Problem 1:

Consider a discrete-time causal, linear, time-invariant system with input $x[n]$, output $y[n]$, and impulse response $h[n]$, where,

$$h[n] \;=\; \left(\frac{1}{2}\right)^n u[n].$$

(a) Find the transfer function $H(z)$ for this system, and state the region of convergence.

(b) Plot the poles and zeros for this system.

(c) Find the Z transform $Y(z)$ of the output and state the region of convergence, given that

$$X(z) \;=\; \frac{1}{1-z^{-1}},\; |z| < 1.$$

Problem 2:

Consider a system described by the following difference equation

$$y[n] = x[n] - \frac{1}{2}\, y[n-1]$$

Suppose that the output of this system is

$$y[n] = \frac{1}{2}\left\{\left(\frac{1}{2}\right)^n + \left(\frac{-1}{2}\right)^n\right\} u[n]$$

(a) Sketch $y[n]$.

(b) Use Z-transform to find $Y(z)$, and determine the region of convergence.

(c) Find the input $x[n]$.

Problem 3:

Consider a DT LTI system described by the following non-recursive difference equation (moving average filter)

$$y[n] = \frac{1}{8}\Big\{x[n]+x[n-1]+x[n-2]+x[n-3]+x[n-4]+x[n-5]+x[n-6]+x[n-7]\Big\}$$

(a) Find the impulse response $h[n]$ for this filter.

(b) Find the transfer function $H(z)$ for this filter.

(c) Sketch the locations of poles and zeros in the complex z-plane.

Hint: To factor $H(z)$, use the geometric series and the fact that the roots of the polynomial $z^N - p_0 = 0$ are given by

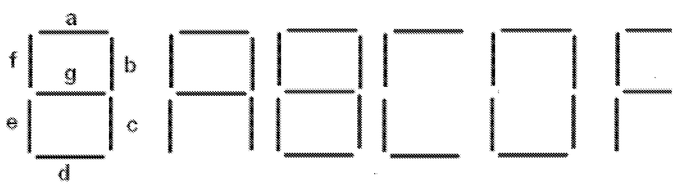
$$z_k = |p_0|^{1/n}\ e^{j[(\arg p_0)/N\ +2\pi k/N]},\ \ k = 0, \ldots .N-1$$

Problem 4:

Suppose that you want to design a grading circuit that allows three teachers to examine one student. Each teacher will ask 4 questions. For each correct answer, the teacher presses a button to increase the grade by one. Therefore, each teacher will give a grade between 0 & 4. The grades of the three teachers are added to get the final grade, N. The grade N is converted into a grade letter and displayed on a 7-segment. A grade is assigned one of five letters A, B, C, D, or F as follows

- If $0 \le N \le 4$, then the LED displays F
- If $N = 5$, then the LED displays D
- If $6 \le N \le 7$, then the LED displays C
- If $8 \le N \le 10$, then the LED displays B
- If $11 \le N \le 12$, then the LED displays A

The 7-segment LED and the five letters look as follows:



(a) Draw a block diagram of the circuit.

(b) Write down the truth table of the 7-segment that converts the grades into letters.

(c) For the output **c** of the 7-segment, find a minimal **product-of-sums** expression.

(d) For the output **d** of the 7-segment, find a minimal **sum-of-products** expression.

Problem 5:

Suppose that you want to design a grading circuit that allows two teachers to examine one student. Each teacher will ask 4 questions. For each correct answer, the teacher presses a button to increase the grade by one. Thus, each teacher will give a grade between 0 & 4 to the student. The grades of the two teachers are added and the result is displayed on a 7-segment.

(a) Draw a block diagram of the grading circuit.

(b) Design a logic circuit with one output R such that:

If the summation of the two grades is ≤ 4, then $R = 0$

If the summation of the two grades is ≥ 5, then $R = 1$
