



UNIT-II

DIFFERENTIAL CALCULUS

TUTORIAL SHEET - 1

1. If $(-1, -\sqrt{3})$ are Cartesian coordinates of a point in plane, the corresponding polar coordinates are _____
Ans: $(2, 4\pi/3)$
2. If $(\sqrt{2}, 5\pi/4)$ are the polar coordinates of a point in plane, the corresponding Cartesian Coordinates are _____
Ans: $(-1, -1)$
3. The circle $x^2 + y^2 - 2ax = 0$ in polar form is _____
Ans: $(r = 2a \cos(\theta))$
4. The polar equation $\theta - k = 0$, geometrically represents _____
Ans: (straight lines)
5. If two polar curves C_1 and C_2 are orthogonal, then value of $\cot(\varphi_1) \cot(\varphi_2) =$ _____ Ans: -1
6. Find the angle of intersection between the polar curves
 $r = \frac{k\theta}{1+\theta}$ and $r = \frac{k}{1+\theta^2}$ Ans: $\tan^{-1}(3)$
7. Show that the angle made by the tangent and the normal at any point $P(r, \theta)$ on the curve Lemniscate $r^2 = a^2 \cos(2\theta)$ with the initial line is '30'.



LINEAR ALGEBRA, CALCULUS & NUMERICAL METHODS

8. Show that the tangents to the cardioid $r = a(1 + \cos\theta)$ at $\theta = \pi/3$ and $\theta = 2\pi/3$ are respectively parallel and perpendicular to the initial line.
9. Show that the circle $r = b$ intersects the curve $r^2 = a^2 \cos(2\theta) + b^2$, at an angle given by $\tan^{-1}\left(\frac{a^2}{b^2}\right)$
10. Find the angle of intersection between the curves $r = a(1 + \sin\theta)$ and $r = a(1 - \sin\theta)$: Ans: $\pi/2$



LINEAR ALGEBRA, CALCULUS & NUMERICAL METHODS

TUTORIAL SHEET - 2

1. The curvature of a circle $s = a\psi$ at any point is _____

Ans: ($\kappa = 1/a$)

2. The radius of curvature for straight line $y = mx + c$ is _____

Ans: ($\rho = \infty$, not defined)

3. The curvature of the curve $y = e^x$ at the point where it crosses the y-axis is _____

Ans: ($\kappa = \frac{1}{2^{3/2}}$)

4. The Taylor series expansion of $\log(x)$ about $x = 1$ up to second degree term is _____

Ans: $\log(x) = (x - 1) - \frac{(x-1)^2}{2} + \dots \infty$

5. The Maclaurin series expansion of $\cos(x)$ is _____

Ans: $\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots \infty$

6. Show that the radius of curvature of the Folium $x^3 + y^3 = 3axy$ at the point $(3a/2, 3a/2)$ is given by $-\frac{3a}{8\sqrt{2}}$.

7. Find the radius of curvature of the curve $y^2 = \frac{4a^2(2a-x)}{x}$ where the curve meets the x-axis.

8. For the curve $y = \frac{ax}{a+x}$, show that $\left(\frac{2\rho}{a}\right)^{\frac{2}{3}} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$

9. Find the radius of curvature of the $x = a \log(\sec t + \tan t)$,
 $y = a \sec t$.

Ans: $\rho = a \sec^2 t$

10. Show that the curvature of the tractrix $x = a[\cos t + \log \tan(\frac{t}{2})]$,
 $y = a \sin t$ at any point is given by $\kappa = \frac{\tan t}{a}$



LINEAR ALGEBRA, CALCULUS & NUMERICAL METHODS

11. Find the coordinates of the centre of curvature at $(at^2, 2at)$ on the parabola $y^2 = 4ax$.

Ans: $((\bar{x}, \bar{y}) = ((2 + 3t)at^2, -4\sqrt{2}at^{3/2})$

12. Find the circle of curvature at the point $(a/4, a/4)$ for the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$.

Ans: $\left(x - \frac{3a}{4}\right) + \left(y + \frac{3a}{4}\right) = \frac{a^2}{2}$

13. Find the radius of curvature of the curve $r^n = a^n \cos(n\theta)$

Ans: $\frac{a^n r^{1-n}}{n+1}$

14. Show that the radius of curvature at any point (r, θ) on the

Cardioid $r = a(1 - \cos \theta)$ varies as \sqrt{r}

15. Find the radius of curvature for the parabola $\frac{2a}{r} = 1 - \cos \theta$ at any point (r, θ)

Ans: $2\sqrt{\frac{r^3}{a}}$



TUTORIAL SHEET -3

1. Match the following:

i)	The angle between radius vector and tangent for the polar curve at any point $P(r, \theta)$ is	a)	$\rho \propto y^2$
ii)	The angle between radius vector and tangent for the Cartesian curve at any point $P(x, y)$ is	b)	$\rho \propto \frac{1}{y^2}$
iii)	The radius of curvature at any point $P(x, y)$ on the catenary $y = c \cdot \cosh\left(\frac{x}{c}\right)$ is	c)	$\cot(\phi) = \frac{1}{r} \cdot \frac{dr}{d\theta}$
		d)	$\tan(\phi) = r \cdot \frac{dr}{d\theta}$
		e)	$\tan(\phi) = \frac{xy' - y}{x + yy'}$
		h)	$\tan(\phi) = \frac{xy' + y}{x - yy'}$

Ans: (i) - (c) (ii) - (e) (iii) - (a)

2. Find the Taylor series expansion of the function $y = \log(\cos x)$ about the point $x = \pi/3$.

Ans: $\log(\cos x) = -\log 2 - \sqrt{3} \left(x - \frac{\pi}{3}\right) - 2 \left(x - \frac{\pi}{3}\right)^2 - \frac{4}{\sqrt{3}} \left(x - \frac{\pi}{3}\right)^3 - \frac{10}{\sqrt{3}} \left(x - \frac{\pi}{3}\right)^4 - \dots$

3. Obtain the expansion of the function $e^{\sin(x)}$ in ascending powers of 'x' up to terms containing x^4 .

Ans: $e^{\sin(x)} = 1 + x + \frac{x^2}{2} - \frac{x^4}{8} \dots$



LINEAR ALGEBRA, CALCULUS & NUMERICAL METHODS

4. Obtain the Maclaurin series expansion for the function $f(x) = \tan^{-1}(x)$ and hence deduce that $\pi = 4 \left[1 - \frac{1}{3} + \frac{1}{5} - + \dots \right]$

Ans: $\tan^{-1}(x) = \left[x - \frac{x^3}{3} + \frac{x^5}{5} - \dots \right]$

5. Using Maclaurin's series, prove that

$$\sqrt{1 + \sin(2x)} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \dots$$

6. Show that $\left(\frac{x}{\sin x} \right) = 1 + \frac{x^2}{6} + \frac{7x^4}{360} + \dots$