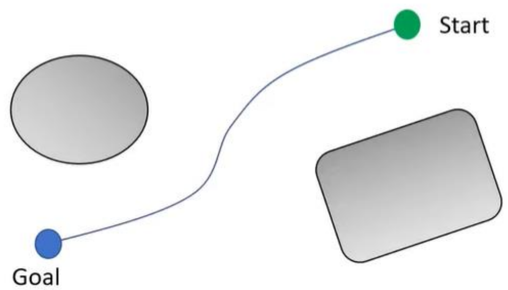
# HOME ASSIGNMENT 3

## Potential field method

Theory:

- Constructing an artificial potential field that draw the robot to the goal and repel it from obstacles in the environment,

- Think of the robot moving from a higher potential energy to a lower level at the goal.




### Potential Energy

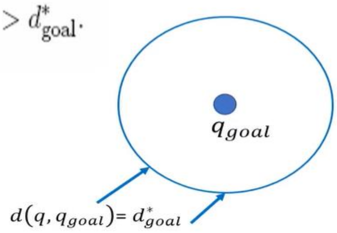
- The robot moves to a lower energy configuration.
- Define a potential function U : $\Re^m \rightarrow \Re$ from the configuration space to the field of real numbers ( scalar field).
- Energy is minimized by following the negative *gradient* of the potential energy function ( gradients points to the maximum rate of change).
- Think of a *vector field* over the space of all configurations 'q'.
  - at every point in time, the robot looks at the negative gradient vector at the point and moves forward in that direction.

$$\nabla U(q) = DU(q)^T = [\frac{\partial U}{\partial q_1}(q), \ldots, \frac{\partial U}{\partial q_m}(q)]^T$$

# Attractive Potential: Composite Definition

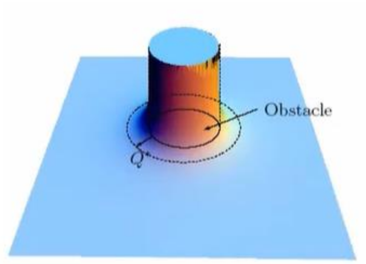
$$U_{\text{att}}(q) = \begin{cases} \frac{1}{2}\zeta d^2(q, q_{\text{goal}}), & d(q, q_{\text{goal}}) \le d^*_{\text{goal}}, \\ \underline{d^*_{\text{goal}}}\zeta d(q, q_{\text{goal}}) - \underline{\frac{1}{2}\zeta (d^*_{\text{goal}})^2}, & d(q, q_{\text{goal}}) > d^*_{\text{goal}}. \end{cases}$$

$$\nabla U_{\text{att}}(q) = \begin{cases} \zeta(q - q_{\text{goal}}), & d(q, q_{\text{goal}}) \le d^*_{\text{goal}}, \\ \frac{d^*_{\text{goal}}\zeta(q - q_{\text{goal}})}{d(q, q_{\text{goal}})}, & d(q, q_{\text{goal}}) > d^*_{\text{goal}}, \end{cases}$$




# Repulsive Potential

$$U_{\text{rep}}(q) = \begin{cases} \frac{1}{2}\eta\left(\frac{1}{D(q)} - \frac{1}{Q^*}\right)^2, & D(q) \le Q^*, \\ 0, & D(q) > Q^*, \end{cases}$$



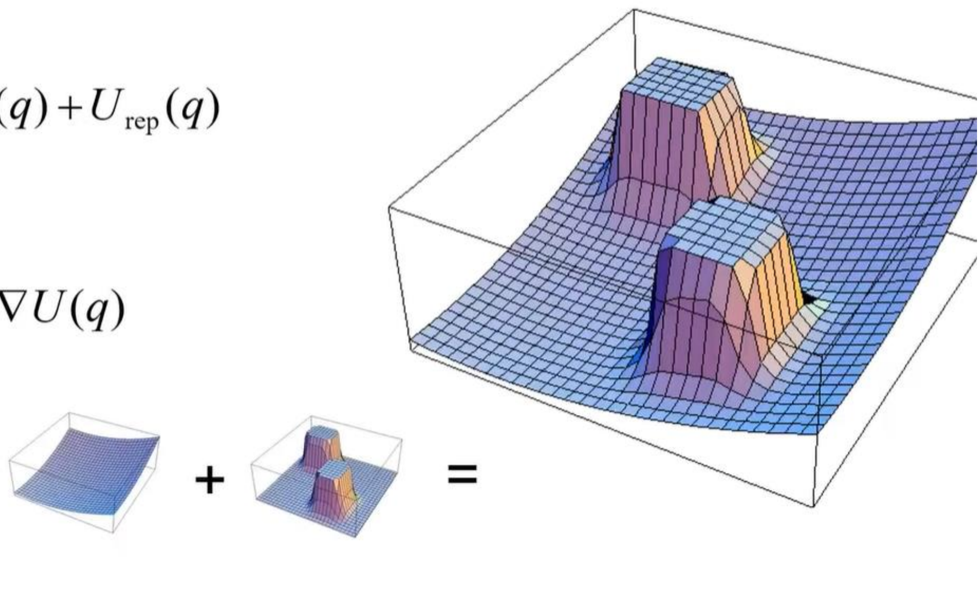

whose gradient is

$$\nabla U_{\text{rep}}(q) = \begin{cases} \eta\left(\frac{1}{Q^*} - \frac{1}{D(q)}\right)\frac{1}{D^2(q)}\nabla D(q), & D(q) \le Q^*, \\ 0, & D(q) > Q^*, \end{cases}$$

# Total Potential Function

$$U(q) = U_{\text{att}}(q) + U_{\text{rep}}(q)$$

$$F(q) = -\nabla U(q)$$

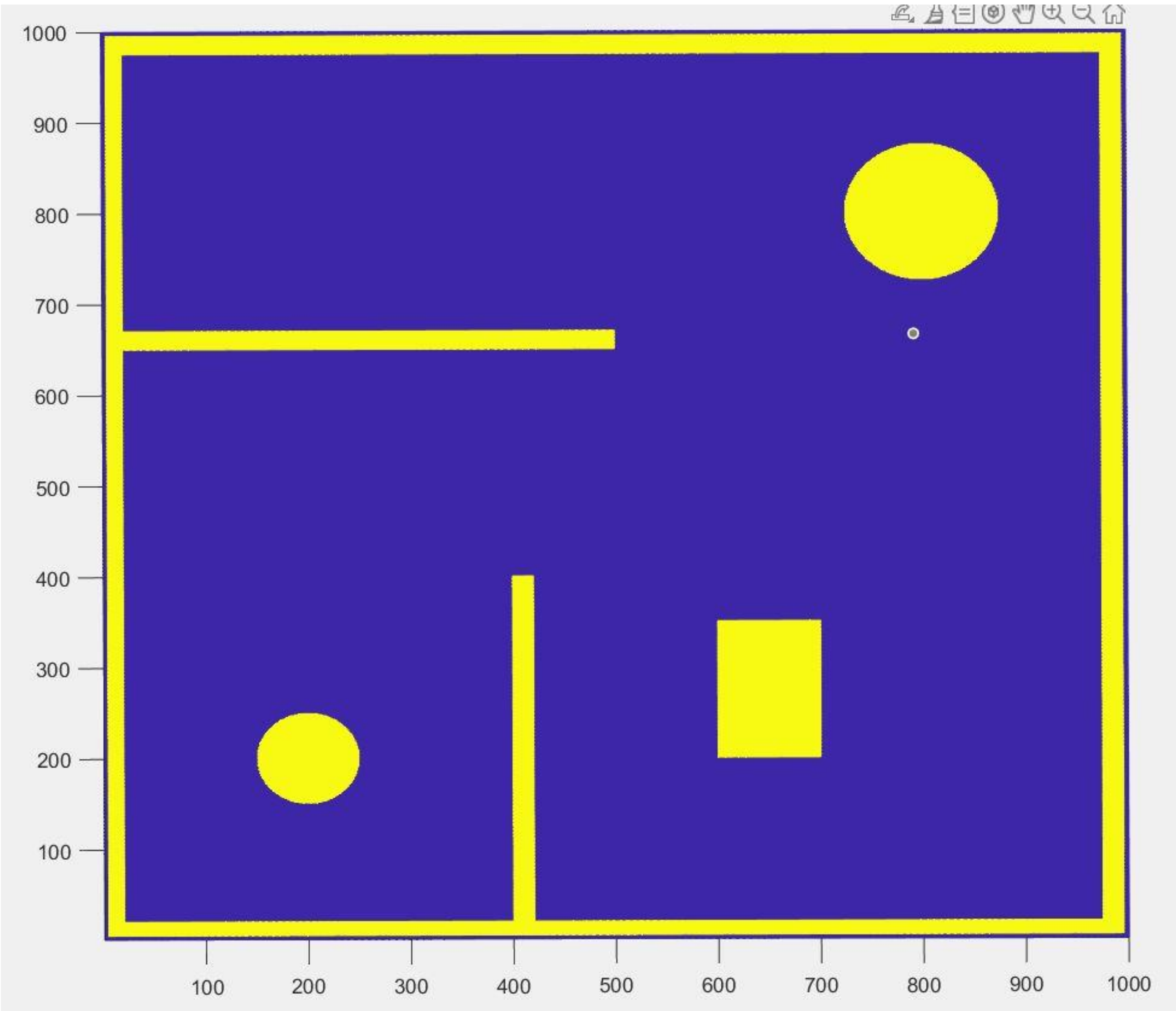


# Gradient Descent Method

- Initialise $q(0) = q_{start}$ , $i = 0$ ;
- Input $threshold$ and gradient descent rate $\alpha$.
- While $\|\nabla U(q(i))\| > threshold$
  - $q(i+1) = q(i) - \alpha \nabla U(q(i))$
  - $i = i + 1$

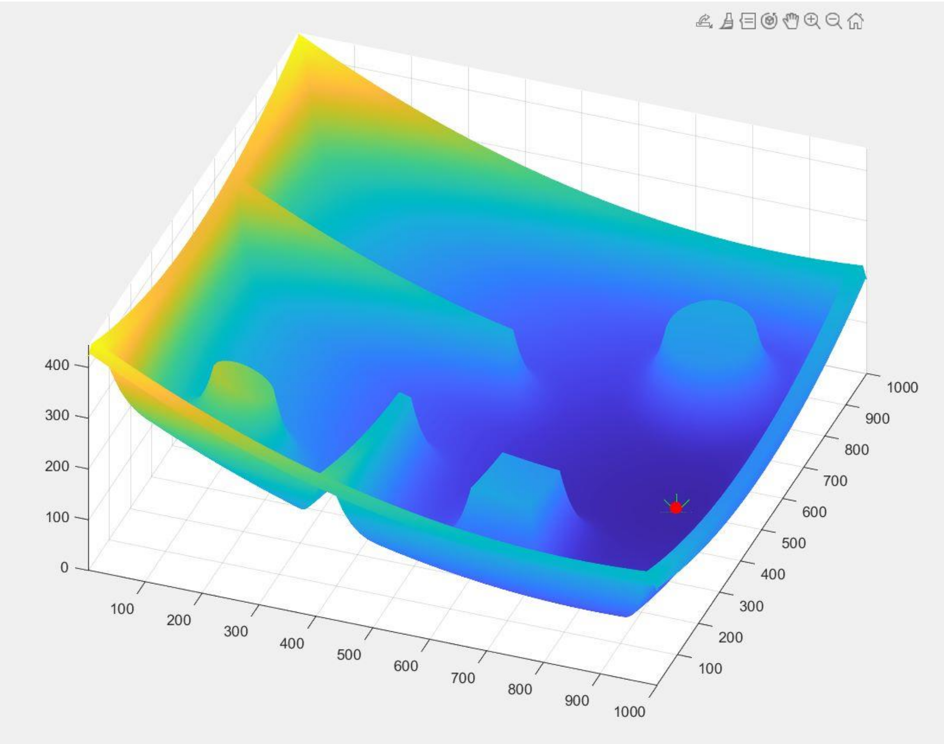
- Here you may also choose a variable $\alpha$ at each step.

*Workspace:*

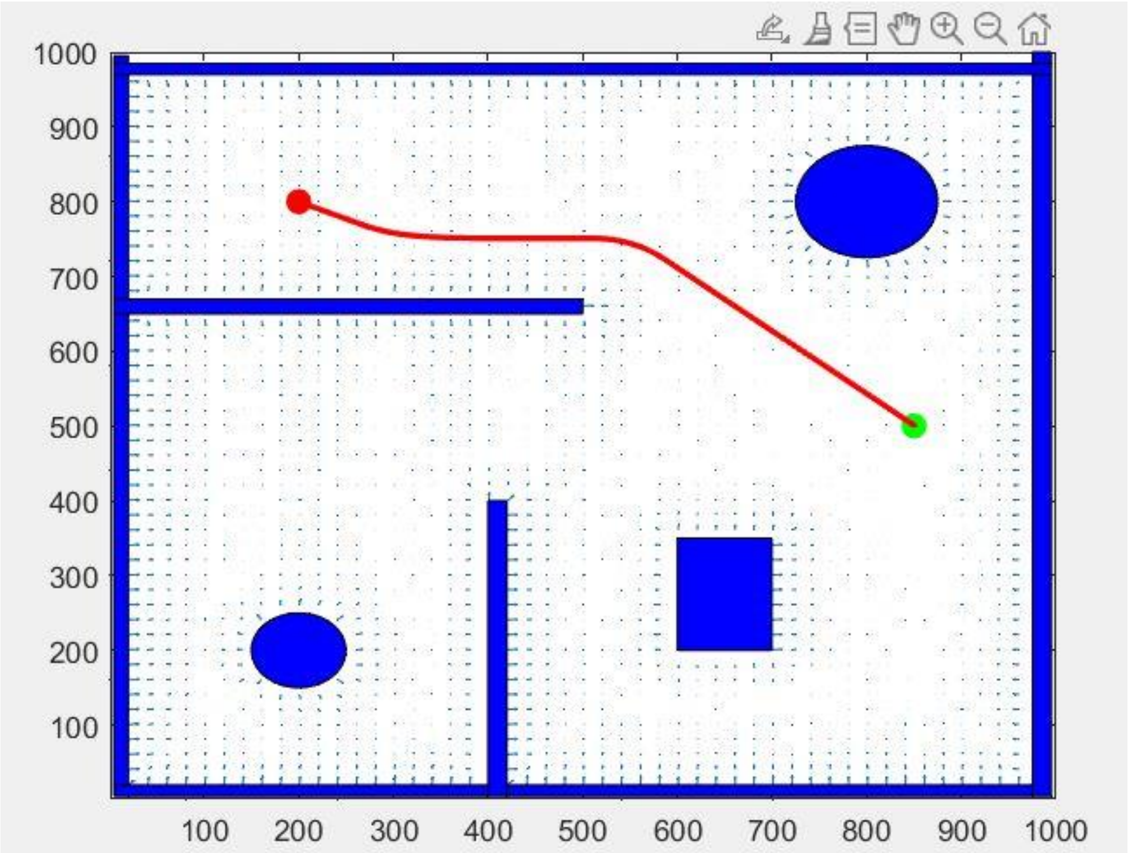


## *For start = [200, 800]; goal = [850, 500]; Eta = 1000;Zeta = 1/1500;*

Total Field



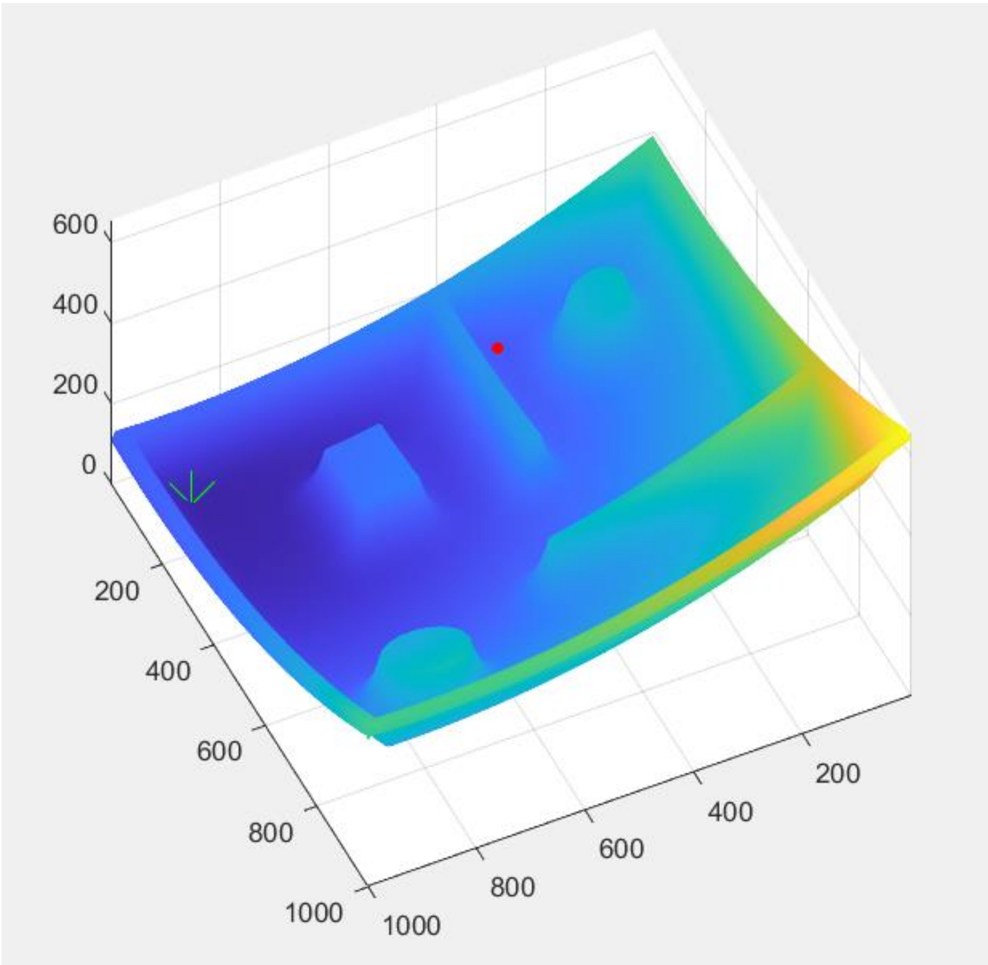

Workspace Results

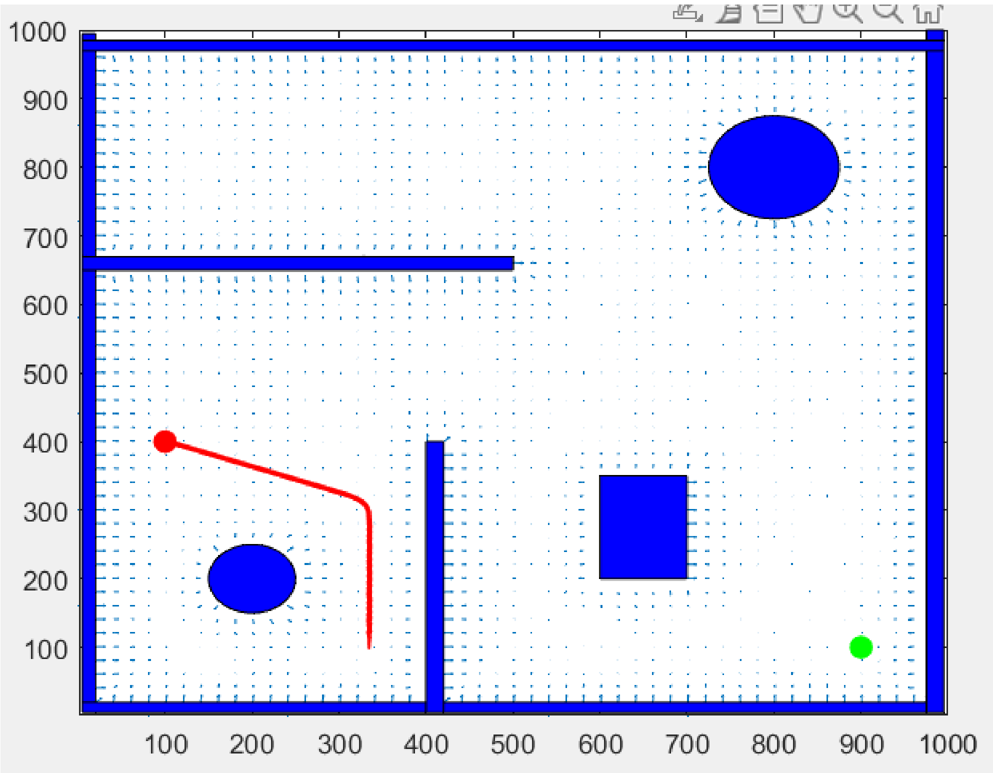


The video of the potential field interaction/visualizing with the graph is there in the code if we run it we can see the ball following through the gradient.

*For Failed Cases : start = [100, 400];*

*goal = [900, 100]; keeping zeta and eta same.*






We can also change zeta and eta with respect to each other to get a failed case etc.

## *Reasons For the failed case:*

1. Local Minima: The potential field method can get stuck in local minima, especially when the obstacles are placed close to the goal or when the robot has limited sensing capabilities. The repulsive potential generated by the obstacles can create multiple local minima, and the robot may get stuck in one of them instead of reaching the goal. ( this is what happened in the above case )

2. Oscillations: When the robot moves towards the goal, it can sometimes oscillate between the attractive and repulsive forces. This can happen when the robot is moving in a narrow corridor or when there are small obstacles in its path. The oscillations can cause the robot to deviate from the planned path or even get stuck.

3. Narrow Passages: The potential field method may not work well when the robot needs to pass through narrow passages or tight corners. In these cases, the repulsive forces generated by the obstacles can be too strong, making it difficult for the robot to navigate through the passage.

4. Scaling: The potential field method can also fail if the scaling factors for the attractive and repulsive potentials are not chosen carefully. If the scaling factor for the repulsive potential is too high, the robot may be too cautious and move too slowly, while if the scaling factor for the attractive potential is too high, the robot may overshoot the goal.

# MATLAB CODE