Contents:

Zero resistance state, Persistent current, Meissner effect, Critical temperature, Critical current (Silsbee Effect) – Derivation for a cylindrical wire using ampere's law, Critical field, Formation of Cooper pairs - Mediation of phonons, Two-fluid model, BCS Theory - Phase coherent state, Limitations of BCS theory, Type-I and Type-II superconductors, High Tc superconductors, Formation of Vortices, Explanation for upper critical field, Josephson junction, Flux quantization, DC Squid,, Superconducting Magnet, MAGLEV, Numerical Problems.

Introduction:

Superconductors are materials that exhibit the state of superconductivity, in which they offer zero electrical resistance when dc current is passed through them, and they do not permit magnetic fields to penetrate them. As a result, an electric current flowing through a superconductor can continue to flow indefinitely without any energy loss or degradation.

Superconductivity was discovered on April 8, 1911, by Heike Kamerlingh Onnes. At the temperature of 4.2K, he observed that the resistance abruptly disappeared. In subsequent decades, superconductivity was observed in several other materials. In 1913, lead was found to be superconductor at 7 K, and in 1941 niobium nitride was found to superconduct at 16K. Before 1980, the critical temperature found cannot be higher than 30K. Those superconductors are called conventional superconductors. Theoretically, if these materials do not possess any disorder, then superconductivity can be fully explained by BCS theory for 3-dimensional samples. Also, the behaviour close to the superconducting transition was well described by the phenomenological Landau-Ginzburg theory.

A perfect superconductor is a material that exhibits two characteristic properties, namely zero electrical resistance and perfect diamagnetism, when it is cooled below a particular temperature Tc, called the critical temperature. At higher temperatures it is a normal metal and ordinarily is not a very good conductor. For example, lead, tantalum, and tin become superconductors, while copper, silver, and gold, which are much better conductors, do not superconduct. In the normal state some superconducting metals are weakly diamagnetic and some are paramagnetic. Below Tc they exhibit infinite electrical conductivity and perfect diamagnetism. Perfect diamagnetism, the second characteristic property, means that a superconducting material does not permit an externally applied magnetic field to penetrate its interior.

Superconductivity has a wide range of applications in various fields of science and technology. Here are some of the applications of superconductivity:

1. Magnetic Resonance Imaging (MRI): Superconducting magnets are used in MRI machines to create strong magnetic fields. These magnets produce a more accurate and detailed image of internal body organs and tissues, making MRI an essential tool in medical diagnosis.

2. Particle Accelerators: Superconducting magnets are used in particle accelerators to produce high-energy particle beams. This is important in high-energy physics research and can help us understand the fundamental properties of matter.

3. Power Transmission: Superconducting wires can transmit electricity with zero resistance, which means less energy is lost during transmission. This makes superconducting power cables more efficient and cost-effective compared to traditional power cables.

4. Levitating Trains: Superconductors can be used to levitate trains above the track, reducing friction and increasing efficiency.

5. Energy Storage: Superconducting energy storage devices can store large amounts of energy for long periods without any significant loss. This technology is important for renewable energy systems, where energy storage is crucial to managing fluctuations in supply and demand.
6. Quantum Computing: Superconducting materials are used in quantum computers to create qubits, which are the basic building blocks of quantum computers. These computers have the potential to perform calculations much faster than traditional computers.

Temperature dependence of resistivity:

The electrical conductivity of metals arises from the presence of free electrons in their outermost shells that can move freely in response to an electric field. When a current flows through a metal, these free electrons move opposite to the applied electric field. Meanwhile, the lattice ions, which are positively charged, vibrate around their equilibrium positions. It is known as lattice vibrations. The resistance of a metal to electrical current flow is due to the scattering of these free electrons by lattice vibrations. As the temperature increases, the amplitude of these lattice vibrations also increases, causing more frequent collisions between the free electrons and lattice vibrations. Consequently, the resistance of the metal increases with temperature. The dependence of resistivity ρ on temperature for a typical metal is shown in Figure 1(Blue curve).

The resistivity of a metal decreases with decreasing temperature and becomes very less close to the absolute zero temperature. This is because at low temperatures, the lattice vibrations are reduced, and the scattering of electrons is minimized. However, at T=0, the resistivity is not zero due to the presence of impurities in the metal. This residual resistivity is denoted by ρ_0.

The temperature dependence of resistivity can be expressed by the Matthiessen's rule, which states that the total resistivity of a metal is the sum of its residual resistivity and its temperature-dependent part, as expressed in Equation 1.

$$\rho = \rho_0 + \rho(T) \text{ --- } (1)$$

Here, ρ is the resistivity of the given metal, ρ_0 is the residual resistivity, and $\rho(T)$ is the temperature-dependent part of the resistivity.

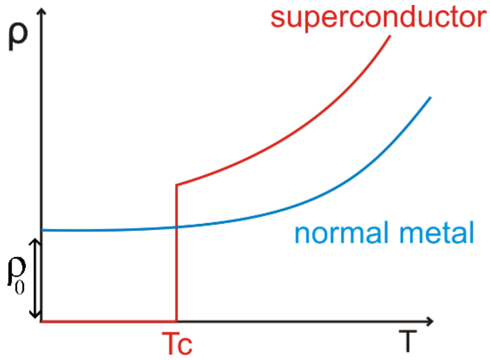


Figure 1: Temperature dependence of resistivity of a metal and a superconductor

Figure 1 (Red curve) shows the temperature dependence of resistivity ρ in a superconductor. In the non-superconducting state, ρ decreases with decreasing temperature, like a normal metal, until it reaches a critical temperature Tc. At Tc, ρ abruptly drops to zero, indicating the transition from the normal state to the superconducting state of the material. The critical temperature varies depending on the superconductor under investigation. For example, mercury becomes a superconductor with zero resistance at 4.2 K. Various superconductors, and their critical temperatures are listed in Table 1.

Table 1: Superconductors and their critical temperatures

Superconductor	Chemical Symbol	Critical Temperature (K)
Mercury	Hg	4.2
Lead	Pb	7.2
Niobium	Nb	9.3
Tin	Sn	3.7
Gallium	Ga	1.5
Indium antimonide	In Sb	1.9
Tungsten Carbide	WC	12.5
Magnesium Diboride	MgB2	39

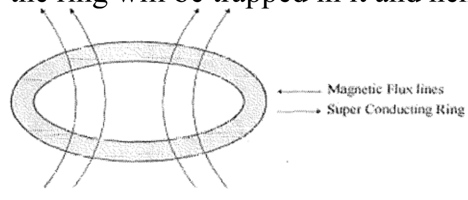
Zero Resistance state

A zero-resistance state (ZRS) refer, to a condition where a material or system conducts electricity with no energy loss due to electrical resistance. The most common example is a superconductor, a material that exhibits zero electrical resistance below a certain critical temperature However, ZRS also occurs in other systems, such as two-dimensional electron systems(2DES) subjected to strong magnetic fields and microwave irradiation, where the phenomenon is thought to be driven by radiation-induced effects that alter the electronic system's dynamics.

A superconductor, below its critical temperature, becomes a zero -resistance state, allowing electric current to flow indefinitely without any power dissipation.

Persistent current

When a current of large magnitude is once induced in a superconducting ring then the current persists in the ring even after the removal of the field as shown in fig. This is due to the diamagnetic property (i.e.,) the magnetic flux inside the ring will be trapped in it and hence the current persists.



Persistaut current

In a superconducting ring, a current induced by magnetic flux remains constant and does not decay with time due to zero resistance. Such a long-lasting current is called a **persistent current.**

Meissner effect

It was observed by Meissner and Ochsenfeld that, when a superconductor is cooled below its critical temperature (T_c), it undergoes a phase transition and enters a superconducting state. In this state, the superconductor exhibits perfect diamagnetism. It repels external magnetic field from its interior. This expulsion of magnetic flux is a characteristic feature of the Meissner effect.

Inside material, we know that

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) \text{ ---- (2)}$$

Where, $\vec{B}$ represents the magnetic induction, $\vec{H}$ is the magnetic field intensity, $\vec{M}$ is the magnetization vector and $\mu_0 = 4\pi \times 10^{-7}$H/m, is the permittivity of the free space.

We can express magnetization vector $\vec{M}$ (the net dipole moment per unit volume) in the following form, $\vec{M} = \chi \vec{H}$ where, χ is magnetic susceptibility.

Substituting the value of magnetization vector in eq. 2, one gets the following expression:

$$\vec{B} = \mu_0\left(\vec{H} + \chi\vec{H}\right) = \mu_0(1+\chi)\vec{H}$$

As we have observed for a superconducting sample, the magnetic field is zero inside the specimen, that is,

$$0 = \mu_0\left(\vec{H} + \chi\vec{H}\right)$$

$$\Rightarrow \chi = -1$$

Thus, a superconductor is an ideal diamagnetic material. For regular diamagnetic material,

$$|\chi| \ll 1$$

According to Maxwell's equation,

$$\vec{\nabla} \times \vec{B} = \mu_0\vec{J}$$

$$\Rightarrow \vec{J} = \vec{0}$$

> Magnetic flux density is given by $\vec{B} = \mu_0(\vec{H} + \vec{M})$
> For a superconducting material, $\vec{B}=\vec{0}$
> $\vec{0} = \mu_0\left(\vec{H} + \vec{M}\right)$
> $\vec{M} = -\vec{H}$
> $\chi = -\frac{|\overrightarrow{M}|}{|\overrightarrow{H}|} = -1$

Thus, the current density is zero in the interior of a superconductor. Surface currents, however, can exist in a superconductor.

In a superconducting pure metal, the magnetic flux is expelled from the sample, irrespective of its geometry and sequence in which the magnetic field is applied. The induction field decays inside exponentially with distance from the surface of the sample. This characteristic decay-length is called the penetration depth (λ) and is generally estimated to be a few hundred angstroms (10^{-7} m).

The Meissner effect occurs due to the formation of Cooper pairs in the superconductor. Cooper pairs are composed of two electrons with opposite spins and momenta that form a bound state due to the attractive interaction mediated by lattice vibrations (phonons). These Cooper pairs collectively behave as a single entity, which extends throughout the entire superconductor (Mechanism of formation of such pair will be discussed afterwards).

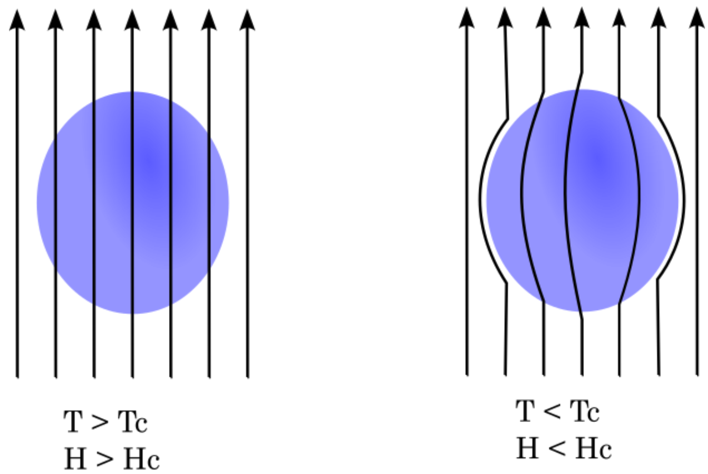


Figure 2. Meissner Effect

When an external magnetic field is applied to a superconductor, the magnetic flux tends to penetrate the material. However, the Cooper pairs actively respond to the magnetic field by forming super currents. These induced currents create a magnetic field that perfectly cancels out the external magnetic field within the bulk of the superconductor. As a result, the magnetic field is expelled from the interior of the superconductor, confining it to the exterior regions as shown in figure 2. The figure shows that above the critical temperature (T_c) or/and above a certain critical magnetic field (H_c), as the superconductivity is destroyed, the material behaves as a normal material allowing field lines through it.

In some superconductors the Meissner effect is limited to specific magnetic field ranges which will be discussed in detail later.

Critical Temperature (Tc).

It is the temperature below which the material changes from normal conductor to superconductor and is denoted as Tc. The critical temperature is also called transition temperature.

Critical Current (*Silsbee Effect*)

Derivation for a cylindrical wire using Ampere's law

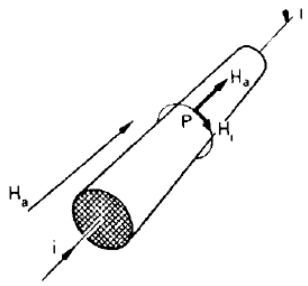


Figure 3. A superconducting current carrying wire in an applied magnetic field of H_a

Consider a current of *I* ampere is applied to a superconducting wire as shown in figure 3. The application of the current induces a magnetic field H_i. When the sum of this H_i and the applied field H_a crosses critical field H_c, the material no longer acts as a superconductor. This is called Silsbee effect.

Let us consider that the applied field is zero. The superconducting wire has a cylindrical cross-section of radius *R*. The current is flowing parallel to axis of the wire. The current is uniformly distributed over its cross-section. We will apply Ampere's circuital law to obtain the magnetic field generated at the surface of the wire due to this current. The law says: *The line integral of the magnetic field $\vec{B}$ around a closed loop is equal to the permeability of the medium μ times the total current i enclosed by the loop.*

We will consider now a circular loop of radius *R* around the wire which encloses a current *i.*

$$\oint \vec{B} \cdot \vec{dl} = \mu I$$

$\therefore B \oint dl = \mu I$ ($\vec{B}$ and $\vec{dl}$ are parallel vectors here (encircling the wire). Also, due to the symmetry of the wire, B is constant at all points on the circular loop, i.e., B is function of radial distance only)

For a circular path of radius R around the wire: $\therefore B \oint dl = \mu I$

$$B(2\pi R) = \mu I$$

$$B = \mu H = \frac{\mu I}{2\pi R}$$

When superconductivity breaks down due to the magnetic field reaching the critical field $H_c$at the surface: *H=Hc*

$$\therefore H_C = \frac{I_C}{2\pi R}$$

So, the critical current required to destroy the superconducting property is given by $I_c = 2\pi R H_c$ (when H_a=0)

Example 1. The critical magnetic field at 5 K is 2×10^3 A/m in a superconducting wire of radius 0.02 m. Find the value of critical current.

Solution.

$I_c = 2\pi R H_C = 2\times3.14\times0.02\times2\times10^3$ A/m = **251.4 A**

Example 2. Find the critical current of a long thin superconducting wire with a radius of 0.4 mm when the critical magnetic field is 7 kA/m.

Solution.

$I_c = 2\pi R H_C = 2\times3.14\times7000\times4\times10^{-3}$ A/m = **17.6 A**

Types of superconductors

Superconductors can be categorized as Type 1 or Type 2 superconductors based on their magnetization properties when subjected to an external magnetic field. Type 1 superconductors are also called *Soft superconductors*, while Type 2 superconductors are known as *Hard superconductors*.

Type-1 Superconductors

In soft superconductors, the magnetic field gets totally expelled from the interior of the material, below a certain critical magnetic field H_c. At the critical magnetic field, there is an abrupt loss of superconductivity. A plot of magnetization versus magnetic field for a Type-1 or soft superconductor is shown in figure 4.

The magnetic field can penetrate the material above the critical field H_c. The material above H_c is said to be in its normal state. This type of superconductor demonstrates complete Meissner effect at magnetic fields below H_c where, it becomes an ideal diamagnetic material. Lead, tin and mercury are common examples of soft superconductors. The transition to the superconducting state at H_c is reversible. Most superconducting metals exhibit Type -1 superconductivity at very low H_c values (around 10^{-1} T).

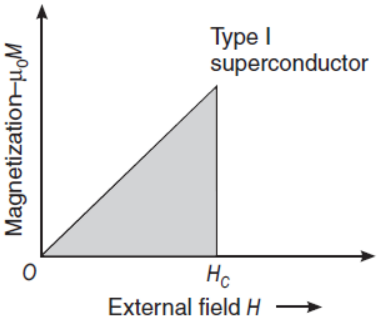


Figure 4. A plot of magnetization versus magnetic field

Type-2 superconductors

Type-2 superconductors show two critical magnetic fields: H_{c1} and H_{c2}. Typical magnetization curves for hard superconductors are shown in figure 5.

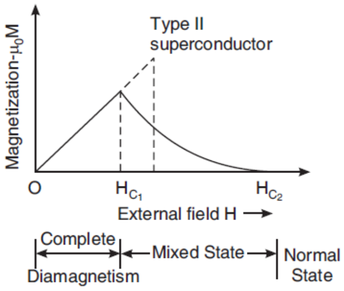


Figure 5. A plot of magnetization versus two critical magnetic fields

Till the first critical field H_{c1}, the material is superconducting and exhibits ideal diamagnetic behaviour. Magnetic flux is expelled completely from within the material. Between the two critical fields H_{c1} and H_{c2} the material exists in a mixed state (Figure 6). The material still demonstrates superconducting property, but exclusion of flux from within the specimen is partial. The bulk of the material is diamagnetic, the flux due to the applied field being opposed by a diamagnetic surface current which circulates around the perimeter of the specimen. This diamagnetic material is threaded by normal cores lying parallel to the applied magnetic field, and within each core is magnetic flux having the same direction as that of the applied magnetic field. These cores are called the vortices. Each vortex contains circulating supercurrent in a direction to support the flux penetration and hence the name 'vortex'. Vortices are normal region inside which the super electron density vanishes. As vortices contain parallel magnetic field lines, they repel each other like bar magnets and arrange themselves in form of a periodic lattice. When magnetic field increases above $H_{c1,}$ the vortex density keeps on increasing. At $H_{c2,}$ the vortex density is such that they tend to touch each other transitioning the material into the normal state.

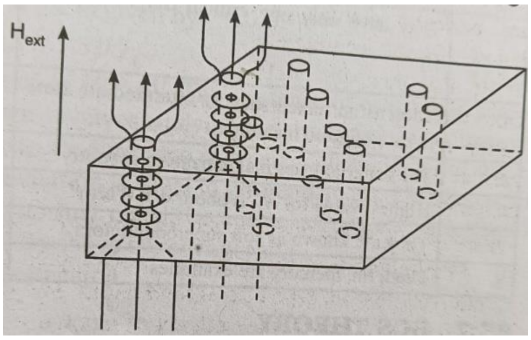


Figure 6. Type 2 Superconductors with Vortices

Alloys or transition metals with high electrical resistivity in the normal state generally demonstrate Type-2 superconductivity. The values of H_{c2} are 100 times more than the critical field values of Type-1 superconductors. H_{c2} values even up to 50 T have been obtained in some materials. As the magnetic field is increased above H_{c1}, magnetization observed in hard superconductors reduces gradually, whereas in soft superconductors, magnetization vanishes abruptly at the critical magnetic field. Hard superconductors are used as magnetizing coils to obtain high magnetic fields (10 T or higher).

Critical field (*Hc*) and Temperature dependence of critical field

The magnetic field plays a significant role in the observation of the phenomenon of superconductivity. When superconductor is placed in a magnetic field, it expels magnetic lines of force completely out of the body and becomes a perfectly diamagnetic material. However, if the strength of the magnetic field is further increased, it is found that for a particular value of the magnetic field, material loses its superconducting property and becomes a normal conductor.

The value of the magnetic field at which superconductivity is destroyed is called the Critical magnetic field, denoted by Hc. It is different for different Superconductors. It was found that by reducing the temperature of the material further, superconducting property of the material could be restored. Thus, critical field does not destroy the superconducting property of the material but only reduces the critical temperature of the material. The variation of critical field with temperature for a given superconductor is shown below.

Temperature dependence of critical field

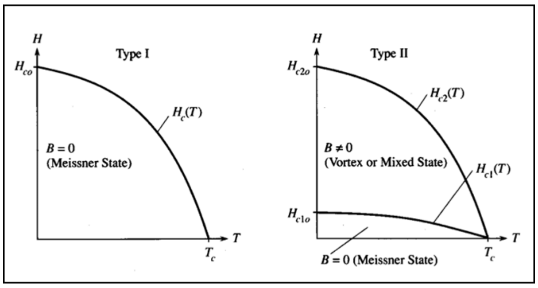


Figure 7. The temperature dependence of the critical field

The magnetic field plays a significant role in superconductivity. Superconductivity disappears at magnetic fields greater than a critical field H_c. The temperature dependence of this critical field is given by

$$H_c = H_0\left[1 - \frac{T^2}{T_c^2}\right]$$

where, H_0 is the field required to destroy the superconducting property at 0 K, H_c is the minimum field required to destroy the superconducting property at T K and T_c the transition temperature of the material at zero field.
This temperature dependence of the critical field is given by Tuyn's law. Variation of the magnetic field with temperature is shown in figure 7.

Example 2. Calculate the critical current for a wire of lead having a diameter of 1 mm at 4.2 *K*. The critical temperature for lead is 7.18 *K* and $H_C(0) = 6.5\times10^4$ *A/m*.
Solution.

$$H_c(T) = H_0(0)\left[1 - \left(\frac{T}{T_C}\right)^2\right] = 6.5 \times 10^4\left[1 - \left(\frac{4.2}{7.18}\right)^2\right] = 4.28 \times 10^4\ A/m$$

The critical current $I_C = 2\pi r H_C = \pi d H_C = 1\times3.14\times10^{-3}\times4.28\times10^4$ = **134.5 A**

Example 3. The transition temperature for Pb is 7.2 *K*. However, at 5 *K* it loses the superconducting property if subjected to magnetic field of 3.3×10^4 *A/m.* Find the maximum value of H which will allow the metal to retain its superconductivity at 0 *K*.
Solution.

$$H_0 = H_c/\left[1 - \frac{T^2}{T_c^2}\right] = \frac{3.3 \times 10^4}{1 - (25/51.28)} = 6.37 \times 10^4\ A/m$$

Example 4. The critical field of niobium is 1×10^5 *A/m* at 8 *K* and 2×10^5 *A/m* at 0 *K*. Calculate the transition temperature of the element.
Solution.

$$T_C = \frac{T}{\left[1 - \frac{H_C(T)}{H_C(0)}\right]^{1/2}} = \frac{8\,K}{\left[1 - \frac{1\times10^5\ A/m}{2\times10^5\ A/m}\right]^{1/2}} = 11.3\,K$$

Example 5. The transition temperature for lead is 7.26 *K*. The maximum critical field for the material is 8×10^5 *A/m*. Lead has to be used as a superconductor subjected to a magnetic field of 4×10^4 *A/m*. What precaution will have to be taken?

Solution.

$$T = T_C\left[1-\frac{H_C(T)}{H_C(0)}\right]^{\frac{1}{2}} = 7.26\ K\left[1-\frac{4\times10^4\ A/m}{8\times10^5\ A/m}\right]^{\frac{1}{2}} = 7.08\ K$$

Therefore, the temperature of the metal should be held below 7.08 *K*.

Example 6. The critical temperature of a superconductor is 8 K. Calculate the ratio of its critical magnetic fields at 6 K and 5 K.

Solution $H_c(T) = H_0(0)\left[1-\left(\frac{T}{T_C}\right)^2\right]$

Ratio of its critical magnetic fields at 6 K and 5 K.

$Hc(6) = H_c(0)\left(1-\frac{36}{64}\right)^2 = H_c(0)\frac{28}{64}$,

$Hc(5) = H_c(0)\left(1-\frac{25}{64}\right) = H_c(0)\frac{39}{64}$

Hc(5)/Hc(6)=39/28=0.72

Example 7. The critical temperature of a superconductor is 10 K. If the critical magnetic field at 0 K is 8×10^4 A m^{-1}, calculate the critical magnetic field at 6 K.

Solution $H_c(T) = H_0(0)\left[1-\left(\frac{T}{T_C}\right)^2\right] = 8\times10^4(1-0.36)$

$= 8\times10^4\times0.64$

$= 5.12\times10^4$ A m^{-1}

Example 8. A superconductor has a critical temperature of 12 K. At what temperature will its critical magnetic field be half of its value at 0 K?

Solution: $\mathrm{Hc(T)}=\frac{1}{2}\mathrm{Hc(0)}$

$$H_c(T) = H_0(0)\left[1-\left(\frac{T}{T_C}\right)^2\right]$$

$$\frac{1}{2} = 1-\left(\frac{T}{12}\right)^2$$

$$\left(\frac{T}{12}\right)^2 = \frac{1}{2}$$

$$T = 12\times\frac{1}{\sqrt{2}} = 8.49\ K$$

Example 9. The critical magnetic field of a superconductor at 4 K is 6×10^4 A m^{-1}. If its critical temperature is 8 K, calculate the critical magnetic field at absolute zero.

Solution:

$$H_0 = H_c/\left[1-\frac{T^2}{T_c^2}\right] = \frac{6\times10^4}{1-(16/64)} = 8\times10^4\ A/m$$

BCS theory

The BCS theory, a microscopic theory proposed by Bardeen, Cooper and Schrieffer in 1957, is based on the formation of ***Cooper pairs***.

A pair of electrons formed by the interaction between the electrons with opposite spin and momenta in a phonon field is called a Cooper pair.

The basic idea that even a very weak attractive potential can result in bound pairs of electrons, known as Cooper pairs, was first formulated by Cooper in 1956. Later, BCS theory could successfully extend the concept for a multiparticle wavefunction representing the superconducting state. This theory explains most of the phenomenon associated with superconductivity. During the flow of current in a superconductor, when an electron approaches a positive ion of the metal lattice, there is Coulomb attraction between the electron and the lattice ion. This produces a distortion in the lattice i.e., the positive ion gets displaced from its mean position. The distortion gives rise to a ***phonon***. Phonons are quanta of lattice vibrations. Smaller the mass of the positive ion core, the greater the distortion will be. This interaction called the electron-phonon interaction leads to a scattering of the electron and causes electrical resistivity. The distortion causes an increase in the density of ions in the region of distortion. The higher density of ions in the distorted regions attracts, in turn, another electron. Thus, a free electron exerts an attractive force on another electron through phonons. This is how the Cooper pairs are formed via lattice mediation overcoming the coulomb repulsion. Suppose an electron of wave vector $\vec{K}$ emits a virtual phonon $\vec{q}$ which is absorbed by another electron having wave vector $\overrightarrow{K'}$. Thus, $\vec{K}$ is scattered as $\vec{K} - \vec{q}$ and $\overrightarrow{K'}$ as $\overrightarrow{K'} + \vec{q}$ as shown in figure 8. The resulting electron-electron interaction depends on the strength of the attractive potential due to the phonons (V_{ph}). As BCS theory considers pairing of electrons with equal and opposite momenta and opposite spins ($\vec{K}_{\uparrow}$ and $\overrightarrow{-K_{\downarrow}}$), the Cooper pair has a total momentum of zero, and total spin of zero. As a result, the electron pairs in a superconductor are bosons.

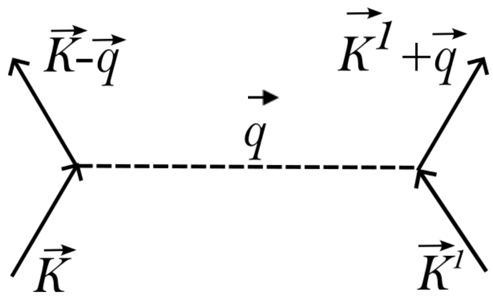


Figure 8. Conservation of momentum for electron-phonon interaction

At higher temperatures, i.e., above the critical temperature, the attractive force is small as the lifetime of the phonons decreases though number of phonons becomes more. Also, thermal energy is too large to let the pairing of electrons happen.

At lower temperatures, i.e., below the critical temperature the apparent force of attraction between electrons exceeds the Columb force of repulsion between them and the electrons form the bound pairs.

In a typical superconductor, the volume of a given pair encompasses as many as 10^6 other pairs. This dense cloud of Cooper pairs from a collective state where strong correlations arise among the motions of all pairs because of which they drift cooperatively through the material. Let us consider that the wavefunction of a Cooper pair is given by $\psi = Ae^{i\theta}$, where A is the amplitude which is proportional to the energy to break the Cooper pair. θ is the phase part of the wavefunction. According to BCS theory, all the Cooper pairs are phase coherent i.e. the phase for all the pairs are same. Thus, superconducting state is an ordered state of the Cooper pairs. Either they are at rest, or if the superconductor carries a current, they drift with identical velocity. Since the density of Cooper pairs is quite high, even large currents require only a small velocity. They do not collide as there are no vacant states available for them to go to after scattering. Because of this, the substance possesses infinite electrical conductivity.

When the electrons flow in the form of Cooper pairs in materials, they do not encounter any scattering and thereby resistance vanishes, or conductivity becomes infinity which is known as superconductivity.

Phase coherent state in superconductors:

The electrons in a cooper pair have opposite spins so that the total spin is zero and the pair behaves as bosons (Bosons means particle having zero spin). Any number of them can exist in the same quantum state at the same time. All the pairs are in the same ground state and can be explained by a single wave function. This is known as Phase coherence state.

In superconductors, the phase-coherent state refers to a macroscopic quantum state where the Cooper pairs formed by two electrons, act as a single entity, behaving like bosons. In this state, the phase of the superconducting order parameter—the collective wave function of the Cooper pairs is well defined and uniform throughout the material. This collective, phase locked behaviour is responsible for superconductivity's defining characteristics, such as dissipation less and super fluidity. The ability of the superconductor to flow without resistance is a direct consequence of this phase-coherent state.

Limitations of BCS Theory:

1. For conventional superconductors: BCS theory is only applicable to low temperature, clean and three-dimensional superconductors. It does not explain high-temperature superconductivity and low-dimensional superconductivity.
2. Weak-coupling limit: It assumes that the electron-phonon interaction is weak. If the electron-phonon coupling constant $\lambda < 1$, the superconductor is said to be in the weak-coupling limit. Aluminum (Tc =1.2 K) with $\lambda = 0.43$ and mercury (Tc =4.2 K) with $\lambda = 0.62$ fits the BCS prediction well. For strong coupling cases where $\lambda > 1$, BCS theory is not suitable. Examples of strong coupling non-BCS superconductors are MgB_2 ($\lambda = 0.9\ to\ 1.1$), High pressure H_3S ($\lambda = 2.0\ to\ 2.5$), LaH_{10} ($\lambda > 2.0$) under high pressure etc.
3. S-wave pairing: It only assumes s-wave pairing, i.e., bound electrons of opposite spins resulting in the isotropic superconducting gap. It does not consider other pairings and complex band structures (topological superconductors).
4. Momentum space picture: BCS theory explains the pairing in momentum space and fails to explain local pairings in real space.
5. Predictive power: The theory does not predict the probability of superconductivity.
6. Time independent model: Ultrafast laser-induced or pressure-driven superconductivity requires time-dependent or non-equilibrium models which are not there in BCS theory.
7. Electron-electron correlation: Strongly correlated systems, i.e., Mott insulators can show superconductivity which is not mediated by phonons. These types of cases are not captured in BCS theory.

Two-fluid model:

The **two-fluid model**, introduced by Gorter and Casimir, is a phenomenological approach used to describe the behavior of superconductors below their critical temperature T_c. The model assumes that the conducting electrons in a superconductor exist in two distinct states: **normal electrons (n_n)**, which contribute to electrical resistance, and **superconducting electrons (n_s)**, which move without resistance as a single coherent quantum fluid. The total electron concentration remains constant, but the proportion of each type changes with temperature. The density of superconducting electrons increases as temperature decreases and is given by $n_s = n(1 - (\frac{T}{T_c})^4)$, while the density of normal electrons follows $n_n = n(\frac{T}{T_c})^4$. Thus, at absolute zero, all electrons behave as superconducting electrons resulting in **perfect conductivity**, whereas at T_c, the density of superconducting electrons becomes zero and superconductivity disappears. This model also explains the **Meissner effect**, since superconducting electrons produce screening currents that expel the applied magnetic field, making the material a perfect diamagnet. Additionally, the transformation of electrons between the two states leads to a noticeable jump in heat capacity near the critical temperature. While the two-fluid model effectively describes the macroscopic properties of superconductors and their temperature dependence, it does not provide a microscopic explanation of electron pairing, which is addressed by the BCS theory.

Quantum tunnelling:

The word 'tunnelling' came into use with the advent of quantum mechanics; it was observed that electrons possess the unique property of sneaking through nano-sized barriers to contribute to the minority carriers in semiconductors. The probability of tunnelling is small; the transmission coefficient is of the order of 10^{-1} to 10^{-3} depending on the barrier height, barrier width and the energy of the particle. Then a similar question came to the mind of the scientists dealing with superconductors. *Do the Cooper pairs have this unique property of tunnelling?* In 1962, B.D Josephson predicted several remarkable phenomena about superconductivity. When two superconductors are joined together by a thin layer of oxide, Josephson explored the probability of tunneling for the newly found cohesive pair of electrons that is the Cooper pairs, through that thin insulating barrier. Cooper pair tunnelling finds many applications related to superconductors like magnetometry, quantum computing etc.

DC Josephson Effect

An insulating material of thickness nearly 1 to 2 nm (much less than the coherence length of the material; it is a length scale over which super electron density falls to zero across a Superconductor – Normal region interface) sandwiched between two different superconducting materials is known as Josephson device (figure 10). In a Josephson device, a dc super current is found to flow from a superconducting electrode that has a higher density of super electrons to the electrode with lower density of super electrons. This phenomenon is said to be DC Josephson effect. The dc current flowing through the device is equal to

$$I = i_c \sin\delta$$

Where, δ is the phase difference between the wave functions of super electrons of the two superconducting electrodes and i_c is a constant for that junction. It represents the maximum current through the insulator which can flow without any voltage drop across the junction (critical current of the junction).

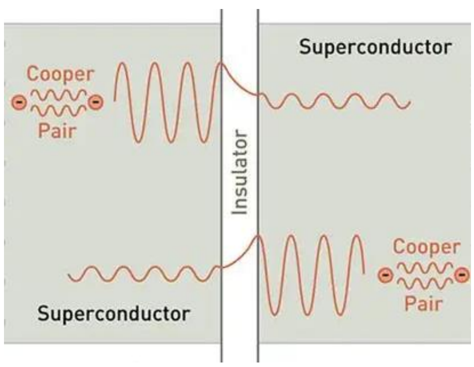


Figure 10. Josephson Junction

The cooper pairs present in a superconducting material are in the same phase. Whenever a Josephson junction is formed by sandwiching an insulator in between two pieces of superconductors, the superconducting electrons present in the two different superconducting electrodes need not be in the same phase. When the width of the insulating barrier is less than the coherence length of the superconducting electrodes, tunnelling of cooper pairs occurs. Wavefunction from one side of the barrier extends to the other side through the barrier even though it decays while crossing the barrier. When we bias the junction with a current source, the number of super electrons tunnelling to the left and that to the right do not balance each other. Thus, this tunnelling results in a net supercurrent or Josephson current depending on the phase difference of the wavefunctions of the cooper pairs of the electrodes.

AC Josephson effect

Consider a dc voltage V applied to a Josephson device. The phase difference δ evolves with time at a rate, $\frac{d\delta}{dt} = \frac{2eV}{\hbar}$. Thus, the current through the junction becomes an alternating current. The Josephson current is given by

$$I = i_c\, sin\left(\frac{2eV}{\hbar}t\right) = i_c \sin\,(\omega t)$$

Where $\omega = \frac{2eV}{\hbar}$ is the angular frequency of the ac signal. The application of a potential difference makes the Josephson current time dependent. This phenomenon is known as AC Josephson effect.

The junction can emit electromagnetic radiation because of this oscillating current. The photon energy of emission at the junction is hν = qV = 2eV.

$$\therefore \nu = \frac{qV}{h} = \frac{2eV}{h} = \frac{2\times1.6\times10^{-19}\times V}{6.625\times10^{-34}} = 483.5\times10^{12} V\ Hz.$$

For an applied voltage of 1μV, the frequency of the ac signal is $\nu = 483.5\times10^{12}$ V = $483.5\times10^{12}\times10^{-6}$ = 483.5 M Hz.

Example 6. A Josephson junction with a voltage difference of 650 μV radiates electromagnetic radiation. Calculate its frequency.

Solution.

$$\nu = \frac{2\ eV}{h}\ Hz = \frac{2(1.6\times10^{-19})(650\times10^{-6})}{6.625\times10^{-34}} = 3\times10^{11} Hz$$

High-T_c Superconductors:

The temperature T_c below which a material shows superconductivity is an important criterion in deciding the application of a superconductor. The earliest known superconductors needed to be cooled to temperature close to 4 K (boiling temperature of helium) to display superconducting properties. Though such temperatures are achievable, cooling to such a low temperature involves enormous expenses.

Continuous research is being carried out to discover superconductors with higher T_c values. Such superconductors are collectively referred to as high-T_c superconductors (HTSCs). But the formal definition of a high-Tc superconductor is the superconductor which shows a Tc above 77 K (table 2).

Many modern high temperature superconductors have a complex unit structure with oxygen called Perovskite structure. Such cells are made of one atom of rare earth metal, two Barium atoms, three Copper atoms and seven oxygen atoms and such compounds are popularly referred to as 1-2-3 super conductors.

In 1986, K.A Muller and J.G Bednorz reported superconducting properties of Lanthanum, Barium and Copper oxides at temperatures higher than 30K. This was followed by the synthesis of Yttrium compounds by C.W Chu in 1987. These Yttrium compounds acquired the superconducting state with a T_c close to 90K. Several HTSCs have since been reported with T_c > 30K. The discovery of these HTSCs rekindled the interest in superconductivity from the application viewpoint.

Table 2: Examples of High T_c Superconductors and their critical temperatures

Material	**T_c(K)**
$LnBa_2Cu_3O_7$	90
$Bi_2(CaSr)_{n+1}Cu_nO_{2n+4}$(n=1-3)	90
$Tl_2Ca_{n-1}Cu_nO_{2n+4}$(n=1-4)	90-110

In 2019 it was discovered that lanthanum hydride (LaH_{10}) becomes a superconductor at 250 K under a pressure of 170 gigapascals. As of 2021, the superconductor with the highest transition temperature at ambient pressure was the cuprate of mercury, barium, and calcium, at around 133 K (−140 °C). Very recently in 2025, single crystal of $HgBa_2Ca_2Cu_3O_{8+\delta}$ (Hg-1223) has shown the highest Tc of 134 K at ambient pressure but the reason is still elusive.

These high temperature superconductors are not commercially viable mainly due to their inherent brittleness, difficulty in forming into wires and inability to carry high current densities. Once these difficulties are overcome, such superconductors find potential applications in zero loss power transmission lines, super strong magnetic materials and as materials for levitating trains.

SQUID:

SQUID is an acronym for Superconducting Quantum Interference Device. It is used as a magnetometer. It is based on the principle of Josephson effect.

DC – SQUID:

Consider two Josephson junctions are connected in parallel as shown in figure 12. Let the current be applied through a common arm T1 and a magnetic field is applied perpendicularly to this device. The current is split into two components to flow through both the Josephson junctions. It is like the splitting of light into two coherent sources in Young's double slit experiment. The current leaving the Josephson junctions (I_a and I_b) are combined at the common arm T2. It is like the two split beams of light who are allowed to interfere with each other in Young's double slit experiment. The current flowing through the arm T2 is

$$I = I_a + I_b = 2i_c \sin(\delta\gamma_0)\, cos\frac{e\varphi}{\hbar}$$

Where $\delta\gamma_0$ is the average phase difference across the junctions, i_c is the critical current of individual junctions (here assumed to be same for both the junctions) and φ is the total magnetic flux.

The maximum current will flow, when $\delta\gamma_0 = 90^o$, $I_{max} = I_c = (I_a + I_b)_{max} = 2i_c cos\frac{e\varphi}{\hbar}$

I_c is critical for the SQUID device. Substituting $\hbar = \frac{h}{2\pi}$, we get

$$I_c = 2i_c cos\frac{2\pi e\varphi}{h} = 2i_c cos\frac{\pi\varphi}{\varphi_o}$$

Where φ_o is known as flux quantum and $\varphi_o = \frac{h}{2e} = \frac{6.626\times10^{-34}}{2\times1.6\times10^{-19}} = 2.07 \times 10^{-15}\ Wb$

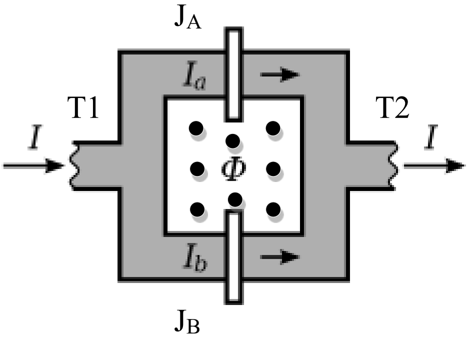


Figure 12 Schematic of a DC SQUID device

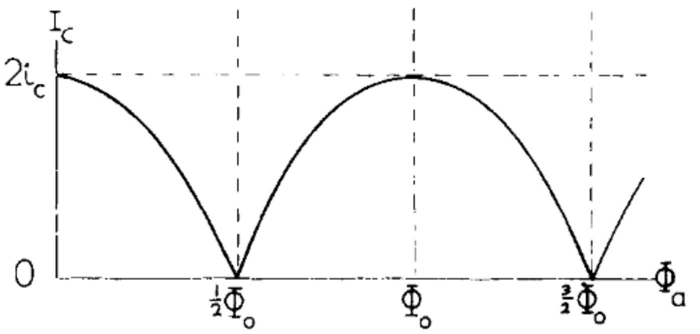


Figure 13 Variation of critical current in a SQUID as a function of applied magnetic field

So, the critical current of the SQUID is an oscillating function of applied magnetic field with a period of φ_o (Figure 13).

Physical Origin of this periodicity:

In a superconductor, the macroscopic wavefunction describing Cooper pairs must be single valued. When a superconducting loop encloses magnetic flux, the phase of the wavefunction changes. To maintain single valuedness of the wavefunction after traversing a full closed path around the superconducting loop, the phase shift must be an integer multiple of 2π ($Ae^{i\theta} = Ae^{i(\theta+2\pi n)}$). This leads to the condition that the magnetic flux enclosed by any superconducting loop must be an integer multiple of flux quantum, φ_o. This is known as ***Flux Quantization***. Usually, the critical current of a Josephson junction is much smaller than that of the individual superconducting electrodes. So, the circulating current in the SQUID is usually very small. However, the circulating current can introduce a significant phase difference across the weak links. As the magnetic field is varied, the circulating supercurrent adjusts itself to maintain integer multiple of φ_o through the SQUID loop. This leads to the oscillation of the critical current with period of φ_o.

The critical current profile as a function of applied field helps us to measure the applied magnetic flux which is of the order of φ_o. Hence, it is possible to measure very small magnetic flux using these arrangements.

DC SQUIDS are used in

- ✓ Measurement of magnetic fields as small as 10^{-21} T produced by biological currents in human heart and brain can be measured.
- ✓ Geophysical measurements (connected with rocks, seismic waves etc)
- ✓ Nondestructive testing (testing the material without damaging it)
- ✓ Readout of qubits
- ✓ Very precise and accurate measurement of voltage.

RF-SQUID or AC SQUID

The RF (Radio Frequency) SQUID is a one-junction SQUID loop and is used as a magnetic field detector. Although it is less sensitive than the DC SQUID, it is cheaper and easier to manufacture and is therefore more commonly used (figure 14).

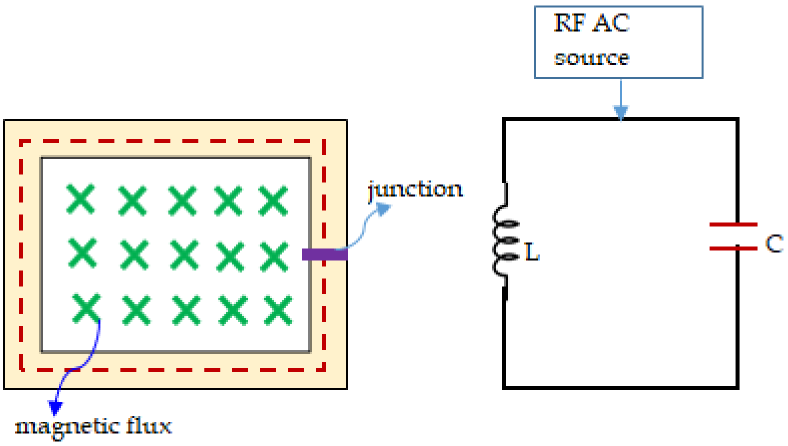


Figure 14 Schematic of an RF SQUID device

RF SQUID loop is usually placed near an LC circuit which is connected to RF AC source. The loop is immersed in a magnetic field whose flux is Φ (to be measured). Now pass an oscillating current (I) through LC circuit from the RF source. It induces magnetic flux Φ_{RF}. This flux is coupled with the loop. The total external flux is

$$\Phi_{ex} = \Phi + \Phi_{RF}$$

The loop is linked to LC circuit through mutual inductance. By chance if the flux in the loop (Φ) changes, there will be corresponding changes in Φ_{ex}. Any changes in the total flux Φ_{ex} will induce emf (according to Faraday's law) in the LC circuit and hence voltage (V) across LC changes. By measuring the change in V one can measure the external magnetic flux (Φ) and its variation with respect to time.

RF-SQUIDS are used in

- ✓ Bio magnetism (i.e., to measure magnetic field produced by different organs of our body)
- ✓ Geophysical measurements (connected with rocks, seismic waves etc.)
- ✓ Nondestructive testing (testing the material without damaging it)

Superconducting Magnet

A superconducting magnet is an electromagnet that uses coils made of superconducting materials which, when cooled below their critical temperature, carry extremely large currents without electrical resistance. Because of zero resistive losses, a persistent current flow indefinitely once established, producing very strong magnetic fields, typically much higher than conventional electromagnets. These magnets rely on cryogenic cooling, such as liquid helium or liquid nitrogen, to maintain superconductivity. They are capable of generating magnetic fields exceeding 10 Tesla with excellent field stability and energy efficiency. Superconducting magnets are widely used in MRI (Magnetic Resonance Imaging), NMR (Nuclear Magnetic Resonance) spectroscopy, particle accelerators, and magnetic confinement in fusion reactors. A key challenge is preventing “quenching,” a sudden loss of superconductivity that converts stored magnetic energy into heat. Despite the need for advanced cooling systems, superconducting magnets are essential in modern science and technology because of their high magnetic field strength and low operational energy cost.

MAGLEV (Magnetic Levitation Train)

MAGLEV (Magnetic Levitation) trains use the principles of superconductivity and magnetic levitation to enable frictionless high-speed transportation. In superconducting MAGLEV systems, superconducting magnets are placed on the train, while the track (guideway) contains coils or permanent magnets. Due to electromagnetic induction and the Meissner effect, strong repulsive magnetic forces lift the train off the track, eliminating rolling friction. This results in extremely high speeds, smooth ride quality, and low maintenance. MAGLEV works mainly on two levitation principles: **Electromagnetic Suspension (EMS)** and **Electrodynamic Suspension (EDS)**, with superconductors primarily used in EDS because they provide strong and stable magnetic fields. The Japanese SCMAGLEV has achieved speeds above 600 km/h, showcasing the potential of the technology. Although infrastructure and cooling costs are high, MAGLEV is seen as a future solution for sustainable, rapid mass transportation

Questions:

1. The dc resistance of a superconductor is practically zero. What about its ac resistance?
2. What are Cooper pairs?
3. How is Josephson tunnelling different from single particle tunnelling?
4. A superconducting wire and a copper wire are connected in parallel. Does the copper wire carry current when a potential difference is applied?
5. What is the significance of critical temperature, critical magnetic field and critical current density for superconductors?
6. What is Meissner effect?
7. Compare the dependence of resistance on temperature of a superconductor with that of a normal conductor.
8. What do you mean by "perfect diamagnetism" in the context of a superconductor?
9. Describe how Cooper pairs are formed and explain the salient features of superconductivity?
10. Explain the term high temperature superconductivity. Give the various applications of superconductors.
11. What are type I and type II superconductors? Explain key points of B.C.S theory.
12. Explain ac and dc Josephson effect.
13. Describe the principle of working of a DC SQUID and mention its applications.
14. Discuss the principle and working of an RF SQUID.