The product of the two signals is:

$$y(t) = \sin(\omega t)\sin(\omega t + heta)$$

We can use the product-to-sum trigonometric identity:

$$\sin(A)\sin(B) = \frac{1}{2}[\cos(A-B) - \cos(A+B)]$$

Applying this to our product, with  $A=\omega t$  and  $B=\omega t+ heta$ :

$$y(t) = rac{1}{2}[\cos(\omega t - (\omega t + heta)) - \cos(\omega t + (\omega t + heta))]$$

$$y(t) = rac{1}{2}[\cos(- heta) - \cos(2\omega t + heta)]$$

Since  $\cos(-\theta) = \cos(\theta)$ , we have:

$$y(t) = rac{1}{2}[\cos( heta) - \cos(2\omega t + heta)]$$

Now, we integrate y(t) over one cycle from t=0 to  $t=T=\frac{2\pi}{\omega}$ :

$$\int_0^{2\pi/\omega} rac{1}{2} [\cos( heta) - \cos(2\omega t + heta)] \, dt$$

$$=rac{1}{2}\int_{0}^{2\pi/\omega}\cos( heta)\,dt-rac{1}{2}\int_{0}^{2\pi/\omega}\cos(2\omega t+ heta)\,dt$$

The first integral is straightforward since  $\cos(\theta)$  is constant with respect to t:

$$rac{1}{2}\int_0^{2\pi/\omega}\cos( heta)\,dt = rac{1}{2}[\cos( heta)\cdot t]_0^{2\pi/\omega} = rac{1}{2}\cos( heta)\cdotrac{2\pi}{\omega} = rac{\pi}{\omega}\cos( heta)$$

For the second integral, let  $u=2\omega t+\theta$ , so  $du=2\omega dt$ , and  $dt=\frac{du}{2\omega}$ . The limits of integration change from t=0 to  $u=\theta$ , and from  $t=\frac{2\pi}{\omega}$  to  $u=2\omega\cdot\frac{2\pi}{\omega}+\theta=4\pi+\theta$ .

$$rac{1}{2}\int_0^{2\pi/\omega}\cos(2\omega t+ heta)\,dt=rac{1}{2}\int_{ heta}^{4\pi+ heta}\cos(u)rac{du}{2\omega}=rac{1}{4\omega}[\sin(u)]_{ heta}^{4\pi+ heta}$$

 $= \frac{1}{4\omega}(\sin(4\pi+\theta)-\sin(\theta))$ \$Since  $\sin(4\pi+\theta)=\sin(\theta)$ , the second integral becomes:

$$\frac{1}{4\omega}(\sin(\theta) - \sin(\theta)) = \frac{1}{4\omega}(0) = 0$$

Combining the results of the two integrals, the integral of the product over one cycle is:

$$\frac{\pi}{\omega}\cos(\theta) - 0 = \frac{\pi}{\omega}\cos(\theta)$$

Final Answer: The final answer is  $\boxed{\frac{\pi}{\omega}\cos(\theta)}$ 

This is the real part

Now correlate with 1.0cos(wt)

The product of the two signals is:

$$y(t) = \cos(\omega t)\sin(\omega t + \theta)$$

We can use the product-to-sum trigonometric identity:

$$\cos(A)\sin(B) = rac{1}{2}[\sin(A+B) - \sin(A-B)]$$

Applying this with  $A=\omega t$  and  $B=\omega t+\theta$ :

$$y(t) = rac{1}{2}[\sin(\omega t + (\omega t + heta)) - \sin(\omega t - (\omega t + heta))]$$

$$y(t) = rac{1}{2}[\sin(2\omega t + heta) - \sin(- heta)]$$

Since  $\sin(- heta) = -\sin( heta)$ , we have:

$$y(t) = rac{1}{2}[\sin(2\omega t + heta) + \sin( heta)]$$

Now, we integrate y(t) over one cycle from t=0 to  $t=T=rac{2\pi}{\omega}$ :

$$\int_0^{2\pi/\omega} rac{1}{2} [\sin(2\omega t + heta) + \sin( heta)] \, dt$$

$$=rac{1}{2}\int_{0}^{2\pi/\omega}\sin(2\omega t+ heta)\,dt+rac{1}{2}\int_{0}^{2\pi/\omega}\sin( heta)\,dt$$

The second integral is straightforward since  $\sin(\theta)$  is constant with respect to t:

$$rac{1}{2}\int_0^{2\pi/\omega}\sin( heta)\,dt = rac{1}{2}[\sin( heta)\cdot t]_0^{2\pi/\omega} = rac{1}{2}\sin( heta)\cdotrac{2\pi}{\omega} = rac{\pi}{\omega}\sin( heta)$$

For the first integral, let  $u=2\omega t+\theta$ , so  $du=2\omega dt$ , and  $dt=\frac{du}{2\omega}$ . The limits of integration change from t=0 to  $u=\theta$ , and from  $t=\frac{2\pi}{\omega}$  to  $u=2\omega\cdot\frac{2\pi}{\omega}+\theta=4\pi+\theta$ .

$$rac{1}{2}\int_0^{2\pi/\omega}\sin(2\omega t+ heta)\,dt=rac{1}{2}\int_ heta^{4\pi+ heta}\sin(u)rac{du}{2\omega}=rac{1}{4\omega}[-\cos(u)]_ heta^{4\pi+ heta}$$

 $= \frac{1}{4\omega}(-\cos(4\pi+\theta)-(-\cos(\theta)))$ \$Since  $\cos(4\pi+\theta)=\cos(\theta)$ , the first integral becomes:

$$\frac{1}{4\omega}(-\cos(\theta)+\cos(\theta)) = \frac{1}{4\omega}(0) = 0$$

Combining the results of the two integrals, the integral of the product over one cycle is:

$$0+rac{\pi}{\omega}\sin( heta)=rac{\pi}{\omega}\sin( heta)$$

Final Answer: The final answer is  $\frac{\pi}{\omega}\sin(\theta)$ 

This is the imaginary part.