a) Union of 2 rectangles

Ans. $VC(R_1) = d_1$
$VC(R_2) = d_2$

Th: $\Pi H_i(2d+c) \le \sum_{k=0}^{d} \binom{2d+c}{k}$ —— (1)

where $d = \max(d_1, d_2)$ & $c > 1$

$$\sum_{k=0}^{d} \binom{2d+c}{k} = 1 + \sum_{k=1}^{d} \binom{2d+c}{k}$$

$$= 1 + \sum_{k=1}^{d} \binom{2d+c-1}{k} + \sum_{k=1}^{d} \binom{2d+c-1}{k-1}$$

Using the formula, $\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$

$$= \sum_{k=1}^{d} \binom{2d+c-1}{k} + \sum_{i=0}^{d} \binom{2d+c-1}{i}$$

$$= \sum_{k=1}^{d} \binom{2d+c-1}{k} + \sum_{i=d+c-1}^{2d+c-1} \binom{2d+c-1}{i}$$

$$\le \sum_{k=1}^{d} \binom{2d+c-1}{k} + \sum_{i=d+1}^{2d+c-1} \binom{2d+c-1}{i}$$

$$= -1 + 2^{2d+c-1}$$

$vc(R_1) \cup vc(R_2)$ is bounded by

$d_1 + d_2 + 1 = 4 + 4 + 1 = 5$.

Thus,

$VC(R_1) \cup V.C(R_2) \leq 5$.

Problem 6

b). V.C dimension for the class of hypothesis of circle.

Ans For 3 non-linear points, we can easily draw a circle that includes
- None of the points
- Any one point
- Any two of the points.
- All of the points.

Thus, 3 non-colinear points could be shattered by the class of circle.

However, consider the set of four points, we observe that they cannot be shattered. We can show this by showing contradiction examples.

- Consider 4 collinear points + - + - (in straight line). It is impossible to shatter them with circle.
- consider convex hull of 4 points in a quadrilateral
- $(x_1, x_2)$ be the points separated by long diagonal & $(y_1, y_2)$ points separated by short diagonal
- $(x_1, x_2)$ + $(y_1, y_2)$ − OR $(x_1, x_2)$ − $(y_1, y_2)$ + would be impossible to shatter with circle.

consider a circle $C_1$ for first labelling and circle $C_2$ for other labelling.

$(x_1, x_2)+ \quad (y_1, y_2)-$
$(x_1, x_2)- \quad (y_1, y_2)+$

Then, symmetric difference of the circles under consideration would be:
$((C_1 \setminus C_2) \cup (C_2 \setminus C_1))$
These would consist of 4 disjoint regions which is not possible.

Thus V.C dimension of circle is 3

c) Triangle:

Ans. Consider 7 points on a circle, one edge of the triangle can be used to cut of each block. There can be maximum of 3 contiguous blocks of -ve (when arranged in circular way). Thus, they can be separated using 3 corners of the triangle. Hence 7 points can be shattered.

Now, if we consider 8 points on circle $x_1, x_2, \ldots x_8$. A triangle can intersect a circle at most at 6 points. Hence it is impossible to have a triangle containing exactly $x_1, x_3, x_5, x_7$ in exterior and $x_2, x_4, x_6, x_8$ in its interior.

Thus V.C. dimention of triangle is 7.

d) multidimensional sphere

Ans A sphere in $m$ space given by
$f(x) = \text{sign}[(x-c)(x-c) - b]$

Consider $m+1$ points consisting of the unit vectors and the origin can be shattered by spheres.

Assume a subset $P$ of $m+1$ points. The centre $b$ of the sphere is the sum of the vectors in $P$.

Unit vector in $P$, distance to centre: $\sqrt{|P|-1}$
Unit vector outside $P$, distance to centre: $\sqrt{|P|+1}$

$\sqrt{|P|}$ = distance of origin to centre.

Thus, if we choose the radius sothe points in $P$ are the sphere, then $m+1$ is the V.C. dimension for sphere in $m$ dimension.