Special Topics
Assignment # 3 - Solution
Due date: June 9, 2025

It is decided to estimate the 60 Hz voltage phasors by using sampled and digitized values of voltage and Least Error Square algorithm. The sampling rate used is 720 Hz.

(a) Determine the filters for obtaining the real and imaginary components of the 60 Hz voltage phasors. Assume that the input voltage signal consists of 60Hz, 180 Hz and a decaying dc component. Use a window size of 9 samples, time reference in the middle of the window and two terms of Taylor series for the decaying dc component. Please show the complete solution.
(b) Plot the frequency response of the filters and discuss them in terms of their effectiveness for eliminating non-60Hz components.
(c) Identify the coefficients of filters that will provide real and imaginary parts of 180Hz phasors. Draw their frequency response and discuss the filters' effectiveness.
(d) Briefly, discuss the design changes you will make to improve the filters from Part (a) for obtaining the real and imaginary components of the 60 Hz voltage phasors. (only a qualitative discussion is needed)

Solution:

(a) LES filters are designed assuming the signal model to consist of 60Hz, 180Hz and a decaying dc component.

Decaying dc component is linearized using first two terms of Taylor series expansion.

Therefore, the equation will contain a total of 6 terms (first two from 60Hz, next two from 180 Hz and the final two from linearized decaying dc)

Using a sampling rate of 720 Hz, window length of 9 samples and time reference in the middle of the window, matrix [A] can be formulated and is shown below:

Matrix [A]

-0.86602295	-0.50000424	-1.46928E-05	1	1	-4
-1	-3.6732E-06	1	1.10196E-05	1	-3
-0.86602663	0.499997879	7.34641E-06	-1	1	-2
-0.50000106	0.866024792	-1	-3.6732E-06	1	-1
0	1	0	1	1	0
0.50000106	0.866024792	1	-3.6732E-06	1	1
0.866026628	0.499997879	-7.34641E-06	-1	1	2
1	-3.6732E-06	-1	1.10196E-05	1	3
0.866022955	-0.50000424	1.46928E-05	1	1	4

Left Pseudo Inverse of Matrix [A] is as below:

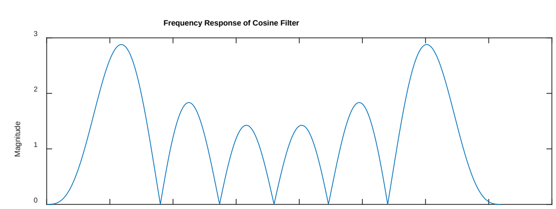
0.674972152	-0.364346074	-0.621244916	-0.364360555	-9.54794E-17	0.364360555	0.621244916	0.364346074	-0.67497215
-0.243878245	-0.143424235	-0.042972332	0.229366476	0.401816673	0.229366476	-0.04297233	-0.143424235	-0.24387825
-0.100263157	0.22984981	-0.009174009	-0.270148784	1.13257E-17	0.270148784	0.009174009	-0.22984981	0.100263157
0.116790202	-0.060968048	-0.238731368	0.038429971	0.288958485	0.038429971	-0.23873137	-0.060968048	0.116790202
0.172165627	0.161423132	0.150681839	0.037214831	-0.04297086	0.037214831	0.150681839	0.161423132	0.172165627
-0.269005946	0.07093654	0.146136299	0.070941566	3.05705E-17	-0.070941566	-0.1461363	-0.07093654	0.269005946

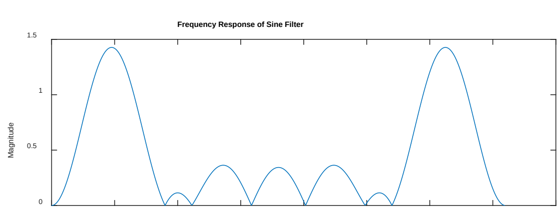
(b) and
(c)

First two rows of the left pseudo inverse matrix provide filter coefficients for estimating the real and imaginary parts of the 60 Hz phasor respectively. Next two rows of the left pseudo inverse matrix provide filter coefficients for estimating the real and imaginary parts of the 180 Hz phasor respectively. Last two rows of the left pseudo inverse provide filter coefficients for estimating the initial value of the decaying dc component and the decay rate respectively.

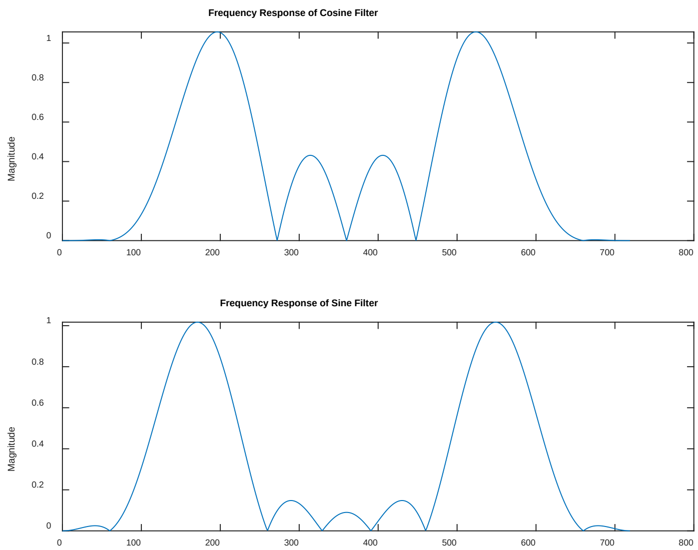
The frequency responses of these filters is given below:

<u>60 Hz filters</u>





180 Hz Filters



(d)

The following changes can be considered to improve filter responses:

1. Increase the window length to more than 9 samples. This will improve frequency response but will increase the response time of the filter to changes.
2. Use three terms of Taylor series expansion for decaying dc component. This will make the filter better eliminate the decaying dc component especially if it is decaying at a fast rate but will add an additional unknown thus window size may have to be increased thus again increasing the response time of the filter.