1. by definition of the sample mean

$$m(x) = \frac{1}{N}\sum_{i=1}^{N} x_i$$

$$m(a + bx) = \frac{1}{N}\sum_{i=1}^{N}(a + bx_i)$$

$$= \frac{1}{N}\left(\sum_{i=1}^{N} a + \sum_{i=1}^{N} bx_i\right)$$

$$= \frac{1}{N}\left(Na + b\sum_{i=1}^{N} x_i\right)$$

$$= \frac{Na}{N} + \frac{b}{N}\sum_{i=1}^{N} x_i)$$

$$= a + bm(x)$$

$$m(a+bx) = a + bm(x)$$

2. by definition of sample covariance

$$\operatorname{cov}(x,y) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(y_i - m(Y))$$

$$\operatorname{cov}(X, a + bY) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))((a+by_i) - m(a+bY))$$

by definition,

$$m(a+bY) = a + bm(Y)$$

so

$$\operatorname{cov}(X, a + bY) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))((a+by_i) - (a + bm(x))).$$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(by_i - bm(Y))$$

$$= \frac{1}{N} \sum_{i=1}^{N} (x_i - m(x))\, b\, (y_i - m(y))$$

$$= b\, \frac{1}{N} \sum_{i=1}^{N} (x_i - m(x))(y_i - m(y))$$

$$\text{cov}(x, a+by) = b \times \text{cov}(x, y)$$

3.

$$\text{cov}(x,x) = \frac{1}{N} \sum_{i=1}^{N} (x_i - m(x))(x_i - m(x))$$

$$= \frac{1}{N} \sum_{i=1}^{N} (x_i - m(x))^2$$

so $\text{cov}(x,x) = s^2$ by given definition

$y = a + bx$, so $y_i = a + bx_i$

covariance with y with itself

$$\text{cov}(a+bx, a+bx) = \frac{1}{N} \sum_{i=1}^{N} [a + bx_i - m(a+bx)]^2$$

we know $m(a+bx) = a + b\,m(x)$

so

$$= \frac{1}{N} \sum_{i=1}^{N} [b(x_i - m(x))]^2$$

$$\text{cov}(a+bx, a+bx) = \frac{1}{N} \sum_{i=1}^{N} b^2 (x_i - m(x))^2$$

$$= b^2 \frac{1}{N} \sum_{i=1}^{N} (x_i - m(x))^2$$

step 2 says $\text{cov}(x,x) = s^2$

$$= b^2\, \text{cov}(x,x)$$

$$\text{cov}(a+bx, a+bx) = b^2 s^2$$