Q3) $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

(i) $\|A\|_2 = $ max $\|Ax\|_2$

$$\|x\| = 1$$

$$\|A\|_2 = \|Ae_1\|_2$$

$$= \sqrt{p^2 + q^2} = \text{max mag}(A) = 10$$

(ii)

min mag $(A) = $ min $\|Ax\|_2$

$$\|x\| = 1$$

$$= \|Ae_2\|$$

$$\text{min mag}(A) = \sqrt{q^2 + s^2}$$

(iii)   Condition number $= 10$

$$\frac{\text{max mag}(A)}{\text{min mg}(A)} = 10$$

$$\text{min mag}(A) = 1 = \sqrt{q^2 + s^2}$$

(iv) $(Ae_1)^T (Ae_2) = 0$

$$pq + rs = 0$$

(V) $\quad (Ae_2)^T \, (e_1) = \dfrac{1}{\sqrt{2}}$

$$q = \dfrac{1}{\sqrt{2}}$$

$p^2 + \gamma^2 = 100 \qquad\qquad - ①$

$q^2 + s^2 = 1 \qquad\qquad - ②$

$\quad pq + \gamma s = 0 \qquad - ③$

$\quad q = \dfrac{1}{\sqrt{2}} \qquad\qquad - ④$

$q = \dfrac{1}{\sqrt{2}} \Rightarrow \qquad \dfrac{1}{2} + s^2 = 1 \Rightarrow s = \dfrac{1}{\sqrt{2}}$

$$(p + \gamma) \dfrac{1}{\sqrt{2}} = 0 \qquad \Rightarrow \qquad p + \gamma = 0$$
$$\Rightarrow p = -\gamma$$

$$p^2 + (-p)^2 = 100$$

$$p^2 = 50$$

$p = 5\sqrt{2} \qquad \text{and} \qquad \gamma = -5\sqrt{2}$

$$o \sigma$$

$p = -5\sqrt{2} \qquad \text{or} \qquad \gamma = 5\sqrt{2}$

$$A = \begin{bmatrix} 5\sqrt{2} & \dfrac{1}{\sqrt{2}} \\ -5\sqrt{2} & \dfrac{1}{\sqrt{2}} \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} -5\sqrt{2} & \dfrac{1}{\sqrt{2}} \\ 5\sqrt{2} & \dfrac{1}{\sqrt{2}} \end{bmatrix}$$