

In [1]:

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import odeint
from scipy.integrate import complex_ode
```

In [2]:

```
def odeintz(func, z0, t, **kwargs):
    """An odeint-like function for complex valued differential equations."""

    # Disallow Jacobian-related arguments.
    _unsupported_odeint_args = ['Dfun', 'col_deriv', 'ml', 'mu']
    bad_args = [arg for arg in kwargs if arg in _unsupported_odeint_args]
    if len(bad_args) > 0:
        raise ValueError("The odeint argument %r is not supported by "
                          "odeintz." % (bad_args[0],))

    # Make sure z0 is a numpy array of type np.complex128.
    z0 = np.array(z0, dtype=np.complex128, ndmin=1)

    def realfunc(x, t, *args):
        z = x.view(np.complex128)
        dzdt = func(z, t, *args)
        # func might return a python list, so convert its return
        # value to an array with type np.complex128, and then return
        # a np.float64 view of that array.
        return np.asarray(dzdt, dtype=np.complex128).view(np.float64)

    result = odeint(realfunc, z0.view(np.float64), t, **kwargs)

    if kwargs.get('full_output', False):
        z = result[0].view(np.complex128)
        infodict = result[1]
        return z, infodict
    else:
        z = result.view(np.complex128)
        return z

#This fuction is from stack overflow as it's difficult for odeint to handle complex num
```



In [16]:

```

if __name__ == "__main__":
    # Generate a solution to:
    #     dC_1/dt = -i*(omega)*(C_2+C_3)
    #     dC_2/dt = -i*((omega)*(C_1+C_4)+delta*(C_2))
    #     dC_3/dt = -i*((omega)*(C_1+C_4)+delta*(C_3))
    #     dC_4/dt = -i*((omega)*(C_2+C_3)+(v+2*delta)*(C_4))

    # Define the right-hand-side of the differential equation.
    #ODE's needed to be solved
    def zfunc(z, t, omega_0, delta_0,v):
        C_1,C_2,C_3,C_4=z
        tau=114.85/omega_0
        #corrected
        omega=omega_0*np.sin((np.pi*t)/tau)**2 / 2
        delta=delta_0*np.cos((np.pi*t)/tau)**2

        new_c1=-1j*(omega)*(C_2+C_3)
        new_c2=-1j*((omega)*(C_1+C_4)+delta*(C_2))
        new_c3=-1j*((omega)*(C_1+C_4)+delta*(C_3))
        new_c4= -1j*((omega)*(C_2+C_3)+((v+2*delta)*(C_4)))

        return [new_c1,new_c2,new_c3,new_c4]

    # Set up the inputs and call odeintz to solve the system.

    #coefficient of the system at the start.
    intial = np.array([1,0,0,0])
    #normalisation.
    intial_1=np.abs(intial)**2
    normalisation=np.sum(intial_1)
    #normalised coeffcient.
    intial=(intial)/(np.sqrt(normalisation))

    #values of the parameters.
    omega_0 = 0.1
    delta_0 = 1.8
    tau=114.85/omega_0
    #corrected
    v=4.0
    t = np.linspace(0, tau, 50000)
    #solution
    z, infodict = odeintz(zfunc, intial, t, args=(omega_0,delta_0,v), full_output=True)

    #Each coeffcient ode solved
    C_1= z[:,0]
    C_2= z[:,1]
    C_3= z[:,2]
    C_4= z[:,3]

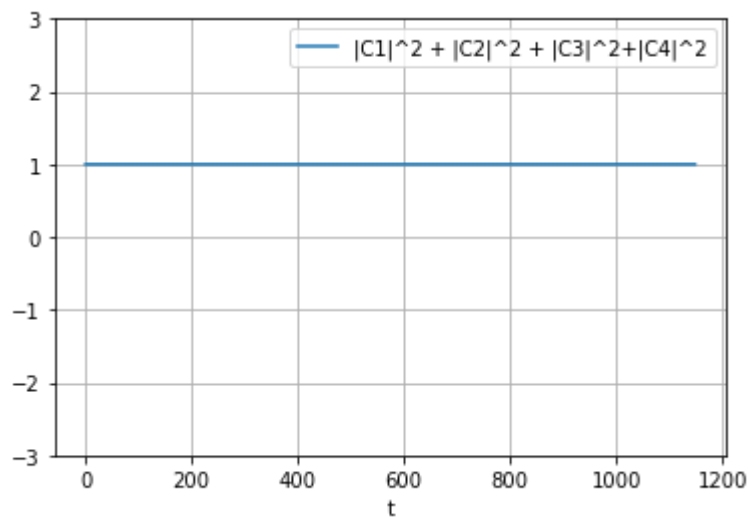
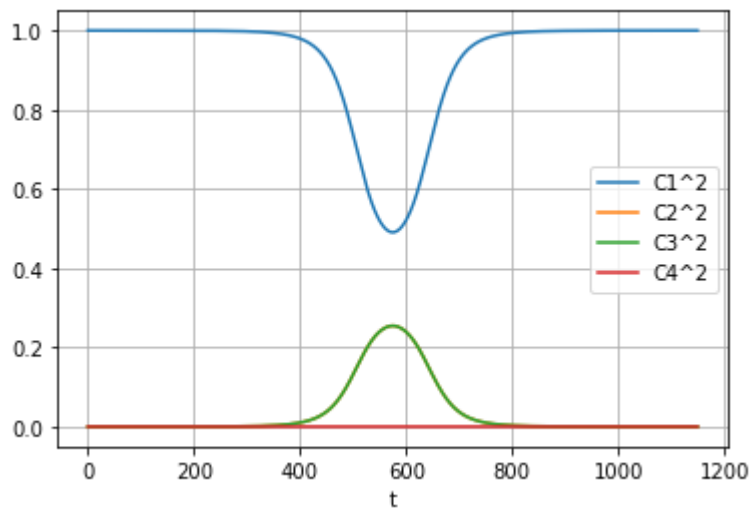
    #check if normalised
    norm=(np.abs(C_1))**2 +(np.abs(C_2))**2 +(np.abs(C_3))**2 +(np.abs(C_4))**2

    #modulus squared of each coefficient
    c1_squared= np.abs(C_1)**2
    c2_squared= np.abs(C_2)**2
    c3_squared= np.abs(C_3)**2
    c4_squared= np.abs(C_4)**2

```

```
import matplotlib.pyplot as plt
#plot coefficient modulus squared against time
plt.clf()
plt.plot(t, c1_squared, label='C1^2')
plt.plot(t, c2_squared, label='C2^2')
plt.plot(t, c3_squared, label='C3^2')
plt.plot(t, c4_squared, label='C4^2')
plt.xlabel('t')
plt.grid(True)
plt.legend(loc='best')
plt.show()
```

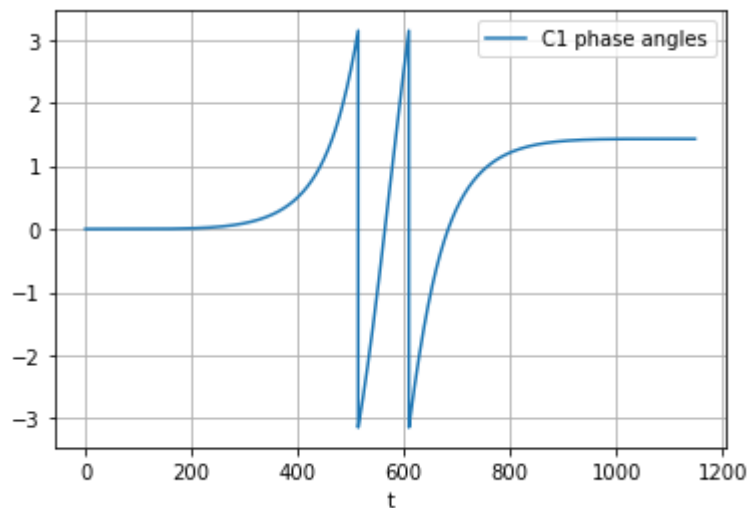
```
plt.clf()
plt.plot(t, norm, label='|C1|^2 + |C2|^2 + |C3|^2+|C4|^2')
plt.xlabel('t')
plt.grid(True)
plt.legend(loc='best')
plt.ylim(-3,3)
plt.show()
```



In [15]:

```
c1_angle=np.angle(C_1)

#plot phase angle against time
plt.clf()
plt.plot(t, c1_angle, label='C1 phase angles')
plt.xlabel('t')
plt.grid(True)
plt.legend(loc='best')
plt.show()
```





In [5]:

```

# now now V=0

if __name__ == "__main__":
    # Generate a solution to:
    #  $dC_1/dt = -i*(\omega)*(C_2+C_3)$ 
    #  $dC_2/dt = -i*((\omega)*(C_1+C_4)+\delta*(C_2))$ 
    #  $dC_3/dt = -i*((\omega)*(C_1+C_4)+\delta*(C_3))$ 
    #  $dC_4/dt = -i*((\omega)*(C_2+C_3)+(v+2*\delta)*(C_4))$ 

    # Define the right-hand-side of the differential equation.
    #ODE's needed to be solved
    def zfunc(z, t, omega_0, delta_0,v):
        C_1,C_2,C_3,C_4=z
        tau=114.85/omega_0
        omega=omega_0*np.sin((np.pi*t)/tau)**2 /2
        delta=delta_0*np.cos((np.pi*t)/tau)**2

        new_c1=-1j*(omega)*(C_2+0.0 * C_3)
        new_c2=-1j*((omega)*(C_1+ 0.0 * C_4)+delta*(C_2))
        new_c3=-1j*(0.0 * (omega)*( C_1+C_4)+delta*(C_3)*0.0) #setting c3 to zero
        new_c4= -1j*( 0.0 * (omega)*(C_2+C_3)+((0.0*v+2*delta)*(C_4))) #setting c4 to zero

        return [new_c1,new_c2,new_c3,new_c4]

    # Set up the inputs and call odeintz to solve the system.

    #coefficient of the system at the start.
    intial = np.array([1,0,0,0])
    #normalisation.
    intial_1=np.abs(intial)**2
    normalisation=np.sum(intial_1)
    #normalised coefficient.
    intial=(intial)/(np.sqrt(normalisation))

    #values of the parameters.
    omega_0 = 0.1
    delta_0 = 1.8
    tau=114.85/omega_0
    ###change V=0
    v=0
    t = np.linspace(0, tau, 50000)
    #solution
    z, infodict = odeintz(zfunc, intial, t, args=(omega_0,delta_0,v), full_output=True)

    #Each coefficient ode solved
    C_1= z[:,0]
    C_2= z[:,1]
    C_3= z[:,2]
    C_4= z[:,3]

    #check if normalised
    norm=(np.abs(C_1))**2 +(np.abs(C_2))**2 +(np.abs(C_3))**2 +(np.abs(C_4))**2

    #modulus squared of each coefficient
    c1_squared= np.abs(C_1)**2
    c2_squared= np.abs(C_2)**2
    c3_squared= np.abs(C_3)**2

```

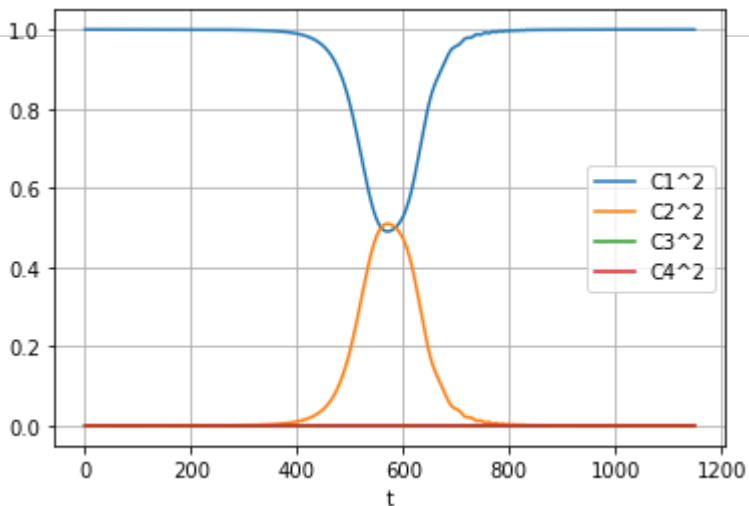
```
c4_squared= np.abs(C_4)**2
```

```
import matplotlib.pyplot as plt
#plot coefficient modulus squared against time
plt.clf()
plt.plot(t, c1_squared, label='C1^2')
plt.plot(t, c2_squared, label='C2^2')
plt.plot(t, c3_squared, label='C3^2')
plt.plot(t, c4_squared, label='C4^2')
plt.xlabel('t')
plt.grid(True)
plt.legend(loc='best')
plt.show()
```

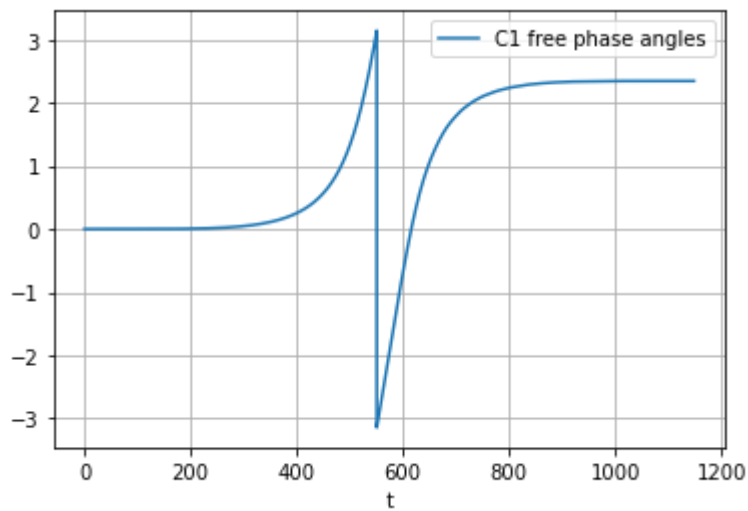
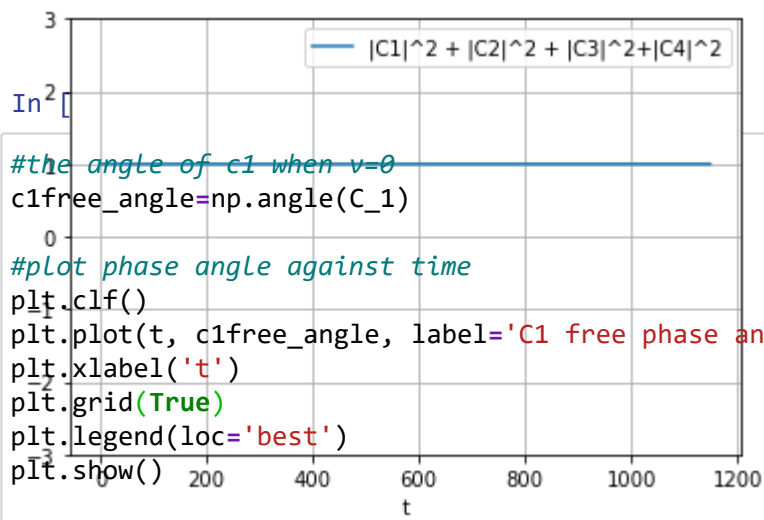
```
plt.clf()
plt.plot(t, norm, label='|C1|^2 + |C2|^2 + |C3|^2+|C4|^2')
plt.xlabel('t')
plt.grid(True)
plt.legend(loc='best')
plt.ylim(-3,3)
plt.show()
```

*#C2 and C3 produce the same line graph*  
*#C2=C3*

*#C3 and C4 have no influence in the ordinary differential equations*







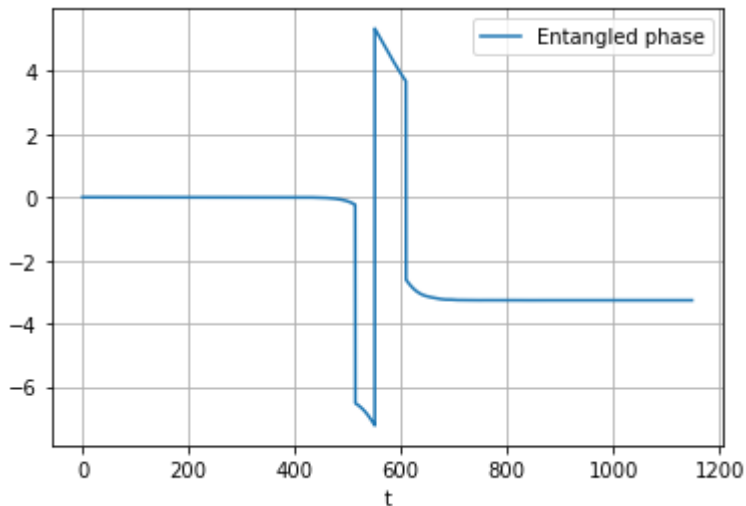
In [7]:

```

phi_1= c1free_angle # when v=0
phi_2=c1_angle # when v=4 and C3 and C4 are included

phi_ent= phi_2 - (2.0*phi_1) #the entangled phase
#plot coefficient modulus squared against time
plt.clf()
plt.plot(t, phi_ent, label='Entangled phase')
plt.xlabel('t')
plt.grid(True)
plt.legend(loc='best')
plt.show()

```



In [8]:

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print(phi_ent[-1]) # the value of the entangled phase at the end of the time
# the expected answer is -pi

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-3.2617186538859135

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In [11]:

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error=-np.pi - (phi_ent[-1])
print('error associated with the value',error)

```

```

error associated with the value 0.12012600029612042

```

In [ ]:

In [ ]: