Numerical Optimization, Fall 2020
Course Project Evaluation Criteria

(Due date: Jan. 3, 2021, before 23:59 (CST))

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1 Criteria for Project Evaluation

As a part of an evaluation of SI152, you should first read the "*Numerical Optimization: Final Project*" material posted in Blackboard. Your project should consist of two parts: one final writeup and the source code (only submit the writeup if you select the third topic). Your writeup should be written in LaTeX and only the hard copy in PDF should be submitted. For the PDF file, both single-column and double-column formats are acceptable in both Chinese and English, but **at least two pages**. According to the selections of different topics, we specify the criteria for the project evaluation as follows.

- **Choose to implement the linear solver.** For both the implementations of a primal simplex method and an interior point method, it is your responsibility to ensure your MATLAB/Python source codes should be
  - **Normative**. Well-organized and necessary code comments.
  - **Executable**. Well-tested and bug-free.
  - **Reused**. Solution information during iteration are capable of being displayed throughout your codes.

  You are provided 3 test instance in §2 to debug your codes. When testing the codes you submit, you should make sure that your codes run with as few input parameters (note: $\boldsymbol{A}$, $\boldsymbol{c}$ and $\boldsymbol{b}$ are necessary) as possible. Please don't make the function interface so complicated that it adds extra workload to the TAs.

- **Choose to discuss the constraint qualification.** You are expected to submit a final writeup that summarizes your findings (e.g., what results presented in this paper?), ideas (e.g., what are your thoughts on this issue?) and discussions (e.g., what have you learned from this paper and what extensions can you make in this field?). Specifically,

  (i) The basic requirement of your project is to complete the survey of constraint qualifications based on [1]. If this is done, you can get an eligible score for your project.

  (ii) A further requirement for the project is to discuss how the qualification conditions would become when considering a general convex-set constrained problem. Alternatively, you could analyze the difficulties of the constraint qualification for the general convex-set constrained problem. If one of these can be done properly, you can get full marks for your project.

It would be awesome if you could complete the analysis of the constraint qualifications for the general convex-set case. A sound and solid analysis will be contributive to be organized into a (nearly-)publishable piece of work.

(iii) If (ii) was too hard to achieve, at least try to read some other papers that cite [1] and figure out how to apply the results developed in [1].

- **Choose to develop an algorithm for the assignment task.** Your writeup should clearly describe your methods for addressing this problem. Additionally, the numerical testing section is emphasized, of which you are expected to demonstrate the efficiency and reliability of your methods. In particular,

  (i) The basic requirement of your project is to propose a mathematical model that can successfully solve the test data we provide. If this is done, you can get an eligible score for your project.

  (ii) If your proposed model's implementation performs equally well (or even better) compared with the existing model, you can get full marks for your project. However, if the implementation of your model does not work well, you can, for example, try to analyze the difficulties of this assignment task or put forward some viewpoints on how to improve your model performance. If this is done, we also give points at our discretion.

2 Test Examples

There are a total of 8 instances for the LP problems: 3 instances (given) for your debug, while the remaining 5 instances are used for testing your project. Please make sure your program could well handle the following (including but not limited to):

- Read the input data directly (the test examples are provided in the attached ".csv" files). The input parameters $\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c}$ as well as the optimal solution $\boldsymbol{x}^*$ are given for each test case.
- The following information should be outputted when finishing the computation:
  - The optimal objective value.
  - The total running time.
  - Numbers of iterations of simplex and interior point methods (**point out the termination criterion**):
    (i) The number of pivots for simplex method.
    (ii) Total number of iterations for interior point method.

The Linear Programming (LP) has been transformed into the standard form, i.e.,

$$\begin{aligned} \min_{\boldsymbol{x}} \quad & \boldsymbol{c}^T \boldsymbol{x} \\ \text{s.t.} \quad & \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \\ & \boldsymbol{x} \geq \boldsymbol{0}. \end{aligned} \tag{1}$$

Example 1:

$$\boldsymbol{A} = \begin{bmatrix} 6 & 1 & -2 & -1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 6 & 4 & -2 & 0 & 0 & -1 \end{bmatrix}, \boldsymbol{c} = \begin{bmatrix} 5 \\ 2 \\ -4 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} 5 \\ 4 \\ 10 \end{bmatrix}. \tag{2}$$

The optimal decision variable is $\boldsymbol{x}^* = [5/3, 4/3, 1]^T$ and the optimal objective takes $z^* = 3$.

Example 2:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 20 & 1 & 0 & 0 & 1 & 0 \\ 200 & 20 & 1 & 0 & 0 & 1 \end{bmatrix}, \boldsymbol{c} = \begin{bmatrix} -100 \\ -10 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \boldsymbol{b} = \begin{bmatrix} 1 \\ 100 \\ 10000 \end{bmatrix}. \tag{3}$$

The optimal decision variable is $\boldsymbol{x}^* = [0, 0, 10000]^T$ and the optimal objective takes $z^* = -10000$.

References

[1] R. Andreani, J. M. MartÍnez, A. Ramos, and P. J. Silva, "A cone-continuity constraint qualification and algorithmic consequences," *SIAM Journal on Optimization*, vol. 26, no. 1, pp. 96–110, 2016.