```
> restart :
```

Solve By PoincareLindstedt method

```
> N := 2; omega := 1 + add(w[k]·ϵ^k, k=1..N);
```

$$N := 2$$

$$\omega := w_2\,\epsilon^2 + w_1\,\epsilon + 1 \tag{1}$$

```
> de := ω^2 · diff(x(t), t, t) + x(t) + epsilon·diff(x(t), t)^3 · ω^3
```

$$de := \left(w_2\,\epsilon^2 + w_1\,\epsilon + 1\right)^2 \left(\frac{d^2}{dt^2}\,x(t)\right) + x(t) + \epsilon\left(\frac{d}{dt}\,x(t)\right)^3 \left(w_2\,\epsilon^2 + w_1\,\epsilon + 1\right)^3 \tag{2}$$

```
> xp := add(x[k](t)·ϵ^k, k=0..N);
```

$$xp := x_0(t) + x_1(t)\,\epsilon + x_2(t)\,\epsilon^2 \tag{3}$$

```
> #de1 := expand(subs(x(t)=xp, de));
> de1 := convert(taylor(expand(subs(x(t)=xp, de)), epsilon, N+1), polynom);
```

$$de1 := \frac{d^2}{dt^2}\,x_0(t) + x_0(t) + \left(\left(\frac{d}{dt}\,x_0(t)\right)^3 + \frac{d^2}{dt^2}\,x_1(t) + 2\left(\frac{d^2}{dt^2}\,x_0(t)\right) w_1 + x_1(t)\right)\epsilon + \left(\frac{d^2}{dt^2}\,x_2(t) + 3\left(\frac{d}{dt}\,x_0(t)\right)^3 w_1 + 3\left(\frac{d}{dt}\,x_0(t)\right)^2\left(\frac{d}{dt}\,x_1(t)\right) + \left(\frac{d^2}{dt^2}\,x_0(t)\right) w_1^2 + 2\left(\frac{d^2}{dt^2}\,x_0(t)\right) w_2 + 2\left(\frac{d^2}{dt^2}\,x_1(t)\right) w_1 + x_2(t)\right)\epsilon^2 \tag{4}$$

```
> x0sol := dsolve({coeff(de1, epsilon, 0), x[0](0) = A, D(x[0])(0) = 0}, x[0](t));
```

$$x0sol := x_0(t) = A\cos(t) \tag{5}$$

```
> xp1 := subs(x0sol, xp);
```

$$xp1 := A\cos(t) + x_1(t)\,\epsilon + x_2(t)\,\epsilon^2 \tag{6}$$

```
> de2 := convert(taylor(expand(subs(x(t)=xp1, de)), epsilon, N+1), polynom);
```

$$de2 := \left(\frac{d^2}{dt^2}\,x_1(t) - A^3\sin(t)^3 + x_1(t) - 2A\cos(t)\,w_1\right)\epsilon + \left(\frac{d^2}{dt^2}\,x_2(t) + 2\left(\frac{d^2}{dt^2}\,x_1(t)\right) w_1 + x_2(t) + 3A^2\sin(t)^2\left(\frac{d}{dt}\,x_1(t)\right) - 3A^3\sin(t)^3\,w_1 - A\cos(t)\,w_1^2 - 2A\cos(t)\,w_2\right)\epsilon^2 \tag{7}$$

```
> e1 := coeff(de2, epsilon, 1); e1 := combine(e1, trig);
```

$$e1 := \frac{d^2}{dt^2}\,x_1(t) - A^3\sin(t)^3 + x_1(t) - 2A\cos(t)\,w_1$$

$$e1 := \frac{d^2}{dt^2}\,x_1(t) + x_1(t) + \frac{A^3\sin(3t)}{4} - \frac{3A^3\sin(t)}{4} - 2A\cos(t)\,w_1 \tag{8}$$

```
>
> e1_sec := coeff(e1, cos(t));
```

$$e1\_sec := -2A\,w_1 \tag{9}$$

```
> ans := solve(%, w[1])
```

(10)

$$ans := 0 \quad \textbf{(10)}$$

> omega := $subs(w[1]=ans$, omega);

$$\omega := \epsilon^2 w_2 + 1 \quad \textbf{(11)}$$

> $de3 := collect(expand(subs(w[1]=ans, de2)),$ epsilon);

$$de3 := \left(\frac{d^2}{dt^2} x_1(t) - A^3 \sin(t)^3 + x_1(t)\right)\epsilon + \left(\frac{d^2}{dt^2} x_2(t) + x_2(t) + 3 A^2 \sin(t)^2 \left(\frac{d}{dt} x_1(t)\right) - 2 A \cos(t) w_2\right)\epsilon^2 \quad \textbf{(12)}$$

> $e1 := coeff(de3,$ epsilon, $1)$;

$$e1 := \frac{d^2}{dt^2} x_1(t) - A^3 \sin(t)^3 + x_1(t) \quad \textbf{(13)}$$

> $e1 := dsolve(\{e1, x[1](0)=0, D(x[1])(0)=0\}, x[1](t))$;

$$e1 := x_1(t) = -\frac{A^3 (12\cos(t)\, t - \sin(3t) - 9\sin(t))}{32} \quad \textbf{(14)}$$

> $xp2 := subs(e1, xp1)$;

$$xp2 := A\cos(t) - \frac{A^3 (12\cos(t)\, t - \sin(3t) - 9\sin(t))\,\epsilon}{32} + x_2(t)\,\epsilon^2 \quad \textbf{(15)}$$

> $de4 := collect(combine(expand(subs(e1, de3)), trig),$ epsilon);

$$de4 := \left(-\frac{9 A^5 t \sin(3t)}{32} + \frac{27 A^5 t \sin(t)}{32} + \frac{27 A^5 \cos(3t)}{128} - \frac{9 A^5 \cos(t)}{64} - \frac{9 A^5 \cos(5t)}{128} - 2 A \cos(t) w_2 + \frac{d^2}{dt^2} x_2(t) + x_2(t)\right)\epsilon^2 \quad \textbf{(16)}$$

> $e2 := coeff(de4, epsilon, 2)$;

$$e2 := -\frac{9 A^5 t \sin(3t)}{32} + \frac{27 A^5 t \sin(t)}{32} + \frac{27 A^5 \cos(3t)}{128} - \frac{9 A^5 \cos(t)}{64} - \frac{9 A^5 \cos(5t)}{128} - 2 A \cos(t) w_2 + \frac{d^2}{dt^2} x_2(t) + x_2(t) \quad \textbf{(17)}$$

> $e\_2 := coeff(e2, \cos(t))$;

$$e\_2 := -\frac{9}{64} A^5 - 2 A w_2 \quad \textbf{(18)}$$

>

> $ans1 := solve(\%, w[2])$;

$$ans1 := -\frac{9 A^4}{128} \quad \textbf{(19)}$$

> omega := $subs(w[2]=ans1$, omega);

$$\omega := -\frac{9 A^4 \epsilon^2}{128} + 1 \quad \textbf{(20)}$$

> omega := subs(w[2]=ans1, omega);

$$\omega := -\frac{9A^4\epsilon^2}{128}+1 \quad \textbf{(21)}$$

> de5 := subs(w[2]=ans1, e2)

$$de5 := -\frac{9A^5 t\sin(3t)}{32}+\frac{27A^5 t\sin(t)}{32}+\frac{27A^5\cos(3t)}{128}-\frac{9A^5\cos(5t)}{128}+\frac{d^2}{dt^2}x_2(t)+x_2(t) \quad \textbf{(22)}$$

> eq2 := dsolve({de5, x[2](0)=0, D(x[2])(0)=0}, x[2](t));

$$eq2 := x_2(t)=\frac{3A^5\left(72\cos(t)t^2-12t\sin(3t)-72t\sin(t)+\cos(t)-\cos(5t)\right)}{1024} \quad \textbf{(23)}$$

> eq_final := subs(eq2, xp2);

$$eq\_final := A\cos(t)-\frac{A^3(12\cos(t)t-\sin(3t)-9\sin(t))\epsilon}{32}+\frac{3A^5\left(72\cos(t)t^2-12t\sin(3t)-72t\sin(t)+\cos(t)-\cos(5t)\right)\epsilon^2}{1024} \quad \textbf{(24)}$$

>

This above more or less shows the same behavior as the simple equation with the non linear in it is not resolved as it remain hidden solution in it.
The first inital stage of w1 is zero, no secular term , so no prediction as the nonlinearity reamains same, on how the frequency works
The Poincaré-Lindstedt method often fails when the nonlinearity in the equation is too strong or when the solution involves resonances that cannot be handled effectively by other perturbation methods.

> restart :

We start doing with method of averging

> de := ω^2 · diff(x(t), t, t) + x(t) + epsilon·diff(x(t), t)^3 · ω^3 = epsilon·sin(t)

$$de := \omega^2\left(\frac{d^2}{dt^2}x(t)\right)+x(t)+\epsilon\left(\frac{d}{dt}x(t)\right)^3\omega^3=\epsilon\sin(t) \quad \textbf{(25)}$$

> xs := A(t)·cos(t + phi(t)); xds := −A(t)·sin(t + phi(t));

$$xs := A(t)\cos(t+\phi(t))$$

$$xds := -A(t)\sin(t+\phi(t)) \quad \textbf{(26)}$$

> xdds := diff(xds, t); xds1 := diff(xs, t);

$$xdds := -\left(\frac{d}{dt}A(t)\right)\sin(t+\phi(t))-A(t)\left(1+\frac{d}{dt}\phi(t)\right)\cos(t+\phi(t))$$

$$xds1 := \left(\frac{d}{dt}A(t)\right)\cos(t+\phi(t))-A(t)\left(1+\frac{d}{dt}\phi(t)\right)\sin(t+\phi(t)) \quad \textbf{(27)}$$

> eq1 := ω^2 · xdds + xs + epsilon·(xds1)^3·ω^3 − epsilon·sin(t)

(28)

$$eq1 := \omega^2 \left(-\left(\frac{d}{dt} A(t) \right) \sin(t+\phi(t)) - A(t) \left(1 + \frac{d}{dt} \phi(t)\right) \cos(t+\phi(t)) \right) + A(t) \cos(t + \phi(t)) + \epsilon \left(\left(\frac{d}{dt} A(t) \right) \cos(t+\phi(t)) - A(t) \left(1+\frac{d}{dt}\phi(t)\right) \sin(t+\phi(t)) \right)^3 \omega^3 - \epsilon \sin(t) \quad \textbf{(28)}$$

> eq2 := simplify(xds1 − xds) ;

$$eq2 := -A(t) \sin(t+\phi(t)) \left(\frac{d}{dt}\phi(t)\right) + \left(\frac{d}{dt} A(t)\right) \cos(t+\phi(t)) \quad \textbf{(29)}$$

>

> eq11 := omega^2 * (−Adot* sin(t + phi) − A * (1 + phidot) * cos(t + phi) + A * cos(t + phi)) + epsilon * (Adot* cos(t + phi) − A * (1 + phidot) * sin(t + phi))^3 * omega^3 − epsilon* sin(t);

$$eq11 := \omega^2 \left(-Adot \sin(t+\phi) - A\,(1+phidot) \cos(t+\phi) + A\cos(t+\phi) \right) + \epsilon \left(Adot \cos(t+\phi) - A\,(1+phidot) \sin(t+\phi) \right)^3 \omega^3 - \epsilon \sin(t) \quad \textbf{(30)}$$

> eq22 := −A * sin(t + phi) * phidot + Adot* cos(t + phi);

$$eq22 := -A \sin(t+\phi)\, phidot + Adot \cos(t+\phi) \quad \textbf{(31)}$$

> sol := solve({eq11, eq22}, {Adot, phidot});

$$sol := \left\{ Adot = -\frac{\epsilon \sin(t+\phi) \left(A^3 \omega^3 \sin(t+\phi)^3 + \sin(t)\right)}{\omega^2}, phidot = -\frac{\epsilon \left(A^3 \omega^3 \sin(t+\phi)^3 + \sin(t)\right) \cos(t+\phi)}{\omega^2 A} \right\} \quad \textbf{(32)}$$

> adotav := simplify$\left(\frac{integrate(subs(sol, Adot), t=0..2\cdot \mathrm{Pi})}{2\cdot \mathrm{Pi}}\right)$;

$$adotav := -\frac{\epsilon\left(3 A^3 \omega^3 + 4\cos(\phi)\right)}{8\,\omega^2} \quad \textbf{(33)}$$

> phidotav := simplify$\left(\frac{integrate(subs(sol, phidot), t=0..2\cdot \mathrm{Pi})}{2\cdot \mathrm{Pi}}\right)$;

$$phidotav := \frac{\epsilon \sin(\phi)}{2\,\omega^2 A} \quad \textbf{(34)}$$

> phi_ := simplify(integrate(phidotav, phi = 0..phi));

$$phi\_ := -\frac{\epsilon\left(\cos(\phi) - 1\right)}{2\,\omega^2 A} \quad \textbf{(35)}$$

>

> Amp := simplify(integrate(adotav, A = 0..a));

$$Amp := -\frac{3\,\epsilon \left(\omega^3 a^3 + \frac{16\cos(\phi)}{3}\right) a}{32\,\omega^2} \quad \textbf{(36)}$$

>

Hence we got the neccesary amplitude and the phase component of the equation