Full marks: 50   Time: 2 hours

NOTE:

(a) Answer all questions. Question 1 carries 20 marks and each other question, 10 marks.

(b) All parts of a particular question should be answered together.

(c) Credits will be given for neat and to-the-point answering.

1. Answer to the point. Unnecessary or redundant words are liable to negative marking. $(2\frac{1}{2} \times 8 = 20)$

   (a) Write the basic principle that leads to $O(n^{\log_2 3})$ time complexity for integer multiplication.

   (b) Draw a flow network with integer capacities in which Ford-Fulkerson algorithm is inefficient without Edmonds-Karp strategy.

   (c) At most how many flow augmentations are required in Edmonds-Karp algorithm? Why?

   (d) In the plane-sweep algorithm to find line intersections [Bentley and Ottmann], what can be the asymptotic orders of the maximum possible size of the event queue and the sweep-line data structure? Why?

   (e) Let $G(V, E, F)$ be the planar graph induced by a set S of n line segments. What is the upper bound of the number of faces in G in terms of n?

   (f) Let R be the smallest axis-parallel rectangle containing a point set S. No four points of S are concyclic. Prove or disprove: "The largest empty circle in R contains 3 points of S on its boundary".

   (g) Let $G(V, E, w)$ be a directed graph with no negative-weight cycle. For any $(i, j) \in V \times V$ and $(i, j) \notin E$, consider all paths from i to j whose intermediate vertices are from the first k vertices of V. Let $d_{ij}^{(k)}$ be the minimum weight of all these paths. Write the recursive definition of $d_{ij}^{(k)}$.

2. Suggest an algorithm to find the transitive closure of an undirected graph. Explain its time complexity.

3. Find the min-cut for the following graph, symbols having their usual meanings.
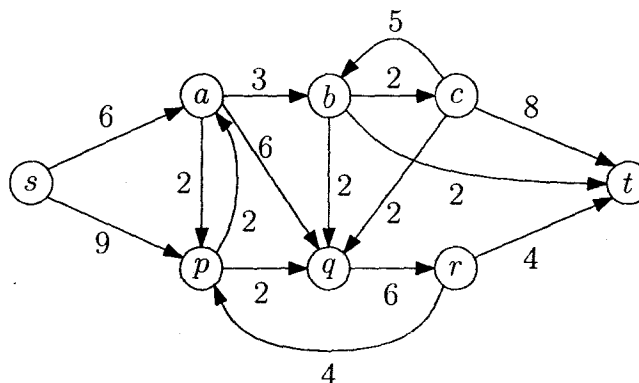
4. Given the DCEL for the Voronoi diagram of a set S of 2D points (sites), suggest an algorithm to obtain the closest pair of sites in S. Explain its time complexity.