Job number
- Slot of execution

Find which job fits into the right slot

Arrange in order of decreasing profit

| Job number | Profit | Slot number |
|------------|--------|-------------|
| 1 | 20 | 2 |
| 2 | 15 | 2 |
| 5 | 10 | 1 |
| 4 | 5 | 3 |
| 3 | 1 | 3 |

Job 1 can execute in slot 1,2 not 3

Job 3 can execute in slot 1,2,3

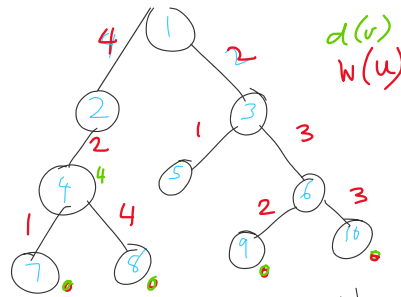
Maximal slot: 3

Draw the graph:

| Job | Slot allotted |
|-----|---------------|
| 1 | 1 |
| 2 | 2 |
| 5 | Reject |
| 4 | 3 |
| 3 | Reject |

| Job | Slot | Job considered | Profit | Slot assigned |
|---------|---------------|----------------|--------|---------------|
| [] | - | 1 | 0 | 0-1 |
| [1] | 0-1 | 2 | 20 | 1-2 |
| [1 2] | 0-1, 1-2 | 5 | 35 | Reject |
| [1 2] | 0-1, 1-2 | 4 | 35 | 2-3 |
| [1 2 4] | 0-1, 1-2, 2-3 | 3 | 40 | reject |

| Job number | Profit | Slot number |
|------------|--------|-------------|
| 1 | 20 | 4 |
| 2 | 15 | 3 |
| 5 | 10 | 2 |
| 4 | 5 | 1 |



Max(w , child nodes)

$d(4) + w(4, x)$

$0 + \max(1, 4) = 4$

If in node 4,

$d(4) + 2 \geq 5$?

Yes split at 4

$d(4) = 0$ after split (leaf node)

$d(7) + 1 \geq 5$?

No, no split at 7

$d(8) + 4 \geq 5$?

No, no split at 8

$d(2) = d(2), d(4) + \max(w(4, 2))$

$= 0 + 0 + (\max(2)) = 2$

For node 2,

$d(2) + w(2, 1) \geq 5$?

Yes, split at 2

For node 1, $d(3)$ to be calculated

POST ORDER

At node 6,

$d(6) = d(6), \max(d(10) + w(10, 6), d(9) + w(9, 6))$

$= 6$

$d(6) + w(6, 3) = 3 + 3 \geq 5$?

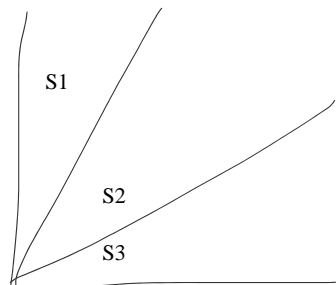
Split at 6

No splitting at root node

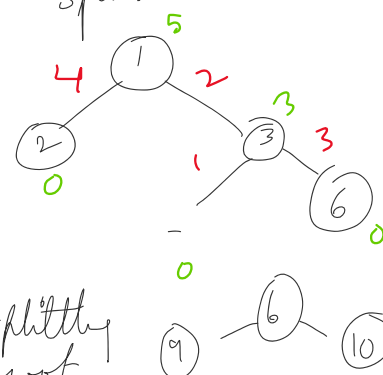
Complexity:

Using arrays : $O(v^2)$

Using linked/adjacency list (singly): $O(v)$



no splitting
at root
node



| | | |
|---|---|---|
| 3 | 1 | 4 |
|---|---|---|

| Job | Slot |
|-----|------------------------|
| 1 | 1 |
| 2 | 2 |
| 3 | 2 (2 pushed to slot 3) |
| 4 | 1 (1 pushed to 4) |