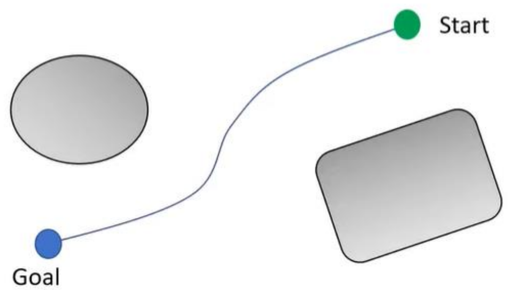
HOME ASSIGNMENT 3

Potential field method

Theory:

- Constructing an artificial potential field that draw the robot to the goal and repel it from obstacles in the environment,
- Think of the robot moving from a higher potential energy to a lower level at the goal.



Potential Energy

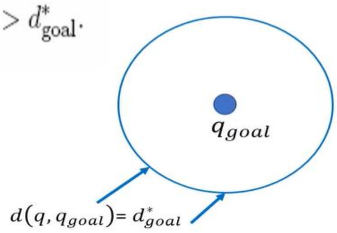
- The robot moves to a lower energy configuration.
- Define a potential function U : $\Re^m \rightarrow \Re$ from the configuration space to the field of real numbers (scalar field).
- Energy is minimized by following the negative *gradient* of the potential energy function (gradients points to the maximum rate of change).
- Think of a *vector field* over the space of all configurations 'q'.
  - at every point in time, the robot looks at the negative gradient vector at the point and moves forward in that direction.

$$\nabla U(q) = DU(q)^T = [\frac{\partial U}{\partial q_1}(q), \ldots, \frac{\partial U}{\partial q_m}(q)]^T$$

Attractive Potential: Composite Definition

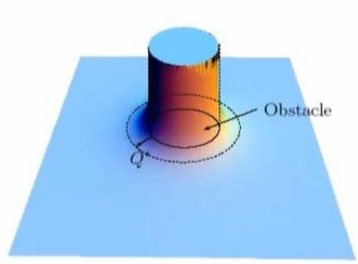
$$U_{\text{att}}(q) = \begin{cases} \frac{1}{2}\zeta d^2(q, q_{\text{goal}}), & d(q, q_{\text{goal}}) \le d^*_{\text{goal}}, \\ \underline{d^*_{\text{goal}}}\zeta d(q, q_{\text{goal}}) - \underline{\frac{1}{2}\zeta (d^*_{\text{goal}})^2}, & d(q, q_{\text{goal}}) > d^*_{\text{goal}}. \end{cases}$$

$$\nabla U_{\text{att}}(q) = \begin{cases} \zeta(q - q_{\text{goal}}), & d(q, q_{\text{goal}}) \le d^*_{\text{goal}}, \\ \frac{d^*_{\text{goal}}\zeta(q - q_{\text{goal}})}{d(q, q_{\text{goal}})}, & d(q, q_{\text{goal}}) > d^*_{\text{goal}}, \end{cases}$$



Repulsive Potential

$$U_{\text{rep}}(q) = \begin{cases} \frac{1}{2}\eta\left(\frac{1}{D(q)} - \frac{1}{Q^*}\right)^2, & D(q) \le Q^*, \\ 0, & D(q) > Q^*, \end{cases}$$



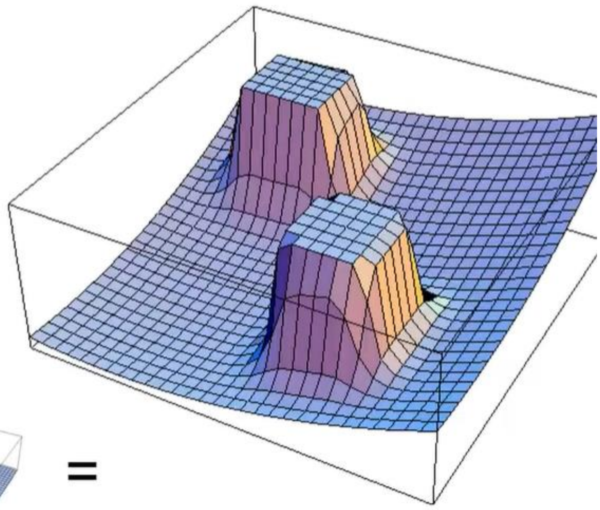
whose gradient is

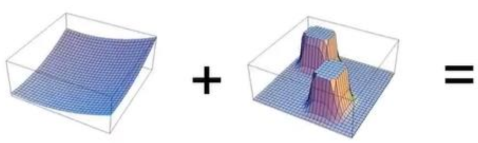
$$\nabla U_{\text{rep}}(q) = \begin{cases} \eta\left(\frac{1}{Q^*} - \frac{1}{D(q)}\right)\frac{1}{D^2(q)}\nabla D(q), & D(q) \le Q^*, \\ 0, & D(q) > Q^*, \end{cases}$$

Total Potential Function

$$U(q) = U_{\text{att}}(q) + U_{\text{rep}}(q)$$

$$F(q) = -\nabla U(q)$$



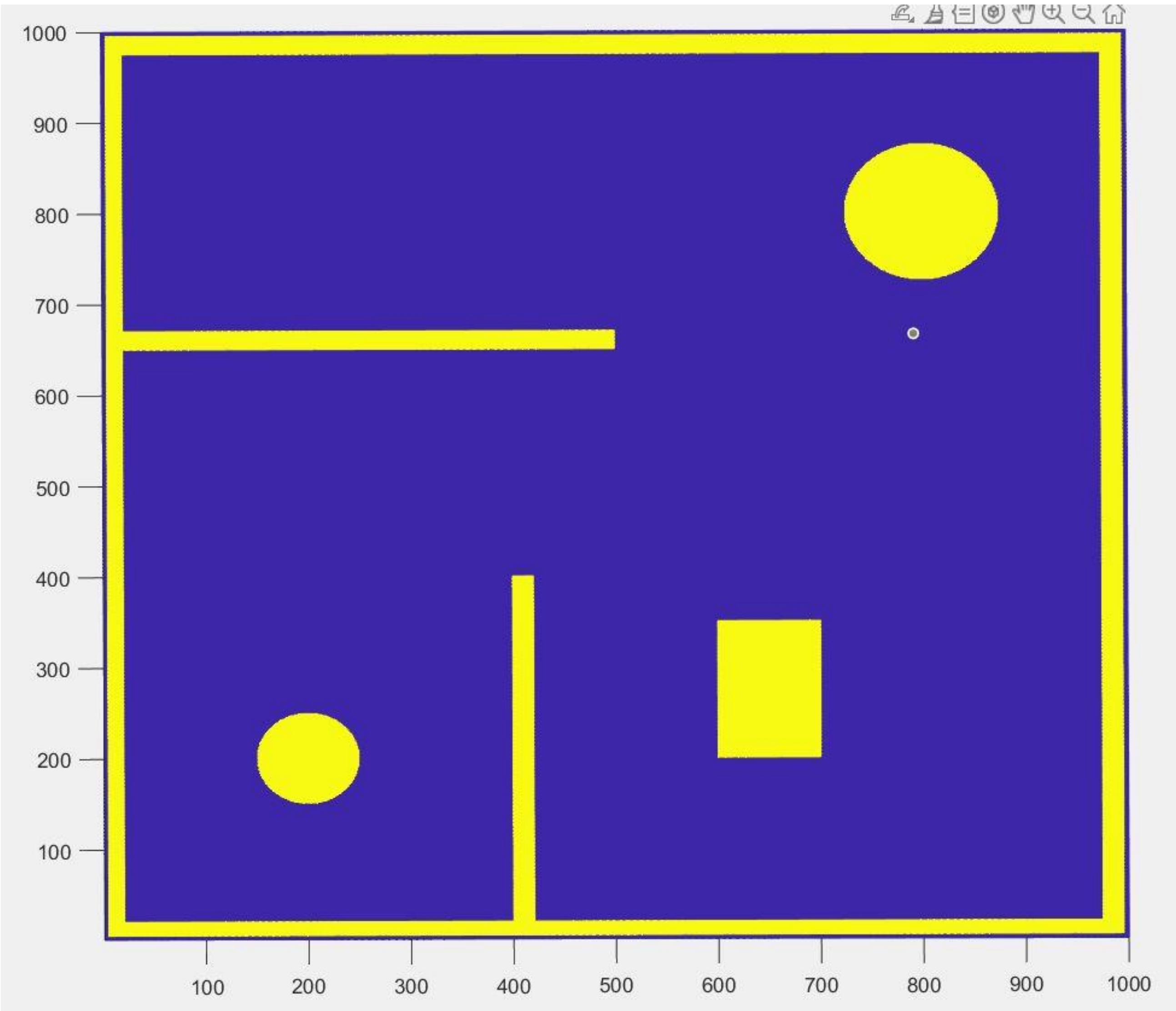


Gradient Descent Method

- Initialise $q(0) = q_{start}$, $i = 0$;
- Input $threshold$ and gradient descent rate α.
- While $\|\nabla U(q(i))\| > threshold$
  - $q(i+1) = q(i) - \alpha \nabla U(q(i))$
  - $i = i + 1$

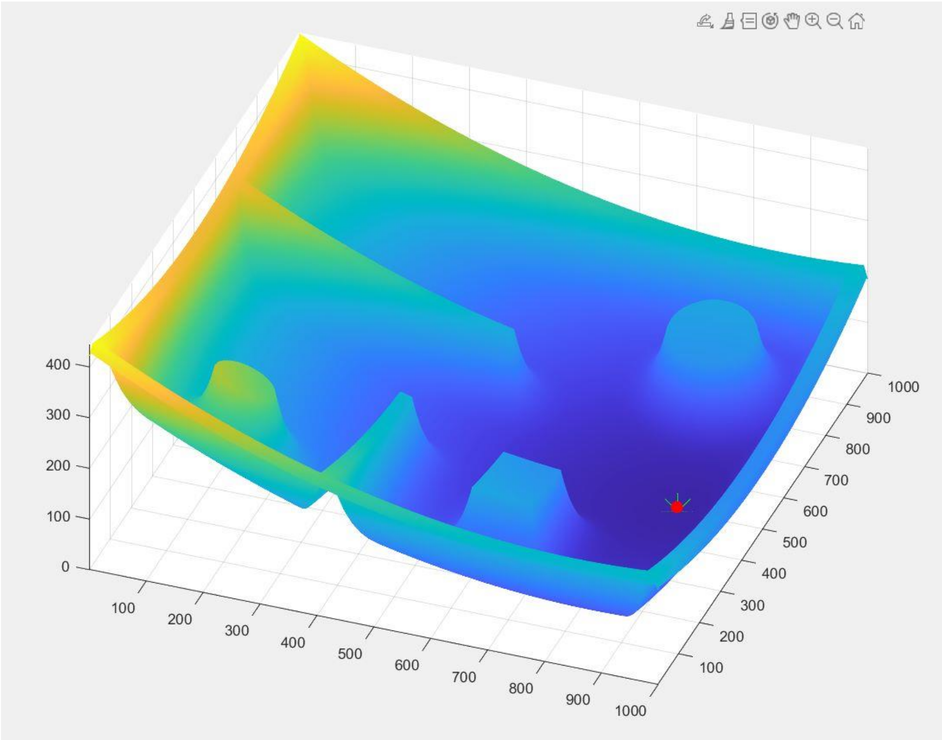
- Here you may also choose a variable α at each step.

Workspace:

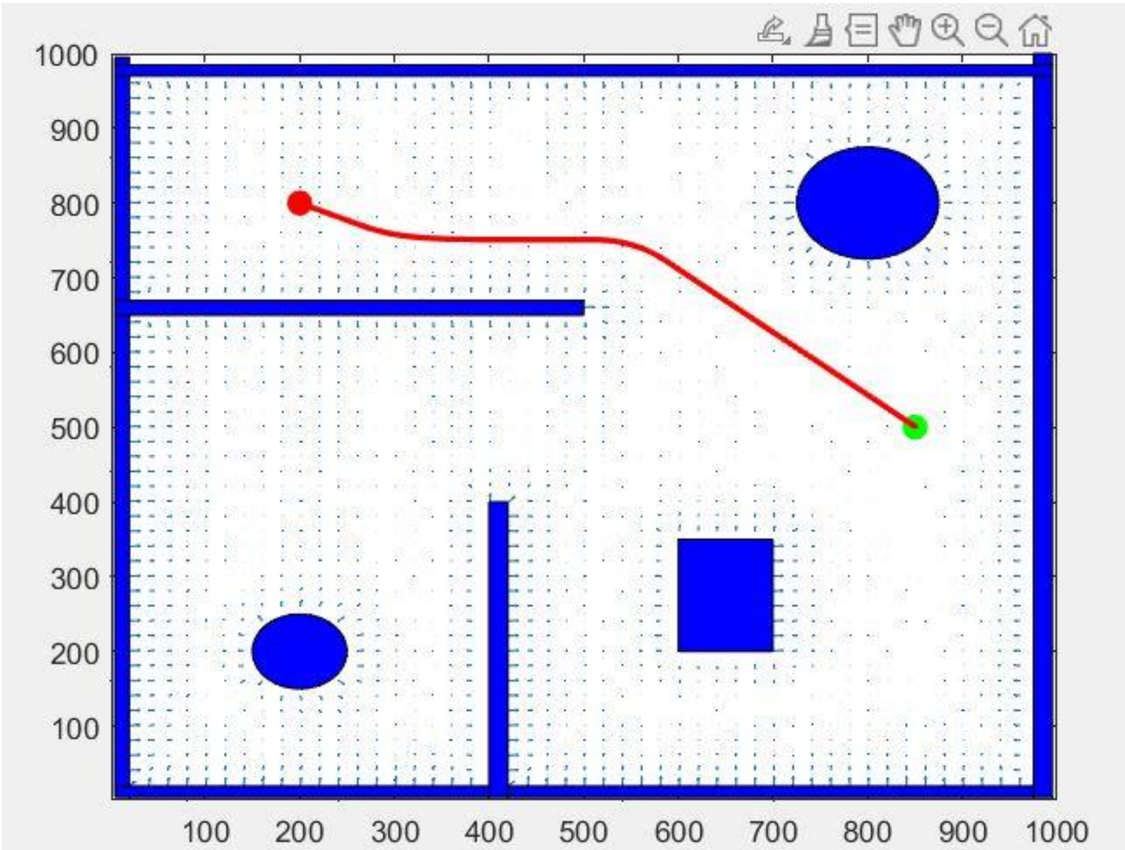


For start = [200, 800]; goal = [850, 500];
Eta = 1000;Zeta = 1/1500;

Total Field



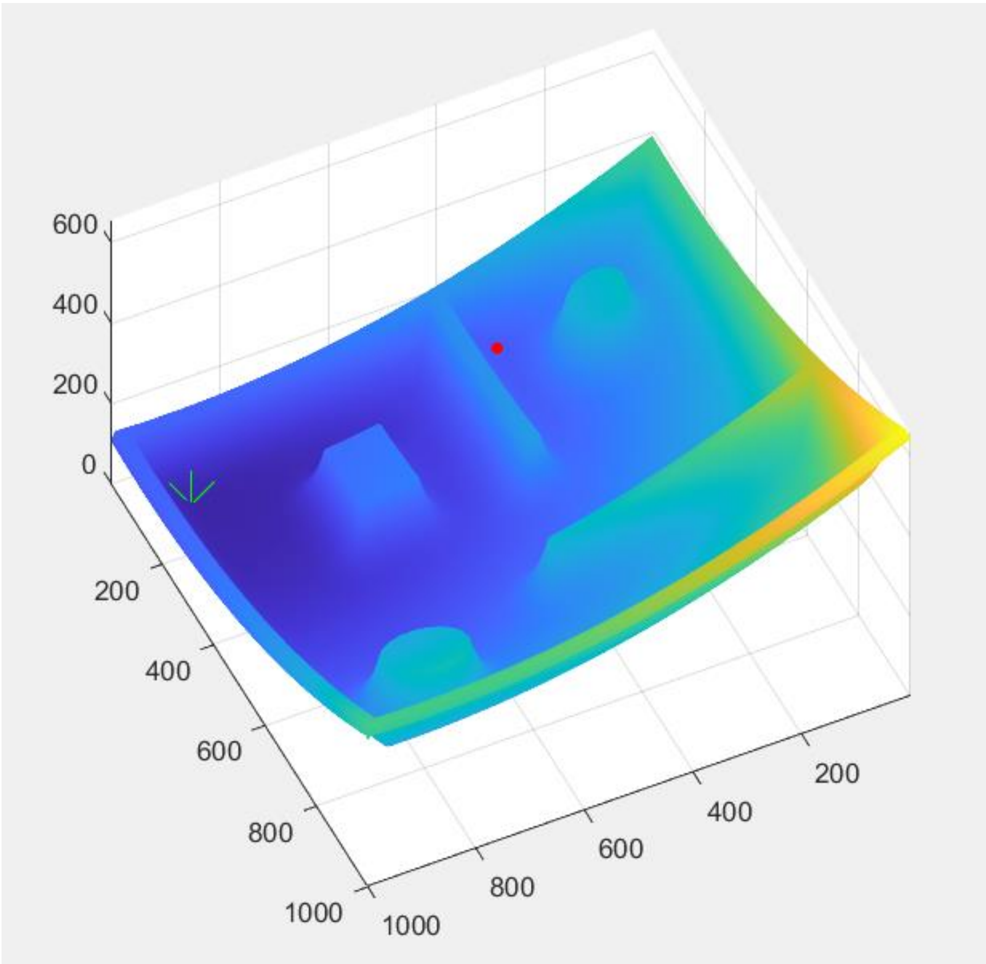
Workspace Results

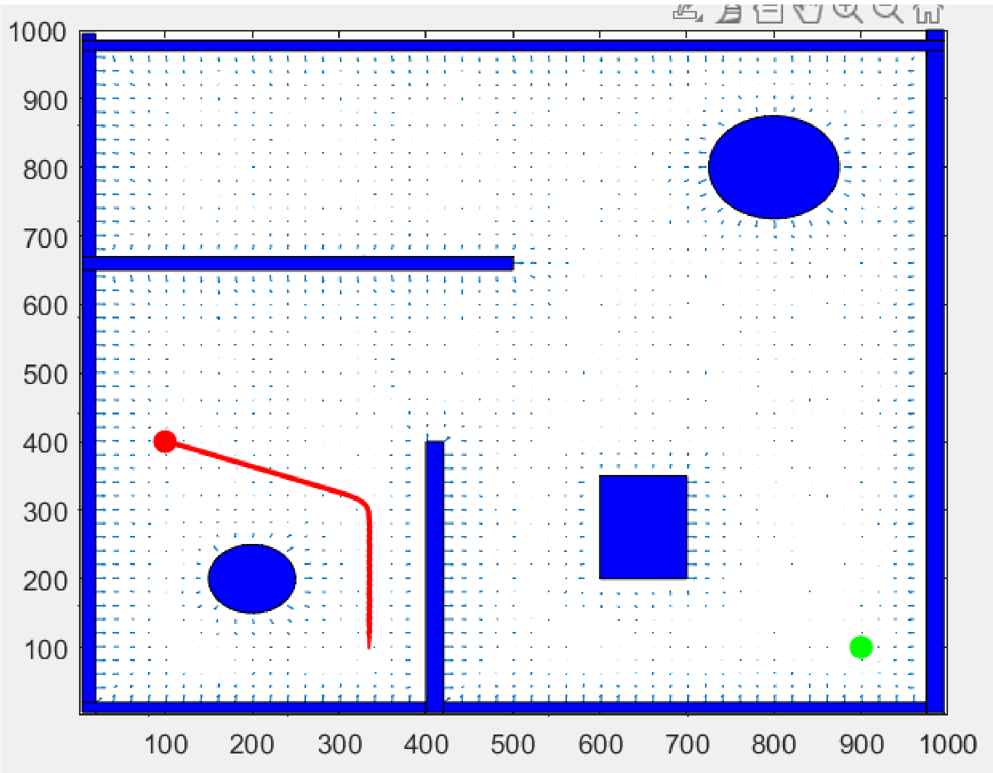


The video of the potential field interaction/visualizing with the graph is there in the code if we run it we can see the ball following through the gradient.

For Failed Cases : start = [100, 400];

goal = [900, 100]; keeping zeta and eta same.





We can also change zeta and eta with respect to each other to get a failed case etc.

Reasons For the failed case:

1. Local Minima: The potential field method can get stuck in local minima, especially when the obstacles are placed close to the goal or when the robot has limited sensing capabilities. The repulsive potential generated by the obstacles can create multiple local minima, and the robot may get stuck in one of them instead of reaching the goal. (this is what happened in the above case)

2. Oscillations: When the robot moves towards the goal, it can sometimes oscillate between the attractive and repulsive forces. This can happen when the robot is moving in a narrow corridor or when there are small obstacles in its path. The oscillations can cause the robot to deviate from the planned path or even get stuck.
3. Narrow Passages: The potential field method may not work well when the robot needs to pass through narrow passages or tight corners. In these cases, the repulsive forces generated by the obstacles can be too strong, making it difficult for the robot to navigate through the passage.
4. Scaling: The potential field method can also fail if the scaling factors for the attractive and repulsive potentials are not chosen carefully. If the scaling factor for the repulsive potential is too high, the robot may be too cautious and move too slowly, while if the scaling factor for the attractive potential is too high, the robot may overshoot the goal.

MATLAB CODE

Contents

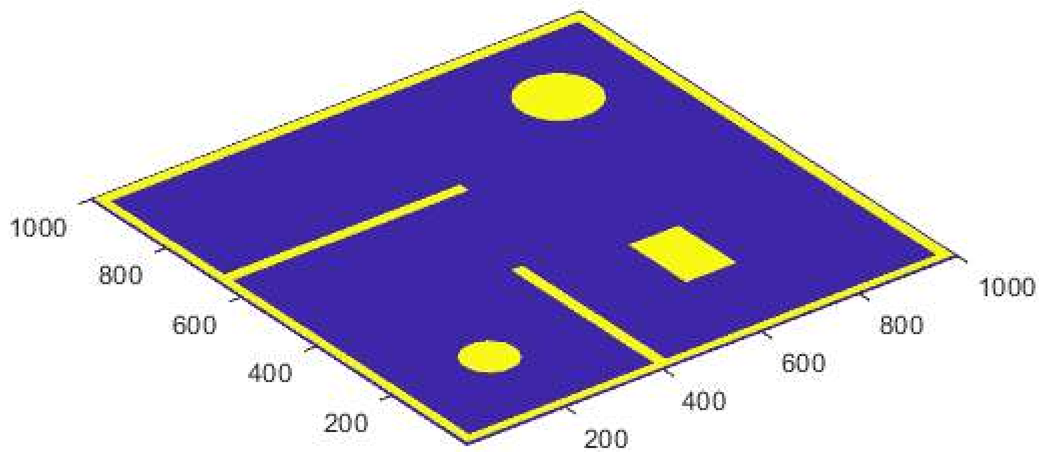
Mohammed sohaib Assignment 3

```
% Potential Potential field method to find a path from a start point to a
% goal point in a 2D workspace with obstacles.

clear all
close all
```

1) Generate a rectangular workspace with three or more obstacles or a

room with walls and passage ways (doors).

```
nrows = 1000;
ncols = 1000;

obstacle = false(nrows,ncols);

[x, y] = meshgrid(1:ncols, 1:nrows);

obstacle(5:20, 5:995) = true; % rectangular obstacle
obstacle(21:995,5:20) = true; % rectangular obstacle
obstacle(975:995,21:995) = true; % rectangular obstacle
obstacle(21:974,975:995) = true; % rectangular obstacle
%--------------------------------------------------
obstacle(650:670,21:500) = true; % rectangular obstacle
obstacle(21:400,400:420) = true; % rectangular obstacle
obstacle(200:350,600:700) = true; % rectangular obstacle

t = ((x-200).^2 + (y-200).^2 < 50^2) ; %circular obstacle
obstacle(t) = true;
t = ((x-800).^2 + (y-800).^2 < 75^2) ; %circular obstacle
obstacle(t) = true; %map every point where the obstacle lies as true.


m = mesh(obstacle);
axis equal
obstacle;
obstacle;
```



Now compute the distance transform

```
%from DistanceFromObstacle script and the scaling factor script

d = bwdist(obstacle); %distance transform assigns a number that is the
% distance between that pixel and the nearest nonzero pixel of BW.
%bwdist is the matlab function which returns distance from any true element
%in ostacle way
```

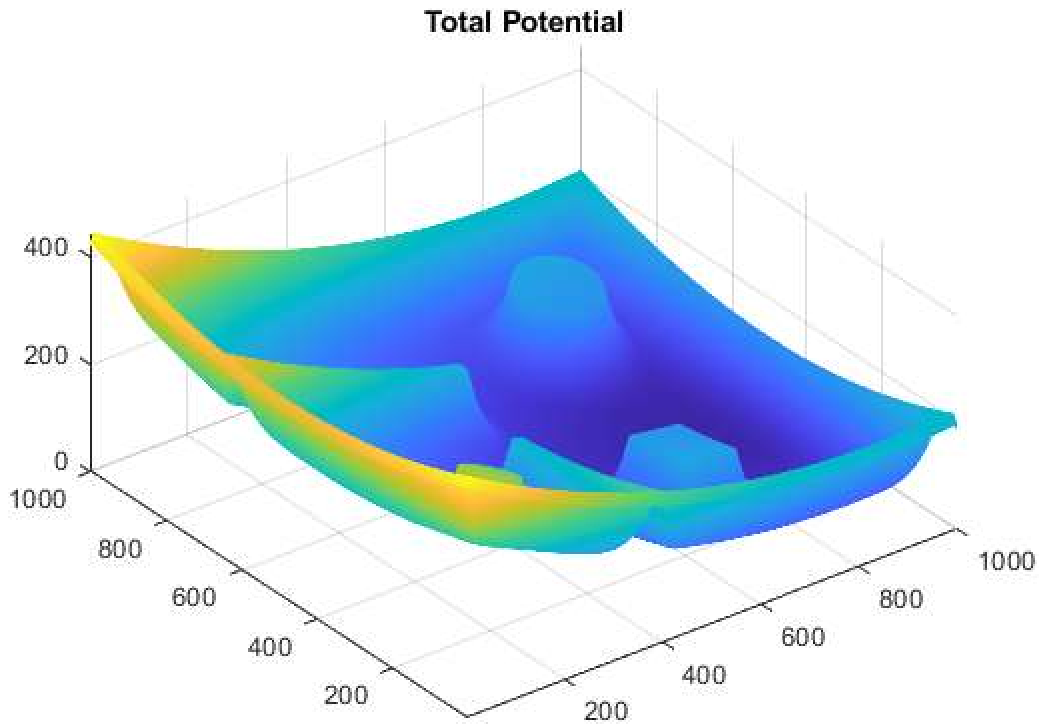
2) Mark the start point and the goal point in the workspace.

```
start = [200, 800];
goal = [850, 500];
```

3) Define the potential field functions for attractive and repulsive fields.

```
% Repulsive Potential

K = 100;
Rho = d/K +1;
%Note some values in d might be 0 will cause problems in calculating the
%repulsive force so we add a addition zero to aviod the division by zero.

d0 = 2; %if any robot away form the obstacle by do unit its repulsive force
%is considered zero

Eta = 1000;%Used to control repulsive force large Eta will cause some balance
%between repulsive and attractive forces we need to make large repulsive
%force so that it dosent gets struck on obstacles.
```

```
repulsive = (Eta/2)*((1./Rho-1/d0).^2);

repulsive(Rho > d0) = 0; % stating the condition.

%plotinng repulsive field
% figure;
% m = mesh(repulsive)
% m.FaceLighting = 'phong';
% axis equal;
% title ('Repulsive Potential');
% hold on
max(max(repulsive));

%Attractive Potential

zeta = 1/1500 ;% used to control the strength of attractivness towards goal if zeta =
% 1/10000, the robot will not reach the location but it 1/10 it will cross
% over the obstacle need to take an optimium solution.

attractive = (zeta/2) * ( (x- goal(1)).^2 + (y-goal(2)).^2);

%plot attractive field
% figure;
% m = mesh(attractive);
% m.FaceLighting = 'phong';
% axis equal;
% title ('Attractive Potential');
max(max(attractive));

% Compbined potential fields.

f = attractive + repulsive;

%plot combined field.
figure;
m = mesh(f);
% m.FaceLighting('phong');
axis equal;
title("Total Potential")
max(max(f));
```



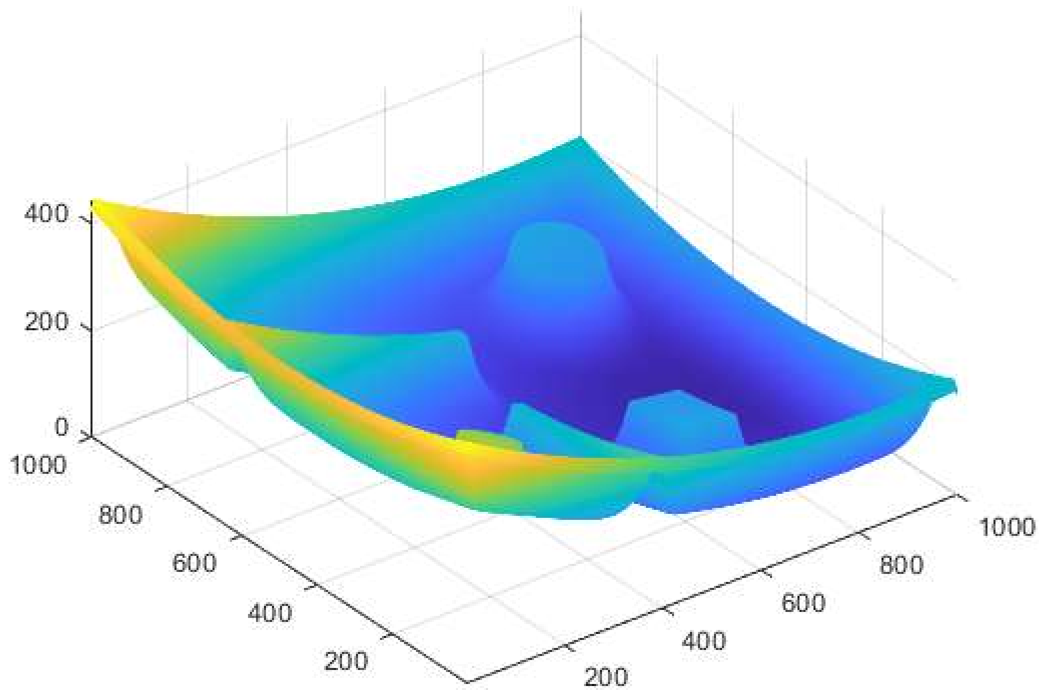
Generate feasible paths for pathplanning

```
%By gradient desent method.

[gx, gy] = gradient(-f);

route = start;
Point_on_route = start;
Speed = 3;
Tolerance = 1;
iterations =1000 ;

while(iterations >0)
    if(norm(goal - Point_on_route)<Tolerance)
        break;
    end
    delta_x = gx(floor(Point_on_route(2)), floor(Point_on_route(1)));
    delta_y = gy(floor(Point_on_route(2)), floor(Point_on_route(1)));

    delta = [delta_x, delta_y];
    %delta vector is both value and direction.

    delta_Direction_x = delta_x/norm(delta);
    delta_Direction_y = delta_y/norm(delta);

    new_route_x = Point_on_route(1) + Speed * delta_Direction_x;
    new_route_y = Point_on_route(2) + Speed * delta_Direction_y;

    Point_on_route = [new_route_x, new_route_y];

    route = [route; Point_on_route];

    iterations = iterations -1 ;
```

```
end
```

Plot the energ surface. as a path planning

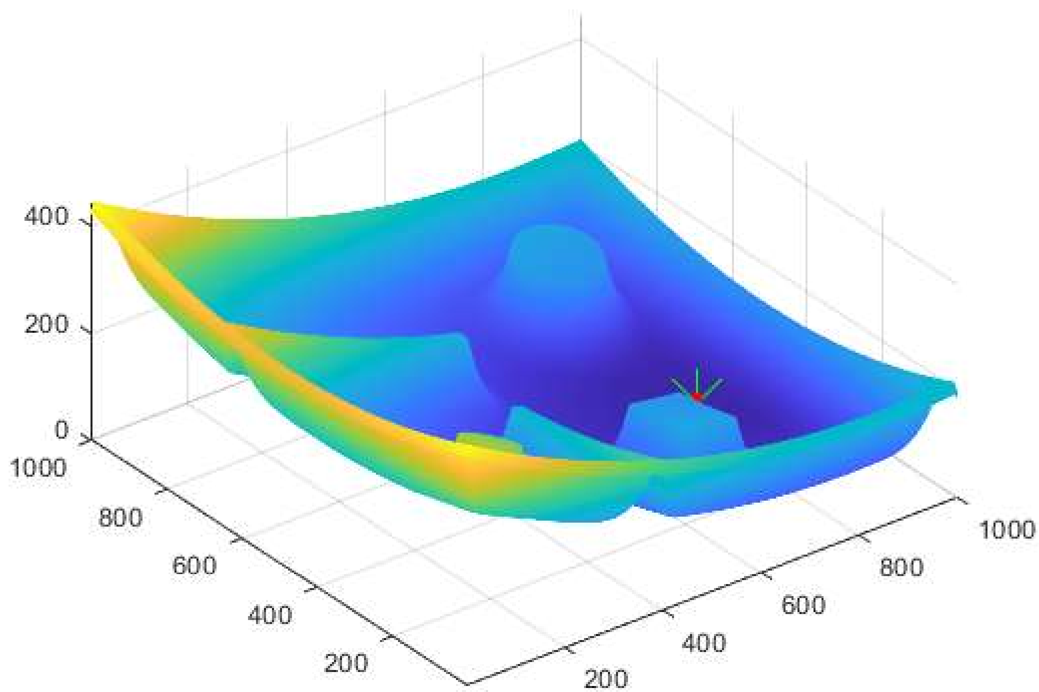
```
figure;
m = mesh(f);
axis equal
```



Plot a ball to vishualize

```
[sx, sy, sz] = sphere();

R = 10;
sx = R*sx;
sy = R*sy;
sz = R*sz +R;
% the lower half will not be vissible if added R

hold on;
p = mesh(sx,sy,sz);
%this will plot the ball at 0,0,0
p.FaceColor = 'red';
p.EdgeColor = 'none';
% p.FaceLighting = 'phong';
hold off;

hold on
plot(goal(1),goal(2),'g*','MarkerSize',25);
hold off
```

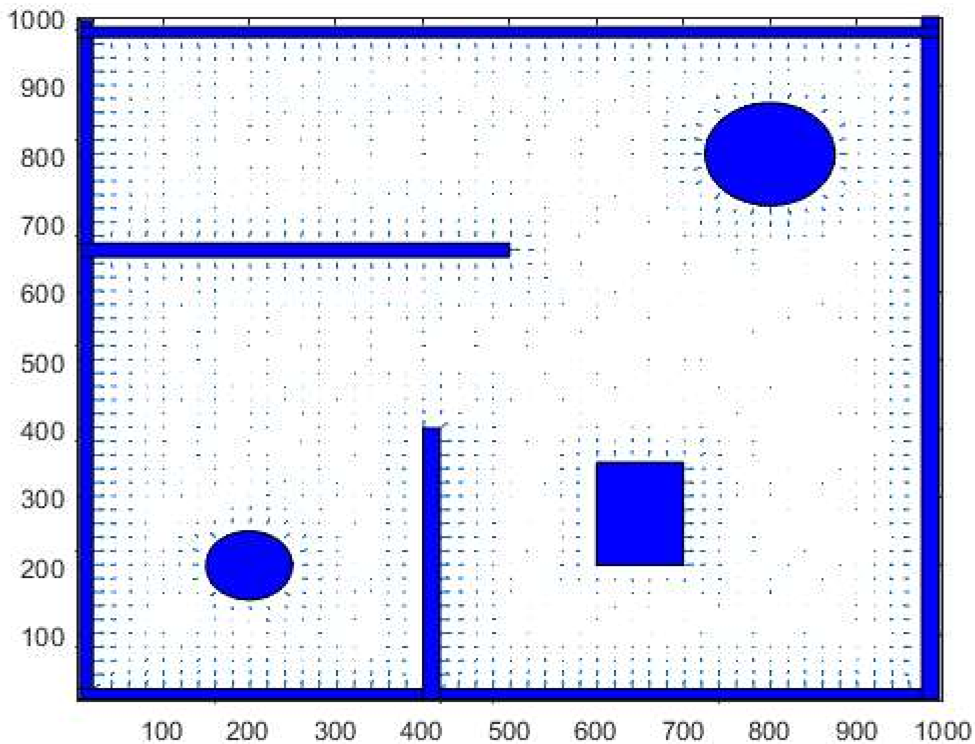
```
%Plot the ball at each point in route from start to goal
for i = 1:size(route,1)
    P = round(route(i,:));
    %P = [x,y]
    z = f(P(2),P(1));
    % z = f(x,y)

    %Draw the ball shifted to the new pos
    p.XData = sx + P(1);
    p.YData = sy + P(2);
    p.ZData = sz + f(P(2),P(1));

    drawnow;

    pause(0.05)
end
```



Quiver plot with obstacles

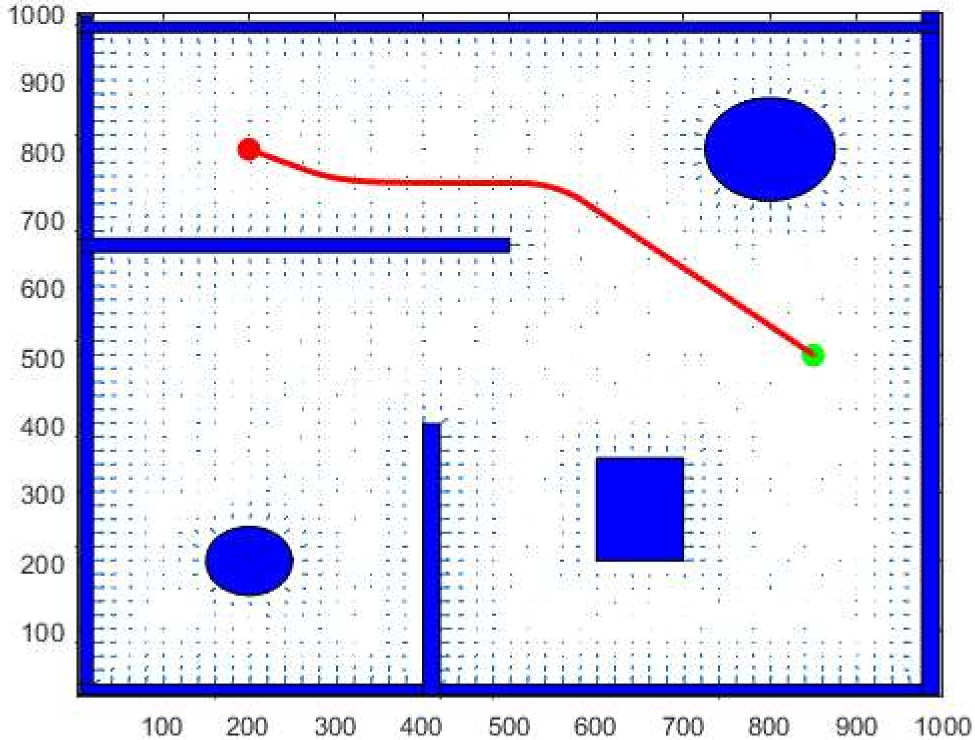
```
[gx, gy] = gradient(-f);
skip = 20 ;
figure ;

xidx = 1:skip:ncols;
yidx = 1:skip:nrows;

quiver(x(yidx,xidx),y(yidx,xidx),gx(yidx,xidx),gy(yidx,xidx),0.4);

axis([1 ncols 1 nrows]);

hold on ;
% Plot the rectangle
```

```
rectangle('Position', [5, 5, 15, 990], 'FaceColor', 'blue', 'EdgeColor', 'black');
rectangle('Position', [5, 5, 990, 15], 'FaceColor', 'blue', 'EdgeColor', 'black');
rectangle('Position', [975,5, 20,995], 'FaceColor', 'blue', 'EdgeColor', 'black');
rectangle('Position', [5,970, 990,15], 'FaceColor', 'blue', 'EdgeColor', 'black');

rectangle('Position', [5,650, 495,20], 'FaceColor', 'blue', 'EdgeColor', 'black');
rectangle('Position', [400,5, 20,395], 'FaceColor', 'blue', 'EdgeColor', 'black');
rectangle('Position', [600,200, 100,150], 'FaceColor', 'blue', 'EdgeColor', 'black');

t = linspace(0,2*pi,100);
x1 = 200 + 50*cos(t);
y1 = 200 +50*sin(t);
x2 = 800 + 75*cos(t);
y2 = 800 +75*sin(t);
hold on

c = [0.8 0.7 0.8];
fill(x1,y1,'b');
fill(x2,y2,'b');
```



Final ploting

```
ps = plot(start(1), start(2), 'r.','MarkerSize',30);
pg = plot(goal(1),goal(2),'g.',MarkerSize=30);
pp3 = plot(route(:,1),route(:,2),'r','LineWidth',2);
```



1000
900
800
700
600
500
400
300
200
100
100
200
300
400
500
600
700
800
900
1000

```
%% Mohammed sohaib Assignment 3

% Potential Potential field method to find a path from a start point to a
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%% 1) Generate a rectangular workspace with three or more obstacles or a
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% distance between that pixel and the nearest nonzero pixel of BW.
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%% 2)  Mark the start point and the goal point in the workspace.
start = [200, 800];
goal = [850, 500];
```

```
%% 3)  Define the potential field functions for attractive and repulsive fields.

% Repulsive Potential

K = 100;
Rho = d/K +1;
%Note some values in d might be 0 will cause problems in calculating the
%repulsive force so we add a addition zero to aviod the division by zero.

d0 = 2; %if any robot away form the obstacle by do unit its repulsive force
%is considered zero

Eta = 1000;%Used to control repulsive force large Eta will cause some balance
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%force so that it dosent gets struck on obstacles.

repulsive = (Eta/2)*((1./Rho-1/d0).^2);

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%plotinng repulsive field
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% m.FaceLighting = 'phong';
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% hold on
max(max(repulsive));

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% Compbined potential fields.
```

```
f = attractive + repulsive;

%plot combined field.
figure;
m = mesh(f);
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max(max(f));

%% Generate feasible paths for pathplanning
%By gradient desent method.

[gx, gy] = gradient(-f);

route = start;
Point_on_route = start;
Speed = 3;
Tolerance = 1;
iterations =1000 ;

while(iterations >0)
    if(norm(goal - Point_on_route)<Tolerance)
        break;
    end
    delta_x = gx(floor(Point_on_route(2)), floor(Point_on_route(1)));
    delta_y = gy(floor(Point_on_route(2)), floor(Point_on_route(1)));

    delta = [delta_x, delta_y];
    %delta vector is both value and direction.

    delta_Direction_x = delta_x/norm(delta);
    delta_Direction_y = delta_y/norm(delta);

    new_route_x = Point_on_route(1) + Speed * delta_Direction_x;
    new_route_y = Point_on_route(2) + Speed * delta_Direction_y;

    Point_on_route = [new_route_x, new_route_y];

    route = [route; Point_on_route];

    iterations = iterations -1 ;

end

%% Plot the energ surface. as a path planning
figure;
m = mesh(f);
```

```
axis equal

%% Plot a ball to vishualize

[sx, sy, sz] = sphere();

R = 10;
sx = R*sx;
sy = R*sy;
sz = R*sz +R;
% the lower half will not be vissible if added R

hold on;
p = mesh(sx,sy,sz);
%this will plot the ball at 0,0,0
p.FaceColor = 'red';
p.EdgeColor = 'none';
% p.FaceLighting = 'phong';
hold off;

hold on
plot(goal(1),goal(2),'g*','MarkerSize',25);
hold off

%Plot the ball at each point in route from start to goal
for i = 1:size(route,1)
    P = round(route(i,:));
    %P = [x,y]
    z = f(P(2),P(1));
    % z = f(x,y)

    %Draw the ball shifted to the new pos
    p.XData = sx + P(1);
    p.YData = sy + P(2);
    p.ZData = sz + f(P(2),P(1));

    drawnow;

    pause(0.05)
end

%% Quiver plot with obstacles

[gx, gy] = gradient(-f);
skip = 20 ;
figure ;

xidx = 1:skip:ncols;
yidx = 1:skip:nrows;
```

```
quiver(x(yidx,xidx),y(yidx,xidx),gx(yidx,xidx),gy(yidx,xidx),0.4);

axis([1 ncols 1 nrows]);

hold on ;
% Plot the rectangle
rectangle('Position', [5, 5, 15, 990], 'FaceColor', 'blue', 'EdgeColor', 'black');
rectangle('Position', [5, 5, 990, 15], 'FaceColor', 'blue', 'EdgeColor', 'black');
rectangle('Position', [975,5, 20,995], 'FaceColor', 'blue', 'EdgeColor', 'black');
rectangle('Position', [5,970, 990,15], 'FaceColor', 'blue', 'EdgeColor', 'black');

rectangle('Position', [5,650, 495,20], 'FaceColor', 'blue', 'EdgeColor', 'black');
rectangle('Position', [400,5, 20,395], 'FaceColor', 'blue', 'EdgeColor', 'black');
rectangle('Position', [600,200, 100,150], 'FaceColor', 'blue', 'EdgeColor', 'black');

t = linspace(0,2*pi,100);
x1 = 200 + 50*cos(t);
y1 = 200 +50*sin(t);
x2 = 800 + 75*cos(t);
y2 = 800 +75*sin(t);
hold on

c = [0.8 0.7 0.8];
fill(x1,y1,'b');
fill(x2,y2,'b');

%%Final ploting
ps = plot(start(1), start(2), 'r.','MarkerSize',30);
pg = plot(goal(1),goal(2),'g.',MarkerSize=30);
pp3 = plot(route(:,1),route(:,2),'r','LineWidth',2);
```