Circuit Analysis Using Sympy
Assignment 7

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1 Introduction

In this assignment, we use Sympy to analytically solve a matrix equation governing an analog circuit. We look at two circuits, an active low pass filter and an active high pass filter. We create matrices using node equations for the circuits in sympy, and then solve the equations analytically. We then convert the resulting sympy solution into a numpy function which can be called. We then use the signals toolbox we studied in the last assignment to understand the responses of the two circuits to various inputs.

Importing required packages

```
In [1]: from sympy import *
        import numpy as np
        import matplotlib.pyplot as plt
        import scipy.signal as sp
        from pylab import *
        from IPython.display import *
```

2 Low pass Filter

where G =1.586 and R1 = R2 = 10kΩ and C1=C2=10pF. This gives a 3dB Butter-worth filter with cutoff frequency of $1/2\pi$MHz.

Circuit Equations are as follows:

$$V_m = \frac{V_o}{G}$$

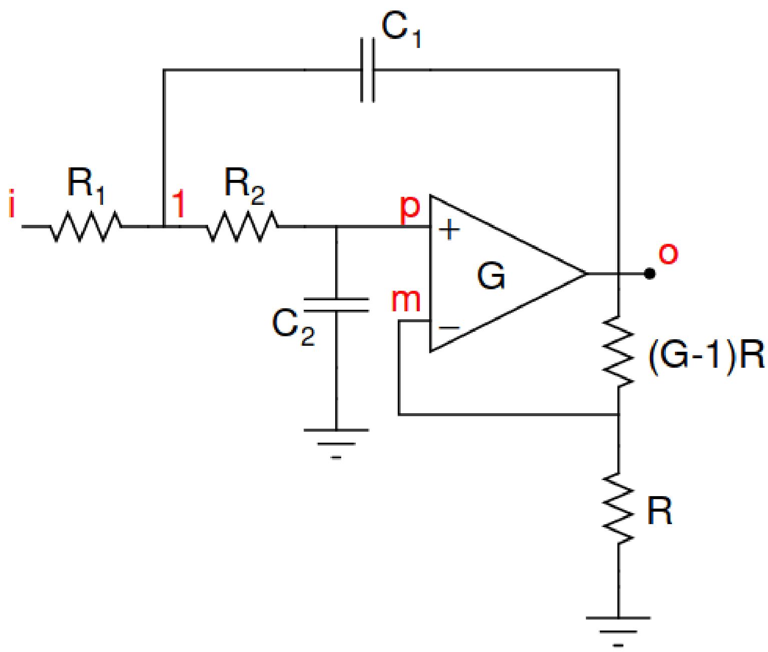
$$V_p = V_1 \frac{1}{1 + sR_2C_2}$$

$$V_o = G(V_p - V_m)$$

$$\frac{V_i - V_1}{R_1} + \frac{V_p - V_1}{R_2} + sC_1(V_0 - V_1) = 0$$

Solving the above equations with approxmtion gives

$$V_o \approx \frac{V_i}{sR_1C_1}$$



Circuit1

We would like to solve this in Python and also get (and plot) the exact result. For this we need the sympy module.

To solve the equtions exactly we use matrix method of solving:

```
In [2]: init_printing()
        R1,R2,C1,C2,G = symbols("R1 R2 C1 C2 G")
        V1,Vp,Vm,Vo,Vi = symbols("V1 Vp Vm Vo Vi")
        s = symbols("s")
        A = Matrix([[0,0,1,-1/G],
                    [-1/(1+s*R2*C2),1,0,0],
                    [0,-G,G,1],
                  [-1/R1-1/R2-s*C1,1/R2,0,s*C1]])
        M = Matrix([V1,Vp,Vm,Vo])
        b = Matrix([0,0,0,Vi/R1])
        display(Eq(MatMul(A,M),b))
```

$$\begin{bmatrix} 0 & 0 & 1 & -\frac{1}{G} \\ -\frac{1}{C_2 R_2 s+1} & 1 & 0 & 0 \\ 0 & -G & G & 1 \\ -C_1 s - \frac{1}{R_2} - \frac{1}{R_1} & \frac{1}{R_2} & 0 & C_1 s \end{bmatrix} \begin{bmatrix} V_1 \\ Vp \\ Vm \\ Vo \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{Vi}{R_1} \end{bmatrix}$$

Solving the above matrix yield exact result

Function defining low pass filter:

```
In [3]: def lowpass(R1=10**4,R2=10**4,C1=10**-11,C2=10**-11,G=1.586,Vi=1):
            s=symbols("s")
```

```
    A=Matrix([[0,0,1,-1/G],
              [-1/(1+s*R2*C2),1,0,0],
              [0,-G,G,1],
              [-1/R1-1/R2-s*C1,1/R2,0,s*C1]])
    b=Matrix([0,0,0,Vi/R1])
    V = A.inv()*b
    return(A,b,V)
```

Function which can take input in laplace domain or time domain and give the output of low pass filter:

```
In [4]: def low_pass_output(laplace_fn = None,time_fn=None,t=np.linspace(0,1e-5,1e5),C=10**-11):
            A,b,V = lowpass(C1=C,C2=C)
            v_low_pass = V[-1]
            temp = expand(simplify(v_low_pass))
            n,d = fraction(temp)
            n,d = Poly(n,s),Poly(d,s)
            num,den = n.all_coeffs(),d.all_coeffs()
            H_v_low_pass = sp.lti([-float(f) for f in num],[float(f) for f in den])
            if laplace_fn !=None:
                temp = expand(simplify(laplace_fn))
                n,d = fraction(temp)
                n,d = Poly(n,s),Poly(d,s)
                num,den = n.all_coeffs(),d.all_coeffs()
                lap = sp.lti([float(f) for f in num],[float(f) for f in den])
                t,u = sp.impulse(lap,None,t)
            else:
                u = time_fn
            t,V_out,svec = sp.lsim(H_v_low_pass,u,t)
            return (t,V_out)
```

```
/home/suhas/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:1: DeprecationWarning: o
  """Entry point for launching an IPython kernel.
```

3 High pass filter

values you can use are R1=R3=10kΩ, C1=C2=1nF, and G=1.586

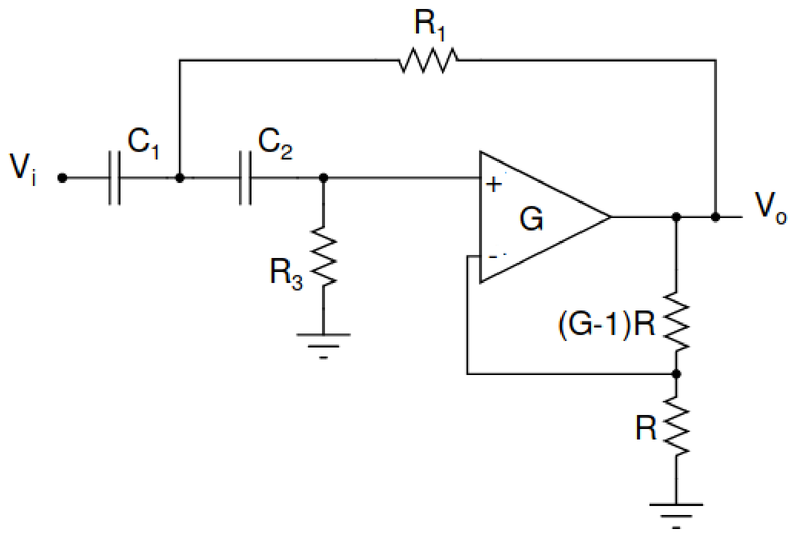
Circuit Equations are as follows:

$$V_n = \frac{V_o}{G}$$

$$V_p = V_1 \frac{sR_3C_2}{1 + sR_3C_2}$$

$$V_o = G(V_p - V_n)$$

$$(V_1 - V_i)sC_1 + \frac{(V_1 - V_o)}{R_1} + (V_i - V_p)sC_2 = 0$$



High pass filter

In [5]:
```
R1, R3, C1, C2, G, Vi = symbols('R_1 R_3 C_1 C_2 G V_i')
V1,Vn,Vp,Vo = symbols('V_1 V_n V_p V_o')
x=Matrix([V1,Vn,Vp,Vo])

A=Matrix([[0,-1,0,1/G],
        [s*C2*R3/(s*C2*R3+1),0,-1,0],
        [0,G,-G,1],
        [-s*C2-1/R1-s*C1,0,s*C2,1/R1]])

b=Matrix([0,0,0,-Vi*s*C1])
init_printing
display(Eq(MatMul(A,x),b))
```

$$\begin{bmatrix} 0 & -1 & 0 & \frac{1}{G} \\ \frac{C_2R_3s}{C_2R_3s+1} & 0 & -1 & 0 \\ 0 & G & -G & 1 \\ -C_1s-C_2s-\frac{1}{R_1} & 0 & C_2s & \frac{1}{R_1} \end{bmatrix} \begin{bmatrix} V_1 \\ V_n \\ V_p \\ V_o \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -C_1V_is \end{bmatrix}$$

Function defining high pass filter:

In [6]:
```
def highpass(R1=10**4,R3=10**4,C1=10**-9,C2=10**-9,G=1.586,Vi=1):
    s= symbols("s")
    A=Matrix([[0,-1,0,1/G],
        [s*C2*R3/(s*C2*R3+1),0,-1,0],
        [0,G,-G,1],
        [-s*C2-1/R1-s*C1,0,s*C2,1/R1]])
```

```
        b=Matrix([0,0,0,-Vi*s*C1])
        V =A.inv() * b
        return (A,b,V)
```

Function which can take input in laplace domain or time domain and give the output of high pass filter:

```
In [7]: def high_pass_output(laplace_fn = None,time_fn=None,t=np.linspace(0,1e-4,1e5),C=10**-11)
            A,b,V = highpass(C1=C,C2=C)
            v_high_pass = V[-1]
            temp = expand(simplify(v_high_pass))
            n,d = fraction(temp)
            n,d = Poly(n,s),Poly(d,s)
            num,den = n.all_coeffs(),d.all_coeffs()
            H_v_high_pass = sp.lti([float(f) for f in num],[float(f) for f in den])
            if laplace_fn !=None:
                temp = expand(simplify(laplace_fn))
                n,d = fraction(temp)
                n,d = Poly(n,s),Poly(d,s)
                num,den = n.all_coeffs(),d.all_coeffs()
                lap = sp.lti([float(f) for f in num],[float(f) for f in den])
                t,u = sp.impulse(lap,None,t)
            else:
                u = time_fn
            t,V_out,svec = sp.lsim(H_v_high_pass,u,t)
            return (t,V_out)
```

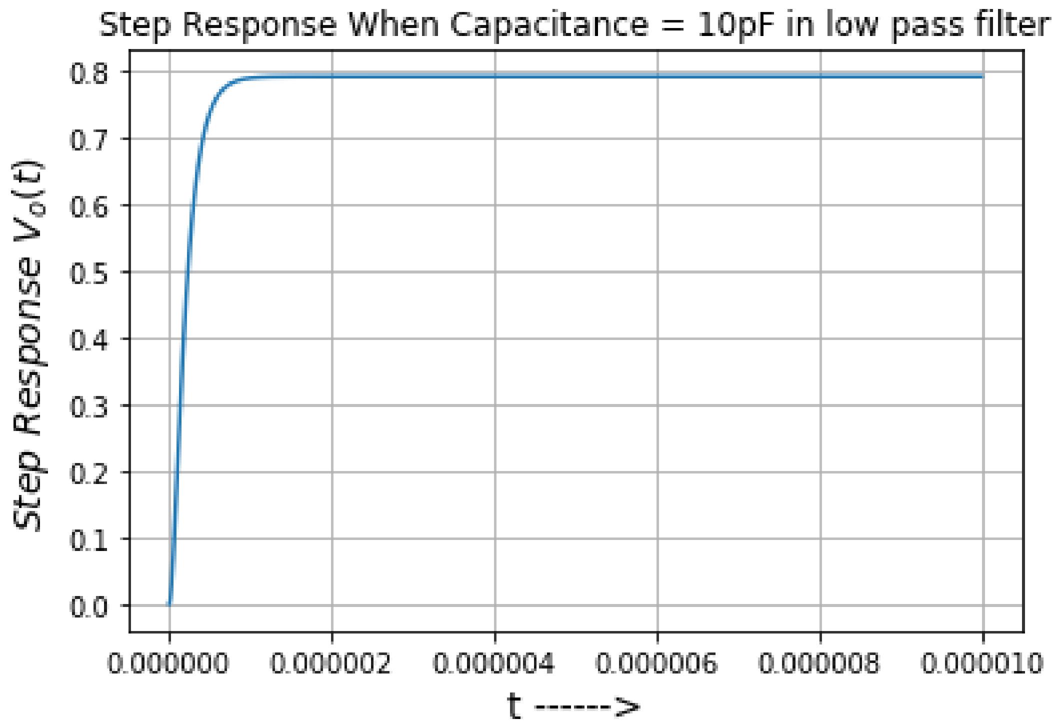
```
/home/suhas/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:2: DeprecationWarning: o
```

4 Question1

Step Response for low pass filter

```
In [8]: t,V_low_step = low_pass_output(laplace_fn=1/s)
```

```
In [9]: plt.plot(t,V_low_step)
        plt.grid(True)
        plt.xlabel("t ------>",size=14)
        plt.ylabel(r"$Step\ Response\ V_{o}(t)$",size=14)
        plt.title("Step Response When Capacitance = 10pF in low pass filter")
        plt.show()
```



Step response is starting from zero and reaching 0.793 at steady state.This is because DC gain oftransfer function is 0.793.Initial value is 0 because AC gain of low pass filter is zero(impulse can be assumed as High frequency signal and we know low pass filter dosen't pass high frequency signal).

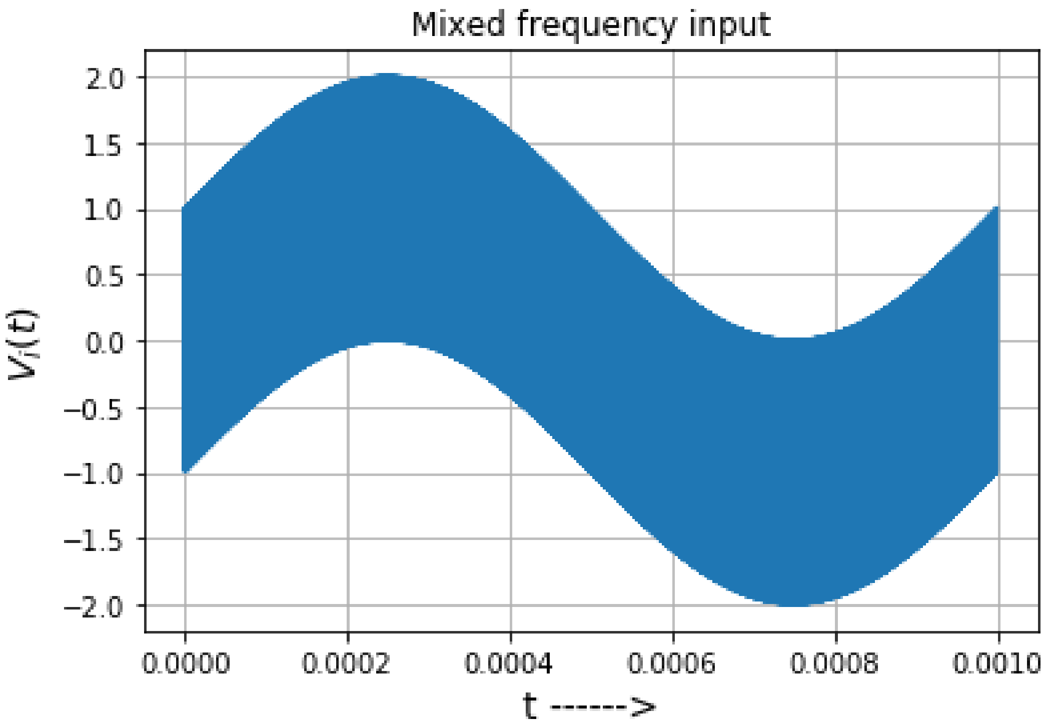
5 Question2

Finding Output when input signal is

$$(sin(2000\pi t) + cos(2 \times 106\pi t))u_o(t)$$

```
In [10]: t = np.linspace(0,1e-3,1e5)
         plt.plot(t,np.sin(2000*np.pi*t)+np.cos(2e6*np.pi*t))
         plt.grid(True)
         plt.xlabel("t ------>",size=14)
         plt.ylabel(r"$V_{i}(t)$",size=14)
         plt.title("Mixed frequency input")
         plt.show()
```

```
/home/suhas/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:1: DeprecationWarning: o
  """Entry point for launching an IPython kernel.
```

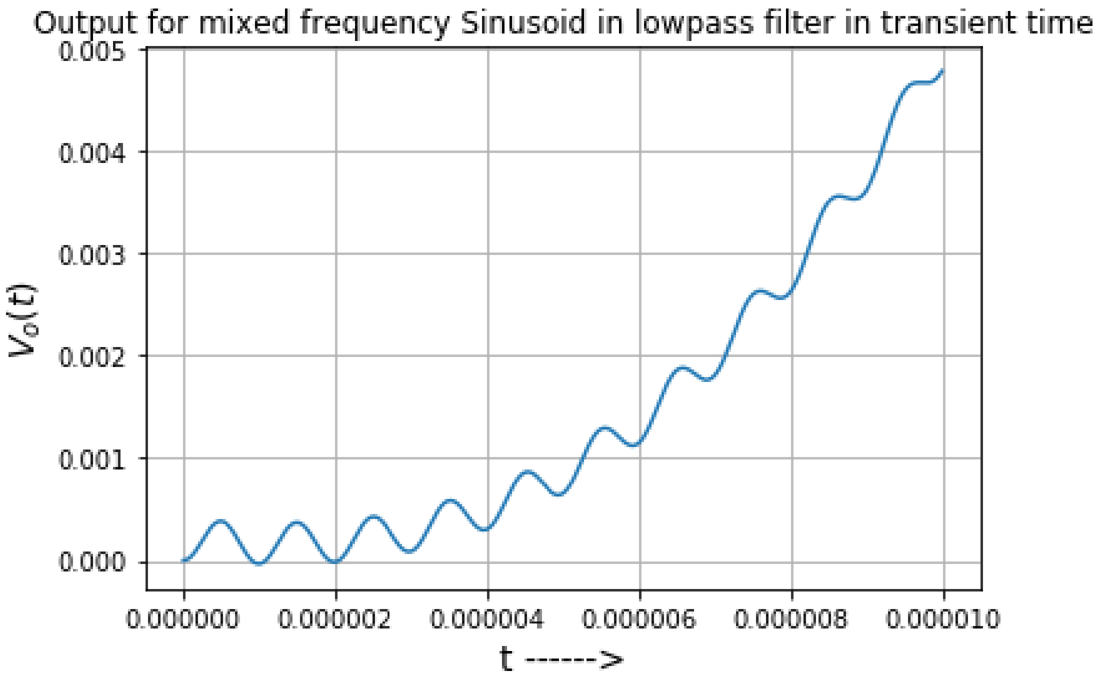


Band is high frequency wave and envolope is the low frequency wave

```
In [11]: t = linspace(0,1e-5,1e5)
         t,vout = low_pass_output(time_fn=np.sin(2000*np.pi*t)+np.cos(2e6*np.pi*t),t=t,C=10**-9)

/home/suhas/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:2: DeprecationWarning: o
```

```
In [12]: plt.plot(t,vout)
         plt.grid(True)
         plt.xlabel("t ------>",size=14)
         plt.ylabel(r"$V_{o}(t)$",size=14)
         plt.title("Output for mixed frequency Sinusoid in lowpass filter in transient time")
         plt.show()
```

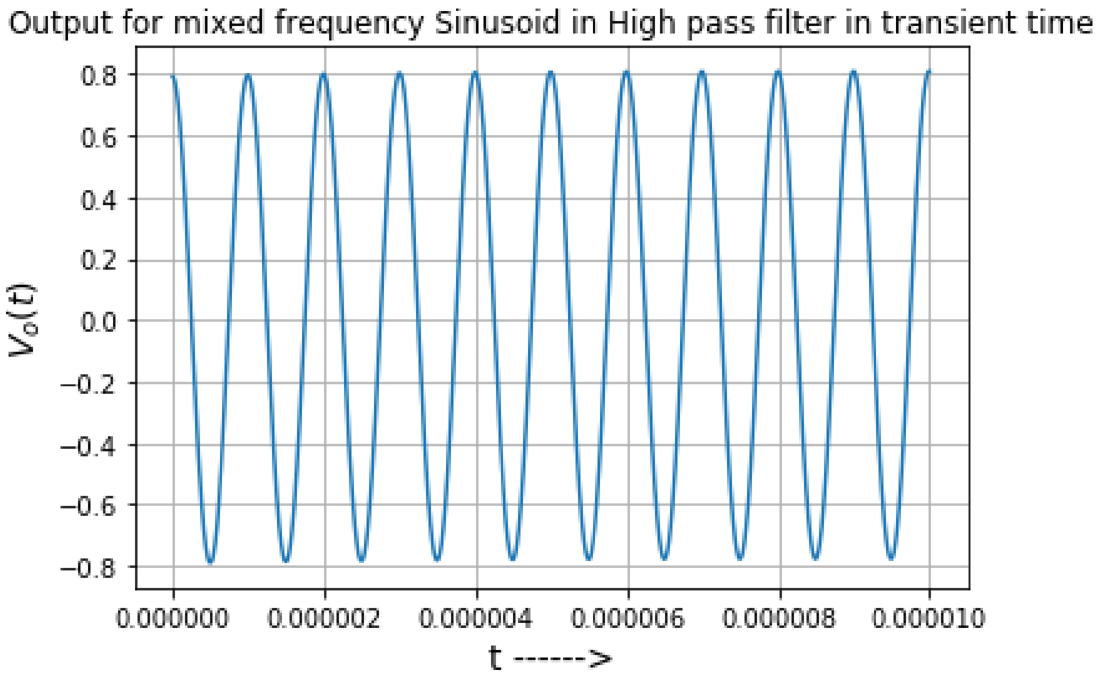


From above we can clearly see that Output is superposition of High Amplitude low frequency wave and Low amplitude High frquency wave(Since Low pass filter attenuates the High frequencies)

```
In [13]: t = linspace(0,1e-5,1e5)
         t,vout = high_pass_output(time_fn=np.sin(2000*np.pi*t)+np.cos(2e6*np.pi*t),t=t,C=10**-9
```

```
/home/suhas/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:2: DeprecationWarning: o
```

```
In [14]: plt.plot(t,vout)
         plt.grid(True)
         plt.xlabel("t ------>",size=14)
         plt.ylabel(r"$V_{o}(t)$",size=14)
         plt.title("Output for mixed frequency Sinusoid in High pass filter in transient time")
         plt.show()
```

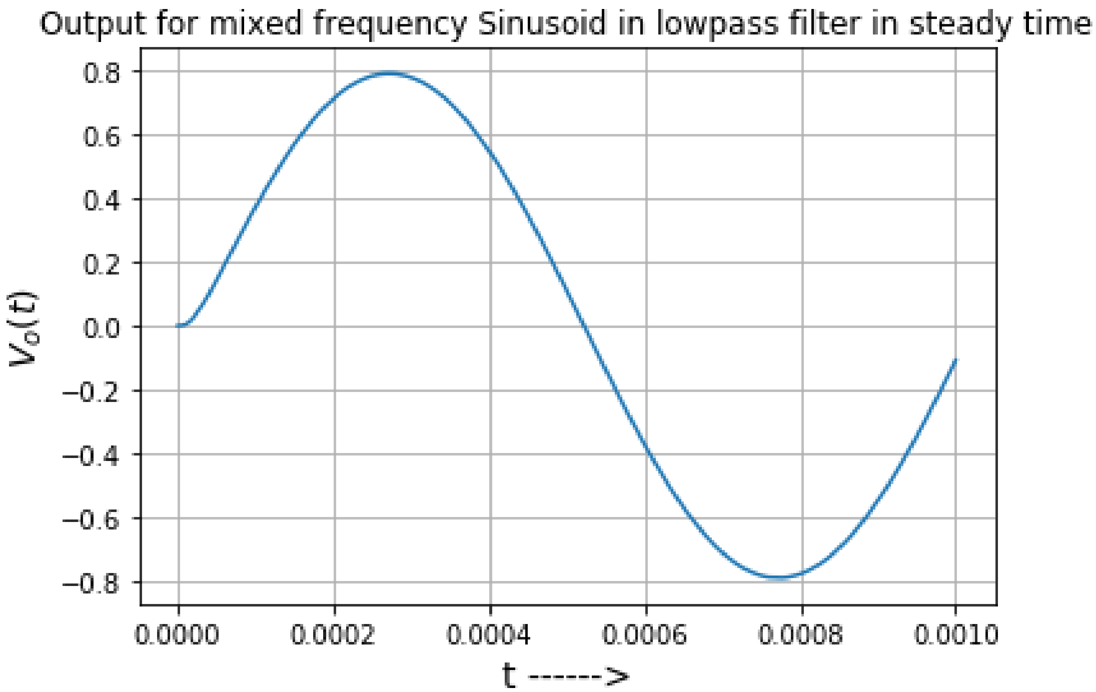


The plot which is appearing to be band(closely placed lines) is superposition of High Amplitude High frequency wave and Low amplitude Low frquency wave(Since High pass filter attenuates the Low frequencies) which inturn appears to be non distorted sine wave.

```
In [15]: t = linspace(0,1e-3,1e5)
         t,vout = low_pass_output(time_fn=np.sin(2000*np.pi*t)+np.cos(2e6*np.pi*t),t=t,C=10**-9)
```

```
/home/suhas/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:2: DeprecationWarning: o
```

```
In [16]: plt.plot(t,vout)
         plt.grid(True)
         plt.xlabel("t ------>",size=14)
         plt.ylabel(r"$V_{o}(t)$",size=14)
         plt.title("Output for mixed frequency Sinusoid in lowpass filter in steady time")
         plt.show()
```

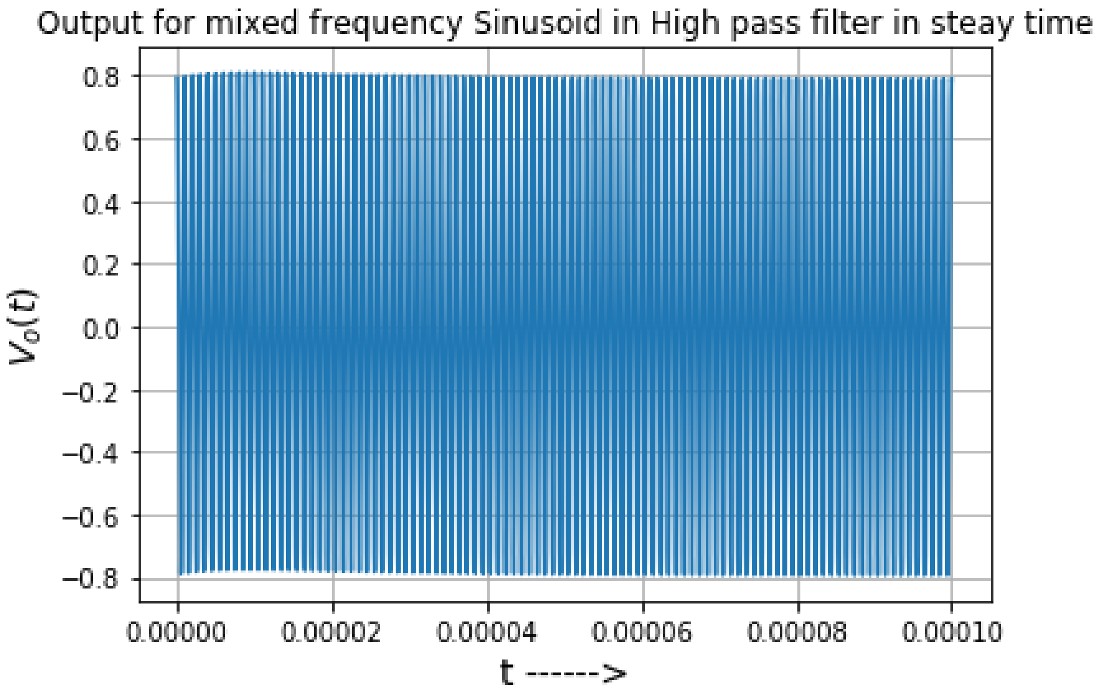


From graph we can see frequency is close to 1000Hz(which is low frquency input)

```
In [17]: t = linspace(0,1e-4,1e5)
         t,vout = high_pass_output(time_fn=np.sin(2000*np.pi*t)+np.cos(2e6*np.pi*t),t=t,C=10**-9
```

```
/home/suhas/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:2: DeprecationWarning: o
```

```
In [18]: plt.plot(t,vout)
         plt.grid(True)
         plt.xlabel("t ------>",size=14)
         plt.ylabel(r"$V_{o}(t)$",size=14)
         plt.title("Output for mixed frequency Sinusoid in High pass filter in steay time")
         plt.show()
```



From graph we can see frequency is close to 1000KHz(which is high frquency input)

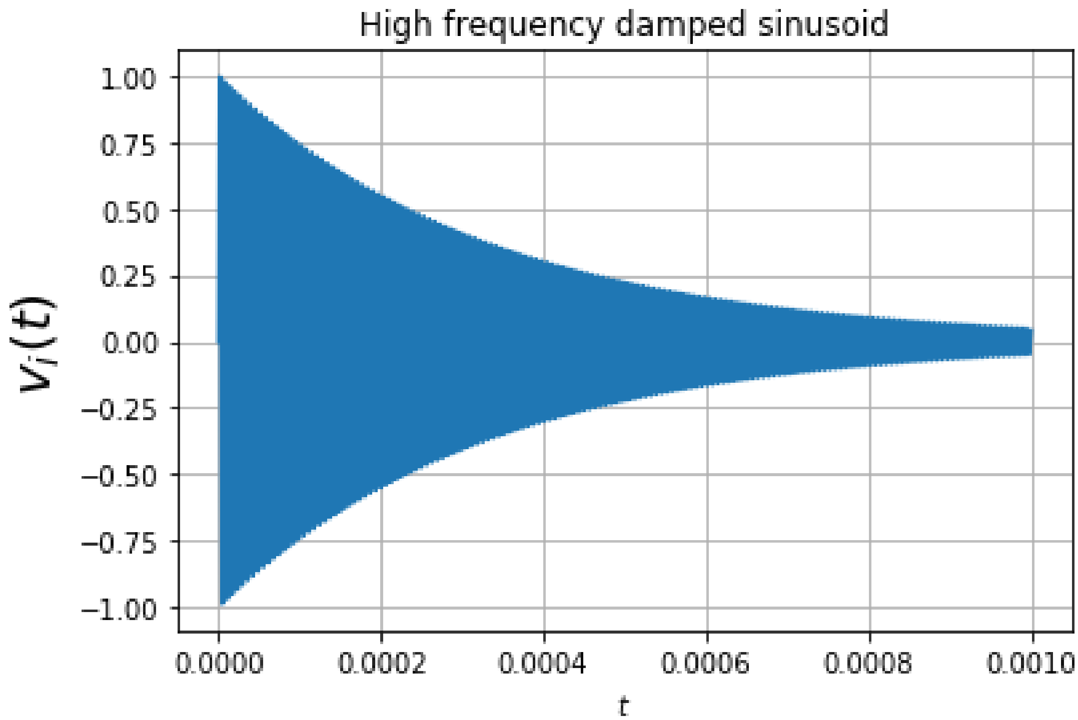
6 Question 3,4

Damped Sinusoid -----> $exp(-300t)sin(10^6t)$

```
In [19]: t = linspace(0,1e-3,1e6)
         f = np.exp(-3000*t) * np.sin(10**6 *t)
```

```
/home/suhas/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:2: DeprecationWarning: o
```

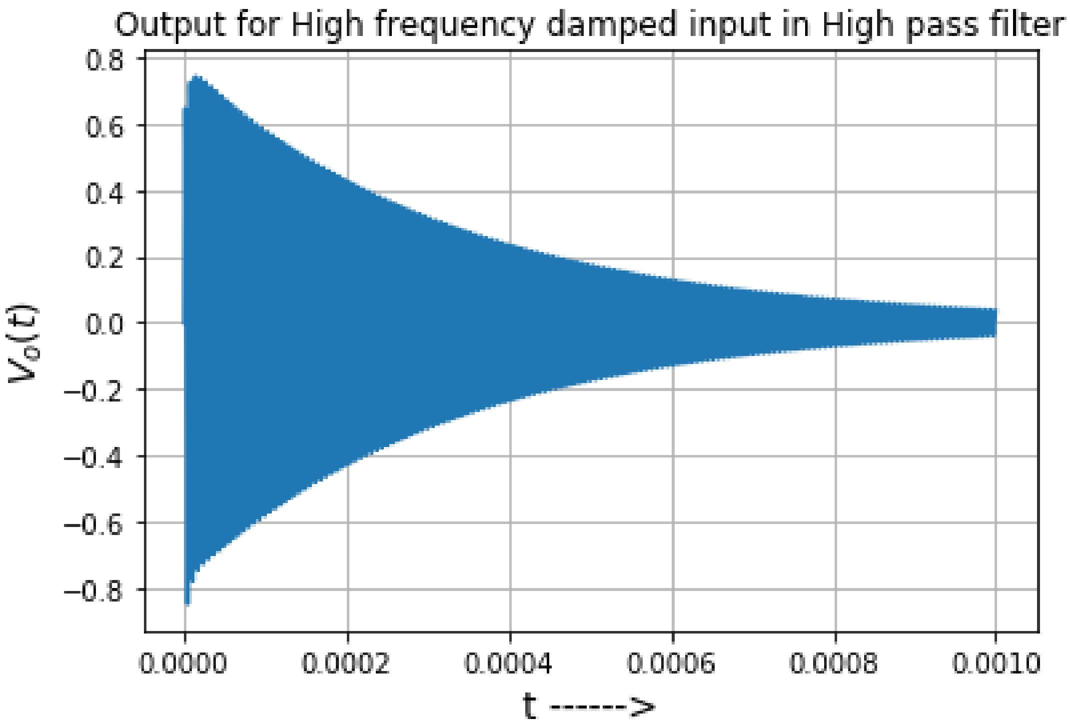
```
In [20]: plt.title("High frequency damped sinusoid")
         plt.xlabel("$t$")
         plt.ylabel("$v_i(t)$",size=20)
         plt.plot(t,f)
         plt.grid()
         plt.show()
```




```
In [21]: t = linspace(0,1e-3,1e6)
         t,vout = high_pass_output(time_fn=f,t=t,C=10**-9)
```

```
/home/suhas/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:2: DeprecationWarning: o
```

```
In [22]: plt.plot(t,vout)
         plt.grid(True)
         plt.xlabel("t ------>",size=14)
         plt.ylabel(r"$V_{o}(t)$",size=14)
         plt.title("Output for High frequency damped input in High pass filter")
         plt.show()
```

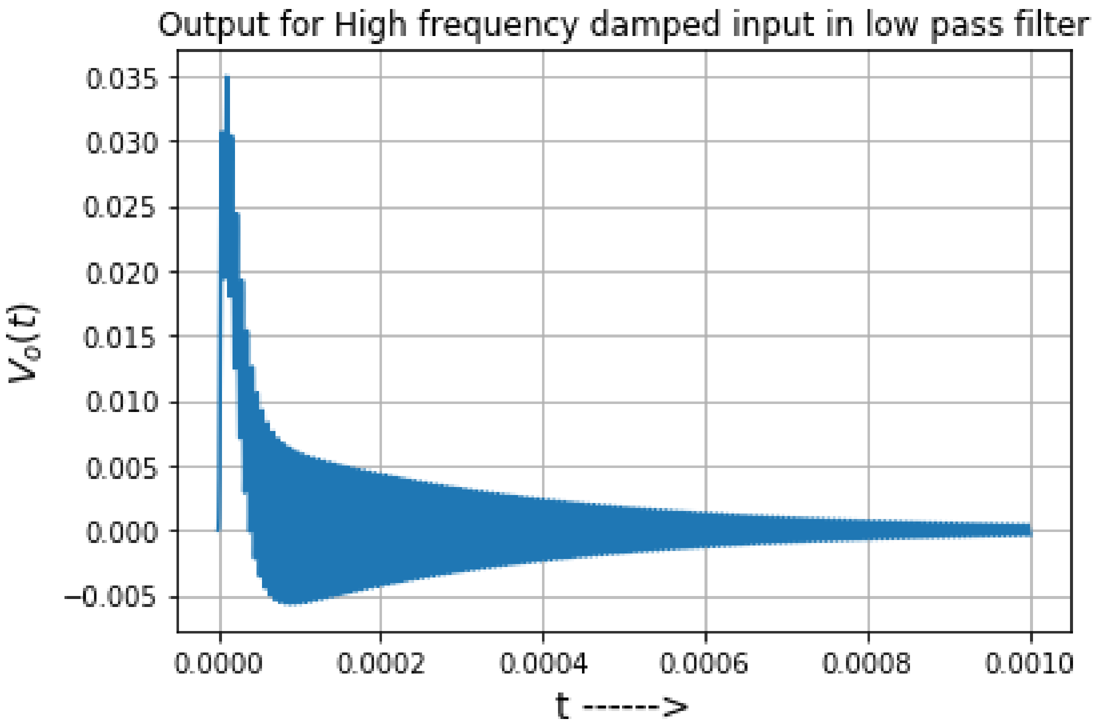


From above graph we can clearly see that High pass filter passed high frequency sinusoid with out attenuating much.(Since property of high pass filter)

```
In [27]: t = linspace(0,1e-3,1e6)
         t,vout = low_pass_output(time_fn=f,t=t,C=10**-9)
```

```
/home/suhas/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:2: DeprecationWarning: o
```

```
In [28]: plt.plot(t,vout)
         plt.grid(True)
         plt.xlabel("t ------>",size=14)
         plt.ylabel(r"$V_{o}(t)$",size=14)
         plt.title("Output for High frequency damped input in low pass filter")
         plt.show()
```

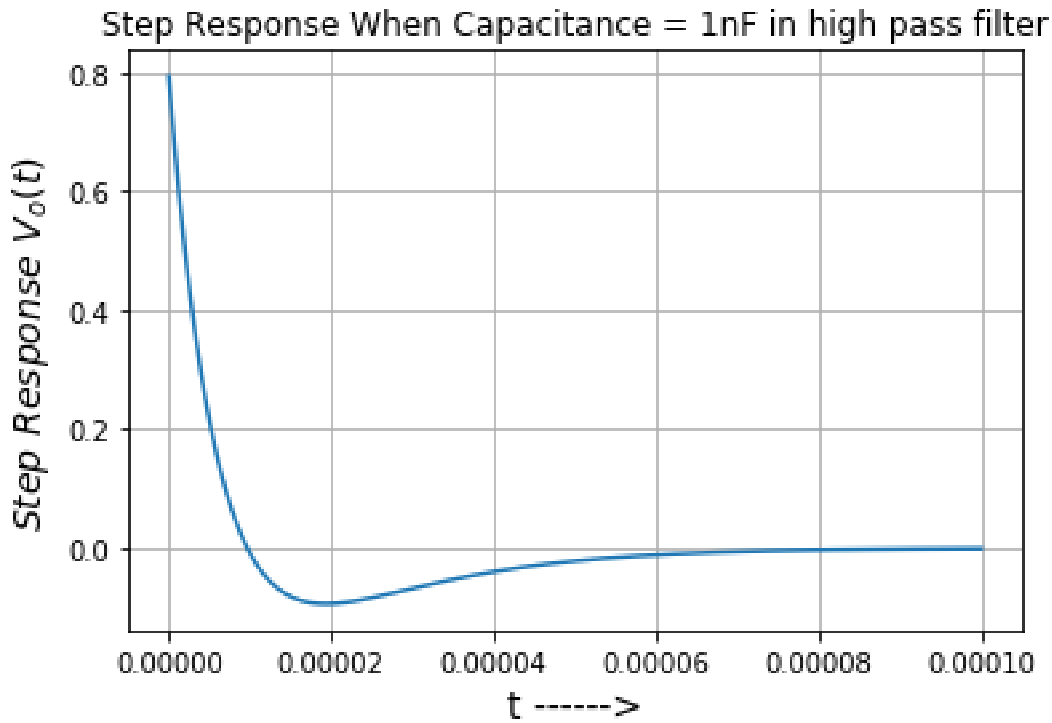


From above graph Low pass filter quickly attenuated the High frequency Sinusoid and gives distorted Output

7 Question 5

In [29]:
```
t,V_high_step = high_pass_output(laplace_fn=1/s,C=10**-9)
```

In [30]:
```
plt.plot(t,V_high_step)
plt.grid(True)
plt.xlabel("t ------>",size=14)
plt.ylabel(r"$Step\ Response\ V_{o}(t)$",size=14)
plt.title("Step Response When Capacitance = 1nF in high pass filter")
plt.show()
```



Step response here saturates at zero and this is because DC gain of High pass filter is 0. We can clearly see from graph that it starts from 0.793 and this because AC gain of transfer function at high frequencies is 0.793(Step can assumed as infinite frequency signal and we know high pass filter only allows high frequency signals)

step response overshoots the steady state value of 0, reaches an extremum, then settles back to 0, unlike the response of the low pass filter which steadily approaches the steady state value with no extrema. This occurs because of the presence of zeros at the origin in the transfer function of the high pass filter(which imply that the DC gain is 0). Since the steady state value of the step response is 0, the total signed area under the curve of the impulse response must also be 0. This means that the impulse response must equal zero at one or more time instants. Since the impulse response is the derivative of the step response, this therefore means that the step response must have at least one extremum. This explains the behaviour of the step response of the high pass filter.

8 Conclusions:

The low pass filter responds by letting the low frequency sinusoid pass through without much additional attenuation. The output decays as the input also decays.

The high pass filter responds by quickly attenuating the input. Notice that the time scales show that the high pass filter response is orders of magnitudes faster than the low pass response. This is because the input frequency is below the cutoff frequency, so the output goes to 0 very fast.

In conclusion, the sympy module has allowed us to analyse quite complicated circuits by analytically solving their node equations. We then interpreted the solutions by plotting time domain responses using the signals toolbox. Thus, sympy combined with the scipy.signal module is a very useful toolbox for analyzing complicated systems like the active filters in this assignment.