

$$y(t) = (1.0 \sin(2\pi \cdot 60 \cdot t)) \cdot (3.5 \sin(2\pi \cdot 60 \cdot t)) = 3.5 \sin^2(2\pi \cdot 60 \cdot t)$$

The frequency of the signals is $f = 60$ Hz. Therefore, the period of one cycle is $T = \frac{1}{f} = \frac{1}{60}$ seconds.

We need to compute the integral of $y(t)$ from $t = 0$ to $t = T$:

$$\int_0^T 3.5 \sin^2(2\pi \cdot 60 \cdot t) dt$$

We can use the trigonometric identity $\sin^2(\theta) = \frac{1 - \cos(2\theta)}{2}$ to simplify the integrand:

$$3.5 \sin^2(2\pi \cdot 60 \cdot t) = 3.5 \cdot \frac{1 - \cos(2 \cdot 2\pi \cdot 60 \cdot t)}{2} = \frac{3.5}{2} (1 - \cos(240\pi t))$$

Now, we can perform the integration:

$$\int_0^{1/60} \frac{3.5}{2} (1 - \cos(240\pi t)) dt = \frac{3.5}{2} \int_0^{1/60} (1 - \cos(240\pi t)) dt$$

$$= \frac{3.5}{2} \left[t - \frac{\sin(240\pi t)}{240\pi} \right]_0^{1/60}$$

$$= \frac{3.5}{2} \left[\left(\frac{1}{60} - \frac{\sin(240\pi \cdot \frac{1}{60})}{240\pi} \right) - \left(0 - \frac{\sin(0)}{240\pi} \right) \right]$$

$$= \frac{3.5}{2} \left[\frac{1}{60} - \frac{\sin(4\pi)}{240\pi} - 0 + 0 \right]$$

Since $\sin(4\pi) = 0$, the integral simplifies to:

$$\frac{3.5}{2} \cdot \frac{1}{60} = \frac{3.5}{120} = \frac{7}{240}$$

Therefore, the integral of the multiplication of the two signals over one cycle is $\frac{7}{240}$.

Now, we need to multiply this result by $\frac{2}{T}$:

$$\frac{2}{T} \cdot \left(\int_0^T 3.5 \sin^2(2\pi \cdot 60 \cdot t) dt \right) = \frac{2}{1/60} \cdot \frac{7}{240}$$

$$= 3.5$$

$$y(t) = (1.0 \cos(2\pi \cdot 60 \cdot t)) \cdot (3.5 \sin(2\pi \cdot 60 \cdot t)) = 3.5 \cos(2\pi \cdot 60 \cdot t) \sin(2\pi \cdot 60 \cdot t)$$

We can use the trigonometric identity $2 \sin(\theta) \cos(\theta) = \sin(2\theta)$ to simplify the integrand:

$$3.5 \cos(2\pi \cdot 60 \cdot t) \sin(2\pi \cdot 60 \cdot t) = \frac{3.5}{2} \sin(2 \cdot 2\pi \cdot 60 \cdot t) = 1.75 \sin(240\pi t)$$

The period of one cycle is $T = \frac{1}{60}$ seconds. Now, we integrate $y(t)$ from $t = 0$ to $t = T$:

$$\begin{aligned} \int_0^{1/60} 1.75 \sin(240\pi t) dt &= 1.75 \int_0^{1/60} \sin(240\pi t) dt \\ &= 1.75 \left[-\frac{\cos(240\pi t)}{240\pi} \right]_0^{1/60} \\ &= 1.75 \left(-\frac{\cos(240\pi \cdot \frac{1}{60})}{240\pi} - \left(-\frac{\cos(0)}{240\pi} \right) \right) \end{aligned}$$

$$= 1.75 \left(-\frac{\cos(4\pi)}{240\pi} + \frac{\cos(0)}{240\pi} \right)$$

Since $\cos(4\pi) = 1$ and $\cos(0) = 1$, the expression becomes:

$$1.75 \left(-\frac{1}{240\pi} + \frac{1}{240\pi} \right) = 1.75 \cdot 0 = 0$$

The integral of the multiplication of the two signals over one cycle is 0.

Finally, we multiply this result by $\frac{2}{T}$:

$$\frac{2}{T} \cdot 0 = 0$$

Therefore, the final result is 0. This makes sense because the sine and cosine functions are orthogonal over a complete cycle.

$$x_1(t) = 1.0 \sin(2\pi \cdot 60 \cdot t)$$

$$x_2(t) = 3.5 \sin(2\pi \cdot 60 \cdot t + \frac{\pi}{6})$$

Their product is:

$$y(t) = x_1(t) \cdot x_2(t) = 1.0 \sin(2\pi \cdot 60 \cdot t) \cdot 3.5 \sin(2\pi \cdot 60 \cdot t + \frac{\pi}{6})$$

$$y(t) = 3.5 \sin(\omega t) \sin(\omega t + \phi)$$

where $\omega = 2\pi \cdot 60$ and $\phi = \frac{\pi}{6}$.

We can use the product-to-sum trigonometric identity:

$$\sin(A) \sin(B) = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$$

Applying this to our product:

$$y(t) = 3.5 \cdot \frac{1}{2} [\cos(\omega t - (\omega t + \phi)) - \cos(\omega t + (\omega t + \phi))]$$

$y(t) = 1.75 [\cos(-\phi) - \cos(2\omega t + \phi)]$ Since $\cos(-\phi) = \cos(\phi)$, we have:

$1.75 [\cos(\phi) - \cos(2\omega t + \phi)]$ Substituting $\phi = \frac{\pi}{6}$:

$$y(t) = 1.75 [\cos(\frac{\pi}{6}) - \cos(120 \cdot 2\pi \cdot t + \frac{\pi}{6})]$$

$$y(t) = 1.75 \left[\frac{\sqrt{3}}{2} - \cos(240\pi t + \frac{\pi}{6}) \right]$$

Now, we need to integrate $y(t)$ over one cycle, $T = \frac{1}{60}$:

$$\begin{aligned} & \int_0^{1/60} 1.75 \left[\frac{\sqrt{3}}{2} - \cos(240\pi t + \frac{\pi}{6}) \right] dt \\ &= 1.75 \left[\int_0^{1/60} \frac{\sqrt{3}}{2} dt - \int_0^{1/60} \cos(240\pi t + \frac{\pi}{6}) dt \right] \end{aligned}$$

The first integral is straightforward:

$$\int_0^{1/60} \frac{\sqrt{3}}{2} dt = \frac{\sqrt{3}}{2} [t]_0^{1/60} = \frac{\sqrt{3}}{2} \cdot \frac{1}{60} = \frac{\sqrt{3}}{120}$$

So, the integral of the product over one cycle is:

$$1.75 \left[\frac{\sqrt{3}}{120} - 0 \right] = \frac{1.75\sqrt{3}}{120} = \frac{7\sqrt{3}}{4 \cdot 120} = \frac{7\sqrt{3}}{480}$$

Finally, we multiply this result by $\frac{2}{T} = \frac{2}{1/60} = 120$:

$$120 \cdot \frac{7\sqrt{3}}{480} = \frac{120 \cdot 7\sqrt{3}}{480} = \frac{840\sqrt{3}}{480} = \frac{84\sqrt{3}}{48} = \frac{21\sqrt{3}}{12} = \frac{7\sqrt{3}}{4}$$

Therefore, the final result is $\frac{7\sqrt{3}}{4}$.

$$x_1(t) = 1.0 \cos(2\pi \cdot 60 \cdot t)$$

$$x_2(t) = 3.5 \sin(2\pi \cdot 60 \cdot t + \frac{\pi}{6})$$

Their product is:

$$y(t) = x_1(t) \cdot x_2(t) = 1.0 \cos(2\pi \cdot 60 \cdot t) \cdot 3.5 \sin(2\pi \cdot 60 \cdot t + \frac{\pi}{6})$$

$$y(t) = 3.5 \cos(\omega t) \sin(\omega t + \phi)$$

where $\omega = 2\pi \cdot 60$ and $\phi = \frac{\pi}{6}$.

We can use the product-to-sum trigonometric identity:

$$\cos(A) \sin(B) = \frac{1}{2} [\sin(A + B) - \sin(A - B)]$$

Applying this to our product:

$$y(t) = 3.5 \cdot \frac{1}{2} [\sin(\omega t + (\omega t + \phi)) - \sin(\omega t - (\omega t + \phi))]$$

$y(t) = 1.75 [\sin(2\omega t + \phi) - \sin(-\phi)]$ Since $\sin(-\phi) = -\sin(\phi)$, we have: $y(t) = 1.75 [\sin(2\omega t + \phi) + \sin(\phi)]$ Substituting $\phi = \frac{\pi}{6}$:

$$y(t) = 1.75 [\sin(120 \cdot 2\pi \cdot t + \frac{\pi}{6}) + \sin(\frac{\pi}{6})]$$

$$y(t) = 1.75\left[\sin\left(240\pi t + \frac{\pi}{6}\right) + \frac{1}{2}\right]$$

Now, we need to integrate $y(t)$ over one cycle, $T = \frac{1}{60}$:

$$\begin{aligned} & \int_0^{1/60} 1.75 \left[\sin\left(240\pi t + \frac{\pi}{6}\right) + \frac{1}{2} \right] dt \\ &= 1.75 \left[\int_0^{1/60} \sin\left(240\pi t + \frac{\pi}{6}\right) dt + \int_0^{1/60} \frac{1}{2} dt \right] \end{aligned}$$

For the first integral, we use the substitution $u = 240\pi t + \frac{\pi}{6}$, so $du = 240\pi dt$, and $dt = \frac{du}{240\pi}$. The limits of integration change from $t = 0$ to $u = \frac{\pi}{6}$, and from $t = \frac{1}{60}$ to $u = 240\pi \cdot \frac{1}{60} + \frac{\pi}{6} = 4\pi + \frac{\pi}{6} = \frac{25\pi}{6}$.

$$\int_0^{1/60} \sin\left(240\pi t + \frac{\pi}{6}\right) dt = \int_{\pi/6}^{25\pi/6} \sin(u) \frac{du}{240\pi} = \frac{1}{240\pi} [-\cos(u)]_{\pi/6}^{25\pi/6}$$

$= \frac{1}{240\pi} \left(-\cos\left(\frac{25\pi}{6}\right) - \left(-\cos\left(\frac{\pi}{6}\right)\right) \right)$ Since $\cos\left(\frac{25\pi}{6}\right) = \cos\left(4\pi + \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$, the first integral becomes:

$$\frac{1}{240\pi} \left(-\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \right) = 0$$

The second integral is straightforward:

$$\int_0^{1/60} \frac{1}{2} dt = \frac{1}{2} [t]_0^{1/60} = \frac{1}{2} \cdot \frac{1}{60} = \frac{1}{120}$$

So, the integral of the product over one cycle is:

$$1.75 \left[0 + \frac{1}{120} \right] = \frac{1.75}{120} = \frac{7}{4 \cdot 120} = \frac{7}{480}$$

Finally, we multiply this result by $\frac{2}{T} = \frac{2}{1/60} = 120$:

$$120 \cdot \frac{7}{480} = \frac{120 \cdot 7}{480} = \frac{840}{480} = \frac{84}{48} = \frac{21}{12} = \frac{7}{4}$$

Therefore, the final result is $\frac{7}{4}$.

$$\arctan \left(\frac{1}{1.732} \right) \approx 30^\circ$$

Magnitude = 3.5