

# Tutorial 1

Monday, February 1, 2021 9:14 AM

1.  $T(n) = n^3 + 20n + 1$   
Is  $O(n^3)$ ?

$$n^3 + 20n + 1 \leq c \cdot n^3$$

Assume  $c = 22$  ( $20 + 1 + 1$ )

$n = 0$ ,  $1 < 0$ , false

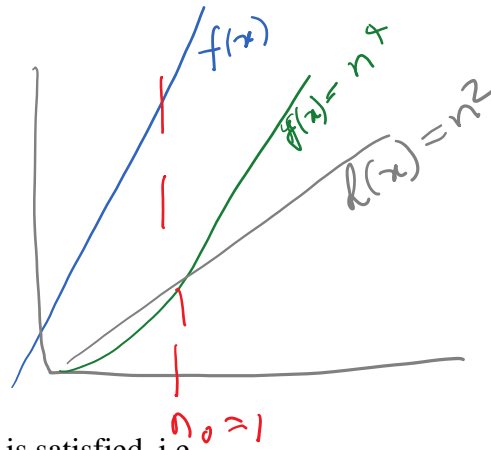
$n = 1$ ,  $22 \leq 22$ , true

$n = 2$ ,  $49 < 176$ , true

Draw graph

When  $n = 1$ ,  $c = 22$ , condition for  $O$  is satisfied, i.e.

$n^3 + 20n + 1 \leq c \cdot n^3$  for all  $n > 0$  and  $c = 22$



Order of growth

$N$	$N, N^2, N^3 \dots$
$N^2$	$N^2, N^3 \dots$
$N^3$	$N^3, N^4 \dots$
Logn	$n, n \log n, N^2, N^3 \dots$

2.  $T(n) = n^3 + 20n + 1$   
Is not  $O(n^2)$ ? True

$$n^3 + 20n + 1 \leq c \cdot n^2$$

3.  $T(n) = n^3 + 20n + 1$   
Is  $O(n^4)$ ?  
True

4.  $T(n) = n^3 + 20n$  Is  $\Omega(n^2)$ ?  
True

5. Sorting algorithm:  
 $O(n \log n)$   
1ms to sort 1000 data items  
 $T(n) = c \cdot n \cdot \log n$  ( $T(N)$  is time to sort  $N$  items)

Find  $T(1,000,000)$

$$T(1000) = c \cdot 1000 \cdot \log 1000 = 1 \text{ ms}$$

$$C = 1/1000 \cdot \log 1000$$

$$C = 1.003 \cdot 10^{-4} = 0.0001$$

$$T(10^6) = 0.0001 \cdot 10^6 \cdot \log(10^6) = 1993.156 \text{ ms} = 2000 \text{ ms}$$

6.  $T(n) = c \cdot n^2$   
 $T(100) = c \cdot 100^2 = 1 \text{ ms}$   
 $C = 1/100^2 = 0.0001$

$$T(5000) = 0.0001 \cdot 5000^2 = 2500.0 \text{ ms}$$

7. 10s  $\rightarrow$  1000 items  
 $T(n) = c \cdot f(n) = c \cdot n$

$$T(1000) = c * 1000 = 10$$

$$C = 10/1000 = 0.01$$

$$T(100,000) = 0.01 * 100000 = 1000.0$$

8. Reflective: all yes

$$f(n) = O(f(n))$$

Symmetric: O and Omega not symmetric

$$f(\log n) = O(n)$$

But  $f(n) \neq O(\log n)$

Theta is symmetric

Transitive: true for all

9. X

a. True

b. False,  $O(a + b) = O(\max(a, b))$

c. False

d. True

e. False,  $O(n^3)$