Q1 B)

$$\frac{\iint_{\text{Surface of Earth}} f(x,y,z)\, dS}{\text{Surface Area of Earth}}$$

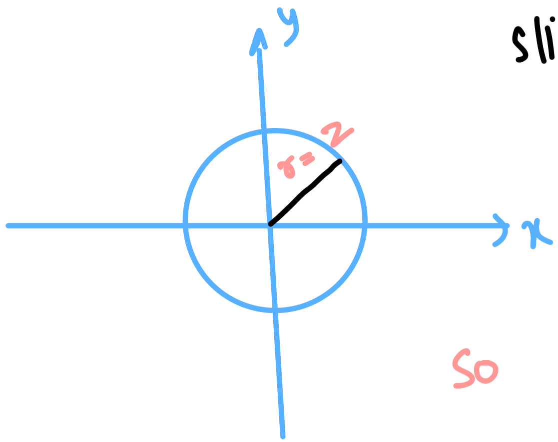
will give the global average surface temperature anomaly. Since $\iint f(x,y,z)\,dS$ is the sum of the total temperature anomalies over the surface. So dividing it by the total surface area gives us the average value.

Q2 a) $\iiint f(x, y, z)\, dV = \iiint (10-z)\, dV$

This integral can be set up in cylindrical coordinates.

we can take slices with z as the outer integral from 2 to 4, and

This is how the slice would look like.



So r is from 0 to 2

and θ from 0 to 2π

dv is $r\,dr\,d\theta\,dz$

$$\int_{2}^{4}\int_{0}^{2\pi}\int_{0}^{2} (10 \cdot z)\, r\,dr\,d\theta\,dz$$

$$= \frac{380\pi}{3}$$

$$= 397.94$$

b) $\iint g(x,y,z)\,dS$

we will use a scalar surface integral since the scalar point is used on the surface

$$\iint g(\vec{r}(u,v))\,|\vec{r}_u \times \vec{r}_v|\,du\,dv$$

the surface can be described by $z = \sqrt{x^2+y^2}$

and $2 \le z \le 4$

$x = u$, $y = v$

$z = \sqrt{u^2+v^2}$

so $\vec{r}(u,v) = \langle u, v, \sqrt{u^2+v^2} \rangle$

so $g(x,y,z) = 6 - z = 6 - \sqrt{u^2+v^2}$

$$\vec{r}_u = \left\langle 1, 0, \frac{u}{\sqrt{u^2+v^2}} \right\rangle$$

$$\vec{r}_v = \left\langle 0, 1, \frac{v}{\sqrt{u^2+v^2}} \right\rangle$$

$$\vec{r}_u \times \vec{r}_v = \left\langle \frac{-u}{\sqrt{u^2+v^2}}, \frac{-v}{\sqrt{u^2+v^2}}, 1 \right\rangle$$

$$|\vec{r}_u \times \vec{r}_v| = \sqrt{\frac{u^2}{u^2+v^2} + \frac{v^2}{u^2+v^2} + 1^2}$$

$$= \sqrt{2}$$

$$= \iint 6 - \sqrt{u^2+v^2}\,(\sqrt{2})\, du\, dv$$

(think of du dv as dx dy)

du dv is nothing but dA and can be written as $r\,dr\,d\theta$ to be converted to polar

$u = r\cos\theta$ $v = r\sin\theta$

r goes from 2 to 4 and θ goes from 0 to 2π so the integral now is

$$= \sqrt{2}\int_0^{2\pi}\int_2^{4} (6-r)\, r\, dr\, d\theta$$

$$= \frac{104\sqrt{2}\,\pi}{3}$$

This is for the cone

Since density is constant for the top and bottom circles:

For the top circle we have

Area = $\pi(4)^2 = 16\pi$

Density = $6-2 = 6-4 = 2$

Mass = 32π

For bottom circle we have

Area = $\pi(2)^2 = 4\pi$

Density = $6-2 = 6-2 = 4$

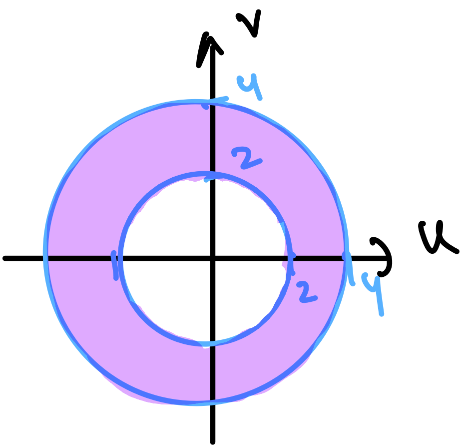
Mass of paint $= 16\pi$

So total mass is

$$\frac{104\sqrt{2}\pi}{3} + 32\pi + 16\pi = 304.816g$$

$$2^2 \leq u^2 + v^2 \leq 4^2$$

So parameter domain looks like:



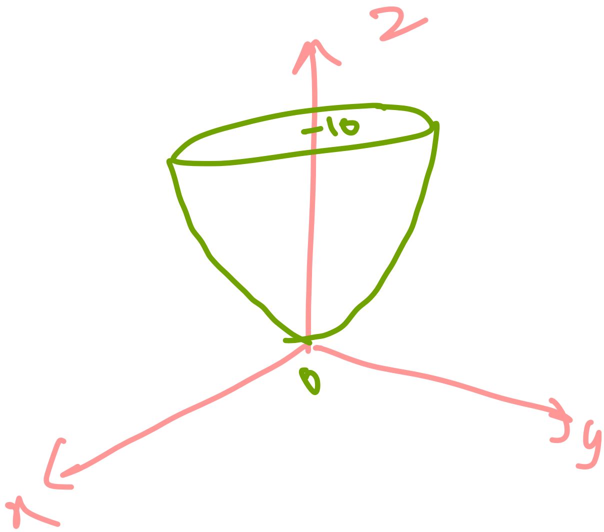
$z = \frac{x^2}{4} + \frac{y^2}{3}$ is an elliptical cone

if it were $\frac{x^2}{4} + \frac{y^3}{3} = k$ then elliptical cylinder

$z = \sqrt{x^2 + y^2}$ is a cone

$k = \sqrt{x^2 + y^2}$ is a cylinder

Q3 $z = x^2+y^2$ $z < 10$ is a paraboloid



between A and B

A, the average value is greater

for

5 as the fn goes from 0 to 10

than

from the bottom to the top, and

since the is more surface area in

the upper half ($z>5$) it makes

the average vale of the function greater than 5.

for B_1 the average value of the fn is less than 5 as the fn goes from 10 to 0 from the base to the top, and size the surface area is more in the upper half $(z>5)$ it makes the average vale of the fn less than 5.

So $A > B$

integral in A and B will be > 0 since

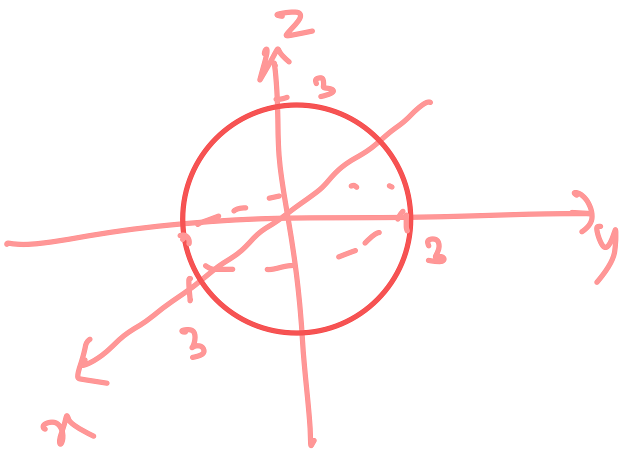
$\iint f(x,y,z)\, dS$ = (average vale)(surface area) for C we see when

$z = 10$

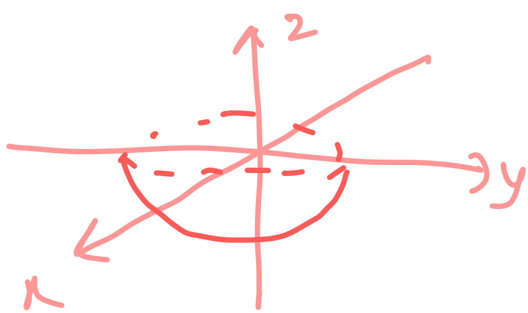
$$\frac{x^2}{40} + \frac{y^2}{30} = 2$$

projection at $z = 10$

for D



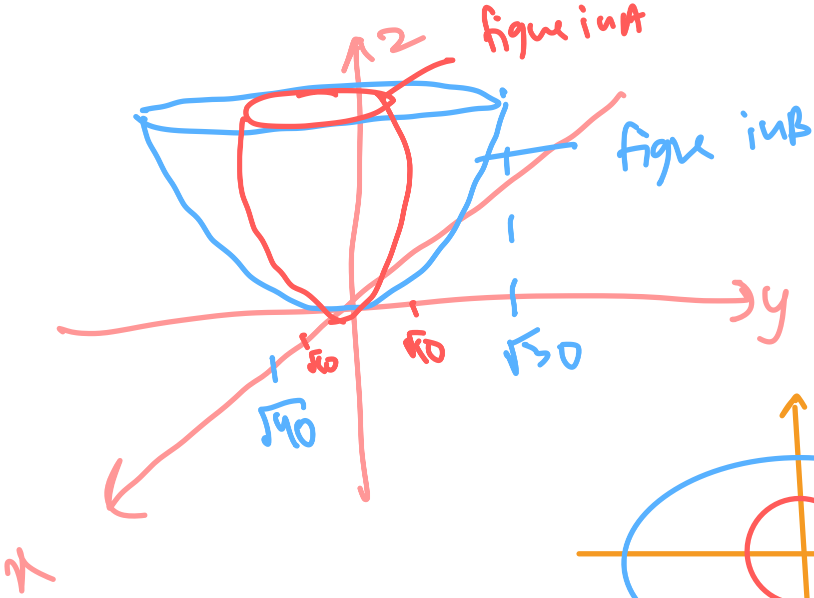
and E

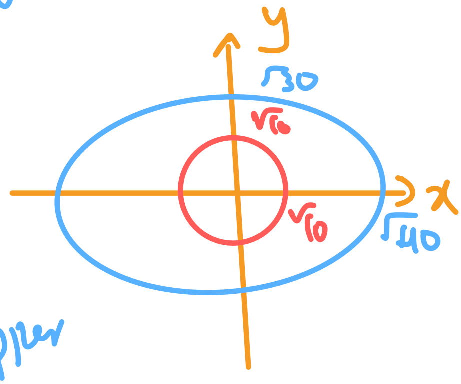


the surface is symmetrical about the yz plane leading to an equal number of negative and positive x values which cancel each other out.

$D = E = 0$

for C and A both the functions are the same so value of the integral depend on the surface area.





for C $\Rightarrow$ upper slice

Surface Area of C is greater than the Surface Area of A so C > A

So overall we get:

$E = D < B < A < C$

Q4 We did not have to set up scalar surface integral because

1. The figure is derived from a cone whose z value has been restricted, so for the upper end we get a bottom face circle whose area is easy to calculate.

2. The density depends on just z and since the z values are constant for

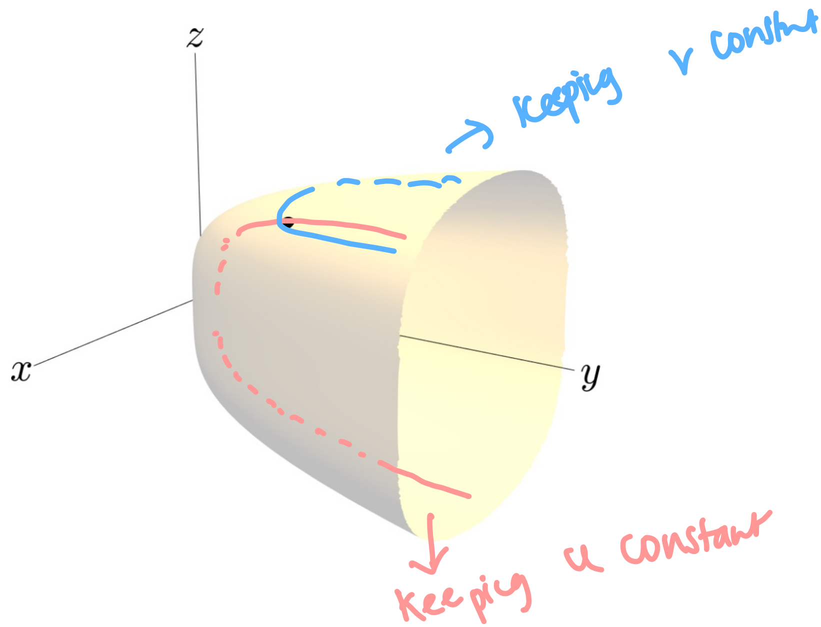
the top and bottom face the density is constant for the top and bottom circular faces and can just be calculated by multiplying density and area to get the amount of paint.

Q5 a) let $x = u$, $z = v$

we get $y = u^2 + \frac{v^4}{4}$

so $\vec{r}(u,v) = \langle u, u^2 + \frac{v^4}{4}, v \rangle$

b)



Keeping u constant means taking slices along x

keeping v constant means taking slices along z

c) To define a plane we need a point on the plane and two vectors parallel to the plane.

We already have a point $(1, 2, \sqrt{2})$ to find the vectors, we can find the tangent to both the curves (fixed u and v values) to find the vectors parallel to the plane. So this is $\vec{r}_u$ and $\vec{r}_v$.

Thus we can use

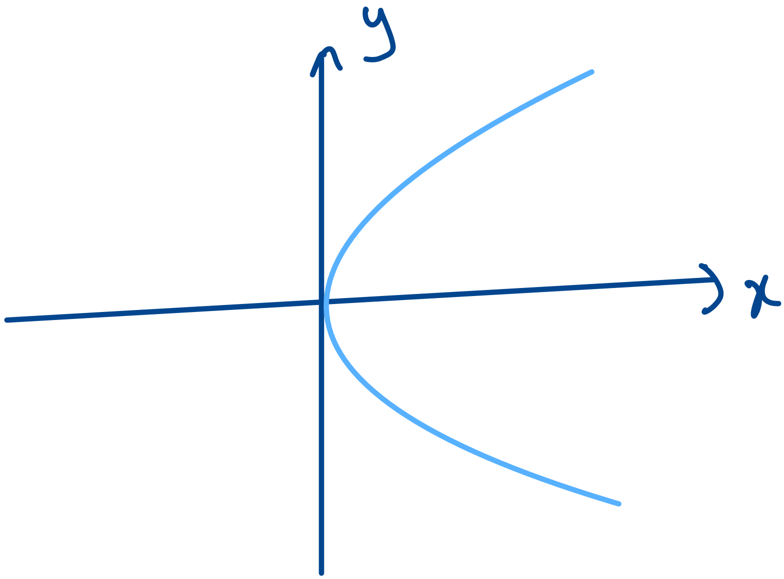
$$\langle x, y, z\rangle = \langle 1, 2, \sqrt{2}\rangle + t\vec{r}_u + s\vec{r}_v$$

To find the implicit equation of the plane we can find the cross product of $\vec{r}_u$ and $\vec{r}_v$ to find a vector $\perp$ to the plane and use the point on the plane.

Q1 $x = t^2$ $y = t$ $z = 1 - t^2$

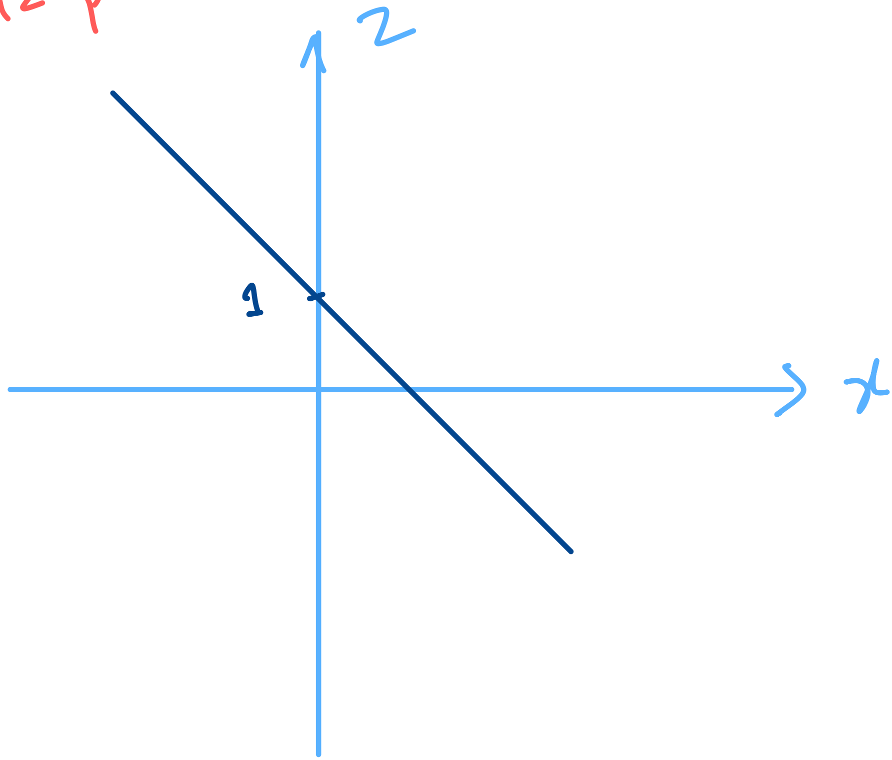
a) i) for the xy

$x = y^2$



ii) $x = t^2 \quad z = 1 - x$

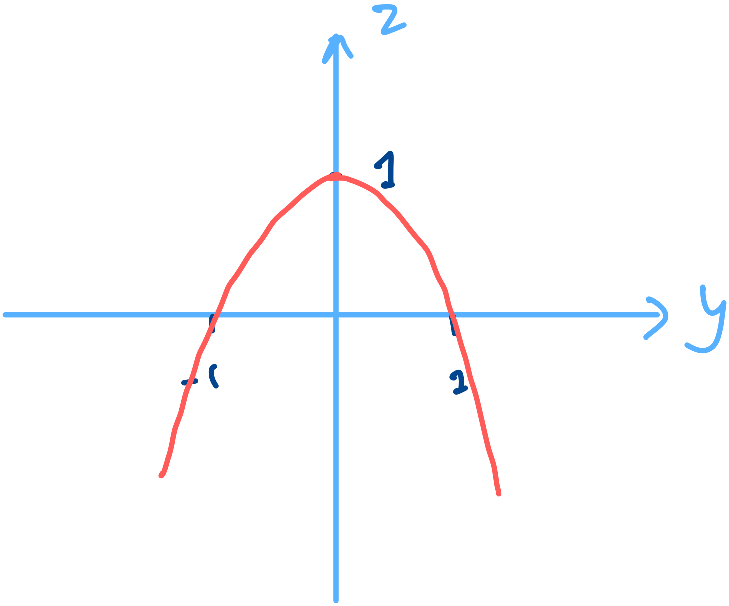
for xz plane



iii) for yz plane

$y = t \quad z = 1 - t^2$

$z = 1 - y^2$



b) $z = 1 - x$ and $x = y^2$

we get this from

the intersection of two

surfaces : xz plane and xy plane.