Problem Set 2 - Dimension, Introduction to Cylindrical Coordinates

You should be able to:

- Identify the dimension of an object. State and use the rule of thumb relating the dimension of an object to the number of equations defining it.
- Explain the difference between the *graph* and *level sets* of a function.
- Write and interpret equations for spheres in $\mathbb{R}^3$.
- Describe the geometric meaning of r and θ in polar and cylindrical coordinates.

In Math 21a, you're welcome to use GeoGebra on your homework unless we specifically ask you to do a problem without using technology. Even then, you're welcome to check your answer using GeoGebra; you just need to explain how you can solve the problem without it. We encourage you to get familiar with GeoGebra; it's a great tool that will help you throughout the semester.

1. In class, we talked about the idea of dimension; please read the "Dimension" handout, which fleshes this idea out.

   (a) Imagine that you're tutoring a student, and the student says to you,

   > "I don't understand why a sphere is considered to be 2-dimensional. After all, a sphere isn't flat. Shouldn't it be 3-dimensional?"

   How would you explain to the student why a sphere is 2-dimensional?[(1)]

   (b) In each part, sketch the object described, and say what its dimension is.

   i. The part of $x^2 + y^2 + z^2 = 3$ below $z = 1$.

   ii. The part of $z = 1$ inside $x^2 + y^2 + z^2 = 3$.

   iii. The intersection of $x^2 + y^2 + z^2 = 3$ and $z = 1$.

   (c) One of the three objects in #1(b) is 1-dimensional; in other words, it's a curve. What's the length of this curve? (We don't know how to calculate length in general, but this curve is a very familiar shape.)

   (d) Two of the three objects in #1(b) are 2-dimensional; in other words, they're surfaces. Although we don't know how to find surface area in general yet, you should be able to find the surface area of one of these two surfaces (because it's a simple geometric shape). Do this.

2. (a) Sketch the shape in $\mathbb{R}^2$ defined by $x^2 + y^2 = 9$.

   (b) In $\mathbb{R}^3$, what shape do you think the equation $x^2 + y^2 = 9$ describes? Write down what you think; you'll get full credit even if it isn't right.

[(1)] Research has found that people learn material better when they teach it (as opposed to simply studying it). In fact, just preparing to teach the material has a beneficial effect on learning (even if you never actually teach it to someone). So, even when we don't explicitly ask you to imagine teaching the material to someone, it's worthwhile to give it a try!

(c) Does your guess in #2(b) agree with the rules of thumb in the "Dimension" handout? Why or why not?

(d) Use GeoGebra to see what $x^2 + y^2 = 9$ looks like, and sketch the result.

(e) Imagine you're tutoring a student, and the student says to you,

> "I don't understand why the shape of $x^2 + y^2 = 9$ in $\mathbb{R}^3$ looks like this."

How would you explain this to the student?

(f) What shape does $y^2 + z^2 = 4$ describe in $\mathbb{R}^3$? Sketch it without using technology, but then feel free to check your answer using GeoGebra. (Remember that the way GeoGebra orients its axes can be confusing; see Problem Set 1, #2(d) for a reminder of how to deal with this.)

3. In class, you talked about ways to visualize functions of 3 variables through their level sets.

(a) Which picture below shows a level set of $f(x, y, z) = 4x^2 + y^2$? (Notice that f is a function of 3 variables!) Explain briefly.

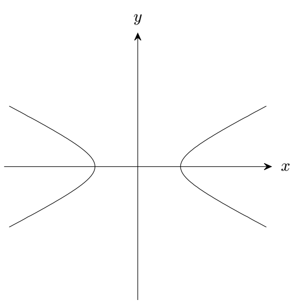
(b) Which picture shows the graph of $g(x, y) = 4x^2 + y^2$? (Notice that g is a function of 2 variables!) Explain briefly.

(c) Which picture shows a level set of $g(x, y) = 4x^2 + y^2$? Explain briefly.

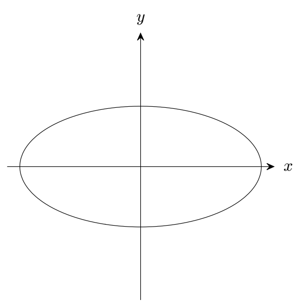
(d) Explain in complete sentences why #3(a) and #3(c) are different.

Choices:

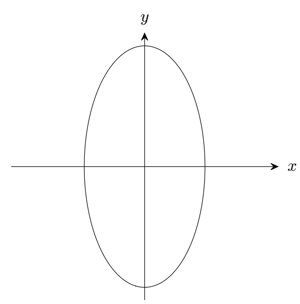
(A)



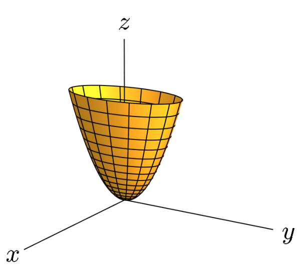
(B)



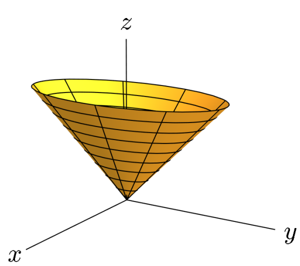
(C)



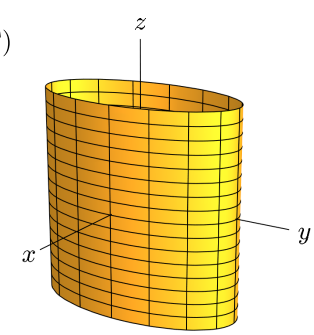
(D)



(E)



(F)



4. Go to `https://www.geogebra.org/m/ejwq7f7f`; you should see a solid. The two curved sides of the solid are parts of cylinders centered around the z-axis.

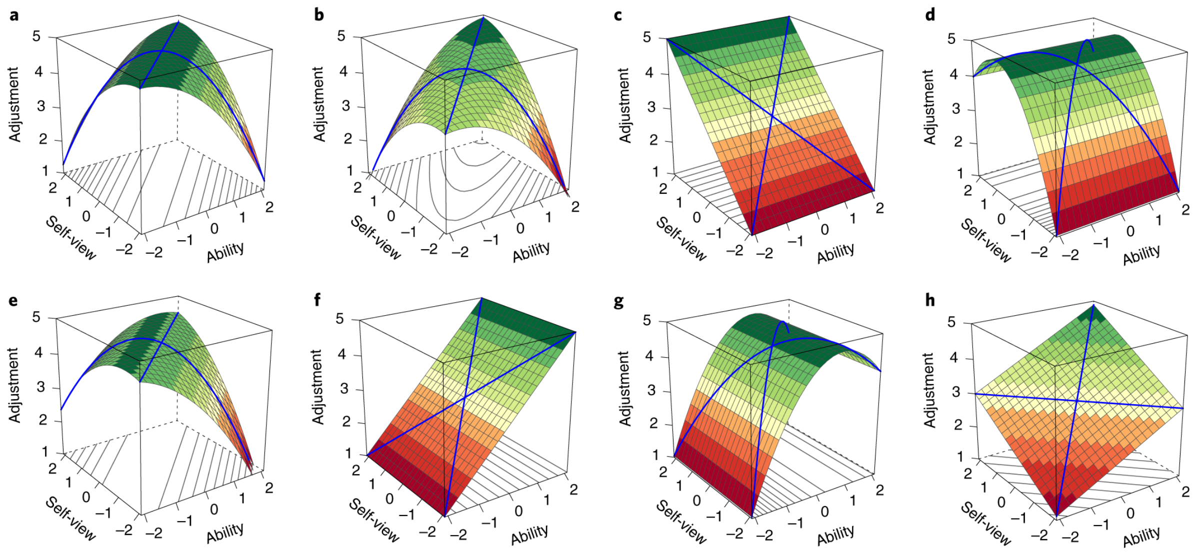
Fill in the blanks in the statement below; no explanation is necessary.

The solid shown can be described in cylindrical coordinates by

$$\_\_\_\_ \le r \le \_\_\_\_, \quad \_\_\_\_ \le \theta \le \_\_\_\_, \quad \_\_\_\_ \le z \le \_\_\_\_.$$

5. *In 21a, we're used to drawing our coordinate axes in a particular orientation, but not all sources use the same orientation. This problem is an example of that.*

   The psychology study tested different hypotheses on how people's psychological "adjustment" (how satisfied they are with their life, career, and relationships) depends on their abilities and "self-view" (how they assess their own abilities). Here's Figure 1 from the paper:

   This figure shows graphical representations of 8 hypotheses the authors considered; look carefully at how the axes are numbered.

   (a) What are the black lines / curves at the bottom of each box? What do the colors on each graph represent?

   (b) One hypothesis the authors considered was that adjustment depends only on self-view, not on actual ability, and higher self-view corresponds to higher adjustment. Which one of the graphs above (a - h) represents this hypothesis?

   (c) Another hypothesis the authors considered was that adjustment depends only on how closely people's abilities and self-view match, and that "individuals are optimally adjusted when their self-views and abilities match." (In this hypothesis, people's actual abilities make no difference.) Which one of the graphs above represents this hypothesis?

6. At the end of the week, we'll start talking about integrals of functions of more than 1 variable. To prepare for this, it's important to really understand single-variable integrals! So, please watch this video, and write a summary of the key points. Also include any questions you have about what the video covers.

For next class, read OpenStax *Calculus Volume 3* - §2.7.