>

We do this by the method of Harmonic balance

> *restart* :

>

> $de := diff(x(t), t, t) - \left(\frac{1}{10} - \frac{10}{3} diff(x(t), t)^2\right) diff(x(t), t) + x(t) - x(t)^3$

$$de := \frac{d^2}{dt^2} x(t) - \left(\frac{1}{10} - \frac{10\left(\frac{d}{dt} x(t)\right)^2}{3}\right)\left(\frac{d}{dt} x(t)\right) + x(t) - x(t)^3 \quad (1)$$

> $xs := x(t) = A_1 \cdot \cos(\text{omega} \cdot t) + A_3 \cos(3\ \text{omega}\ t) + B_3 \sin(3\ \text{omega}\ t)$;

$$xs := x(t) = A_1 \cos(\omega t) + A_3 \cos(3 \omega t) + B_3 \sin(3 \omega t) \quad (2)$$

> $dea := combine(expand(subs(xs, de)), trig)$;

$$\begin{aligned} dea := & -45 A_1 \omega^3 A_3 B_3 \cos(7 \omega t) + 45 A_1 \omega^3 A_3 B_3 \cos(5 \omega t) + \frac{A_1 \omega \sin(\omega t)}{10} + A_1 \cos(\omega t) \quad (3) \\ & + A_3 \cos(3 \omega t) + B_3 \sin(3 \omega t) - A_1 \omega^2 \cos(\omega t) + \frac{B_3^3 \sin(9 \omega t)}{4} - \frac{3 B_3^3 \sin(3 \omega t)}{4} \\ & - \frac{3 A_3^3 \cos(3 \omega t)}{4} - \frac{A_3^3 \cos(9 \omega t)}{4} - \frac{A_1^3 \cos(3 \omega t)}{4} - \frac{3 A_1^3 \cos(\omega t)}{4} \\ & + \frac{3 A_3 B_3^2 \cos(9 \omega t)}{4} - \frac{3 A_1 B_3^2 \cos(\omega t)}{2} - \frac{3 A_1^2 B_3 \sin(5 \omega t)}{4} + \frac{3 A_1 B_3^2 \cos(5 \omega t)}{4} \\ & + \frac{3 A_1 B_3^2 \cos(7 \omega t)}{4} + \frac{5 A_1^3 \omega^3 \sin(3 \omega t)}{6} - \frac{5 A_1^3 \omega^3 \sin(\omega t)}{2} - \frac{135 A_3^3 \omega^3 \sin(3 \omega t)}{2} \\ & + \frac{45 B_3^3 \omega^3 \cos(9 \omega t)}{2} + \frac{135 B_3^3 \omega^3 \cos(3 \omega t)}{2} - 9 B_3 \omega^2 \sin(3 \omega t) - \frac{3 B_3 \omega \cos(3 \omega t)}{10} \\ & - \frac{3 A_3^2 B_3 \sin(3 \omega t)}{4} - 9 A_3 \omega^2 \cos(3 \omega t) - \frac{3 A_1 A_3^2 \cos(\omega t)}{2} - \frac{3 A_1 A_3^2 \cos(7 \omega t)}{4} \\ & - \frac{3 A_1 A_3^2 \cos(5 \omega t)}{4} - \frac{3 A_1^2 A_3 \cos(3 \omega t)}{2} - \frac{3 A_1^2 A_3 \cos(\omega t)}{4} - \frac{3 A_1^2 A_3 \cos(5 \omega t)}{4} \\ & + \frac{45 A_3^3 \omega^3 \sin(9 \omega t)}{2} + \frac{3 A_3 \omega \sin(3 \omega t)}{10} - \frac{3 A_3 B_3^2 \cos(3 \omega t)}{4} - \frac{3 A_3^2 B_3 \sin(9 \omega t)}{4} \\ & - \frac{3 A_1^2 B_3 \sin(3 \omega t)}{2} - \frac{3 A_1^2 B_3 \sin(\omega t)}{4} + 15 A_1^2 \omega^3 B_3 \cos(3 \omega t) \\ & - \frac{15 A_1^2 \omega^3 B_3 \cos(5 \omega t)}{2} - \frac{3 A_1 A_3 B_3 \sin(5 \omega t)}{2} - \frac{3 A_1 A_3 B_3 \sin(7 \omega t)}{2} \\ & + \frac{45 A_1 \omega^3 A_3^2 \sin(7 \omega t)}{2} - 45 A_1 \omega^3 A_3^2 \sin(\omega t) - \frac{45 A_1 \omega^3 A_3^2 \sin(5 \omega t)}{2} \end{aligned}$$

$$+\frac{15 A_1^2 \omega^3 A_3 \sin(5\omega t)}{2}+\frac{15 A_1^2 \omega^3 A_3 \sin(\omega t)}{2}-15 A_1^2 \omega^3 A_3 \sin(3\omega t)$$

$$-\frac{135 A_3^2 \omega^3 B_3 \cos(9\omega t)}{2}-\frac{135 B_3^2 \omega^3 A_3 \sin(3\omega t)}{2}-\frac{135 B_3^2 \omega^3 A_3 \sin(9\omega t)}{2}$$

$$+\frac{135 A_3^2 \omega^3 B_3 \cos(3\omega t)}{2}+\frac{45 B_3^2 \omega^3 A_1 \sin(5\omega t)}{2}-\frac{45 B_3^2 \omega^3 A_1 \sin(7\omega t)}{2}-45$$

$$B_3^2 \omega^3 A_1 \sin(\omega t)-\frac{15 A_1^2 \omega^3 B_3 \cos(\omega t)}{2}$$

>

> $e1 := coeff(dea, \sin(\omega t)) = 0; e2 := coeff(dea, \cos(\omega t)) = 0; e3 := coeff(dea, \cos(3\omega t)); e4 := coeff(dea, \sin(3\, \text{omega}\, t));$

$$e1 := -\frac{5}{2} A_1^3 \omega^3 - \frac{3}{4} A_1^2 B_3 + \frac{1}{10} A_1 \omega + \frac{15}{2} A_1^2 \omega^3 A_3 - 45 B_3^2 \omega^3 A_1 - 45 A_1 \omega^3 A_3^2 = 0$$

$$e2 := -\frac{3}{4} A_1^3 + A_1 - \frac{3}{4} A_1^2 A_3 - \frac{3}{2} A_1 A_3^2 - A_1 \omega^2 - \frac{3}{2} A_1 B_3^2 - \frac{15}{2} A_1^2 \omega^3 B_3 = 0$$

$$e3 := -\frac{3}{4} A_3^3 - \frac{1}{4} A_1^3 + A_3 - \frac{3}{4} A_3 B_3^2 - \frac{3}{2} A_1^2 A_3 - \frac{3}{10} B_3 \omega + \frac{135}{2} B_3^3 \omega^3 - 9 A_3 \omega^2 + 15 A_1^2 \omega^3 B_3 + \frac{135}{2} A_3^2 \omega^3 B_3$$

$$e4 := -\frac{3}{2} A_1^2 B_3 - \frac{135}{2} A_3^3 \omega^3 + \frac{5}{6} A_1^3 \omega^3 - \frac{3}{4} A_3^2 B_3 - \frac{3}{4} B_3^3 + B_3 + \frac{3}{10} A_3 \omega - 9 B_3 \omega^2 - 15 A_1^2 \omega^3 A_3 - \frac{135}{2} B_3^2 \omega^3 A_3 \qquad \textbf{(4)}$$

>

>

>

> $sol := solve(\{e1, e2, e3, e4\}, \{A_1, A_3, B_3, \text{omega}\});$

Heave work been done so hence we go for 1 term cos(wt)

>

> $restart:$

> $de := diff(x(t), t, t) - \left(\frac{1}{10} - \frac{10}{3} diff(x(t), t)^2\right) diff(x(t), t) + x(t) - x(t)^3$

$$de := \frac{d^2}{dt^2} x(t) - \left(\frac{1}{10} - \frac{10\left(\frac{d}{dt} x(t)\right)^2}{3}\right)\left(\frac{d}{dt} x(t)\right) + x(t) - x(t)^3 \qquad \textbf{(5)}$$

> $xs := x(t) = A \cdot \cos(\text{omega} \cdot t);$

$$xs := x(t) = A \cos(\omega t) \qquad \textbf{(6)}$$

> $dea := combine(expand(subs(xs, de)), trig);$

$$dea := -A\,\omega^2 \cos(\omega t) + \frac{A\,\omega \sin(\omega t)}{10} + \frac{5\,A^3\,\omega^3 \sin(3\,\omega t)}{6} - \frac{5\,A^3\,\omega^3 \sin(\omega t)}{2} + A\cos(\omega t) - \frac{A^3 \cos(3\,\omega t)}{4} - \frac{3\,A^3 \cos(\omega t)}{4} \tag{7}$$

> $e1 := coeff(dea, \sin(\omega t)) = 0;$

$$e1 := \frac{1}{10}\,A\,\omega - \frac{5}{2}\,A^3\,\omega^3 = 0 \tag{8}$$

> $e2 := coeff(dea, \cos(\omega t)) = 0\,;$

$$e2 := -A\,\omega^2 + A - \frac{3}{4}\,A^3 = 0 \tag{9}$$

> $e3 := coeff(dea, \cos(3\,\omega t));$

$$e3 := -\frac{A^3}{4} \tag{10}$$

> $e4 := coeff(dea\,, \sin(3\ \mathrm{omega}\, t));$

$$e4 := \frac{5\,A^3\,\omega^3}{6} \tag{11}$$

> $sol := solve(\{e1, e2\}, \{A, \mathrm{omega}\});$

$$sol := \{A=0, \omega=\omega\}, \{A = 2\,RootOf(3\,\_Z^2 - 1), \omega = 0\}, \Big\{A = -\frac{20\,RootOf(100\,\_Z^4 - 100\,\_Z^2 + 3)^3}{3} + \frac{20\,RootOf(100\,\_Z^4 - 100\,\_Z^2 + 3)}{3}, \omega = RootOf(100\,\_Z^4 - 100\,\_Z^2 + 3)\Big\}, \Big\{A = \frac{20\,RootOf(100\,\_Z^4 - 100\,\_Z^2 + 3)^3}{3} - \frac{20\,RootOf(100\,\_Z^4 - 100\,\_Z^2 + 3)}{3}, \omega = RootOf(100\,\_Z^4 - 100\,\_Z^2 + 3)\Big\} \tag{12}$$

>