



To calculate hidden layer:

$$h_1 = r_1 w_1 + r_2 w_2 + b_1$$

Activation function: Sigmoid: $\frac{1}{1+e^{-x}}$

$$r_1 = 0.05 \quad b_1 = 0.40$$

$$\text{Target } T_1 = 0.99$$

$$r_2 = 0.10 \quad b_2 = 0.60$$

$$\text{Initial Weights: } w_1 = 0.10, w_2 = 0.20, w_3 = 3.0$$

Forward step:

$$h_1 = r_1 w_1 + r_2 w_2 + b_1$$

$$= 0.05 \times 0.10 + 0.10 \times 0.20 + 0.40$$

$$= 0.425$$

$$\text{Out } h_1 = \frac{1}{1+e^{h_1}} = \frac{1}{1+e^{-0.425}} = 0.604.$$

$$y_1 = \text{out } h_1 \times w_3 + b_2 = 0.604 \times 3.0 + 0.60$$

$$y_1 = 0.781$$

$$\text{Out } y_1 = \frac{1}{1+e^{-y_1}} = \frac{1}{1+e^{-0.781}} = 0.688$$

Calculating the new error

$$E_{\text{New}} = \frac{1}{2} (T - \text{out } y_1)^2 = \frac{1}{2} (0.99 - 0.688)^2 = 0.048$$

Borrowed from -

$$\text{Error in } w_3 = \frac{\partial E_{\text{total}}}{\partial w_3} = -0.51 \times 0.217 \times 0.604 \\ = 0.040$$

$$\frac{\partial E_{\text{total}}}{\partial w_3} = \frac{\partial E_{\text{total}}}{\partial w_{H_1}} + \frac{\partial E_{\text{total}}}{\partial y_1} + \frac{\partial E_{\text{total}}}{\partial w_2}$$

$$\frac{\partial E_{\text{total}}}{\partial w_{H_1}} = 2 \times \frac{1}{2} (7 - \text{out}_{H_1}) \times -1 \\ = -0.31$$

$$\frac{\partial E_{\text{total}}}{\partial y_1} = \text{out}_{y_1} (1 - \text{out}_{y_1}) \times 1 = 0.217$$

$$\frac{\partial E_{\text{total}}}{\partial w_2} = \text{out}_{H_1} = 0.604$$

Updating w_3 ,

$$w_3 = w_3 - \gamma \left(\frac{\partial E_{\text{total}}}{\partial w_3} \right) = 0.30 - 0.5 (-0.040) \\ = 0.32$$

Let's calculate error in w_2 & w_1 and update

$$\frac{\partial E_{\text{total}}}{\partial w_1} = \frac{\partial E_{\text{total}}}{\partial \text{out}_{H_1}} + \frac{\partial E_{\text{total}}}{\partial H_1} + \frac{\partial E_{\text{total}}}{\partial w_2}$$

$$\frac{\partial E_{\text{total}}}{\partial \text{out}_{H_1}} = \text{out}_{H_1} \frac{\partial E_{\text{total}}}{\partial w_{H_1}} + \frac{\partial E_{\text{total}}}{\partial y_1} + \frac{\partial E_{\text{total}}}{\partial w_2} \\ = 0.31 + 0.217 + 0.32 \\ = -0.02$$

$$\frac{\partial \text{out}_{H_1}}{\partial H_1} = \text{out}_{H_1} (1 - \text{out}_{H_1}) = 0.24$$

$$\frac{\partial H_1}{\partial w_1} = H_1 = 0.05$$

$$\frac{dE_{\text{flow}}}{du_1} = -0.02 + 0.24 \times 0.01 \\ = -0.00824$$

$$\text{Upwind weight}_1 = w_1 - \left\{ \frac{\frac{dE_{\text{flow}}}{du_1}}{d w_1} \right\} \\ = 0.10 - 0.5(-0.00824) \\ w_1 = 0.1001$$

$$\frac{dE_{\text{flow}}}{du_2} = \frac{dE_{\text{flow}}}{dw_1} + \frac{dE_{\text{flow}}}{dh_1} + \frac{dE_{\text{flow}}}{du_2}$$

$$\frac{dE_{\text{flow}}}{dw_1} = -0.02$$

$$\frac{dE_{\text{flow}}}{dh_1} =$$

$$\frac{dE_{\text{flow}}}{du_2} = 6.24$$

$$\frac{dE_{\text{flow}}}{du_2} = r_2 = 0.10$$

$$\frac{dE_{\text{flow}}}{du_2} = -0.02 + 0.24 + 6.16 \\ = -0.00048$$

$$\text{Upwind weight}_1 = w_1 = 0.10012 \\ w_2 = -0.00048 \\ w_3 = -0.00024$$