1.

$$SE = \sum_{i=1}^{N} (y_i - b_0 - b_1 z_{i1} - b_2 z_{i2})^2$$

2.

$$\frac{\partial SSE}{\partial SSt} = -2 \sum_{i=1}^{N} (y_i - b_0 - b_1 z_{i1} - b_2 z_{i2})^2$$

$$\frac{\partial SSE}{\partial b_1} = -2 \sum_{i=1}^{N} z_{i1} (y_i - b_0 - b_1 z_{i1} - b_2 z_{i2})^2$$

$$\frac{\partial SSE}{\partial b_2} = -2 \sum_{i=1}^{N} z_{i2} (y_i - b_0 - b_1 z_{i1} - b_2 z_{i2})^2$$

3. Verify average error is 0, and $e \cdot z = 0$ at the optimum

$$\frac{\partial SSE}{\partial b_0} = \sum_{i=1}^{N} -2 (y_i - b_0 - b_1 z_{i1} - b_2 z_{i2}) = 0$$

divide by $-2$

$$\sum_{i=1}^{N} (y_i - b_0 - b_1 z_{i1} - b_2 z_{i2}) = 0$$

$$\sum_{i=1}^{N} (y_i - \hat{y}_i) = 0$$

$$\sum_{i=1}^{N} e_i = 0$$

so therefore

$$\frac{1}{N} \sum_{i=1}^{N} e_i = \bar{e} = 0$$

average error is 0 at the optimum

$$\frac{\partial SSE}{\partial b_1} = \sum_{i=1}^{N} -2z_{i1}(y_1 - b_0 - b_1 z_{i1} - b_2 z_{i2}) = 0$$

$$\sum_{i=1}^{N} z_{i1}(y_1 - b_0 - b_1 z_{i1} - b_2 z_{i2}) = 0$$

$$\sum_{i=1}^{N} z_{i1}(y_i - \hat{y}_1) = 0$$

$$\sum_{i=1}^{N} e_i \cdot z_{i1} = 0$$

therefore $e \cdot z_1 = 0$

$$\frac{\partial SSE}{\partial b_2} = \sum_{i=1}^{N} -2z_{i2}(y_1 - b_0 - b_1 z_{i1} - b_2 z_{i2}) = 0$$

$$\sum_{i=1}^{N} z_{i2}(y_1 - b_0 - b_1 z_{i1} - b_2 z_{i2}) = 0$$

$$\sum_{i=1}^{N} z_{i2}(y_i - \hat{y}_i) = 0$$

$$\sum_{i=1}^{N} e_i \cdot z_{i2} = 0$$

therefore $e \cdot z_2 = 0$

so at the optimum, we verified that average error is zero

$$\bar{e} = 0$$

4. First order condition respect to $b_0$

$$-2\sum_{i=1}^{N}(y_1 - b_0 - b_1 z_{i1} - b_2 z_{i2}) = 0$$

rearrange

$$\sum_{i=1}^{N} y_i - Nb_0 - b_1 \sum_{i=1}^{N} z_{i1} - b_2 \sum_{i=1}^{N} z_{i2} = 0$$

predictors are centered:

$$\sum z_{i1} = 0 \quad \text{and} \quad \sum z_{i2} = 0$$

So, $$\sum_{i=1}^{N} y_i - Nb_0 = 0$$

$$Nb_0 = \sum_{i=1}^{N} y_i \Rightarrow b^*_0 = \bar{y}$$

Substitute:

respect to $b'$

$$\sum_{i=1}^{N} z_{i1}(y_i - b_0^* - b_1^* z_{i1} - b_2^* z_{i2}) = 0$$

$b^*_0 = \bar{y}$

so, $$\sum_{i=1}^{N} z_{i1}(y_i - \bar{y} - b_1^* z_{i1} - b_2^* z_{i2}) = 0$$

after distrubuting and seperating sums

$$\sum_{i=1}^{N} y_i z_{i1} - \underbrace{\bar{y} \sum_{i=1}^{N} z_{i1}}_{= \text{ to } 0} - b^*_1 \sum_{i=1}^{N} (z_{i1})^2 - b_2^* \sum_{i=1}^{N} z_{i1} z_{i2} = 0$$

$$\boxed{\sum_{i=1}^{N} y_i z_{i1} = b^*_1 \sum_{i=1}^{N} (z_{i1})^2 + b^*_2 \sum_{i=1}^{N} z_{i1} z_{i2}}$$

and for respect to $b_2$

$$\sum_{i=1}^{N} z_{i2}(b_0 - b^*_0 - b^*_1 z_{i1} - b^*_2 z_{i2}) = 0$$

Substitute:

$$\sum_{i=1}^{N} z_{i2}\left(b_0 - \bar{y} - b_1^* z_{i1} - b_2^* z_{i2}\right) = 0$$

after distribution and seperating sums

$$\sum_{i=1}^{N} y_i z_{i2} - b_1^* \sum_{i=1}^{N} z_{i1} z_{i2} - b_2^* \sum_{i=1}^{N} (z_{i2})^2 = 0$$

$$= \boxed{\sum_{i=1}^{N} y_i z_{i2} = b_1^* \sum_{i=1}^{N} z_{i1} z_{i2} + b_2^* \sum_{i=1}^{N} (z_{i2})^2}$$

5.

$$\underbrace{\begin{bmatrix} \sum (z_{i1})^2 & \sum z_{i1} z_{i2} \\ \sum z_{i1} z_{i2} & \sum (z_{i2})^2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}}_{B} = \underbrace{\begin{bmatrix} \sum z_{i1} y_i \\ \sum z_{i2} y \end{bmatrix}}_{C}$$