```
> restart;
```

```
>
```

```
>
```

Useing pointicare we first find the frequency of the system

```
> N := 2 ; omega := 1 + add(w[k] · ϵ^k, k = 1 .. N); alpha1 = 0.01;
```

$$N := 2$$

$$\omega := w_2\,\epsilon^2 + w_1\,\epsilon + 1$$

$$\alpha 1 = 0.01 \tag{1}$$

```
> de := diff(x(t), t, t) + epsilon·diff(x(t), t) · (x(t)^2 − 1) + x(t) − 0.01 ·x(t)^2
```

$$de := \frac{d^2}{dt^2}\,x(t) + \epsilon\left(\frac{d}{dt}\,x(t)\right)\left(x(t)^2 - 1\right) + x(t) - 0.01\,x(t)^2 \tag{2}$$

```
> xp := add(x[k](t) ·ϵ^k, k = 0 ..N);
```

$$xp := x_0(t) + x_1(t)\,\epsilon + x_2(t)\,\epsilon^2 \tag{3}$$

```
> de1 := convert(taylor(expand(subs(x(t) = xp, de)), epsilon, N + 1), polynom);
```

$$de1 := \frac{d^2}{dt^2}\,x_0(t) - 0.01\,x_0(t)^2 + x_0(t) + \left(-0.02\,x_0(t)\,x_1(t) + \left(\frac{d}{dt}\,x_0(t)\right)x_0(t)^2 + x_1(t) + \frac{d^2}{dt^2}\,x_1(t) - 1.\,\frac{d}{dt}\,x_0(t)\right)\epsilon + \left(-0.02\,x_0(t)\,x_2(t) + \left(\frac{d}{dt}\,x_1(t)\right)x_0(t)^2 + x_2(t) + \frac{d^2}{dt^2}\,x_2(t) - 0.01\,x_1(t)^2 - 1.\,\frac{d}{dt}\,x_1(t) + 2.\left(\frac{d}{dt}\,x_0(t)\right)x_0(t)\,x_1(t)\right)\epsilon^2 \tag{4}$$

```
> x0sol := dsolve({coeff(de1, epsilon, 0), x[0](0) = x_0, D(x[0])(0) = 0}, x[0](t));
```

$$x0sol := x_0(t) = RootOf\left(30\left(\int_{\_Z}^{x_0}\frac{1}{\sqrt{6\,\_a^3 - 6\,x_0^3 - 900\,\_a^2 + 900\,x_0^2}}\,d\_a\right) + t\right), x_0(t) \tag{5}$$

$$= RootOf\left(30\left(\int_{x_0}^{\_Z}\frac{1}{\sqrt{6\,\_a^3 - 6\,x_0^3 - 900\,\_a^2 + 900\,x_0^2}}\,d\_a\right) + t\right)$$

```
>
```

the roots are giveing no information so we have discard it .

```
>
```

Now trying my Multiple scaling method

```
> de := xddot + x + epsilon·xdot · (x^2 − 1) − alpha · x^2 ;
```

$$de := xddot + x + \epsilon\,xdot\left(x^2 - 1\right) - \alpha\,x^2 \tag{6}$$

```
> N := 2 ; ts := seq(T[k], k = 0 ..N);
```

$$N := 2$$

(7)

$$ts := T_0, T_1, T_2 \tag{7}$$

> $xp := add(X[k](ts)\cdot\epsilon^k, k=0..N);$

$$xp := X_0(T_0, T_1, T_2) + X_1(T_0, T_1, T_2)\,\epsilon + X_2(T_0, T_1, T_2)\,\epsilon^2 \tag{8}$$

> $xpds := add(diff(xp, T[k])\cdot \text{epsilon}^k, k=0..N);$

$$xpds := \frac{\partial}{\partial T_0} X_0(T_0, T_1, T_2) + \left(\frac{\partial}{\partial T_0} X_1(T_0, T_1, T_2)\right)\epsilon + \left(\frac{\partial}{\partial T_0} X_2(T_0, T_1, T_2)\right)\epsilon^2 + \left(\frac{\partial}{\partial T_1} X_0(T_0, T_1, T_2) + \left(\frac{\partial}{\partial T_1} X_1(T_0, T_1, T_2)\right)\epsilon + \left(\frac{\partial}{\partial T_1} X_2(T_0, T_1, T_2)\right)\epsilon^2\right)\epsilon + \left(\frac{\partial}{\partial T_2} X_0(T_0, T_1, T_2) + \left(\frac{\partial}{\partial T_2} X_1(T_0, T_1, T_2)\right)\epsilon + \left(\frac{\partial}{\partial T_2} X_2(T_0, T_1, T_2)\right)\epsilon^2\right)\epsilon^2 \tag{9}$$

> $xpdds := add(diff(xpds, T[k])\cdot\epsilon^k, k=0..N);$

$$\begin{aligned} xpdds := {} & \frac{\partial^2}{\partial T_0^2} X_0(T_0, T_1, T_2) + \left(\frac{\partial^2}{\partial T_0^2} X_1(T_0, T_1, T_2)\right)\epsilon + \left(\frac{\partial^2}{\partial T_0^2} X_2(T_0, T_1, T_2)\right)\epsilon^2 \\ & + \left(\frac{\partial^2}{\partial T_0 \partial T_1} X_0(T_0, T_1, T_2) + \left(\frac{\partial^2}{\partial T_0 \partial T_1} X_1(T_0, T_1, T_2)\right)\epsilon + \left(\frac{\partial^2}{\partial T_0 \partial T_1} X_2(T_0, T_1, T_2)\right)\epsilon^2\right)\epsilon \\ & + \left(\frac{\partial^2}{\partial T_0 \partial T_2} X_0(T_0, T_1, T_2) + \left(\frac{\partial^2}{\partial T_0 \partial T_2} X_1(T_0, T_1, T_2)\right)\epsilon + \left(\frac{\partial^2}{\partial T_0 \partial T_2} X_2(T_0, T_1, T_2)\right)\epsilon^2\right)\epsilon^2 \\ & + \left(\frac{\partial^2}{\partial T_0 \partial T_1} X_0(T_0, T_1, T_2) + \left(\frac{\partial^2}{\partial T_0 \partial T_1} X_1(T_0, T_1, T_2)\right)\epsilon + \left(\frac{\partial^2}{\partial T_0 \partial T_1} X_2(T_0, T_1, T_2)\right)\epsilon^2\right) \\ & + \left(\frac{\partial^2}{\partial T_1^2} X_0(T_0, T_1, T_2) + \left(\frac{\partial^2}{\partial T_1^2} X_1(T_0, T_1, T_2)\right)\epsilon + \left(\frac{\partial^2}{\partial T_1^2} X_2(T_0, T_1, T_2)\right)\epsilon^2\right)\epsilon \\ & + \left(\frac{\partial^2}{\partial T_1 \partial T_2} X_0(T_0, T_1, T_2) + \left(\frac{\partial^2}{\partial T_1 \partial T_2} X_1(T_0, T_1, T_2)\right)\epsilon + \left(\frac{\partial^2}{\partial T_1 \partial T_2} X_2(T_0, T_1, T_2)\right)\epsilon^2\right)\epsilon^2 \\ & + \left(\frac{\partial^2}{\partial T_0 \partial T_2} X_0(T_0, T_1, T_2) + \left(\frac{\partial^2}{\partial T_0 \partial T_2} X_1(T_0, T_1, T_2)\right)\epsilon + \left(\frac{\partial^2}{\partial T_0 \partial T_2} X_2(T_0, T_1, T_2)\right)\epsilon^2\right)\epsilon \\ & + \left(\frac{\partial^2}{\partial T_1 \partial T_2} X_0(T_0, T_1, T_2) + \left(\frac{\partial^2}{\partial T_1 \partial T_2} X_1(T_0, T_1, T_2)\right)\epsilon + \left(\frac{\partial^2}{\partial T_1 \partial T_2} X_2(T_0, T_1, T_2)\right)\epsilon^2\right)\epsilon^2 \\ & + \left(\frac{\partial^2}{\partial T_2^2} X_0(T_0, T_1, T_2) + \left(\frac{\partial^2}{\partial T_2^2} X_1(T_0, T_1, T_2)\right)\epsilon + \left(\frac{\partial^2}{\partial T_2^2} X_2(T_0, T_1, T_2)\right)\epsilon^2\right)\epsilon^2 \end{aligned} \tag{10}$$

> $deq := convert(taylor(expand(subs(xddot=xpdds, xdot=xpds, x=xp, de)), \text{epsilon}, N+1), polynom);$

$$deq := -\alpha X_0(T_0, T_1, T_2)^2 + \frac{\partial^2}{\partial T_0^2} X_0(T_0, T_1, T_2) + X_0(T_0, T_1, T_2) + \left(\left(\frac{\partial}{\partial T_0} X_0(T_0, T_1, T_2)\right) X_0(T_0, T_1, T_2)^2 - \frac{\partial}{\partial T_0} X_0(T_0, T_1, T_2) + 2\frac{\partial^2}{\partial T_0 \partial T_1} X_0(T_0, T_1, T_2) + \frac{\partial^2}{\partial T_0^2} X_1(T_0, T_1, T_2)\right) \tag{11}$$

$$+ X_1(T_0, T_1, T_2) - 2\,\alpha X_0(T_0, T_1, T_2)\, X_1(T_0, T_1, T_2)\Big)\,\epsilon + \Big(\Big(\frac{\partial}{\partial T_1} X_0(T_0, T_1,$$

$$T_2)\Big)\, X_0(T_0, T_1, T_2)^2 + \Big(\frac{\partial}{\partial T_0} X_1(T_0, T_1, T_2)\Big)\, X_0(T_0, T_1, T_2)^2 - \alpha X_1(T_0, T_1, T_2)^2 - \frac{\partial}{\partial T_1}$$

$$X_0(T_0, T_1, T_2) - \frac{\partial}{\partial T_0} X_1(T_0, T_1, T_2) + 2\,\frac{\partial^2}{\partial T_0 \partial T_1} X_1(T_0, T_1, T_2) + 2\,\frac{\partial^2}{\partial T_0 \partial T_2} X_0(T_0, T_1, T_2)$$

$$+ \frac{\partial^2}{\partial T_1^2} X_0(T_0, T_1, T_2) + \frac{\partial^2}{\partial T_0^2} X_2(T_0, T_1, T_2) + X_2(T_0, T_1, T_2) - 2\,\alpha X_0(T_0, T_1, T_2)\, X_2(T_0,$$

$$T_1, T_2) + 2\Big(\frac{\partial}{\partial T_0} X_0(T_0, T_1, T_2)\Big)\, X_0(T_0, T_1, T_2)\, X_1(T_0, T_1, T_2)\Big)\,\epsilon^2$$

```
> te := X0(T0, T1, T2) = A(T[1], T[2]) *sin(T[0] + phi(T[1], T[2]));
```

$$te := X_0(T_0, T_1, T_2) = A(T_1, T_2)\,\sin\big(T_0 + \phi(T_1, T_2)\big) \tag{12}$$

```
> xp1 := subs(te, xp);
```

$$xp1 := A(T_1, T_2)\,\sin\big(T_0 + \phi(T_1, T_2)\big) + X_1(T_0, T_1, T_2)\,\epsilon + X_2(T_0, T_1, T_2)\,\epsilon^2 \tag{13}$$

```
> deq1 := collect(expand(subs(te, deq)), epsilon);
```

$$deq1 := \Big(\Big(\frac{\partial^2}{\partial T_1^2} A(T_1, T_2)\Big)\cos(T_0)\sin\big(\phi(T_1, T_2)\big) + \Big(\frac{\partial^2}{\partial T_1^2} A(T_1, T_2)\Big)\sin(T_0)\cos\big(\phi(T_1, \tag{14}$$

$$T_2)\big) - \Big(\frac{\partial}{\partial T_1} A(T_1, T_2)\Big)\sin(T_0)\cos\big(\phi(T_1, T_2)\big) - \Big(\frac{\partial}{\partial T_1} A(T_1, T_2)\Big)\cos(T_0)\sin\big(\phi(T_1,$$

$$T_2)\big) + 2\Big(\frac{\partial}{\partial T_2} A(T_1, T_2)\Big)\cos(T_0)\cos\big(\phi(T_1, T_2)\big) - 2\Big(\frac{\partial}{\partial T_2} A(T_1,$$

$$T_2)\Big)\sin(T_0)\sin\big(\phi(T_1, T_2)\big) + 3\,A(T_1, T_2)^2\Big(\frac{\partial}{\partial T_1} A(T_1, T_2)\Big)\sin(T_0)^2\cos\big(\phi(T_1,$$

$$T_2)\big)^2\cos(T_0)\sin\big(\phi(T_1, T_2)\big) + 3\,A(T_1, T_2)^2\Big(\frac{\partial}{\partial T_1} A(T_1, T_2)\Big)\sin(T_0)\cos\big(\phi(T_1,$$

$$T_2)\big)\cos(T_0)^2\sin\big(\phi(T_1, T_2)\big)^2 - 2\,A(T_1, T_2)^3\sin(T_0)^2\Big(\frac{\partial}{\partial T_1}\phi(T_1, T_2)\Big)\sin\big(\phi(T_1,$$

$$T_2)\big)^2\cos\big(\phi(T_1, T_2)\big)\cos(T_0) + 2\,A(T_1, T_2)^3\cos(T_0)^2\Big(\frac{\partial}{\partial T_1}\phi(T_1, T_2)\Big)\cos\big(\phi(T_1,$$

$$T_2)\big)^2\sin(T_0)\sin\big(\phi(T_1, T_2)\big) + 2\Big(\frac{\partial}{\partial T_0} X_1(T_0, T_1, T_2)\Big)\,A(T_1, T_2)^2\sin(T_0)\cos\big(\phi(T_1,$$

$$T_2)\big)\cos(T_0)\sin\big(\phi(T_1, T_2)\big) - \alpha X_1(T_0, T_1, T_2)^2 + 2\,\frac{\partial^2}{\partial T_0 \partial T_1} X_1(T_0, T_1, T_2) + \frac{\partial^2}{\partial T_0^2} X_2(T_0,$$

$$T_1, T_2) - \frac{\partial}{\partial T_0} X_1(T_0, T_1, T_2) + X_2(T_0, T_1, T_2) + A(T_1, T_2)\cos(T_0)\Big(\frac{\partial^2}{\partial T_1^2}\phi(T_1,$$

$$T_2\Big)\Big)\cos\big(\phi(T_1,T_2)\big)+A(T_1,T_2)\sin(T_0)\left(\frac{\partial}{\partial T_1}\,\phi(T_1,T_2)\right)\sin\big(\phi(T_1,T_2)\big)-A(T_1,$$

$$T_2)\cos(T_0)\left(\frac{\partial}{\partial T_1}\,\phi(T_1,T_2)\right)\cos\big(\phi(T_1,T_2)\big)+A(T_1,T_2)^2\left(\frac{\partial}{\partial T_1}\,A(T_1,$$

$$T_2)\right)\sin(T_0)^3\cos\big(\phi(T_1,T_2)\big)^3+A(T_1,T_2)^2\left(\frac{\partial}{\partial T_1}\,A(T_1,T_2)\right)\cos(T_0)^3\sin\big(\phi(T_1,T_2)\big)^3$$

$$+\left(\frac{\partial}{\partial T_0}\,X_1(T_0,T_1,T_2)\right)A(T_1,T_2)^2\sin(T_0)^2\cos\big(\phi(T_1,T_2)\big)^2+\left(\frac{\partial}{\partial T_0}\,X_1(T_0,T_1,\right.$$

$$T_2)\Big)A(T_1,T_2)^2\cos(T_0)^2\sin\big(\phi(T_1,T_2)\big)^2-2\,A(T_1,T_2)\cos(T_0)\left(\frac{\partial}{\partial T_2}\,\phi(T_1,\right.$$

$$T_2)\Big)\sin\big(\phi(T_1,T_2)\big)-2\,A(T_1,T_2)\sin(T_0)\left(\frac{\partial}{\partial T_2}\,\phi(T_1,T_2)\right)\cos\big(\phi(T_1,T_2)\big)-2\left(\frac{\partial}{\partial T_1}\right.$$

$$A(T_1,T_2)\Big)\sin(T_0)\left(\frac{\partial}{\partial T_1}\,\phi(T_1,T_2)\right)\sin\big(\phi(T_1,T_2)\big)+2\left(\frac{\partial}{\partial T_1}\,A(T_1,T_2)\right)\cos(T_0)\left(\frac{\partial}{\partial T_1}\right.$$

$$\phi(T_1,T_2)\Big)\cos\big(\phi(T_1,T_2)\big)-A(T_1,T_2)\cos(T_0)\left(\frac{\partial}{\partial T_1}\,\phi(T_1,T_2)\right)^2\sin\big(\phi(T_1,T_2)\big)-A(T_1,$$

$$T_2)\sin(T_0)\left(\frac{\partial^2}{\partial T_1^{\,2}}\,\phi(T_1,T_2)\right)\sin\big(\phi(T_1,T_2)\big)-A(T_1,T_2)^3\sin(T_0)^3\left(\frac{\partial}{\partial T_1}\,\phi(T_1,\right.$$

$$T_2)\Big)\sin\big(\phi(T_1,T_2)\big)\cos\big(\phi(T_1,T_2)\big)^2-A(T_1,T_2)^3\sin(T_0)\left(\frac{\partial}{\partial T_1}\,\phi(T_1,\right.$$

$$T_2)\Big)\sin\big(\phi(T_1,T_2)\big)^3\cos(T_0)^2+A(T_1,T_2)^3\cos(T_0)\left(\frac{\partial}{\partial T_1}\,\phi(T_1,T_2)\right)\cos\big(\phi(T_1,$$

$$T_2)\big)^3\sin(T_0)^2+A(T_1,T_2)^3\cos(T_0)^3\left(\frac{\partial}{\partial T_1}\,\phi(T_1,T_2)\right)\cos\big(\phi(T_1,T_2)\big)\sin\big(\phi(T_1,T_2)\big)^2$$

$$-A(T_1,T_2)\sin(T_0)\left(\frac{\partial}{\partial T_1}\,\phi(T_1,T_2)\right)^2\cos\big(\phi(T_1,T_2)\big)-2\,\alpha A(T_1,T_2)\,X_2(T_0,T_1,$$

$$T_2)\sin(T_0)\cos\big(\phi(T_1,T_2)\big)-2\,\alpha A(T_1,T_2)\,X_2(T_0,T_1,T_2)\cos(T_0)\sin\big(\phi(T_1,T_2)\big)$$

$$+2\,A(T_1,T_2)^2X_1(T_0,T_1,T_2)\cos(T_0)\cos\big(\phi(T_1,T_2)\big)^2\sin(T_0)+2\,A(T_1,T_2)^2X_1(T_0,T_1,$$

$$T_2)\cos(T_0)^2\cos\big(\phi(T_1,T_2)\big)\sin\big(\phi(T_1,T_2)\big)-2\,A(T_1,T_2)^2X_1(T_0,T_1,$$

$$T_2)\sin(T_0)^2\sin\big(\phi(T_1,T_2)\big)\cos\big(\phi(T_1,T_2)\big)-2\,A(T_1,T_2)^2X_1(T_0,T_1,$$

$$T_2)\sin(T_0)\,\sin\big(\phi(T_1,T_2)\big)^2\cos(T_0)\Big)\epsilon^2+\left(A(T_1,T_2)^3\cos(T_0)\cos\big(\phi(T_1,T_2)\big)^3\sin(T_0)^2\right.$$

$$+2A(T_1,T_2)^3\cos(T_0)^2\cos(\phi(T_1,T_2))^2\sin(T_0)\sin(\phi(T_1,T_2))+A(T_1,$$

$$T_2)^3\cos(T_0)^3\cos(\phi(T_1,T_2))\sin(\phi(T_1,T_2))^2-A(T_1,T_2)^3\sin(T_0)^3\sin(\phi(T_1,$$

$$T_2))\cos(\phi(T_1,T_2))^2-2A(T_1,T_2)^3\sin(T_0)^2\sin(\phi(T_1,T_2))^2\cos(\phi(T_1,T_2))\cos(T_0)$$

$$-A(T_1,T_2)^3\sin(T_0)\sin(\phi(T_1,T_2))^3\cos(T_0)^2-A(T_1,T_2)\cos(T_0)\cos(\phi(T_1,T_2))$$

$$+A(T_1,T_2)\sin(T_0)\sin(\phi(T_1,T_2))+2\left(\frac{\partial}{\partial T_1}A(T_1,T_2)\right)\cos(T_0)\cos(\phi(T_1,T_2))$$

$$-2A(T_1,T_2)\cos(T_0)\left(\frac{\partial}{\partial T_1}\phi(T_1,T_2)\right)\sin(\phi(T_1,T_2))-2\left(\frac{\partial}{\partial T_1}A(T_1,\right.$$

$$\left.T_2)\right)\sin(T_0)\sin(\phi(T_1,T_2))-2A(T_1,T_2)\sin(T_0)\left(\frac{\partial}{\partial T_1}\phi(T_1,T_2)\right)\cos(\phi(T_1,T_2))$$

$$+\frac{\partial^2}{\partial T_0^2}X_1(T_0,T_1,T_2)+X_1(T_0,T_1,T_2)-2\alpha A(T_1,T_2)X_1(T_0,T_1,T_2)\sin(T_0)\cos(\phi(T_1,$$

$$T_2))-2\alpha A(T_1,T_2)X_1(T_0,T_1,T_2)\cos(T_0)\sin(\phi(T_1,T_2))\Big)\epsilon-\alpha A(T_1,$$

$$T_2)^2\cos(T_0)^2\sin(\phi(T_1,T_2))^2-2\alpha A(T_1,T_2)^2\sin(T_0)\cos(\phi(T_1,T_2))\cos(T_0)\sin(\phi(T_1,$$

$$T_2))-\alpha A(T_1,T_2)^2\sin(T_0)^2\cos(\phi(T_1,T_2))^2$$

> $tems := combine(coeff(deq1, \text{epsilon}, 1), trig);$

$$tems := \frac{\partial^2}{\partial T_0^2}X_1(T_0,T_1,T_2)+X_1(T_0,T_1,T_2)-2A(T_1,T_2)\left(\frac{\partial}{\partial T_1}\phi(T_1,T_2)\right)\sin(T_0+\phi(T_1,\quad\textbf{(15)}$$

$$T_2))-2\alpha A(T_1,T_2)\sin(T_0+\phi(T_1,T_2))X_1(T_0,T_1,T_2)$$

$$-\frac{A(T_1,T_2)^3\cos(3T_0+3\phi(T_1,T_2))}{4}+\frac{A(T_1,T_2)^3\cos(T_0+\phi(T_1,T_2))}{4}-A(T_1,$$

$$T_2)\cos(T_0+\phi(T_1,T_2))+2\left(\frac{\partial}{\partial T_1}A(T_1,T_2)\right)\cos(T_0+\phi(T_1,T_2))$$

> $e1 := coeff(tems, \cos(T_0+\text{phi}(T_1,T_2)))$; $solve\left(e1, \frac{\partial}{\partial T_1}A(T_1,T_2)\right)$; $e1a := \frac{\partial}{\partial T_1}A(T_1,T_2) = \%$;

$$e1 := \frac{A(T_1,T_2)^3}{4}-A(T_1,T_2)+2\frac{\partial}{\partial T_1}A(T_1,T_2)$$

$$-\frac{A(T_1,T_2)^3}{8}+\frac{A(T_1,T_2)}{2}$$

$$e1a := \frac{\partial}{\partial T_1} A(T_1, T_2) = -\frac{A(T_1, T_2)^3}{8} + \frac{A(T_1, T_2)}{2} \tag{16}$$

```
> e2 := coeff(tems, sin(T_0 + phi(T_1, T_2)));
```

$$e2 := -2\,A(T_1, T_2)\left(\frac{\partial}{\partial T_1}\,\phi(T_1, T_2)\right) - 2\,\alpha\,A(T_1, T_2)\,X_1(T_0, T_1, T_2) \tag{17}$$

```
> tems1 := combine(tems - e1·cos(T_0 + phi(T_1, T_2)) - e2·sin(T_0 + phi(T_1, T_2)), trig);
```

$$tems1 := \frac{\partial^2}{\partial T_0^{\,2}}\,X_1(T_0, T_1, T_2) + X_1(T_0, T_1, T_2) - \frac{A(T_1, T_2)^3\cos(3\,T_0 + 3\,\phi(T_1, T_2))}{4} \tag{18}$$

```
> dsolve(tems1, X_1(T_0, T_1, T_2));
```

$$X_1(T_0, T_1, T_2) = \sin(T_0)\,f_2(T_1, T_2) + \cos(T_0)\,f_1(T_1, T_2) - \frac{A(T_1, T_2)^3\cos(3\,T_0 + 3\,\phi(T_1, T_2))}{32} \tag{19}$$

```
> tems2 := subs(_F2(T_1, T_2) = 0, _F1(T_1, T_2) = 0, %);
```

$$tems2 := X_1(T_0, T_1, T_2) = -\frac{A(T_1, T_2)^3\cos(3\,T_0 + 3\,\phi(T_1, T_2))}{32} \tag{20}$$

```
> deq2 := expand(subs(tems2, deq1)) : dea := combine(collect(expand(subs(phi(T_1, T_2)
      = phi1(T[2]), e1a, deq1)), epsilon), trig);
```

$$\begin{aligned} dea := {} & \frac{\sin(3\,T_0 + 3\,\phi 1(T_2))\,A(T_1, T_2)^5\,\epsilon^2}{32} - \frac{3\sin(T_0 + \phi 1(T_2))\,A(T_1, T_2)^5\,\epsilon^2}{32} \\ & - \frac{\sin(3\,T_0 + 3\,\phi 1(T_2))\,A(T_1, T_2)^3\,\epsilon^2}{8} + \frac{\sin(T_0 + \phi 1(T_2))\,A(T_1, T_2)^3\,\epsilon^2}{2} \\ & + \frac{A(T_1, T_2)^2\left(\frac{\partial}{\partial T_0}\,X_1(T_0, T_1, T_2)\right)\epsilon^2}{2} - \frac{\sin(T_0 + \phi 1(T_2))\,A(T_1, T_2)\,\epsilon^2}{2} \\ & + \frac{\sin(T_0 + \phi 1(T_2))\left(\frac{\partial}{\partial T_1}\,A(T_1, T_2)\right)\epsilon^2}{2} + 2\cos(T_0 + \phi 1(T_2))\left(\frac{\partial}{\partial T_2}\,A(T_1, T_2)\right)\epsilon^2 \\ & - \frac{A(T_1, T_2)^3\cos(3\,T_0 + 3\,\phi 1(T_2))\,\epsilon}{4} - \frac{\alpha\,A(T_1, T_2)^2}{2} \\ & + \frac{\alpha\,A(T_1, T_2)^2\cos(2\,T_0 + 2\,\phi 1(T_2))}{2} - \epsilon^2\,\alpha\,X_1(T_0, T_1, T_2)^2 + \left(\frac{\partial^2}{\partial T_0^{\,2}}\,X_1(T_0, T_1, T_2)\right)\epsilon \\ & + \left(\frac{\partial^2}{\partial T_0^{\,2}}\,X_2(T_0, T_1, T_2)\right)\epsilon^2 + 2\,\epsilon^2\left(\frac{\partial^2}{\partial T_0 \partial T_1}\,X_1(T_0, T_1, T_2)\right) - \epsilon^2\left(\frac{\partial}{\partial T_0}\,X_1(T_0, T_1, T_2)\right) \\ & + X_1(T_0, T_1, T_2)\,\epsilon + X_2(T_0, T_1, T_2)\,\epsilon^2 \end{aligned} \tag{21}$$

$$-\frac{3\sin\left(T_0+\phi l\left(T_2\right)\right)A\left(T_1,T_2\right)^2\left(\frac{\partial}{\partial T_1}A\left(T_1,T_2\right)\right)\epsilon^2}{8}$$

$$-\frac{A\left(T_1,T_2\right)^2\cos\left(2\,T_0+2\,\phi l\left(T_2\right)\right)\left(\frac{\partial}{\partial T_0}X_1\left(T_0,T_1,T_2\right)\right)\epsilon^2}{2}+A\left(T_1,T_2\right)^2X_1\left(T_0,T_1,$$

$$T_2\right)\sin\left(2\,T_0+2\,\phi l\left(T_2\right)\right)\epsilon^2-2\sin\left(T_0+\phi l\left(T_2\right)\right)A\left(T_1,T_2\right)\left(\frac{\mathrm{d}}{\mathrm{d}T_2}\phi l\left(T_2\right)\right)\epsilon^2-2\sin\left(T_0$$

$$+\phi l\left(T_2\right)\right)A\left(T_1,T_2\right)X_2\left(T_0,T_1,T_2\right)\alpha\,\epsilon^2-2\sin\left(T_0+\phi l\left(T_2\right)\right)A\left(T_1,T_2\right)X_1\left(T_0,T_1,T_2\right)\alpha\,\epsilon$$

> *collect*(*dea*, epsilon);

$$\left(\frac{\sin\left(3\,T_0+3\,\phi l\left(T_2\right)\right)A\left(T_1,T_2\right)^5}{32}-\frac{3\sin\left(T_0+\phi l\left(T_2\right)\right)A\left(T_1,T_2\right)^5}{32}\right. \tag{22}$$

$$-\frac{\sin\left(3\,T_0+3\,\phi l\left(T_2\right)\right)A\left(T_1,T_2\right)^3}{8}+\frac{\sin\left(T_0+\phi l\left(T_2\right)\right)A\left(T_1,T_2\right)^3}{2}$$

$$+\frac{A\left(T_1,T_2\right)^2\left(\frac{\partial}{\partial T_0}X_1\left(T_0,T_1,T_2\right)\right)}{2}-\frac{\sin\left(T_0+\phi l\left(T_2\right)\right)A\left(T_1,T_2\right)}{2}$$

$$+\frac{\sin\left(T_0+\phi l\left(T_2\right)\right)\left(\frac{\partial}{\partial T_1}A\left(T_1,T_2\right)\right)}{2}+2\cos\left(T_0+\phi l\left(T_2\right)\right)\left(\frac{\partial}{\partial T_2}A\left(T_1,T_2\right)\right)$$

$$-\alpha X_1\left(T_0,T_1,T_2\right)^2+\frac{\partial^2}{\partial T_0{}^2}X_2\left(T_0,T_1,T_2\right)+2\frac{\partial^2}{\partial T_0\partial T_1}X_1\left(T_0,T_1,T_2\right)-\frac{\partial}{\partial T_0}X_1\left(T_0,T_1,T_2\right)$$

$$+X_2\left(T_0,T_1,T_2\right)-\frac{3\sin\left(T_0+\phi l\left(T_2\right)\right)A\left(T_1,T_2\right)^2\left(\frac{\partial}{\partial T_1}A\left(T_1,T_2\right)\right)}{8}$$

$$-\frac{A\left(T_1,T_2\right)^2\cos\left(2\,T_0+2\,\phi l\left(T_2\right)\right)\left(\frac{\partial}{\partial T_0}X_1\left(T_0,T_1,T_2\right)\right)}{2}+A\left(T_1,T_2\right)^2X_1\left(T_0,T_1,$$

$$T_2\right)\sin\left(2\,T_0+2\,\phi l\left(T_2\right)\right)-2\sin\left(T_0+\phi l\left(T_2\right)\right)A\left(T_1,T_2\right)\left(\frac{\mathrm{d}}{\mathrm{d}T_2}\phi l\left(T_2\right)\right)-2\sin\left(T_0$$

$$+\phi l\left(T_2\right)\right)A\left(T_1,T_2\right)X_2\left(T_0,T_1,T_2\right)\alpha\left.\right)\epsilon^2+\left(-\frac{A\left(T_1,T_2\right)^3\cos\left(3\,T_0+3\,\phi l\left(T_2\right)\right)}{4}\right.$$

$$+\frac{\partial^2}{\partial T_0^{\,2}}\,X_1(T_0,T_1,T_2)+X_1(T_0,T_1,T_2)-2\sin\left(T_0+\phi l(T_2)\right)A(T_1,T_2)\,X_1(T_0,T_1,T_2)\,\alpha\Bigg)\epsilon$$

$$-\frac{\alpha A(T_1,T_2)^2}{2}+\frac{\alpha A(T_1,T_2)^2\cos\left(2\,T_0+2\,\phi l(T_2)\right)}{2}$$

> $tems3 := subs(e1a, coeff(dea, \text{epsilon}, 2));$

$$tems3 := \frac{\sin\left(3\,T_0+3\,\phi l(T_2)\right)A(T_1,T_2)^5}{32}-\frac{3\sin\left(T_0+\phi l(T_2)\right)A(T_1,T_2)^5}{32} \tag{23}$$

$$-\frac{\sin\left(3\,T_0+3\,\phi l(T_2)\right)A(T_1,T_2)^3}{8}+\frac{\sin\left(T_0+\phi l(T_2)\right)A(T_1,T_2)^3}{2}$$

$$+\frac{A(T_1,T_2)^2\left(\frac{\partial}{\partial T_0}\,X_1(T_0,T_1,T_2)\right)}{2}-\frac{\sin\left(T_0+\phi l(T_2)\right)A(T_1,T_2)}{2}$$

$$+\frac{\sin\left(T_0+\phi l(T_2)\right)\left(-\frac{A(T_1,T_2)^3}{8}+\frac{A(T_1,T_2)}{2}\right)}{2}+2\cos\left(T_0+\phi l(T_2)\right)\left(\frac{\partial}{\partial T_2}\right.$$

$$\left.A(T_1,T_2)\right)-\alpha X_1(T_0,T_1,T_2)^2+\frac{\partial^2}{\partial T_0^{\,2}}\,X_2(T_0,T_1,T_2)+2\,\frac{\partial^2}{\partial T_0\partial T_1}\,X_1(T_0,T_1,T_2)-\frac{\partial}{\partial T_0}$$

$$X_1(T_0,T_1,T_2)+X_2(T_0,T_1,T_2)$$

$$-\frac{3\sin\left(T_0+\phi l(T_2)\right)A(T_1,T_2)^2\left(-\frac{A(T_1,T_2)^3}{8}+\frac{A(T_1,T_2)}{2}\right)}{8}$$

$$-\frac{A(T_1,T_2)^2\cos\left(2\,T_0+2\,\phi l(T_2)\right)\left(\frac{\partial}{\partial T_0}\,X_1(T_0,T_1,T_2)\right)}{2}+A(T_1,T_2)^2\,X_1(T_0,T_1,$$

$$T_2)\sin\left(2\,T_0+2\,\phi l(T_2)\right)-2\sin\left(T_0+\phi l(T_2)\right)A(T_1,T_2)\left(\frac{\mathrm{d}}{\mathrm{d}T_2}\,\phi l(T_2)\right)-2\sin\left(T_0\right.$$

$$\left.+\phi l(T_2)\right)A(T_1,T_2)\,X_2(T_0,T_1,T_2)\,\alpha$$

> $e3 := coeff\left(tems3, \cos\left(T_0+\phi l(T_2)\right)\right);$

$$e3 := 2\,\frac{\partial}{\partial T_2}\,A(T_1,T_2) \tag{24}$$

> $e4 := coeff\left(tems3\,, \sin\left(T_0+\phi l(T_2)\right)\right);$

$$e4 := -\frac{3\,A(T_1,T_2)^5}{32}+\frac{7\,A(T_1,T_2)^3}{16}-\frac{A(T_1,T_2)}{4} \tag{25}$$

$$-\frac{3\,A(T_1,T_2)^2\left(-\frac{A(T_1,T_2)^3}{8}+\frac{A(T_1,T_2)}{2}\right)}{8}-2\,A(T_1,T_2)\left(\frac{\mathrm{d}}{\mathrm{d}T_2}\,\phi l(T_2)\right)-2\,A(T_1,$$

$T_2)\ X_2(T_0, T_1, T_2)\ \alpha$

> ela ;

$$\frac{\partial}{\partial T_1}\ A(T_1, T_2) = -\frac{A(T_1, T_2)^3}{8} + \frac{A(T_1, T_2)}{2} \qquad \textbf{(26)}$$

Hence we got till the amplitude derivative ;

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