R V COLLEGE OF ENGINEERING

(An autonomous institution affiliated to VTU, Belgaum)

DEPARTMENT OF MATHEMATICS

FUNDAMENTALS OF LINEAR ALGEBRA, CALCULUS & NUMERICAL METHODS (MAT211TA)

UNIT-1: ELEMENTARY LINEAR ALGEBRA

TUTORIAL SHEET-1

I. Objective type questions:

1. If A is a 3×4 matrix then rank of A cannot exceed _____.
2. Rank of the matrix $\begin{bmatrix}1&0&0&0\\0&0&0&1\\0&0&0&0\\0&0&1&0\end{bmatrix}$ is _____.
3. Rank of identity matrix of order 4 is _____ .
4. If the rank of the transpose matrix A is 3 then the rank of matrix A is _____.
5. Rank of singular matrix of order 5 is_____.

II. Find the rank of the following matrices

1) $A=\begin{bmatrix}1&2&3&4\\-2&-3&1&2\\-3&-4&5&8\\1&3&10&14\end{bmatrix}$ **Answer: rank of A=2**

2) $A=\begin{bmatrix}1&1&-1&3\\2&-2&6&8\\3&5&-7&3\end{bmatrix}$ **Answer: rank of A=2**

3) $A=\begin{bmatrix}1&0&2&-2\\2&-1&0&-1\\2&0&4&-2\\12&-3&9&-3\end{bmatrix}$ **Answer: rank of A=4**

4) Find the values of k such that the rank of the matrix A is 3, where

$A=\begin{bmatrix}1&2&-1&3\\4&1&2&1\\3&-1&1&2\\1&2&0&k\end{bmatrix}$ **Answer: k=1**

5) For which value of b the rank of the matrix

$A=\begin{bmatrix}1&5&4\\0&3&2\\b&13&10\end{bmatrix}$ is 2 **b=2.**

6) Find the rank of A, B, A+B, BA and AB if

$A=\begin{bmatrix}1&1&-1\\2&-3&4\\3&-2&3\end{bmatrix}$ and $B=\begin{bmatrix}-1&-2&-1\\6&12&6\\5&10&5\end{bmatrix}$

Answer: rank of A=2, rank of B=1, rank of (A+B)=2, rank of (AB)=0, rank of (BA)=1.

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UNIT-1: ELEMENTARY LINEAR ALGEBRA
TUTORIAL SHEET-2

1. Test the consistency of the following system of equations
   2x+6y=-11
   6x+20y-6z=-3
   6y-18z=-1
   Answer: Inconsistent

2. Test the consistency of the following system and solve if the system is consistent
   $x_1 + 2x_2 + x_3 = 2$
   $3x_1 + x_2 - 2x_3 = 1$
   $4x_1 - 3x_2 - x_3 = 3$
   $2x_1 + 4x_2 + 2x_3 = 4$
   Answer: Consistent. $x_1 = 1, x_2 = 0, x_3 = 1.$

3. Find the value of k such that the following system of equations posses a non-trivial solution. Also find the solution of the system
   $4x_1 + 9x_2 + x_3 = 0$
   $kx_1 + 3x_2 + kx_3 = 0$
   $x_1 + 4x_2 + 2x_3 = 0$
   Answer: $k = 1, x_1 = 2k, y = -k, z = k.$

4. Investigate the values of λ and μ so that the equations
   2x+3y+5z=9
   7x+3y-2z=8
   2x+3y+ λ z= μ
   have (a) Unique solution (b) Infinite number of Solutions (c) No Solution
   Answer: (a) $\lambda \neq 5$ (b) $\lambda = 5, \mu = 9$ (c) $\lambda = 5, \mu \neq 9$

5. Solve the system of equations by Gauss elimination method
   x-2y+3z=2
   3x-y+4z=4
   2x+y-2z=5
   Answer: $x = \frac{11}{5}, y = -\frac{7}{5}, z = -1$

6. Solve the system of equations by Gauss elimination method
   $6x_1 - 2x_2 + 2x_3 + 4x_4 = 16$
   $12x_1 - 8x_2 + 6x_3 + 10x_4 = 26$
   $3x_1 - 13x_2 + 9x_3 + 3x_4 = -19$
   $-6x_1 + 4x_2 + x_3 - 18x_4 = -34$
   Answer: $x_1 = 3, x_2 = 1, x_3 = -2, x_4 = 1.$

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UNIT-1: ELEMENTARY LINEAR ALGEBRA

TUTORIAL SHEET-3

1. Solve the following system of equations by Gauss –Jordon method
   2x+y+z=10
   3x+2y+3z=18
   x+4y+9z=16
   Answer: x=7, y=-9, z=5.
2. Find the inverse of a matrix A=$\begin{bmatrix}2 & 3 & 4\\4 & 3 & 1\\1 & 2 & 4\end{bmatrix}$ using Gauss-Jordan method.

   Answer: $A^{-1}=\begin{bmatrix}-2 & \frac{4}{5} & \frac{9}{5}\\3 & -\frac{4}{5} & -\frac{14}{5}\\-1 & \frac{1}{5} & \frac{6}{5}\end{bmatrix}$

3. Solve the system of equations by Gauss elimination method
   9x+2y+4z=20
   x+10y+4z=6
   2x-4y+10z=-15
   Answer: x=2.7372, y=0.9872, z=-1.6525
4. Find the eigenvalues and eigenvectors of the matrix $A=\begin{bmatrix}11 & -4 & -7\\7 & -2 & -5\\10 & -4 & -6\end{bmatrix}$.

   Answer: $\lambda = 0,1,2$ and $X_1=\begin{bmatrix}1\\1\\1\end{bmatrix}, X_2=\begin{bmatrix}1\\-1\\2\end{bmatrix}, X_3=\begin{bmatrix}2\\1\\2\end{bmatrix}$

5. Find the eigenvalues and eigenvectors of the matrix $A=\begin{bmatrix}5 & -2 & 0\\-2 & 6 & 2\\0 & 2 & 7\end{bmatrix}$.

   Answer: $\lambda = 3,6,9$ and $X_1=\begin{bmatrix}2\\2\\-1\end{bmatrix}, X_2=\begin{bmatrix}2\\-1\\2\end{bmatrix}, X_3=\begin{bmatrix}1\\-2\\-2\end{bmatrix}$

6. The sum and product of the eigenvalues of the matrix $A=\begin{bmatrix}2 & -3\\4 & -2\end{bmatrix}$ are
   Answer: 0 and 8.
7. If two eigenvalues of $\begin{bmatrix}8 & -6 & 2\\-6 & 7 & -4\\2 & -4 & 3\end{bmatrix}$ are 3 and 15, then the third eigenvalue is _____.
   Answer: 0
8. If $A=\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}$, then the eigenvalues of A^{-1} are
   Answer: 1 and $\frac{1}{3}$.

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TUTORIAL SHEET-4

1. Find the largest eigenvalue and the corresponding eigenvector of the matrix $A = \begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}$ by Rayleigh power method. (Perform 5 iterations)

   Answer: $AX^{(4)} = 6.941 \begin{bmatrix} 0.341 \\ 0.039 \\ 1 \end{bmatrix}$

2. Find the largest eigenvalue and the corresponding eigenvector of the matrix

   $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by Rayleigh power method taking initial eigenvector as

   $[1 \quad 1 \quad 1]^T$. (Perform 5 iterations)

   Answer: $AX^{(4)} = 6.941 \begin{bmatrix} 0.341 \\ 0.039 \\ 1 \end{bmatrix}$