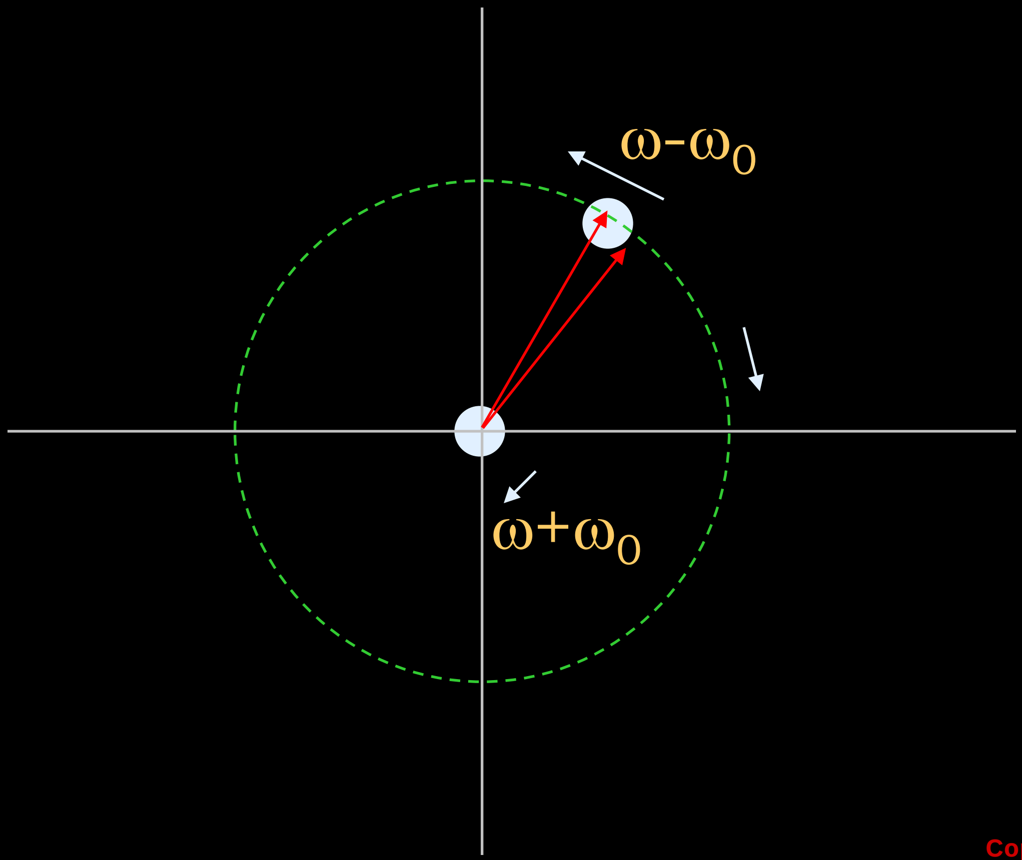
Frequency Estimation



Why measure frequency

Performance of power system during steady state and transient states

Performance of relays during steady state and transient conditions

Frequency as a papremeter for protecting equipment

Controlling sampling rate in relays

Techniques

Estimated from voltage signals

Numerical algorithms affected by

- Signal pollution
- Signal distortion
- Rapidly changing frequency

Principles of estimation

- Zero crossing
- Phasor based techniques

Zero crossing technique

$$f(t_M) = \frac{M-1}{2}\frac{1}{t_M - t_1}$$

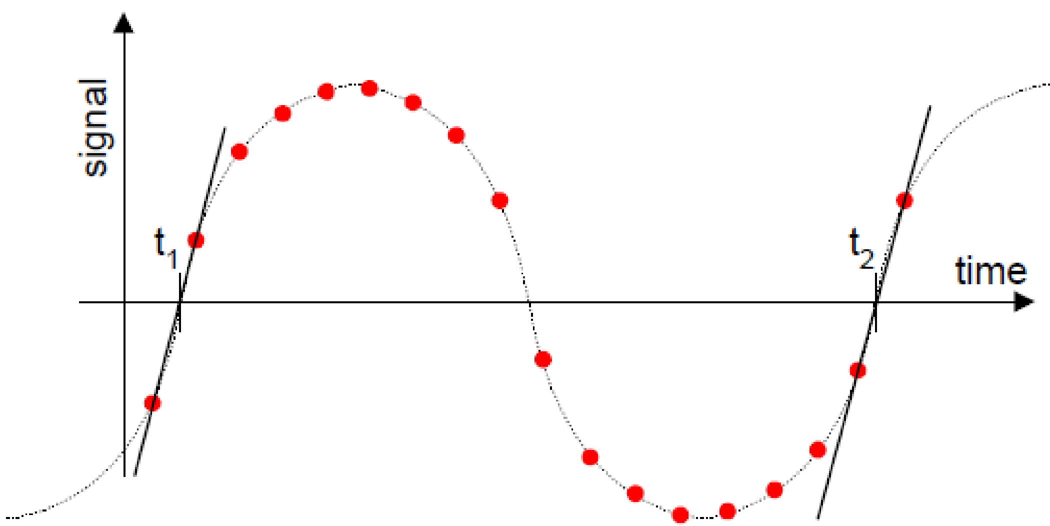
where,

M is the number of zero crossings

t_M time of the mth zero crossing

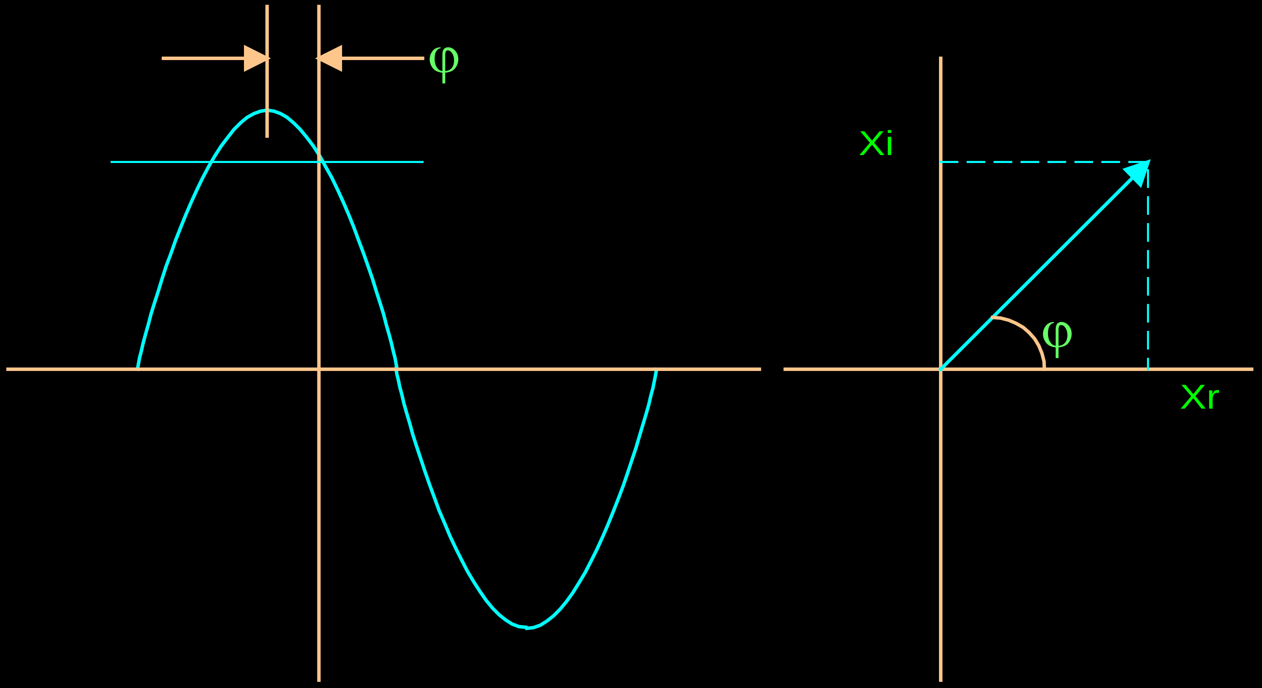
t_1 is the time of the first zero crossing

f is the estimate of the frequency



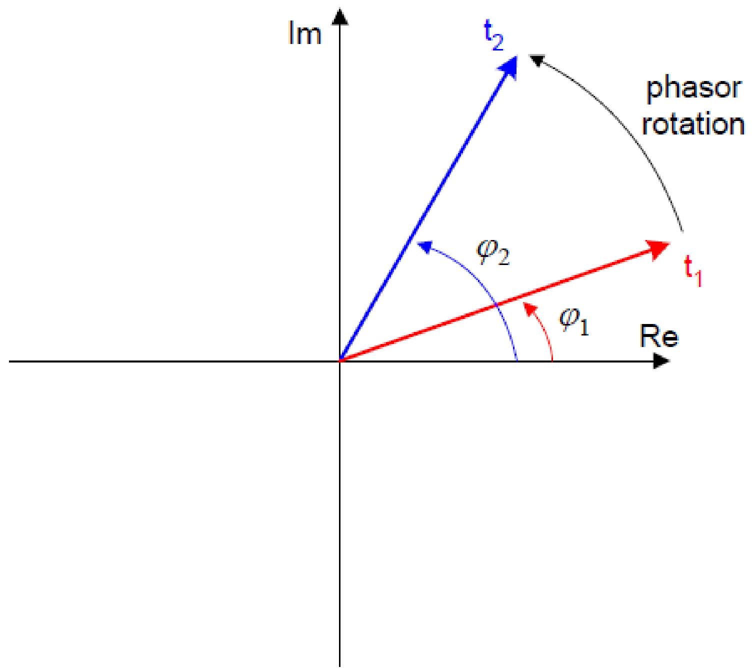
Phasor Based Techniques

A sinusoidal quantity and its phasor representation



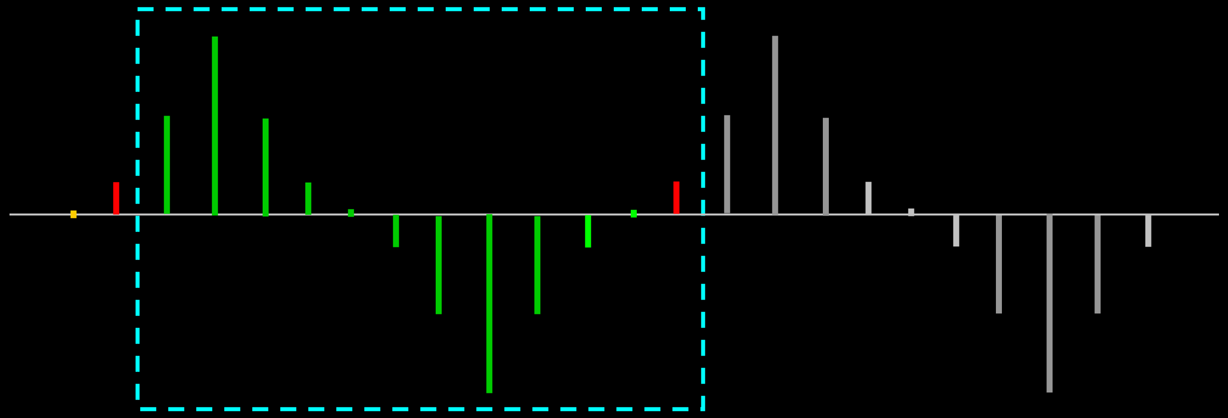
Phasor rotation

$$f(t) = \frac{1}{2\pi}\frac{d\varphi}{dt}$$



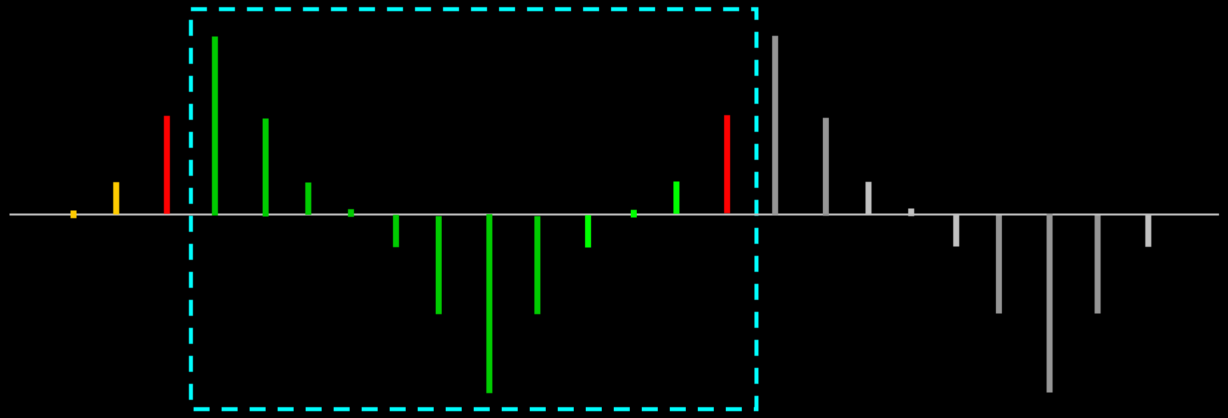
DFT Algorithm

Input waveform



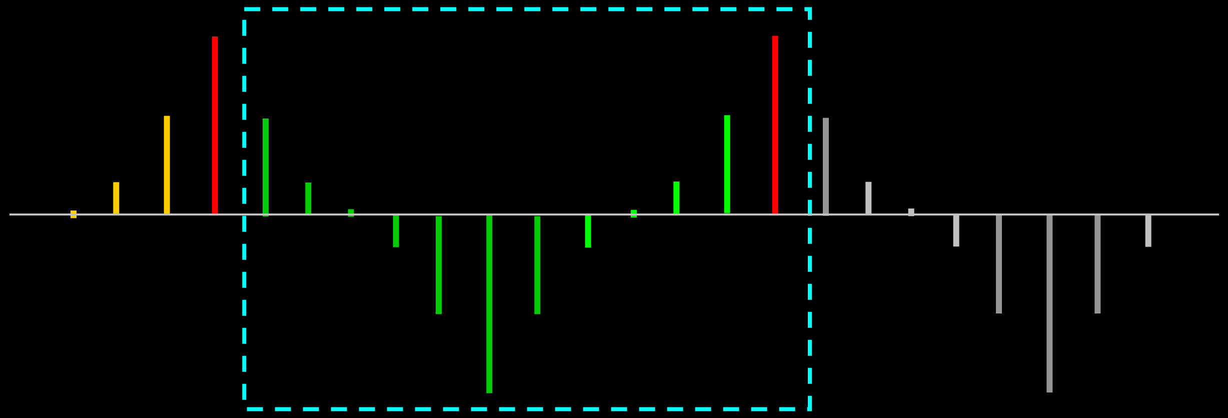
DFT Algorithm

Input waveform

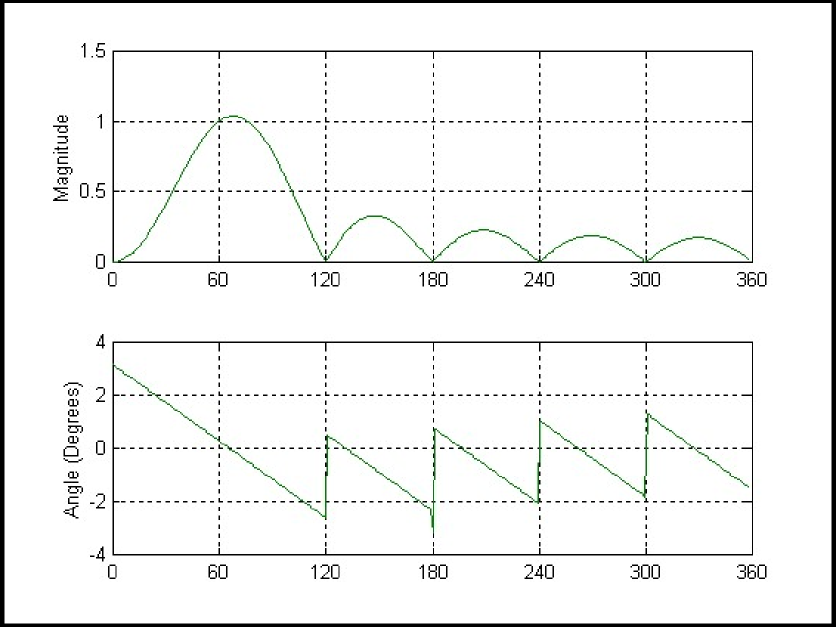


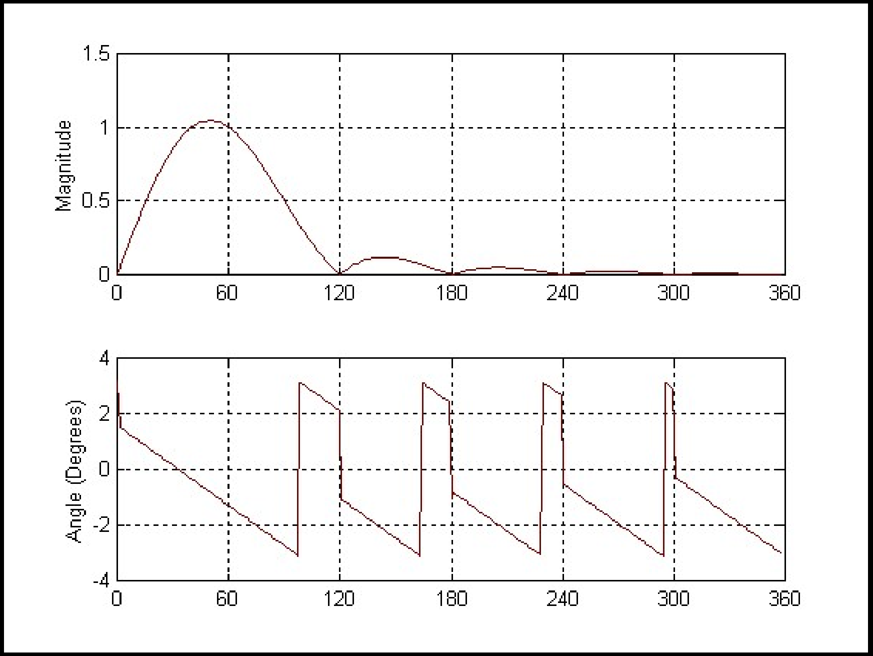
DFT Algorithm

Input waveform

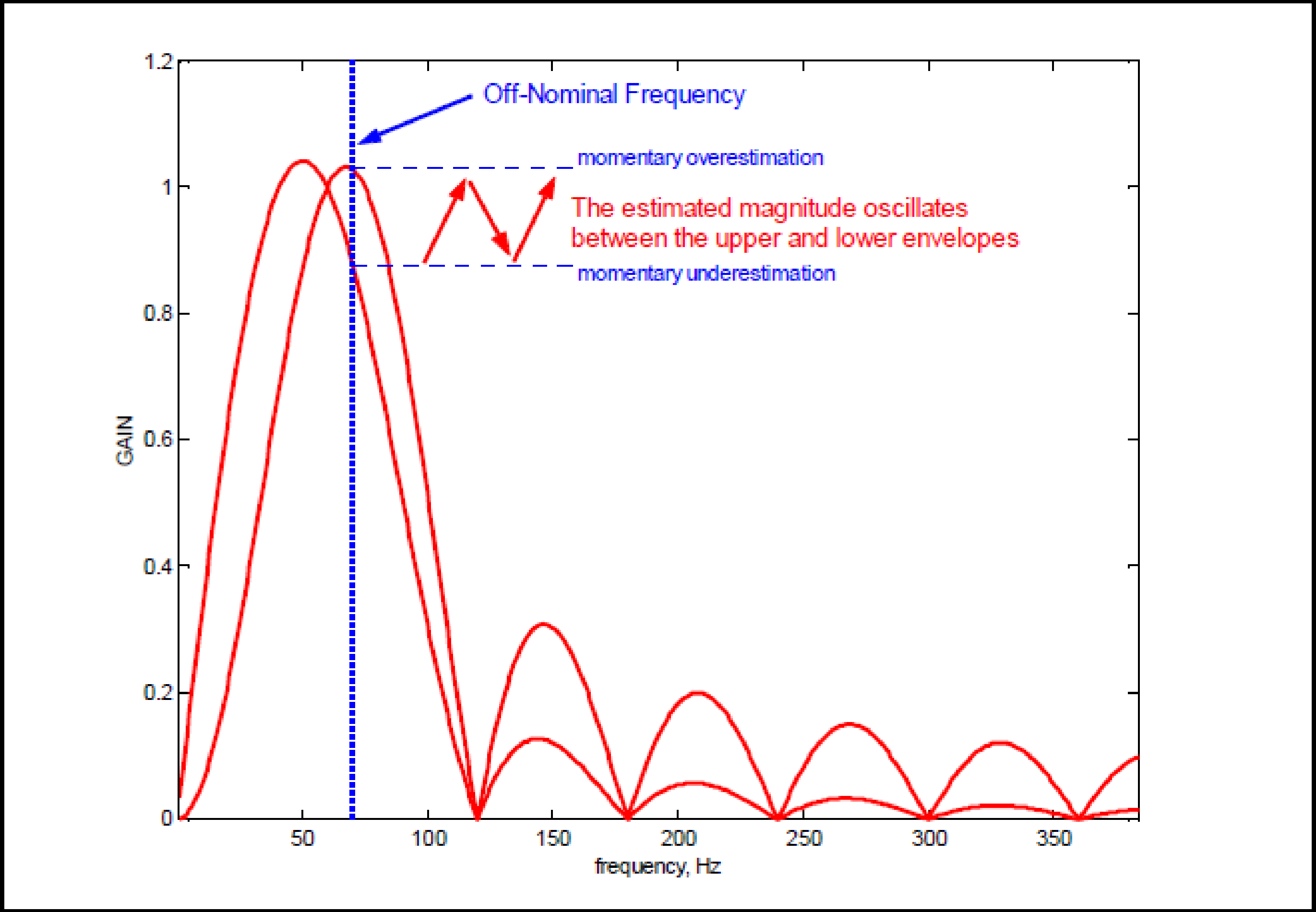


DFT Algorithm

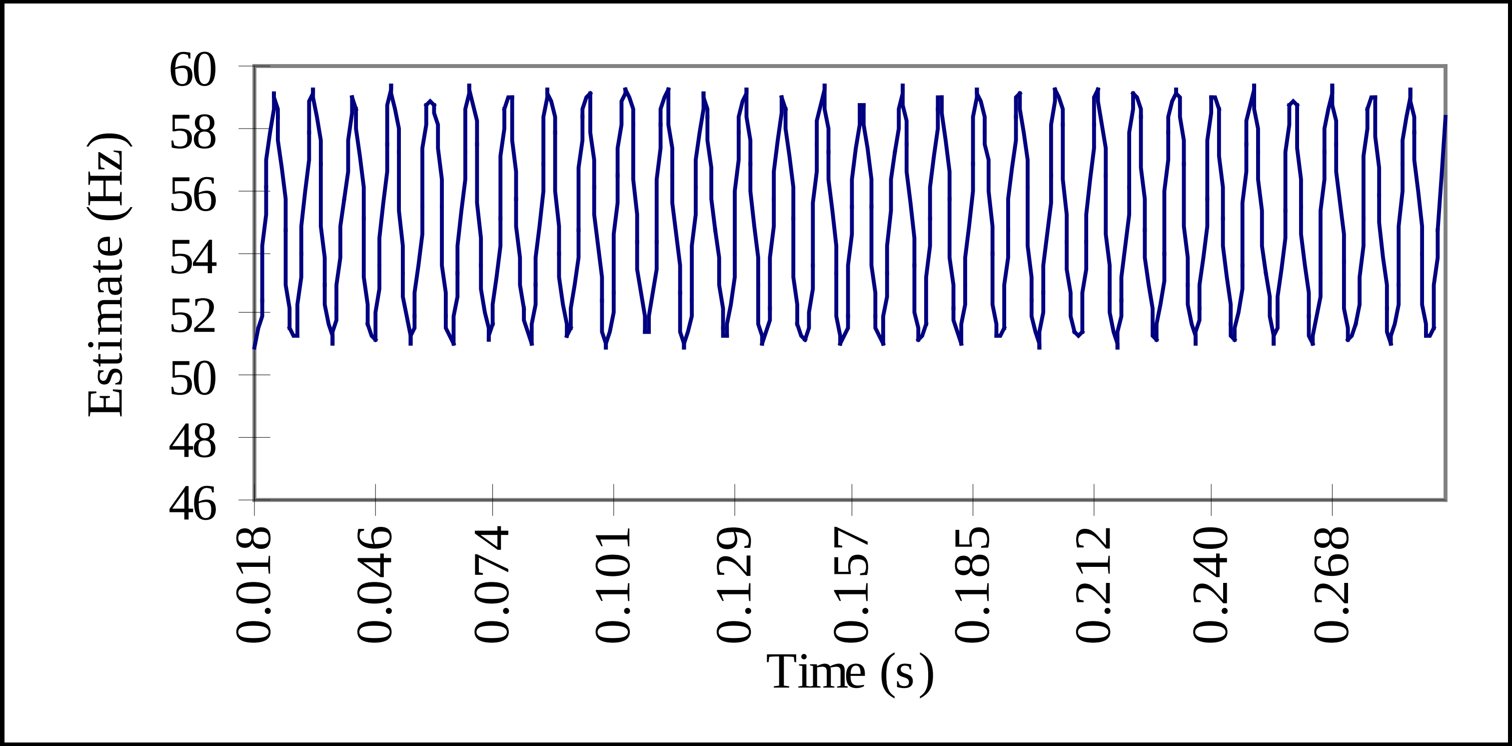




DFT Algorithm



55 Hz signal's frequency estimation



DFT Algorithm

Take average value or some other fit to eliminate variations

Adjust sampling frequency

Fixed sampling frequency but adjust sampled values in the window

FREQUENCY ESTIMATION

LEAST ERROR SQUARE TECHNIQUE

LES Technique

Assumption

- Voltage waveforms are sinusoids of single frequency

$$\boxed{v(t) = V_P \sin(2\pi ft + \theta)} \quad (1)$$

- where,
  - $v(t)$ is the instantaneous value of the voltage at time t
  - V_P is the peak value of the voltage
  - f is the frequency of the voltage waveform
  - θ is the arbitrary phase angle

LES Technique

Equation 1 is written as

$$v(t) = V_P \cos\theta \sin(2\pi ft) + V_P \sin\theta \cos(2\pi ft) \qquad (2)$$

Replace sin(2πft) and cos(2πft) by their first three terms of the Taylor series expansion

At arbitrary time t_1

- $V(t_1) = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 + a_{16}x_6 \qquad (3)$

LES Technique

Where,

$x_1 = V_P \cos\theta$

$x_2 = (\Delta f)\, V_P \cos\theta$

$x_3 = V_P \sin\theta$

$x_4 = (\Delta f)\, V_P \sin\theta$

$x_5 = (\Delta f)^2\, V_P \cos\theta$

$x_6 = (\Delta f)^2\, V_P \sin\theta$

$a11 = \sin(2\pi f_0 t_1)$

$a12 = 2\pi t_1 \cos(2\pi f_0 t_1)$

$a13 = \cos(2\pi f_0 t_1)$

$a14 = -2\pi t_1 \sin(2\pi f_0 t_1)$

$a15 = -2(\pi t_1)^2 \sin(2\pi f_0 t_1)$

$a16 = -2\, (\pi t_1)^2 \cos(2\pi f_0 t_1)$

$\Delta f = f - f_0$

LES Technique

- Equation 3 LHS is known when the voltage is sampled at time t_1
- “a” co-efficients in equation 3 are unknown since t_1 can be assigned a value
- Six unknowns – 5 more equations are needed to solve
- If input is sampled at ΔT seconds, six consecutive samples provide six equations in six unknowns
- More than 6 equations provide reasonable result

LES Technique

- m such equations in n unknowns in the matrix form is given by $[A] [x] = [v]$ (4)
- where,
  - $[v]$ is the vector of voltage measurements
  - $[x]$ is the vector of unknowns from which the frequency can be estimated
  - $[A]$ is the co-efficient matrix whose elements are known
- For $m>n$, the least error squares solution is given by,

  $[x] = [A]^{+}[v]$ (5)
- where,
  - $[A]^{+}$ is the left pseudo-inverse of $[A]$ given by$[[A]^{T}[A]]^{-}$

LES Technique

- Factors that affect the suitability of the LES technique
  - Size of the data window
  - Sampling frequency
  - Truncation of the Taylor series expansions of the sine and cosine terms
- Equation 5 shows that multiplying the row elements of the pseudo-inverse matrix digitised voltage sample provides the elements of the [x] vector
- The elements of each row of the pseudo-inverse matrix are the co-efficients of non-recursive filters that are used to calculate the unknowns, [x]

LES Technique

Estimation of phasor and frequency

- Unknowns are
  - $V_P \sin\theta$, $V_P \cos\theta$
  - $(\Delta f)\, V_P \sin\theta$, $(\Delta f) V_P \cos\theta$
- From [x], the phasor and frequency are estimated

$$V_P = \sqrt{(V_P \sin\theta)^2 + (V_P \cos\theta)^2}$$

LES Technique

Unknown frequency is given by two equations

$$\Delta f = \frac{x_2}{x_1} = \frac{\Delta f V_P \cos\theta}{V_P \cos\theta}$$

$$\Delta f = \frac{x4}{x3} = \frac{\Delta f V_P \sin\theta}{V_P \sin\theta}$$

Ist Equation not suitable when $V_P \cos\theta$ is small

2nd Equation not suitable when $V_P \sin\theta$ is small

Iterative LES Technique

Technique

- Iterative technique for frequency estimation
- Provides accurate estimate in 20 ms
- Modest computations
- Frequency estimation over a wide operating range
- Proposed technique was tested using voltage signals obtained from a dynamic frequency source and from SaskPower system

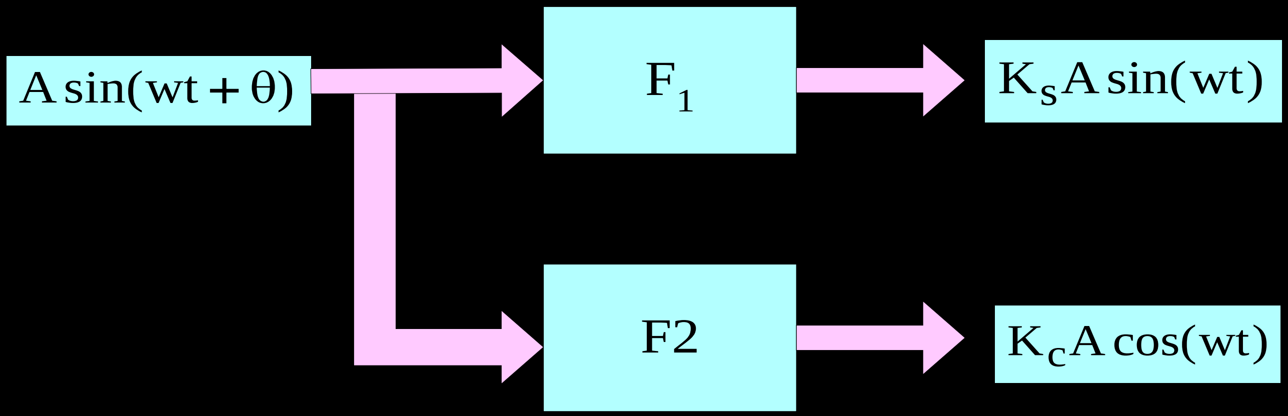
Basis

Error Analysis

- Real and Imaginary parts of the fundamental frequency component are used to compute its peak value and phase angle
- Consecutive phase angle estimates for computing frequencies
- Orthogonal filters are used to estimate, the real and imaginary parts
- Normally assumed to be 60 Hz
- Errors in phasor and angle when fundamental frequency deviates from the normal

Error Analysis

Orthogonal Filters



Error Analysis

The estimated peak value is given by

$$A_e = \{[K_s A \sin(wt)]^2 + [K_c A \cos(wt)]^2\}^{1/2}$$

This can be further reduced to

$$A_e = \left\{ \begin{array}{l} \left[\dfrac{(K_s^2 A^2 + K_c^2 A^2)}{2} \right] + \\ \dfrac{1}{2}(K_c^2 A^2 - K_s^2 A^2)\cos(2wt) \end{array} \right\}^{1/2}$$

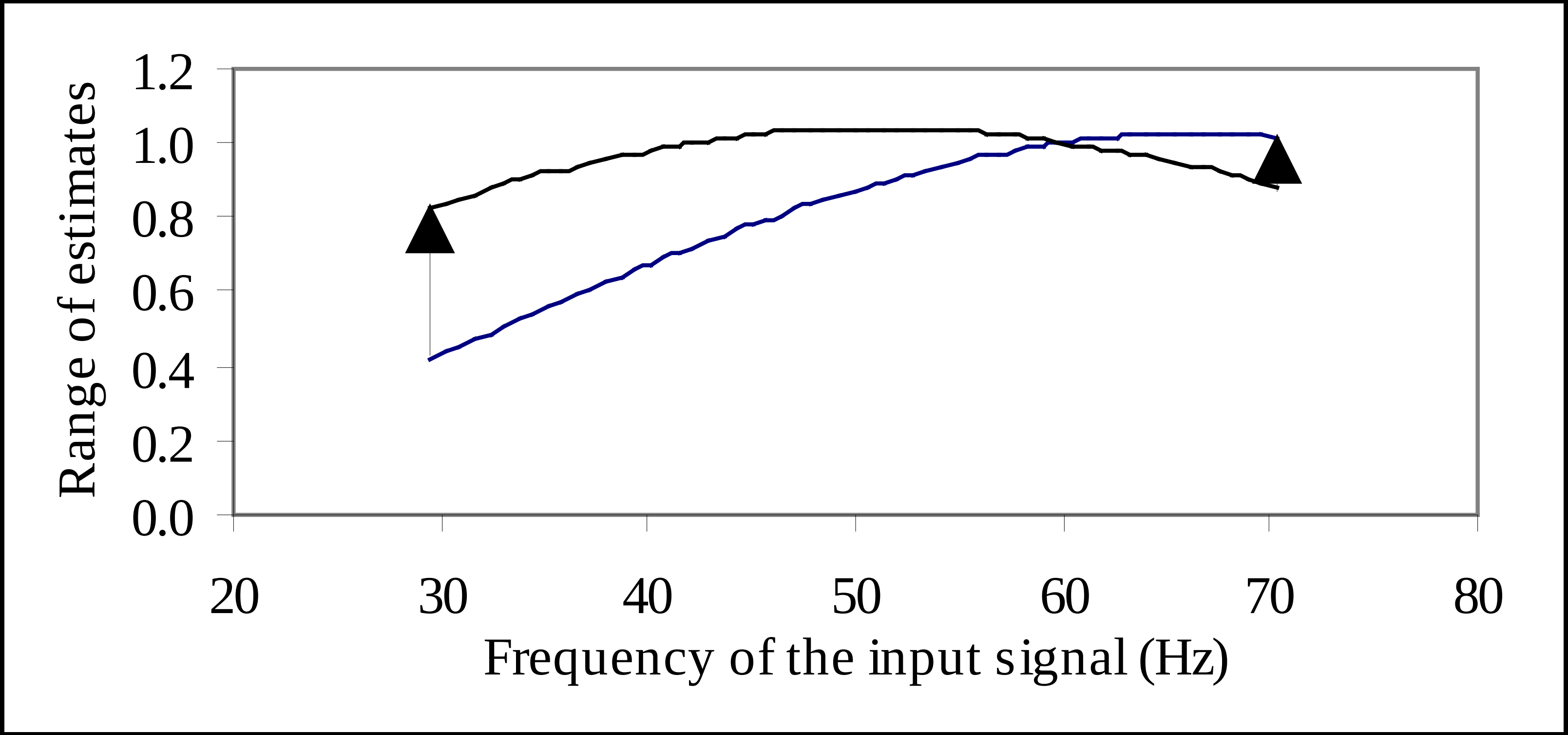
Error Analysis

Estimates

- The estimated peak value is correct for frequencies at which $K_S = K_C = 1$
- At other frequencies, the outputs are time variant
- Estimates will be in the range of $K_S A$ and $K_C A$
- Orthogonal filters are designed to have gains equal to 1 at the nominal frequency
- Estimates for signals having non-nominal frequencies will have error

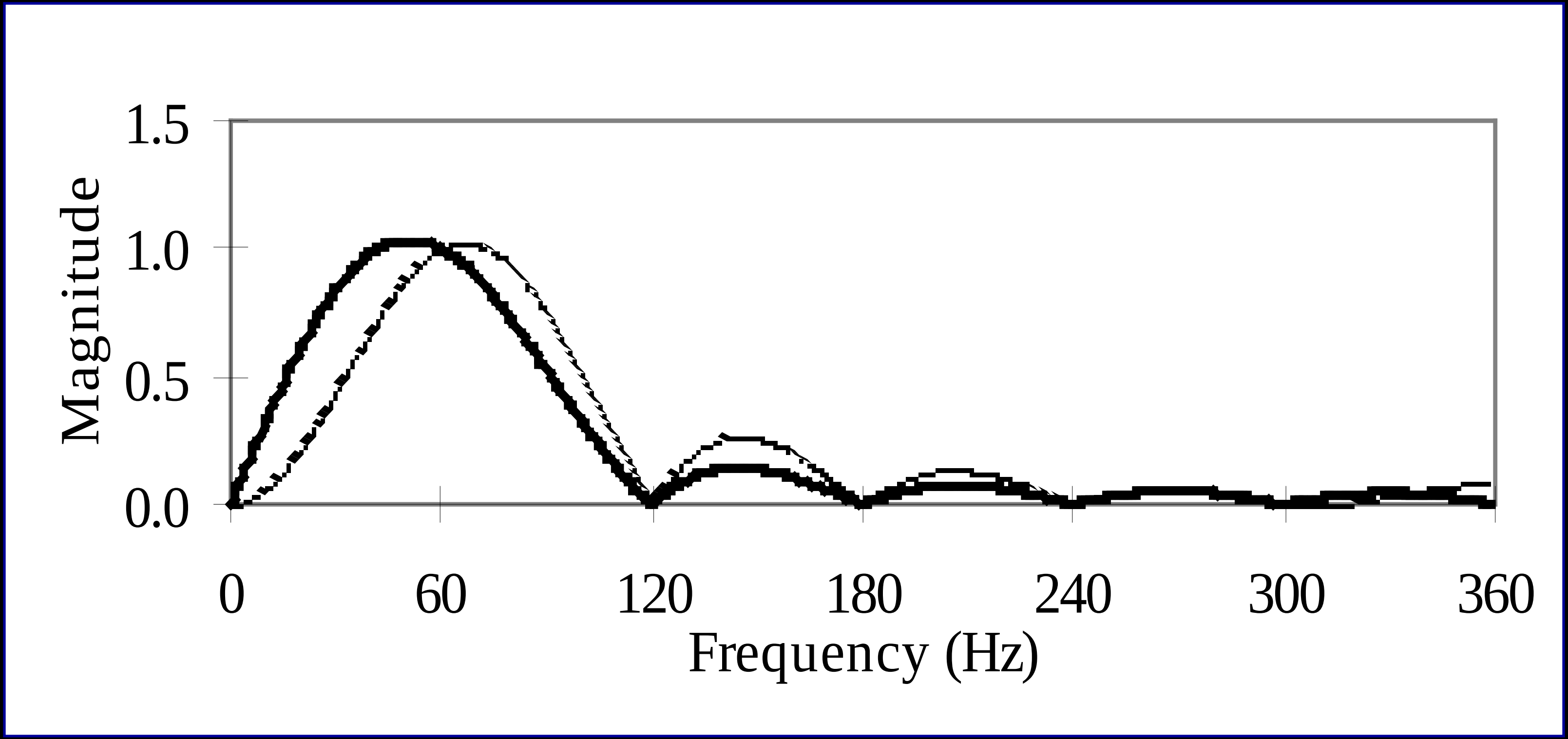
Error Analysis

Range of Estimates



Error Analysis

Frequency response of orthogonal filters



Error Analysis

Phase Angle Estimate

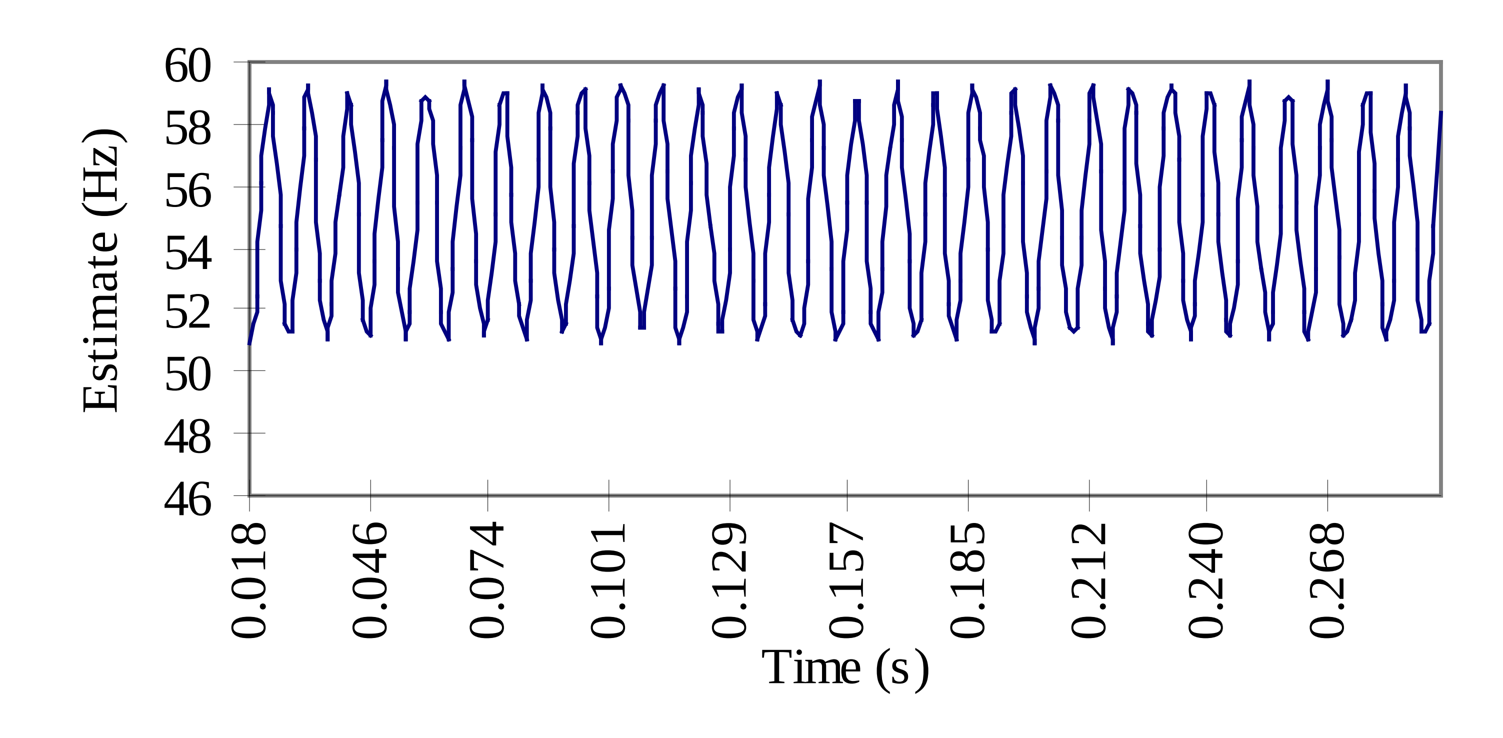
- The estimated phase angle, θ_e from the orthogonal filter output is

$$\theta_e = \tan^{-1}\left[\frac{K_s A}{K_c A}\tan(wt)\right] = \tan^{-1}\left[\frac{K_s}{K_c}\tan(wt)\right]$$

- Phase angle estimate is correct only when $K_S = K_C$
- Non-Nominal frequency estimates will have error

Error Analysis

Frequency estimates of a 55 Hz input signal by using the orthogonal filters



Error Analysis

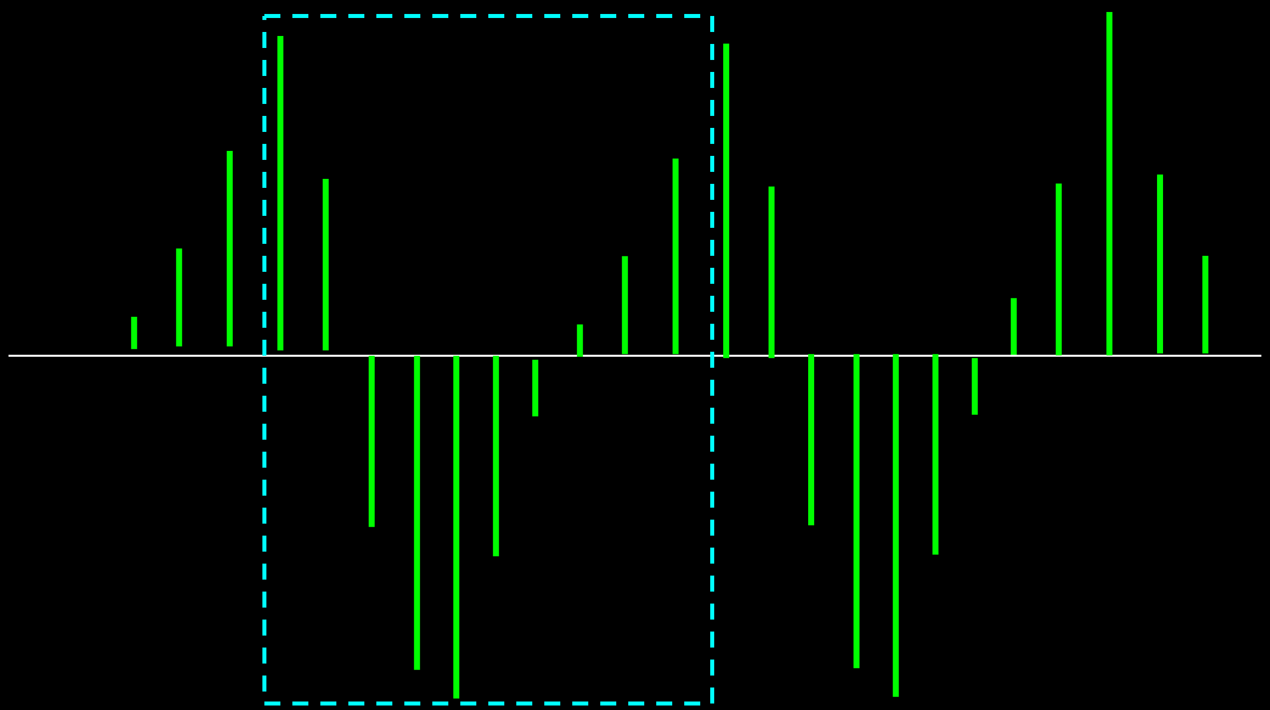
Disadvantages

- Harmonics will distort estimates
- When frequency deviates from nominal frequency, harmonics of non-nominal frequency are not eliminated by these orthogonal filters – errors in estimates

Iterative Technique

The Technique

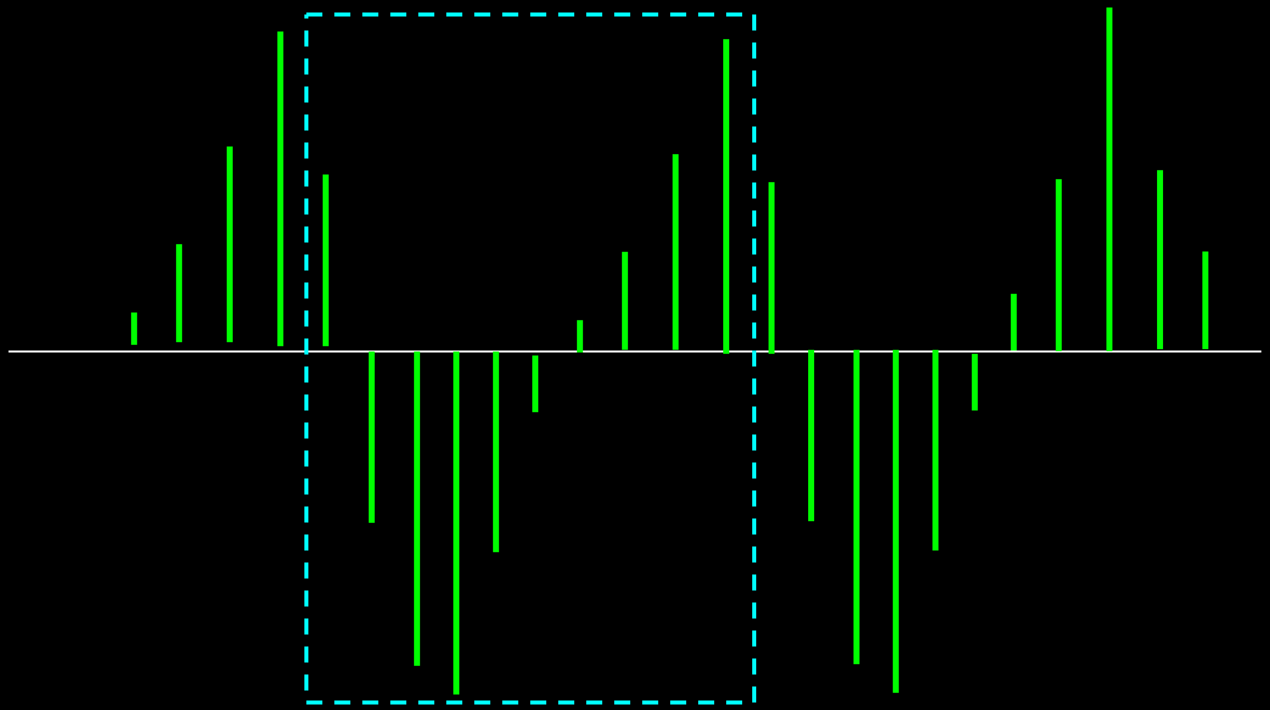
n^{th} data window



$$\theta_n = \tan^{-1}(V_{in} / V_{rn})$$

The Technique

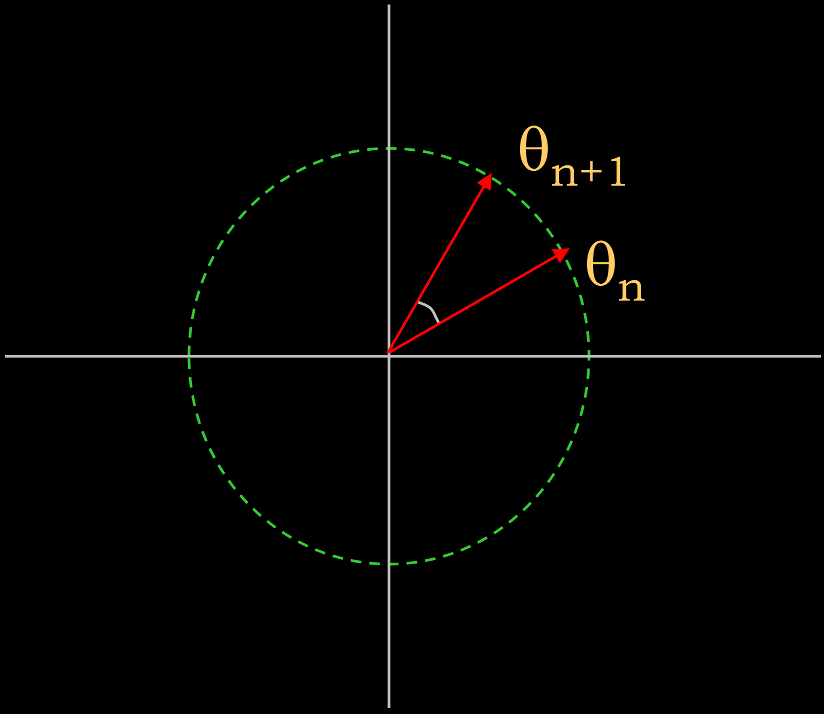
Next sample - data window shifted by one sample



$$\theta_{n+1} = \tan^{-1}(V_{in} / V_{rn})$$

The Technique

Phasor Rotation



The rotation for a phasor having a fundamental frequency of f_0 will equal to

$$\theta_{n+1} - \theta_n = 2\pi f_0 / f_s$$

If system frequency changes, then the angle between the samples changes

Estimate of frequency $f^{\wedge}$

$$f^{\wedge} = (\theta_{n+1} - \theta_n) / (2\pi / f_s)$$

The Technique

Two Situations arise

- Estimated frequency equal to fundamental frequency assumed for designing the orthogonal filters that used to compute the phase angles θ_n and θ_{n+1}
- Estimated frequency not equal to fundamental frequency assumed for orthogonal filters
  - Means that the estimated frequency not the fundamental one
  - However to achieve situation 1 we an iterative procedure has to be followed

The Technique

Iterative Procedure

- **Step 1** : Design new orthogonal filters by assuming the fundamental frequency of the signal being equal to the latest estimate of the frequency using

$$f\hat{} = (\theta_{n+1} - \theta_n)/(2\pi/f_s)$$

- **Step 2:** Compute phase angles θ_n and θ_{n+1} by using the orthogonal filters designed in step 1 and the samples corresponding to data windows n and n+1

The Technique

Iterative Procedure

- **Step 3**: Estimate the frequency using $\hat{f} = (\theta_{n+1} - \theta_n)/(2\pi/f_s)$ and the phase angles computed in step 2
- **Step 4**: Check if the estimated frequency from step 3 is equal to the fundamental frequency assumed for designing filters in step 1.
- If it is, estimated frequency in step 1 is the fundamental frequency of the signal. Otherwise revert to step 1
- An initial estimate of the frequency is assumed for starting the procedure for the first time

The Technique

Practical Issues - I

- Estimation process may require design of new orthogonal filters at every iteration
- Design of filters requires considerable computations
- Computation not possible within one sampling interval
- Filters to be designed off-line and their co-efficient be stored for use in estimating the frequency

The Technique

Practical Issues 2

- Iterative procedure terminated when estimated frequency is equal to fundamental frequency assumed for designing the orthogonal filters
- A margin should be allowed to account for errors arising from truncations during calculations, data acquisitions etc.,

Practical Issues 3

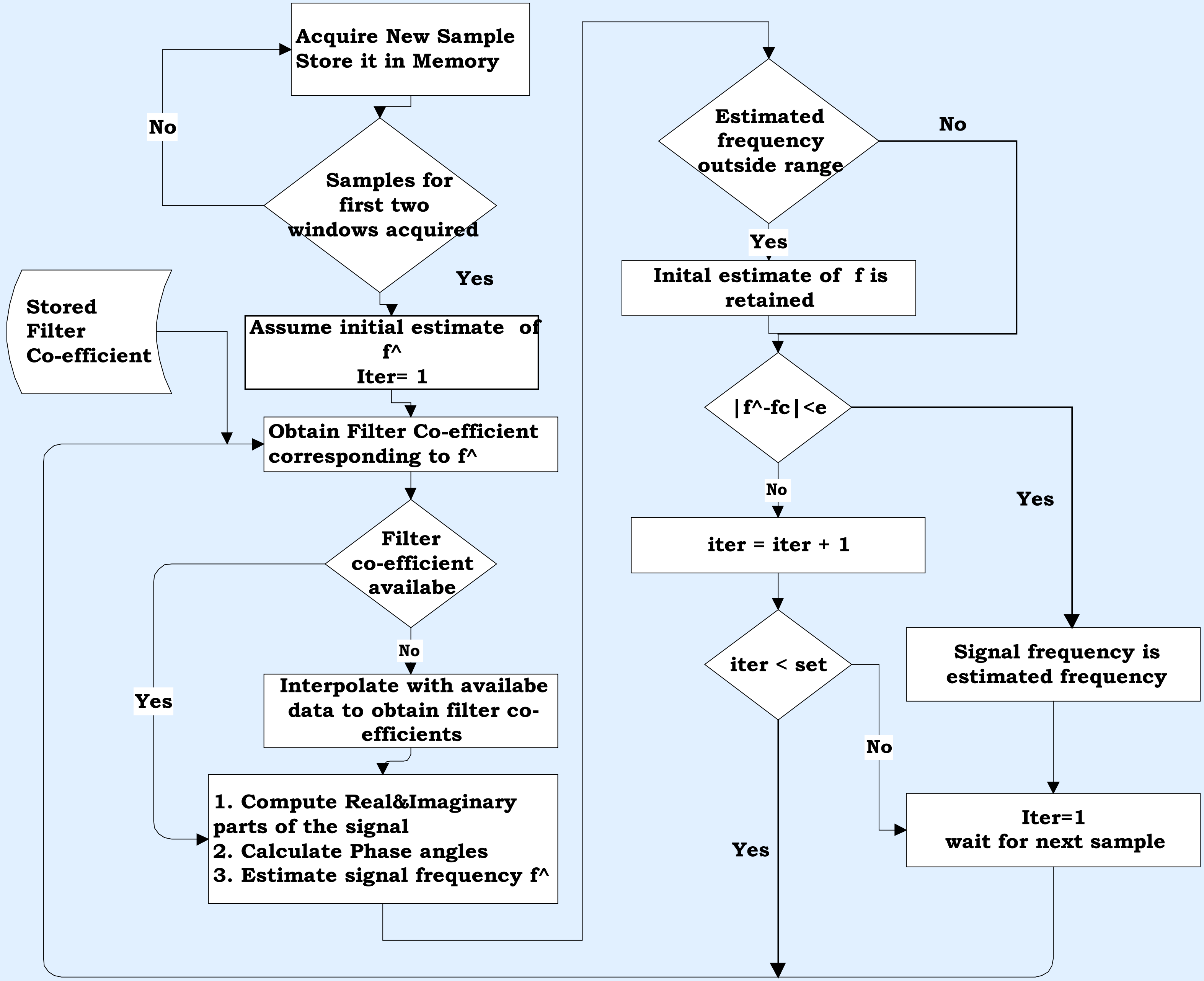
- Number of iterations limited by the digital processor capabilities

Computations

Off-Line Computations

- Design of orthogonal filters – 34 Hz to 75 Hz in steps of 1 Hz
- Co-efficients of resulting sine and cosine filters stored in a look-up table for on-line calculations
- Filter co-efficients corresponding to other frequencies – interpolation must be performed
- This arrangement estimates frequencies in the range 40-70Hz with this technique

On-Line Computations


Acquire New Sample
Store it in Memory
No
Samples for first two windows acquired
Yes
Stored Filter Co-efficient
Assume initial estimate of f^
Iter= 1
Obtain Filter Co-efficient corresponding to f^
Filter co-efficient availabe
No
Yes
Interpolate with availabe data to obtain filter co-efficients
1. Compute Real&Imaginary parts of the signal
2. Calculate Phase angles
3. Estimate signal frequency f^
Estimated frequency outside range
No
Yes
Inital estimate of f is retained
|f^-fc|<e
No
Yes
iter = iter + 1
iter < set
No
Yes
Signal frequency is estimated frequency
Iter=1
wait for next sample

Parameter Selection

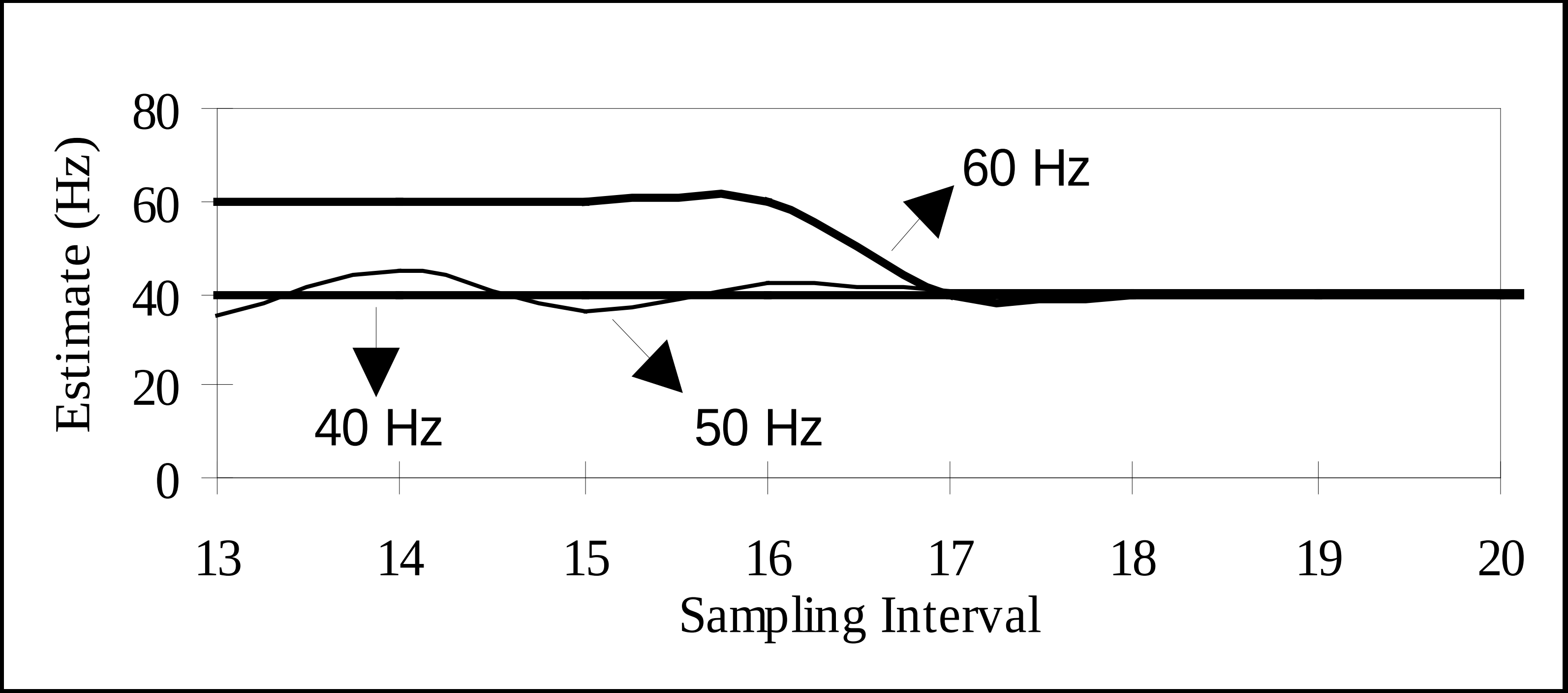
Design Parameter

- Orthogonal filters designed using LES
- Sampling rate 720 Hz
- Window length 13 samples
- Time reference coinciding with the middle of the window
- Signal assumed to contain DC, Fundamental frequency and harmonic components up to 5^{th} of the fundamental

Parameter Selection

Initial Estimate

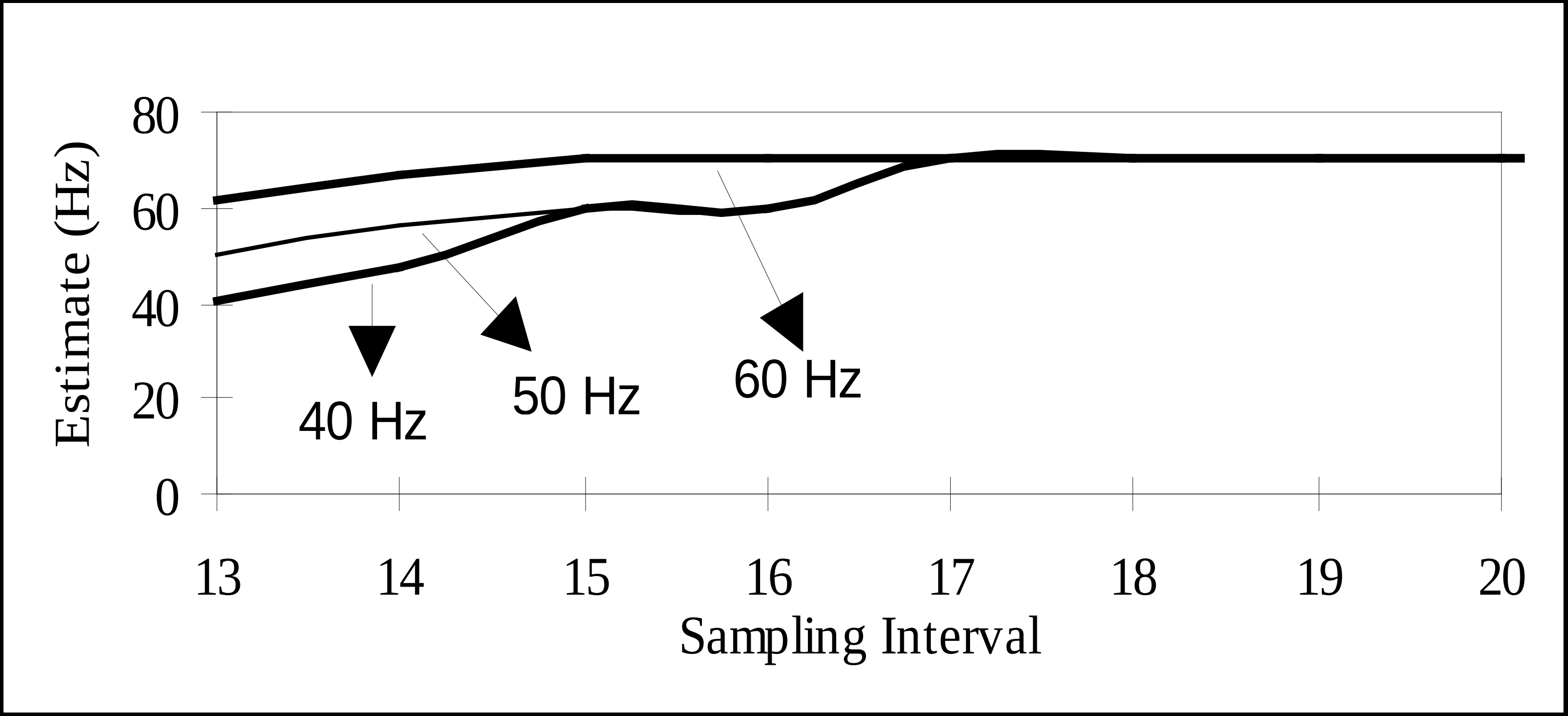
Frequency estimate of an input signal of 40 Hz when initial estimates of 40-,50,- and 60-Hz are used



Parameter Selection

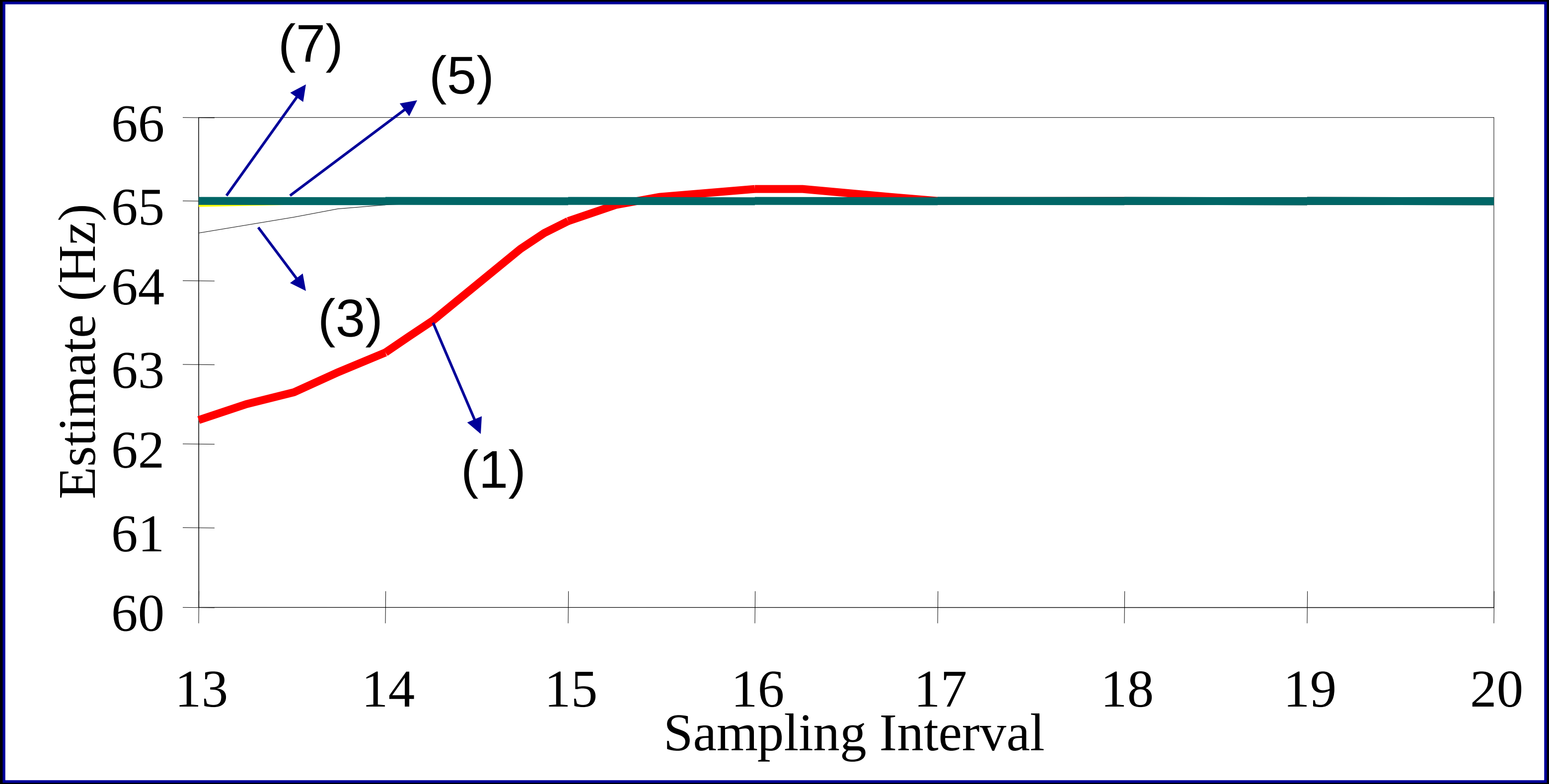
Initial Estimate

Frequency estimate of an input signal of 70 Hz when initial estimates of 40-,50,- and 60-Hz are used



Parameter Selection

Frequency estimate for an input signal of 65 Hz when number of iterations per sampling interval are varied



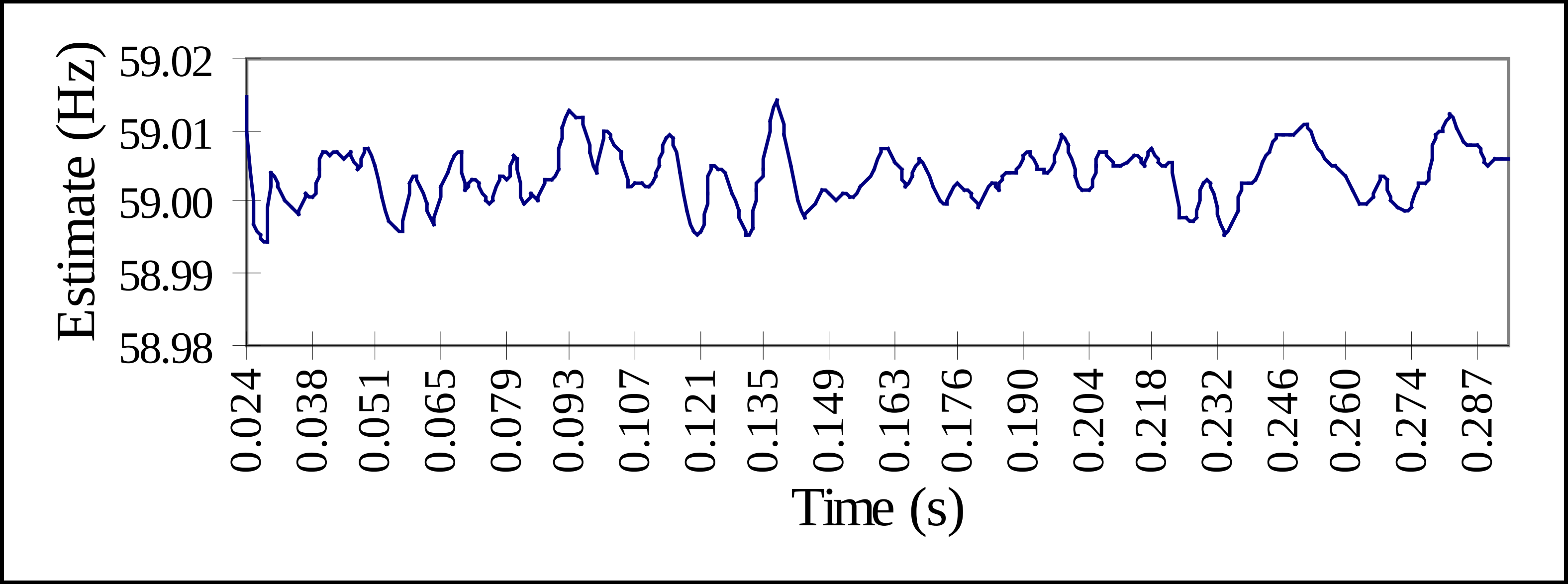
Parameter Selection

Recommended Values

- Convergence achieved faster and less number of iterations are needed when initial estimate is closer to the frequency of the signal
- Without any prior knowledge – nominal frequency recommended
- Number of iteration should be kept less than five – depends on the processor capability

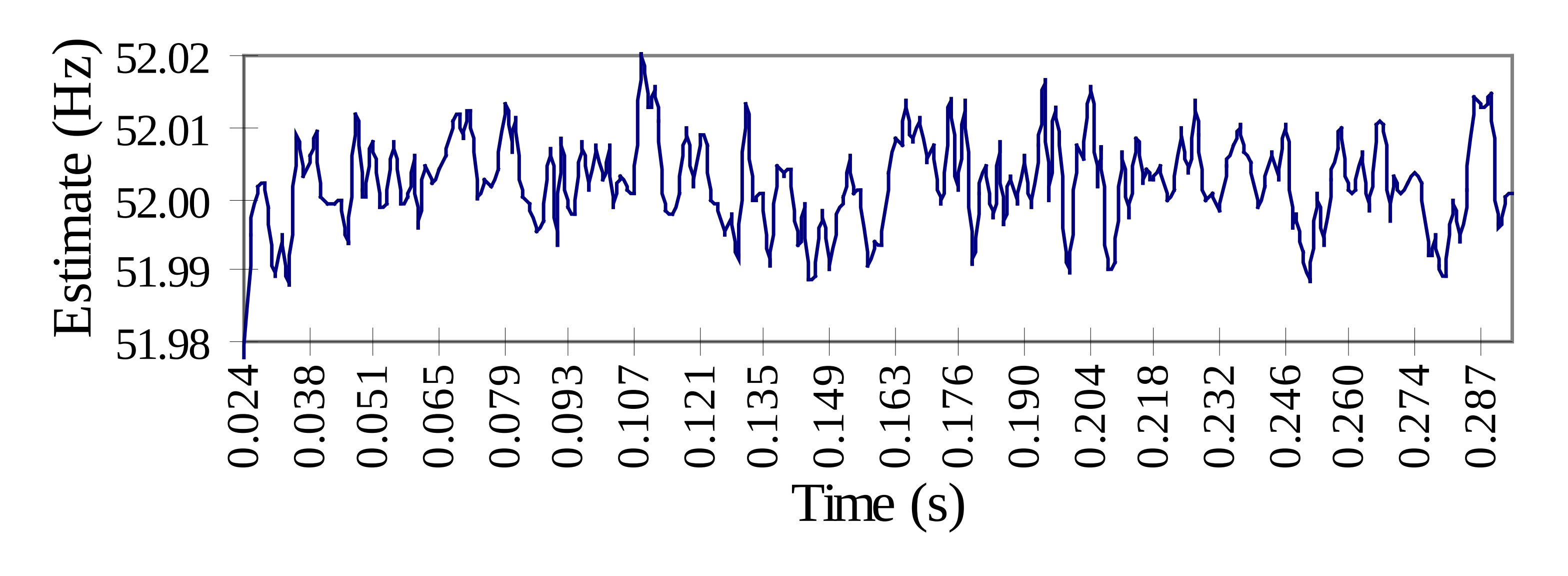
Test Results

Frequency estimate of input signals of 59 Hz



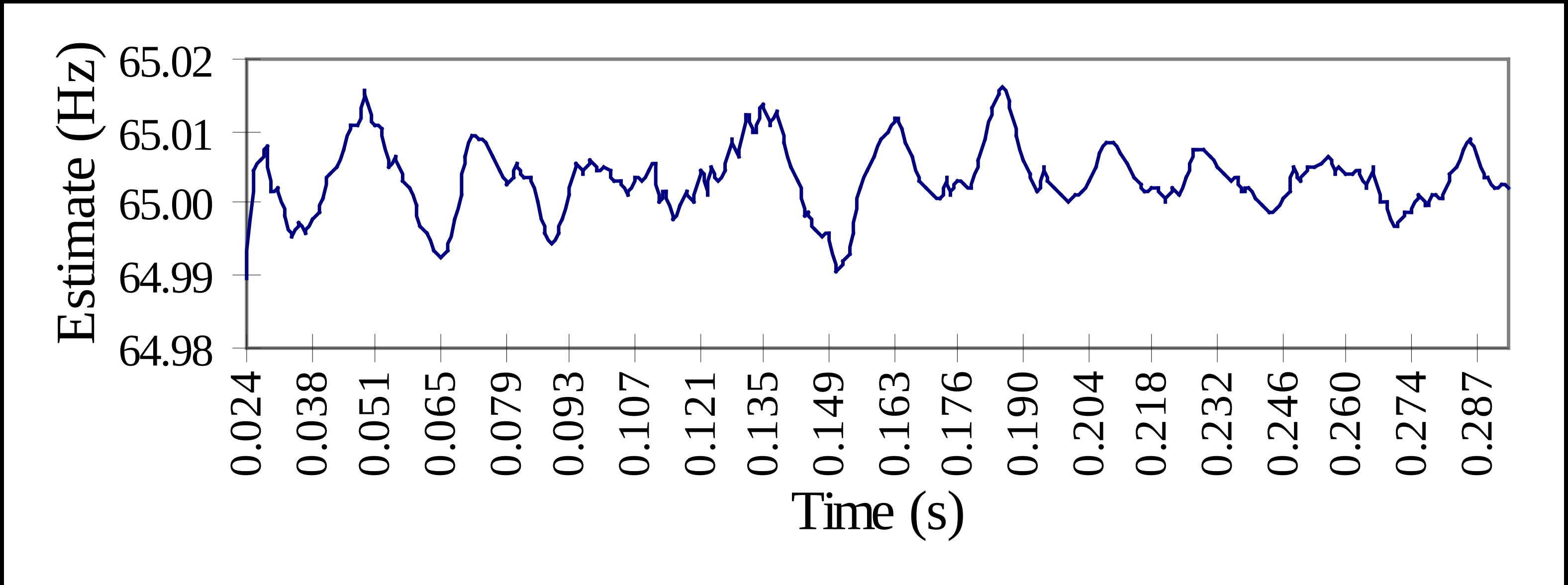
Test Results

Frequency estimate of input signals of 52 Hz



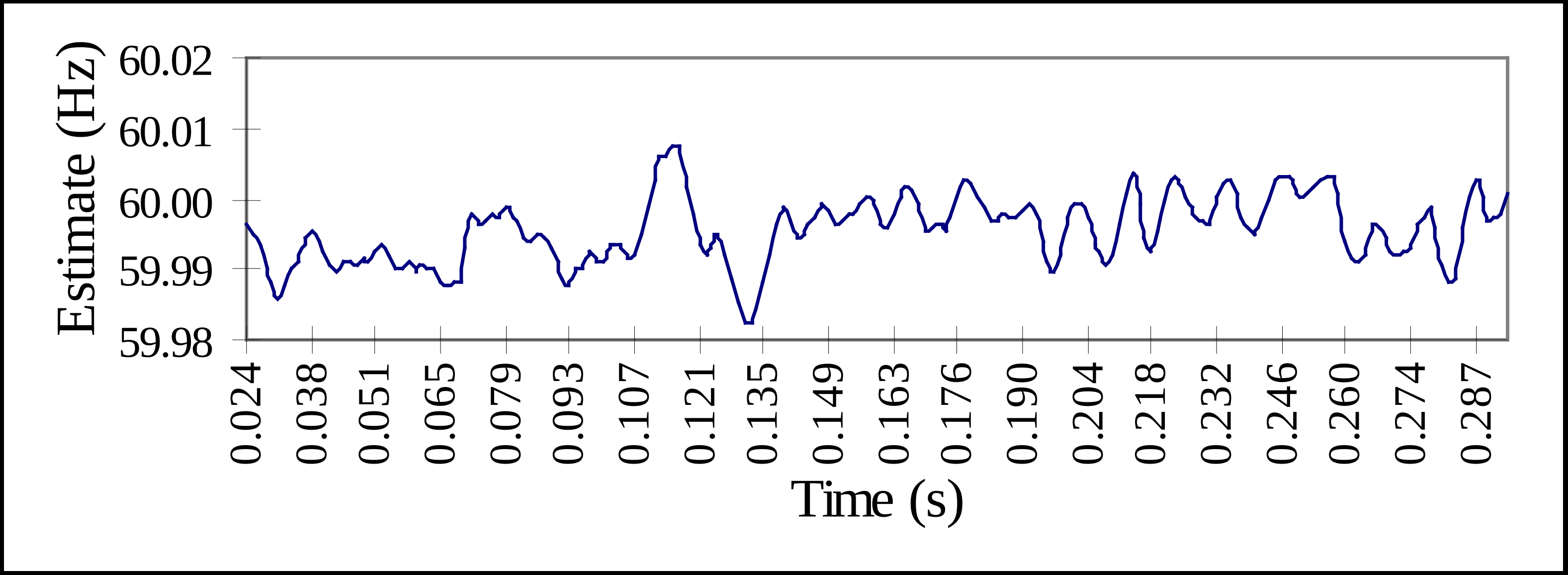
Test Results

Frequency estimate of input signals of 65 Hz



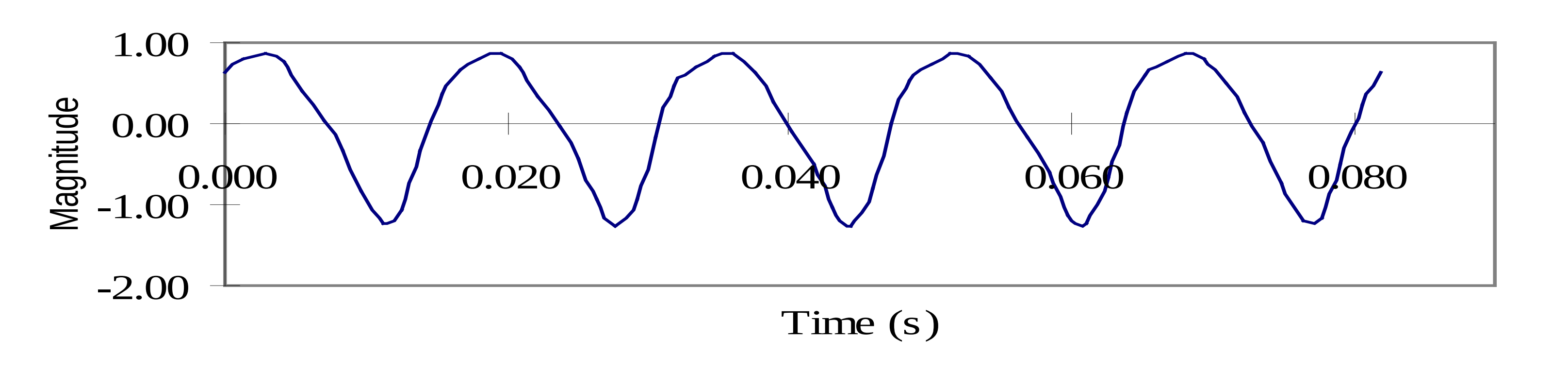
Test Results Test

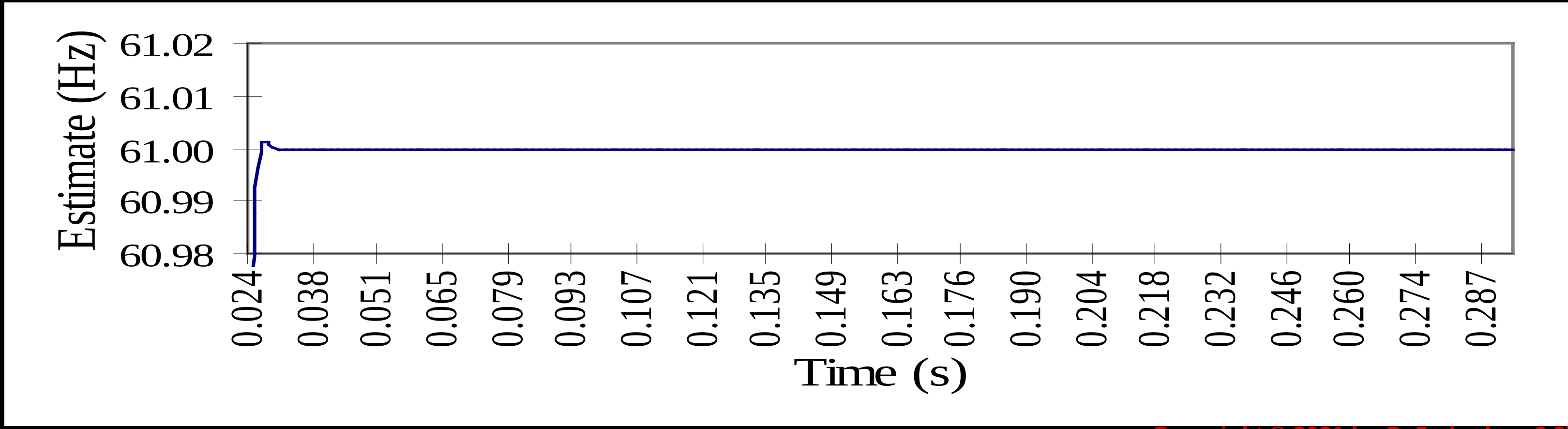
Frequency estimate of a voltage signal recorded from the SaskPower system



Test Results Test

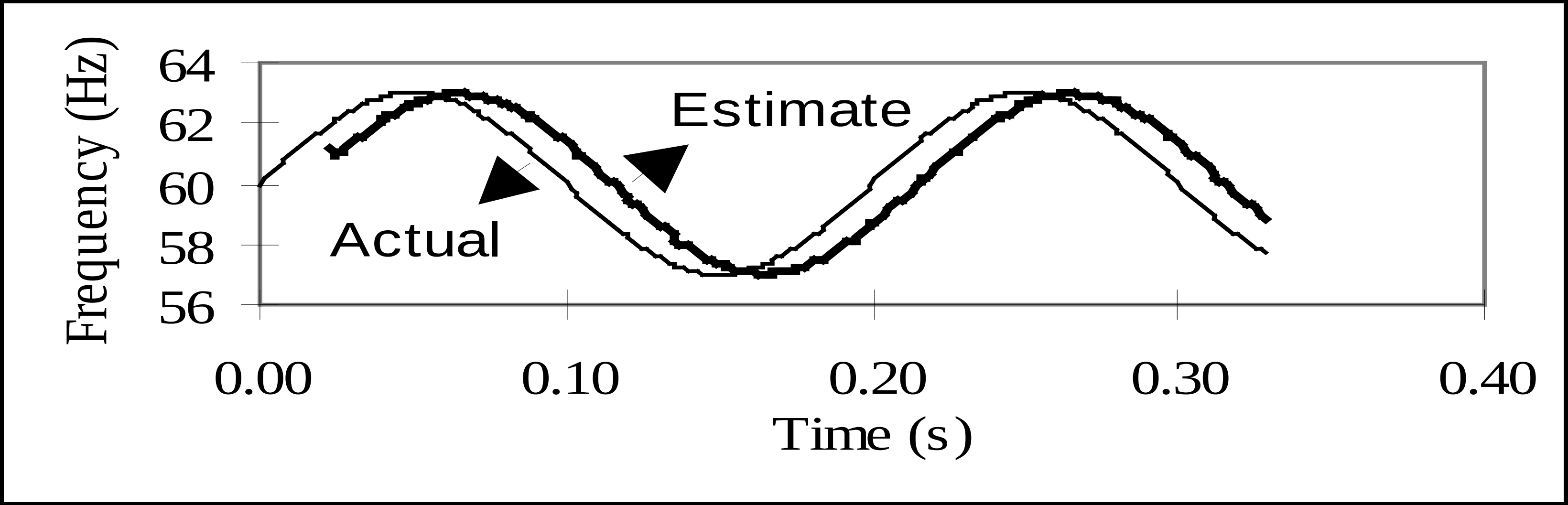
A 60 Hz voltage signal containing second and third harmonics and its frequency estimate





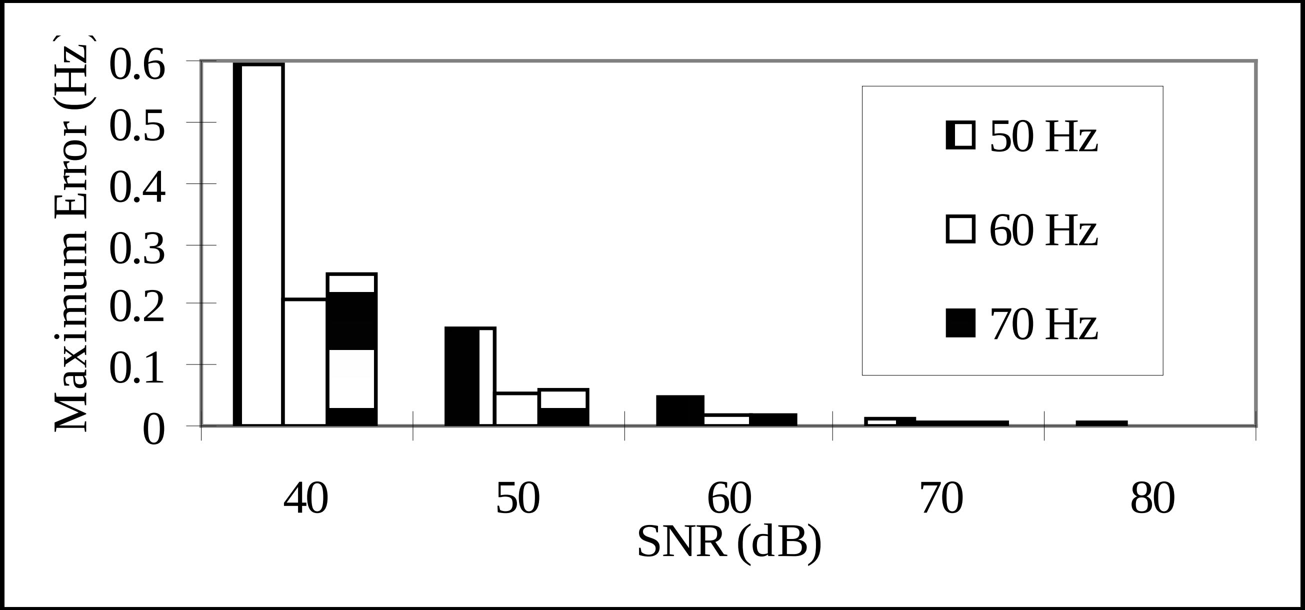
Test Results Test

Actual and estimated frequency of an input signal having dynamic frequency oscillating at 5 Hz



Test Results Test

Maximum estimation errors for input signals with varying degree of noise



Conclusions

- Proposed technique is suitable for measuring near-nominal, nominal and off-nominal frequencies
- Beneficial features
  - Fixed sampling rate – useful for synchronized phasor measurement
  - Fixed data window size
  - Easy implementation
- Accurate estimates within 20 msec
- Maximum estimation error
  - 0.01 Hz for nominal frequency
  - 0.02 Hz for off-nominal frequency
- Modest Computations

Rate of change of frequency

$\text{Ø} = a_0 + a_1 t + a_2 t^2$

For multiple estimations, taken at intervals of ΔT, an overdetermined set of equations can be written as:

$$\begin{bmatrix} \text{Ø}_0 \\ \text{Ø}_0 \\ . \\ . \\ \text{Ø}_{n-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & \Delta T & \Delta T^2 \\ & & \\ & & \\ 1 & (n-1)\,\Delta T & (n-1)^2 \Delta T^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$$

Angles are known, Matrix A is known as times are known (note time reference here is taken at the start of the window); We can now estimate a’s using LES

Rate of change of frequency

Now we can compute frequency and rate of change of frequency

Differentiation of the polynomial gives the frequency:

Frequency = $f_0 + \Delta f = f_0 + (1/2\pi)(\alpha_1 + 2\, a_2\, t)$

Rate of change of frequency = $df/dt = (1/2\pi)(2\, \alpha_2)$