19CS30048

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6) If $XA = I$, then X is called left inverse of A.

$A \in R^{m \times n}$

$X \in R^{n \times m}$

$I \in R^{n \times n}$ (Identity Matrix)

for existance of left inverse columns of A should be linearly independent.

(a) $A = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ A only has one column thus columns are linearly independent.

Let x be some left inverse of A.

$x = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5]$

$xA = [1]$

$[x_1 + x_4] = [1]$

$x_1 + x_4 = 0$

$x = [x_1 \quad x_2 \quad x_3 \quad 1-x_1 \quad x_5]$ $x_i \in R$

is left inverse of A.

In general :-

- Let x be a solution of equation $xA = I$ (x is a left inverse)
- Let Y be set of solution of the equation $yA = 0$.
- Then set of all inverse is given by. $\{x + y \mid y \in Y\}$.

In $A = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

$yA = 0$ $\quad y = [y_1 \ y_2 \ y_3 \ y_4 \ y_5]$

$y_1 + y_4 = 0$

$Y = [y_1 \ y_2 \ y_3 \ -y_1 \ y_5], \ y_i \in R$

$x = [0 \ 0 \ 0 \ 1 \ 0]$

~~left inverse = x+y | for all y~~

left inverse $= \{x + y \mid y \in Y\}$

(b) $A = \begin{bmatrix} 2 & 0 \\ 0 & -2 \\ 3 & 3 \end{bmatrix}$

$$\begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} a + \begin{bmatrix} 0 \\ -2 \\ 3 \end{bmatrix} b = \begin{bmatrix} 2a \\ -2b \\ 3(a+b) \end{bmatrix}$$

$\begin{bmatrix} 2a \\ -2b \\ 3(a+b) \end{bmatrix}$ is 0 when $a=0$, $b=0$,

So columns are linearly independent.

$$\underbrace{\begin{bmatrix} a & c & e \\ b & d & f \end{bmatrix}} \begin{bmatrix} 2 & 0 \\ 0 & -2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$2a + 3e = 1.$ $\qquad -2c + 3e = 0$

$-2d + 3f = 1$ $\qquad 2b + 3f = 0$

$$X = \begin{bmatrix} a & \frac{1-2a}{2} & \frac{1-2a}{3} \\ b & \frac{-2b-1}{2} & \frac{-2b}{3} \end{bmatrix}$$

It can be written as a sum of.

$$\underbrace{\begin{bmatrix} a & -a & \frac{-2a}{3} \\ b & -b & \frac{-2b}{3} \end{bmatrix}}_{Y} \text{ and } \underbrace{\begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{3} \\ 0 & \frac{-1}{2} & 0 \end{bmatrix}}_{x.}$$

Left inverse $= \{x + y \mid y \in Y\}$