Quiz 1: DIC on Discrete Structures

20 marks, 50min

1 Sep 2023

Instructions:

- Attempt *all* questions. Write all answers and proofs carefully. If you are *making any assumptions or using results proved in class, state them clearly.*
- All sets considered below are general sets that can be infinite. Hence, you must not assume that they are finite or countable, unless clearly specified otherwise.
- In this course, one of our aims is to learn how to write good proofs, hence considerable weightage will be given to *clarity and completeness* of proofs.
- Do <u>not</u> copy or use any other unfair means. Offenders will be reported to the Disciplinary Action Committee.

---

1. [2 marks] Prove that, if a, b are both prime numbers (> 1), and $a \neq b$, then $\log_b a$ is irrational.
2. [2 marks] Answer True or False. Also give a 1-line justification for each.
   (a) Every non-empty subset of integers has a smallest element.
   (b) If a proposition is true, then its converse must also be true.
3. [3 marks] Prove by induction that for all $n \in \mathbb{N}$, 3^n is odd.
4. [4 marks] Prove or disprove: For any two sets A, B, there exists a surjection from A to B iff there exists an injection from B to A.
5. Which of the following sets are countable? Justify with formal proof. You may assume that countable union of countable sets is countable.
   (a) [4 marks] Set of all functions from $\mathbb{N}$ to $\mathbb{N}$.
   (b) [5 marks] Set of all non-increasing functions from $\mathbb{N}$ to $\mathbb{N}$. A function $f : \mathbb{N} \to \mathbb{N}$ is said to be non-increasing if for all $x, y \in \mathbb{N}$, if $x \leq y$ then $f(x) \geq f(y)$.