Moments of the value of single payments and annuities with iid returns

4.1 Let i_t be the rate of return over the peiord $(t-1,\ t)$ and suppose $\{i_t :\ t = 1, \ldots 15\}$ is a series of independent random variables all distributed $U\,(0.06, 0.12)$. Let A_{15} reperesent the accumulation, at time 15, of 1 invested at time 0.

(a) Calculate the mean and standard deviation of A_{15}.

(b) Making such theoretical assumptions as you feel might be appropriate, calculate the probability that $A_{15} > 4$.

4.2 Make the same assumptions in relation to $\{i_t :\ t = 1, \ldots 15\}$ as the previous exercise and let $\ddot{s}_{15}$ represent the accumulation, at time 15, of 1 invested at times 0, 1, ...and 14. Calculate the mean and standard deviation of $\ddot{s}_{15}$. (Hint: use spreadsheet)

4.3 Show by induction that $\left(\sum_{j=1}^{n} V_j\right)^2 = \sum_{j=1}^{n} V_j^2 + 2\sum_{j=2}^{n}\sum_{i=1}^{j-1} V_i V_j.$

4.4 Consider a sequence of n payments of $1, (1+s), (1+s)^2, \ldots (1+s)^{n-1}$ at unit intervals with the first payment due immediately where s is a scalar constant.

Let i_t be a random variable denoting the return over the time period $(t-1, t)$ where $\{i_t : t = 1, \ldots n\}$ is a set of independent and identically distributed random variables.

Justifying your steps, paying particular attention to independence, derive recursive formulae for calculating the first two moments of

(i) $\ddot{z}_n$, the accumulation of this cash-flow at time n and

(ii) (hard) $\ddot{u}_n$ the present value of this cash-flow.