5. Tutorial 4

Monday, March 1, 2021 10:20 AM

① Series : $a, a+c^1, a+c^2 \dots a+c^m$

$a>0, c>0, m>0$

$a=1$, $c=2$	1	3	5	9	17	33	65	127	257
	1	3	5	9	17	33	127	257	
	0	1	2	3	4	5	6	7	

in less than linear time:

1) (A, l, h) $(A, 0, 7)$ $mid = 0+7/2 = 3$

2) check $a[3] = a+c^3 = 1+2^3 = 9$
error in right half

$(A, 4, 7)$ $mid = \frac{4+7}{3} = 6$

check $a[6] = a+c^6 = 1+2^6 = 67 \neq 127$
error in left half

$O(\log n)$

② Sorting DOB

1.	05	04	1986
2.	06	03	2011
3.	03	02	1925
4.	22	04	1986
⋮			

1. Sort by year, then month and day
2. Use bucket / radix sort
3. $O(n\log n)$ to sort buckets (lesser than $n\log n$ because many numbers repeat)
4. 3 set of buckets : bit by bit
   ⓐ for year
   ⓑ for month within year bucket
   ⓒ for day within month bucket

$O(k \cdot n) \rightarrow O(8 \cdot n)$

k → number of times you sort

$k \neq n$

$O(n^2)$ is never reached.

③ Array A of M+N elements

first N sorted

first N sorted
last M unsorted

1. fully sorting using insertion sort

$\underbrace{1 \quad 2 \quad 3 \quad 4}_{N} \qquad \underbrace{8 \quad 7 \quad 6 \quad 5}_{M}$

5 8 7 6

5 6 8 7

5 6 7 8

1	3	5	7	2	8	6	4
1	2	3	5	7	8	6	4
1	2	3	4	5	7	8	6
1	2	3	5	5	6	7	8

Shifting happens only n times, n+m elements shifted

Complexity = O(m(m*n))

If m = O(1):
Using insertion sort, O(n)

If m=O(logn)
Using insertion sort, O(nlogn)

If m=O(n)
Using insertion sort, O(n^2)

4. T(n) = 4T(n/2) + n

Let $a \geq 1$ and $b > 1$ be constants, let $f(n)$ be a function, and let $T(n)$ be defined on the nonnegative integers by the recurrence

$$T(n) = aT(n/b) + f(n) ,$$

where we interpret n/b to mean either $\lfloor n/b \rfloor$ or $\lceil n/b \rceil$. Then $T(n)$ has the following asymptotic bounds:

1. If $f(n) = O(n^{\log_b a - \epsilon})$ for some constant $\epsilon > 0$, then $T(n) = \Theta(n^{\log_b a})$.
2. If $f(n) = \Theta(n^{\log_b a})$, then $T(n) = \Theta(n^{\log_b a} \lg n)$.
3. If $f(n) = \Omega(n^{\log_b a + \epsilon})$ for some constant $\epsilon > 0$, and if $af(n/b) \leq cf(n)$ for some constant $c < 1$ and all sufficiently large n, then $T(n) = \Theta(f(n))$. ∎

$T(N) = aT(N/b) + f(n)$

$T(N) = 4T(N/2) + n$

$a = 4,\ b = 2,\ f(n)$

$f(n) \leq n^{\log_2 4}$

$n \leq n^2$, true $\Rightarrow f(n) \in n^{\log_2 4}$ $\Rightarrow$

$T(n) \in \Theta(n^2)$

5.

a) $T(n) = 5T(n/3) + n\log n$

$n\log n \leq n^{\log_3 5} \approx n^{1.5}$ → deduced from graph

$T(n) = \Theta(n^{1.5})$

b) c) cannot be solved (unequal division)

d) $T(n) = 16T(n/4) + n^2 \Rightarrow \Theta(n^2 \log n)$

e) cannot be solved (not dinding)