!) Determine the values of r at which the 1-cycle and the 2-cycle are super stable.

```
%FOR Cycle 1 refer written one

syms x r

p = r*(1-x)*(1-r*x+r*x^2) - 1
```

p = $-r\,(x-1)\,(r\,x^2 - r\,x + 1) - 1$

```
roots = solve(p,x,"MaxDegree",3)
```

roots =

$$\begin{pmatrix} \sigma_1 - \frac{\sigma_2}{\sigma_1} + \frac{2}{3} \\ \frac{\sigma_2}{2\,\sigma_1} - \frac{\sigma_1}{2} + \frac{2}{3} - \frac{\sqrt{3}\,\left(\frac{\sigma_2}{\sigma_1} + \sigma_1\right)\,\mathrm{i}}{2} \\ \frac{\sigma_2}{2\,\sigma_1} - \frac{\sigma_1}{2} + \frac{2}{3} + \frac{\sqrt{3}\,\left(\frac{\sigma_2}{\sigma_1} + \sigma_1\right)\,\mathrm{i}}{2} \end{pmatrix}$$

where

$$\sigma_1 = \left(\frac{r-1}{2\,r^2} - \frac{r^2+r}{3\,r^2} + \sqrt{{\sigma_2}^3 + \left(\frac{r-1}{2\,r^2} - \frac{r^2+r}{3\,r^2} + \frac{8}{27}\right)^2} + \frac{8}{27}\right)^{1/3}$$

$$\sigma_2 = \frac{r^2+r}{3\,r^2} - \frac{4}{9}$$

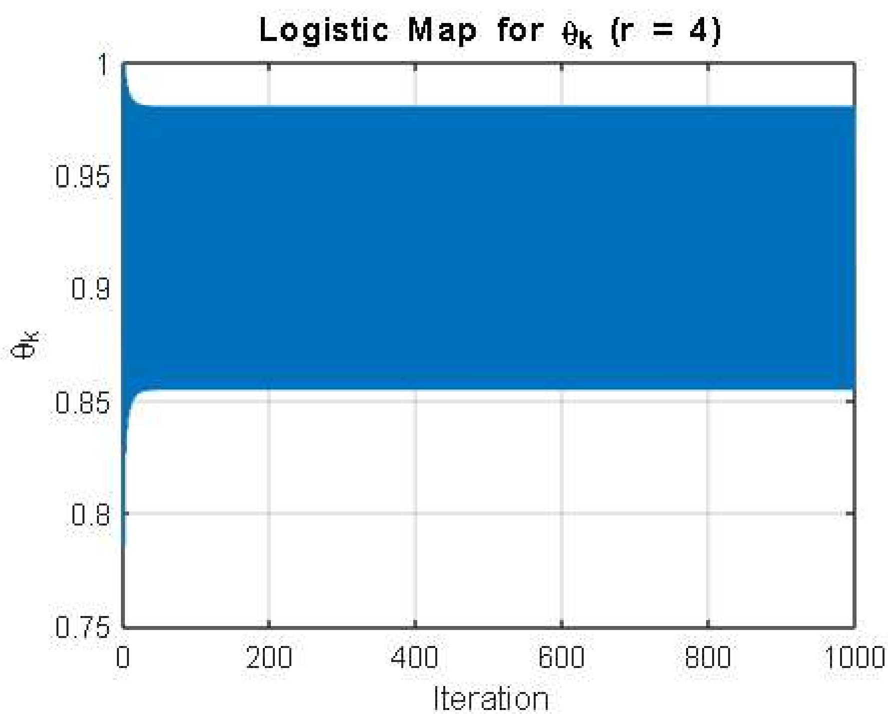
```
pdash = diff(p)
```

pdash = $r\,(r - 2\,r\,x)\,(x-1) - r\,(r\,x^2 - r\,x + 1)$

2) Consider the logistic map xk+1 = r xk (1 − xk) , 0 ≤ x ≤ 1 . (a) Determine the values of r at which the 1-cycle and the 2-cycle are super stable. (b) Next consider the special case of r = 4, i.e., the map xk+1 = 4 xk (1 − xk) , 0 ≤ x ≤ 1 . What are the various types of solutions possible for this special case? Hint: You might like to use the substitution xk = sin2 (θk) with an appropriate interval for θk for uniqueness of the mapping, and obtain the map for θk as θk+1 = g (θk) . If you proceed along these lines, also sketch the map for θk. Calculate the Lyapunov exponent to justify presence of chaotic solutions if any.

```
% Define the logistic map function
g = @(theta) 4 * sin(theta).^2 .* cos(theta).^2;
```

```
% Initial condition
theta0 = pi / 4; % Choose some initial angle

% Number of iterations
n_iterations = 1000;

% Preallocate array to store trajectory
theta_traj = zeros(1, n_iterations);

% Initialize trajectory
theta_traj(1) = theta0;

% Iterate the logistic map
for i = 2:n_iterations
    theta_traj(i) = g(theta_traj(i-1));
end

% Plot the map
figure;
plot(1:n_iterations, theta_traj);
xlabel('Iteration');
ylabel('\theta_k');
title('Logistic Map for \theta_k (r = 4)');
grid on;
```




```
% Calculate Lyapunov exponent
delta_theta = abs(diff(theta_traj));
lyapunov_exp = mean(log(abs(delta_theta(2:end) ./ delta_theta(1:end-1))));
fprintf('Lyapunov Exponent: %f\n', lyapunov_exp);
```

```
Lyapunov Exponent: -0.000534
```