4. Dynamic programming

Wednesday, February 24, 2021 12:05 PM

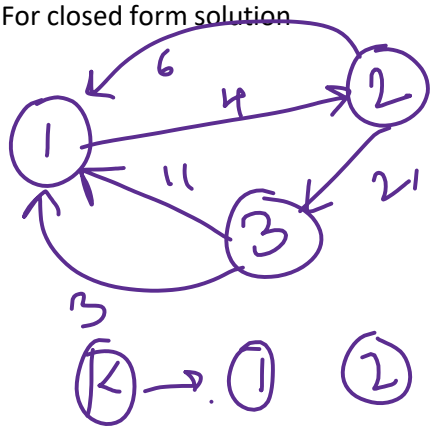
Dynamic programming is always iterative

1. All pair shortest path
2. Single source shortest path Bellman Ford Algorithm

APSP
Objective
Shortest node from every node to every other node

Prerequisite
Cycles are allowed, directed graph
Negative cycles are not allowed

Property
PoO Principle of Optimality
Only holds for shortest path
For closed form solution



$$A^0 = \begin{bmatrix} 0 & 4 & 11 \\ 6 & 0 & 2 \\ 3 & \infty & 0 \end{bmatrix}$$

K → 1 2 3

$$A^1[1,2] = \min\{A^0(1,2),\ A^1(1,1) + A^0(1,2)\}$$

$$= \min\{4, 0+4\} = 4$$

$$A^1[1,3] = \{A^0(1,3),\ A^0(1,1) + A^0(1,3)\}$$

$$A^1 = \begin{bmatrix} 0 & 4 & 11 \\ 6 & 0 & 2 \\ 3 & 7 & 0 \end{bmatrix}$$

Cost of the shortest path from i to j via k
Ak(I,j) = min{Ak-1(I,j), Ak-1(I,k) + Ak-1(k,j)

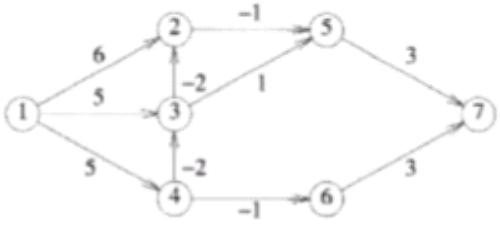
$$A^2 = \begin{bmatrix} 0 & 4 & 6 \\ 6 & 0 & 2 \\ 3 & 7 & 0 \end{bmatrix} \quad A^3 = \begin{bmatrix} 0 & & 6 \\ 5 & 0 & 2 \\ & & 0 \end{bmatrix}$$

SSSP (BF)
Objective: given a node x, find the shortest path from x to all other nodes
Prerequisite: no cycles, negative values are allowed (DAG)

Cost of reaching node u via k edges

Number of edges	Node 1	Node 2	Node 3
1			
2			
3			

$$dist^k[u] = \min \{dist^{k-1}[u], \min_i \{dist^{k-1}[i] + cost[i,u]\}\}$$



Single source: node 1

Via	1	2	3	4	5	6	7
1	0	6	5	5	Inf	Inf	Inf
2	0	3	3				
3	0						
4	0						
5	0						
6	0						
7	0						

d2(2) = min {d1[2], min{d1[3]+c(3,2)}
= min(6,5-2) = 3