

The product of the two signals is:

$$y(t) = \sin(\omega t) \sin(\omega t + \theta)$$

We can use the product-to-sum trigonometric identity:

$$\sin(A) \sin(B) = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$$

Applying this to our product, with $A = \omega t$ and $B = \omega t + \theta$:

$$y(t) = \frac{1}{2} [\cos(\omega t - (\omega t + \theta)) - \cos(\omega t + (\omega t + \theta))]$$

$$y(t) = \frac{1}{2} [\cos(-\theta) - \cos(2\omega t + \theta)]$$

Since $\cos(-\theta) = \cos(\theta)$, we have:

$$y(t) = \frac{1}{2} [\cos(\theta) - \cos(2\omega t + \theta)]$$

Now, we integrate $y(t)$ over one cycle from $t = 0$ to $t = T = \frac{2\pi}{\omega}$:

$$\int_0^{2\pi/\omega} \frac{1}{2} [\cos(\theta) - \cos(2\omega t + \theta)] dt$$

$$= \frac{1}{2} \int_0^{2\pi/\omega} \cos(\theta) dt - \frac{1}{2} \int_0^{2\pi/\omega} \cos(2\omega t + \theta) dt$$

The first integral is straightforward since $\cos(\theta)$ is constant with respect to t :

$$\frac{1}{2} \int_0^{2\pi/\omega} \cos(\theta) dt = \frac{1}{2} [\cos(\theta) \cdot t]_0^{2\pi/\omega} = \frac{1}{2} \cos(\theta) \cdot \frac{2\pi}{\omega} = \frac{\pi}{\omega} \cos(\theta)$$

For the second integral, let $u = 2\omega t + \theta$, so $du = 2\omega dt$, and $dt = \frac{du}{2\omega}$. The limits of integration change from $t = 0$ to $u = \theta$, and from $t = \frac{2\pi}{\omega}$ to $u = 2\omega \cdot \frac{2\pi}{\omega} + \theta = 4\pi + \theta$.

$$\frac{1}{2} \int_0^{2\pi/\omega} \cos(2\omega t + \theta) dt = \frac{1}{2} \int_{\theta}^{4\pi+\theta} \cos(u) \frac{du}{2\omega} = \frac{1}{4\omega} [\sin(u)]_{\theta}^{4\pi+\theta}$$

$= \frac{1}{4\omega} (\sin(4\pi + \theta) - \sin(\theta))$ Since $\sin(4\pi + \theta) = \sin(\theta)$, the second integral becomes:

$$\frac{1}{4\omega} (\sin(\theta) - \sin(\theta)) = \frac{1}{4\omega} (0) = 0$$

Combining the results of the two integrals, the integral of the product over one cycle is:

$$\frac{\pi}{\omega} \cos(\theta) - 0 = \frac{\pi}{\omega} \cos(\theta)$$

Final Answer: The final answer is $\boxed{\frac{\pi}{\omega} \cos(\theta)}$

This is the real part

Now correlate with $1.0\cos(\omega t)$

The product of the two signals is:

$$y(t) = \cos(\omega t) \sin(\omega t + \theta)$$

We can use the product-to-sum trigonometric identity:

$$\cos(A) \sin(B) = \frac{1}{2} [\sin(A + B) - \sin(A - B)]$$

Applying this with $A = \omega t$ and $B = \omega t + \theta$:

$$y(t) = \frac{1}{2} [\sin(\omega t + (\omega t + \theta)) - \sin(\omega t - (\omega t + \theta))]$$

$$y(t) = \frac{1}{2} [\sin(2\omega t + \theta) - \sin(-\theta)]$$

Since $\sin(-\theta) = -\sin(\theta)$, we have:

$$y(t) = \frac{1}{2} [\sin(2\omega t + \theta) + \sin(\theta)]$$

Now, we integrate $y(t)$ over one cycle from $t = 0$ to $t = T = \frac{2\pi}{\omega}$:

$$\begin{aligned} & \int_0^{2\pi/\omega} \frac{1}{2} [\sin(2\omega t + \theta) + \sin(\theta)] dt \\ &= \frac{1}{2} \int_0^{2\pi/\omega} \sin(2\omega t + \theta) dt + \frac{1}{2} \int_0^{2\pi/\omega} \sin(\theta) dt \end{aligned}$$

The second integral is straightforward since $\sin(\theta)$ is constant with respect to t :

$$\frac{1}{2} \int_0^{2\pi/\omega} \sin(\theta) dt = \frac{1}{2} [\sin(\theta) \cdot t]_0^{2\pi/\omega} = \frac{1}{2} \sin(\theta) \cdot \frac{2\pi}{\omega} = \frac{\pi}{\omega} \sin(\theta)$$

For the first integral, let $u = 2\omega t + \theta$, so $du = 2\omega dt$, and $dt = \frac{du}{2\omega}$. The limits of integration change from $t = 0$ to $u = \theta$, and from $t = \frac{2\pi}{\omega}$ to $u = 2\omega \cdot \frac{2\pi}{\omega} + \theta = 4\pi + \theta$.

$$\frac{1}{2} \int_0^{2\pi/\omega} \sin(2\omega t + \theta) dt = \frac{1}{2} \int_{\theta}^{4\pi+\theta} \sin(u) \frac{du}{2\omega} = \frac{1}{4\omega} [-\cos(u)]_{\theta}^{4\pi+\theta}$$

$= \frac{1}{4\omega} (-\cos(4\pi + \theta) - (-\cos(\theta)))$ Since $\cos(4\pi + \theta) = \cos(\theta)$, the first integral becomes:

$$\frac{1}{4\omega} (-\cos(\theta) + \cos(\theta)) = \frac{1}{4\omega} (0) = 0$$

Combining the results of the two integrals, the integral of the product over one cycle is:

$$0 + \frac{\pi}{\omega} \sin(\theta) = \frac{\pi}{\omega} \sin(\theta)$$

Final Answer: The final answer is $\boxed{\frac{\pi}{\omega} \sin(\theta)}$

This is the imaginary part.