Quiz 2: DIC on Discrete Structures

25 marks, 55min

27 Oct 2023

Instructions:

- Attempt *all* questions. Write all answers and proofs carefully. If you are *making any assumptions or using results proved in class, state them clearly.*
- In this course, one of our aims is to learn how to write good proofs, hence considerable weightage will be given to *clarity and completeness* of proofs.
- Do _not_ copy or use any other unfair means. Offenders will be reported to the Disciplinary Action Committee.

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1. [2+2=4 marks] True or False. If True write a 2-3 line justification. If False, give a counter-example or a justification as applicable. Recall that a simple graph has at most one edge between two vertices and does not have self-loops.
   (a) If two simple graphs have same number of vertices and edges, then they must be isomorphic.
   (b) If simple graph G has 50 vertices, each having degree 5, then the number of edges of G is 100.
2. [3+3 = 6 marks] A piece of wire is 120 cm long.
   (a) Can one bend it to form the edges of a cube, each of whose edges is 10cm? Why or why not?
   (b) What is the smallest number of **cuts** one must make in the wire, so as to be able to form the required cube? Justify. (Note that the question asks for number of cuts, not number of pieces.)
3. [2 + 4 + 4 = 10 marks] A flag consists of n horizontal stripes, where each stripe can be any one of three colors, red, blue and green, such that no two adjacent stripes have same color.
   (a) How many such flags are possible (as a function of n)? Why?

   Now suppose that, in order to avoid flying the flag upside down, the top and bottom stripes are required to be of different colors. Let a_n denote the number of such flags with n stripes.

   (b) Write a recurrence relation for a_n, along with initial conditions.
   (c) Solve the recurrence to obtain an expression for a_n in terms of n. Simplify it as much as possible.
4. [5 marks] Suppose we take a complete (simple) graph on 6 vertices where all edges are colored red or blue. We saw in class that it must have one monochromatic triangle (i.e., triangle whose all sides are red or all are blue). Show that it must have 2 monochromatic triangles! That is, it must be that there must be two triangles, such that their edges are all red or all blue. (It may be that one of them has all red and other has all blue or both triangles are all blue or both are all red).