COMPARISON BASED SORTING ALGORITHMS

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Data Structures used:

- ## Insertion sort:

  Insertion sort uses only the input array that is passed as a parameter. No other data structures are used.

- ## Merge sort:

  Similar to Insertion sort, merge sort uses the input array passed as a parameter. Additionally, the merging operation uses two arrays for the left and right subtree. I also use the pop(0) operation on both these arrays which is similar to removing the first element of a queue.

- ## Heap sort:

  Heap sort uses arrays in several places. However, some of the operations performed on such arrays are similar to the operations of different data structures. For instance, I use the append operation on an array in the function heap_insert which is similar to adding an element at the end of the list Similarly, I use the pop operation in the function heap_remove which is similar to removing the last element of a stack.

- ## In-place Quicksort and Modified Quicksort:

  In-place quicksort and modified quicksort use only array as its data structure. No special operations typical of other data structures are used here.

Complexity Analysis:

- ## Insertion sort:

  Since there are two loops in this algorithm, we are iterating twice over each element. Hence the worst-case time complexity is $O(n^2)$

- ## Merge sort:

  Dividing one single array into two has a time complexity of O(log n). Merging the arrays has a time complexity of O(n). Thus, the worst-case time complexity of this algorithm is O(n log n).

- ## Heap sort:

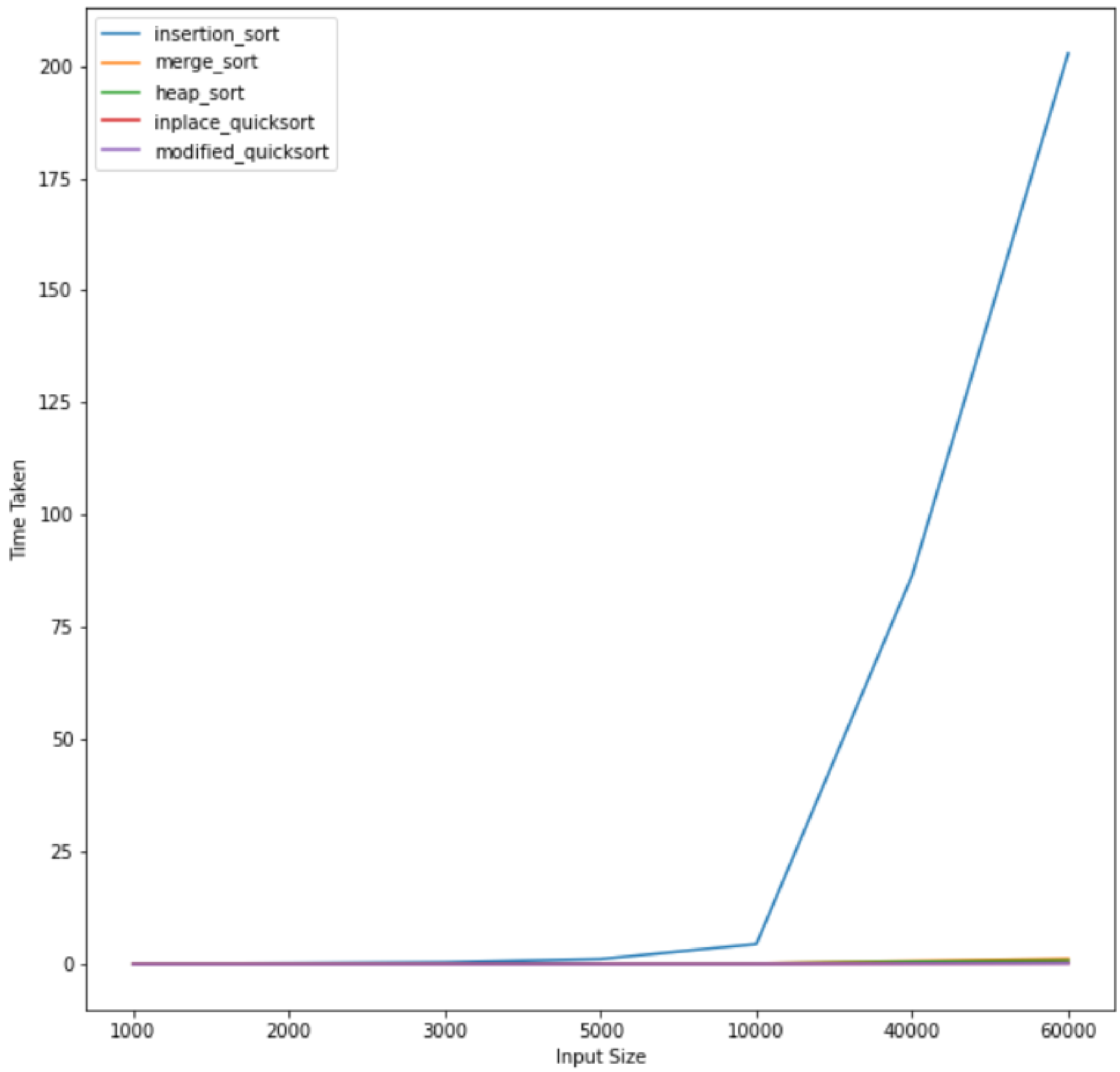
  Inserting an item into a heap and the resulting heapify operation has a time complexity of O(log n). Since we are inserting and removing n items from the heap, the worst-case time complexity of heap sort is O(n log n).

- ## In-place Quicksort and Modified Quicksort:

  Both, in-place quicksort and modified quicksort have a worst-case time complexity of $O(n^2)$. However, their complexity would be O(n log n) in the average case.

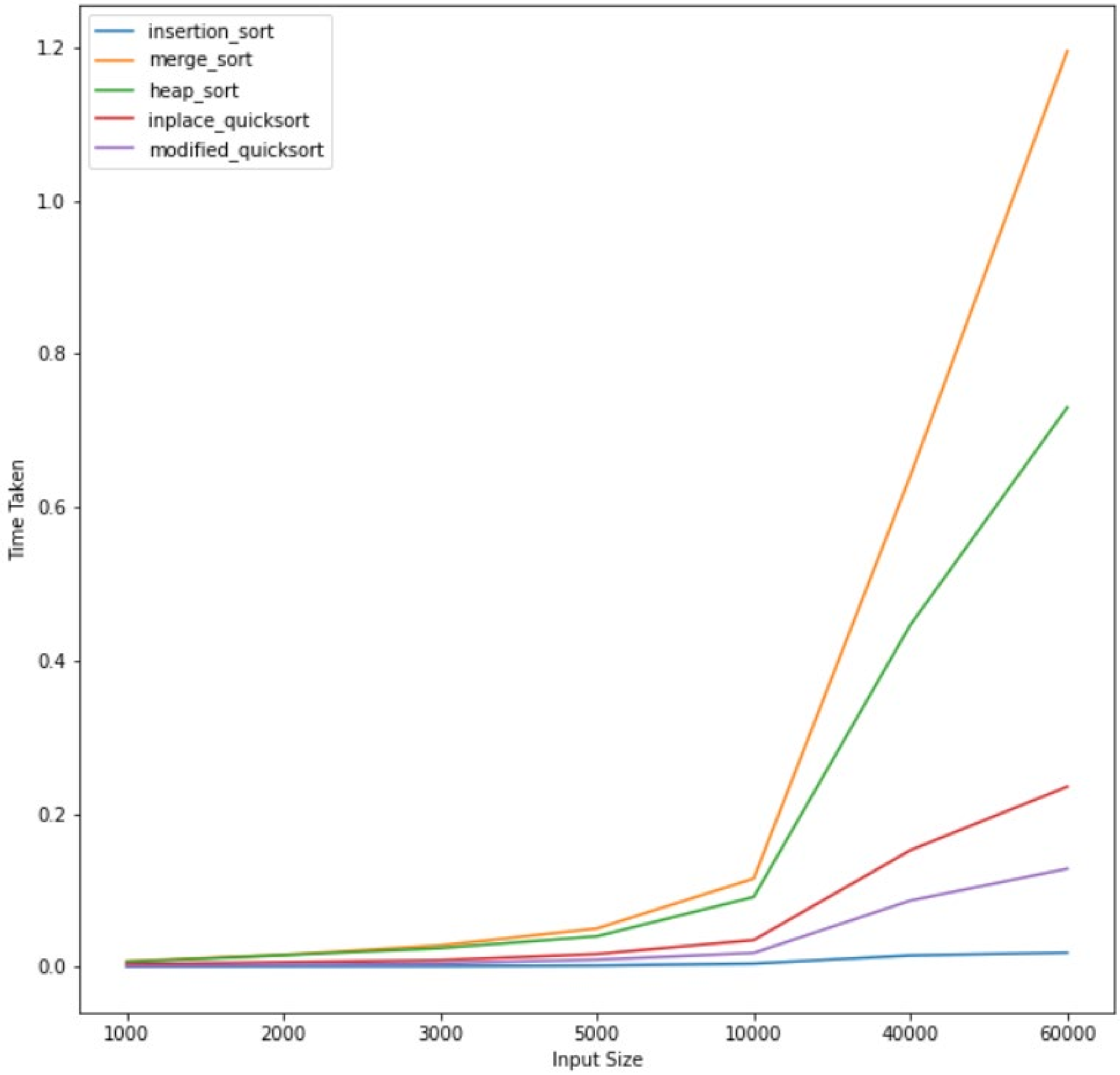
Results:

- ## Unsorted Array:



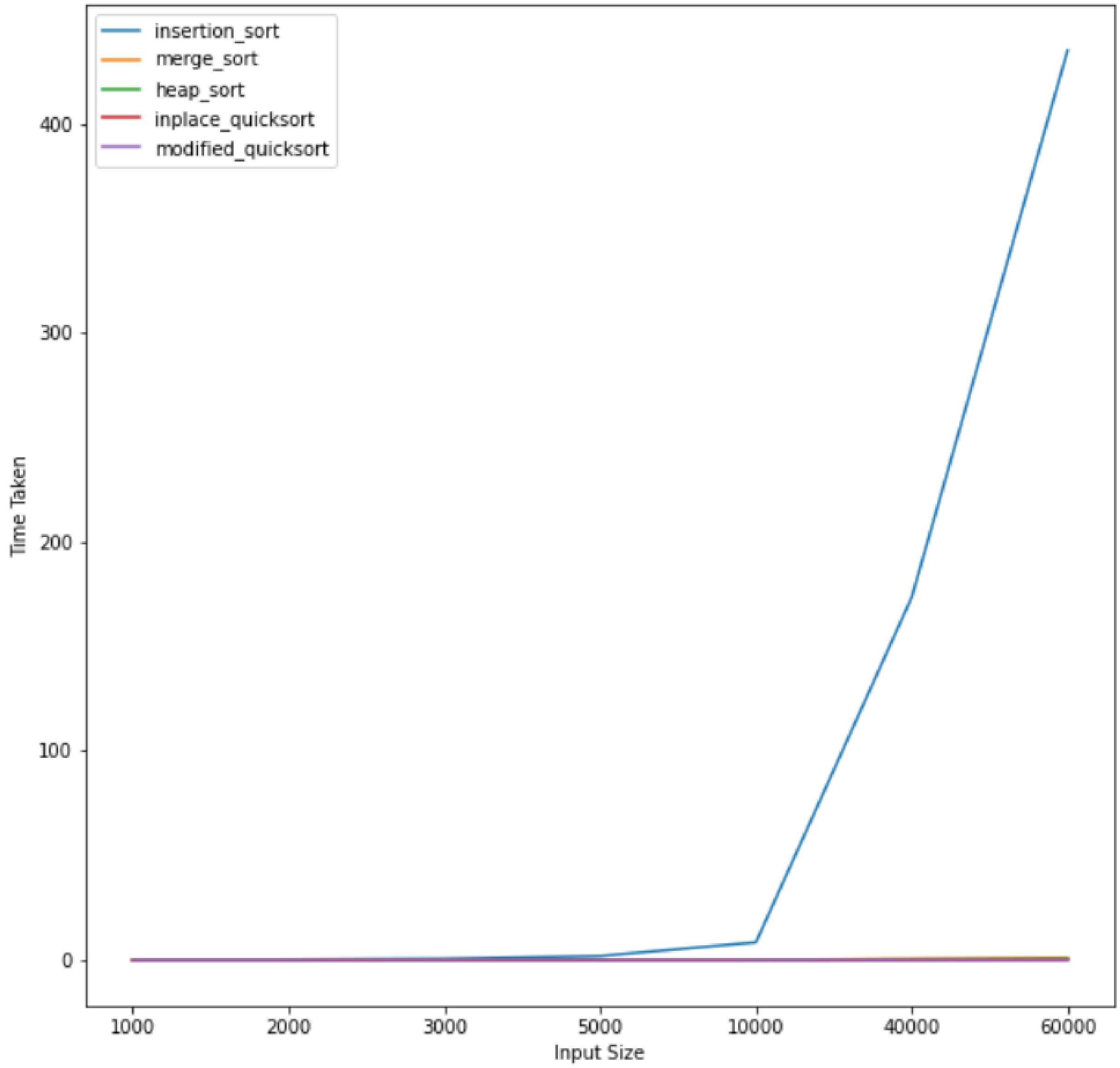
For unsorted arrays, Insertion sort performs worse than the other algorithms as the input size increases. This is due to its complexity being $O(n^2)$. Although the worst-case complexity of in-place and modified quicksort is $O(n^2)$, their average complexity is O(n log n) which enables them to have a lower execution time than insertion sort. In fact, their execution time is similar to that of merge sort and heap sort – both of which have a worst-case time complexity of O(n log n).

- Sorted Array:



The result shown for unsorted array above is reversed when the array is sorted. In this case, insertion sort performs better than the other algorithms on higher input sizes. Similarly, merge sort performs significantly worse than the other algorithms on higher input sizes.

- Reverse Sorted Array:



Similar to unsorted array, insertion sort performs significantly worse than the other algorithms. However, here insertion sort takes a much longer time to finish sorting. For instance, it takes insertion sort about 220 seconds to finish sorting an unsorted array of size 60,000. But insertion sort takes about 450 seconds to finish sorting a reversely sorted array of the same size.

Code:

```
import random, time
import matplotlib.pyplot as plt

def insertion_sort(arr, left=0, right=None):
    if right is None: right= len(arr)-1
    for i in range(left+1, right+1):
        key = arr[i]
        j = i-1
        while j >= 0 and key < arr[j]:
            arr[j+1] = arr[j]
            j -= 1
        arr[j+1] = key
    return arr

def merge_sort(arr):
    if len(arr) > 1:
        mid = len(arr)//2
        left_sublist = arr[:mid]
        right_sublist = arr[mid:]
        merge_sort(left_sublist)
        merge_sort(right_sublist)

        i = 0
        while len(left_sublist) > 0 or len(right_sublist) > 0:
            if len(left_sublist) == 0: arr[i] = right_sublist.pop(0)
            elif len(right_sublist) == 0: arr[i] = left_sublist.pop(0)
            else:
                if left_sublist[0] < right_sublist[0]: arr[i] = left_sublist.pop(0)
                else: arr[i] = right_sublist.pop(0)
            i += 1
    return arr
```

```python
def heap_insert(arr, item):
    arr.append(item)
    i = len(arr) - 1
    while i > 1 and arr[i//2] > arr[i]:
        arr[i//2], arr[i] = arr[i], arr[i//2]
        i = i//2

def heap_remove(arr):
    if len(arr) <= 2: return arr.pop()
    temp = arr[1]
    arr[1] = arr.pop()
    i = 1
    while i < len(arr)-1:
        if 2*i+1 <= len(arr)-1:
            if arr[i] <= arr[2*i] and arr[i] <= arr[2*i+1]: return temp
            else:
                j = 2*i if arr[2*i] < arr[2*i+1] else 2*i+1
                arr[i], arr[j] = arr[j], arr[i]
                i = j
        else:
            if 2*i <= len(arr)-1:
                if arr[i] > arr[2*i]: arr[i], arr[2*i] = arr[2*i], arr[i]
            return temp
    return temp

def heap_sort(arr):
    heap_arr = [None]
    sorted_arr = []
    for item in arr: heap_insert(heap_arr, item)
    for _ in range(len(arr)): sorted_arr.append(heap_remove(heap_arr))
    return sorted_arr
```

```python
def inplace_quicksort(arr, left=0, right=None):
    if right is None: right = len(arr)-1
    if left >= right: return arr
    i = random.randint(left, right)
    x = arr[i]
    arr[i], arr[right] = arr[right], arr[i]

    l = left
    r = right-1
    while True:
        while arr[l] < x: l += 1
        while r > left and x < arr[r]: r -= 1
        if l < r:
            arr[l], arr[r] = arr[r], arr[l]
            l += 1
            r -= 1
        else:
            arr[l], arr[right] = arr[right], arr[l]
            break

    inplace_quicksort(arr, left, l-1)
    inplace_quicksort(arr, l+1, right)
    return arr
```

```python
def modified_quicksort(arr, l=0, r=None):
    if r is None: r = len(arr)-1
    if l+10 < r:
        i = (l+r)//2
        if arr[l] > arr[i]: arr[l], arr[i] = arr[i], arr[l]
        if arr[i] > arr[r]: arr[r], arr[i] = arr[i], arr[r]
        if arr[l] > arr[i]: arr[l], arr[i] = arr[i], arr[l]

        x = arr[i]
        arr[i], arr[r-1] = arr[r-1], arr[i]
        j = l+1
        k = r-2
        while True:
            while j <= r and arr[j] < x: j += 1
            while k > l and x < arr[k]: k -= 1
            if j < k:
                arr[j], arr[k] = arr[k], arr[j]
                j += 1
                k -= 1
            else: break

        arr[r-1], arr[j] = arr[j], arr[r-1]
        modified_quicksort(arr, l, j-1)
        modified_quicksort(arr, j+1, r)
        return arr

    else: return insertion_sort(arr, l, r)
```

```python
def generate_input(n, sorted, reverse=False):
    '''
    Function to generate input array
    :param n: size of array
            sorted: whether array needs to be sorted in as
    :return: array of size n
    '''
    arr = [random.randint(0, 1000) for _ in range(n)]
    if sorted: arr = modified_quicksort(arr)
    return arr[::-1] if reverse else arr

class execution_time():
    def start(self):
        self.program_start = time.time()

    def end(self):
        return round(time.time() - self.program_start, 5)

def time_taken(arr, fun):
    time = execution_time()
    test_arr = arr.copy()
    time.start()
    fun(test_arr)
    return time.end()
```

```python
def generate_plots(sorted, reverse=False):
    methods = [insertion_sort, merge_sort, heap_sort, inplace_quicksort, modified_quicksort]

    input_sizes = [1000, 2000, 3000, 5000, 10000, 40000, 60000]
    input_sizes_str = [str(x) for x in input_sizes]
    input_dict = {}
    for input_size in input_sizes:
        input_dict[input_size] = generate_input(input_size, sorted, reverse)

    for i in range(len(methods)):
        time = []
        for key in input_dict.keys(): time.append(time_taken(input_dict[key], methods[i]))
        plt.plot(input_sizes_str, time, label= methods[i].__name__)

    plt.xlabel('Input Size')
    plt.ylabel('Time Taken')
    plt.legend()
    plt.show()
```

```python
# Generating plots for instruction 1
generate_plots(False)
```

```python
#%%

# Generating plots when input array is already sorted
generate_plots(sorted=True)
```

```python
#%%

# Generating plots when input array is reverse sorted
generate_plots(sorted=True, reverse=True)
```