19CS30048
SUMAS JAIN

7) $A \in R^{n \times n}$, A is invertible

Goal : write A as product of L and U

L → Lower triangular matrix

U → Upper triangular matrix

⇒ We can change A into a ~~A~~ upper triangular matrix if we convert all the elements ~~above~~ below diagonal in each column to 0. We can do this sequentially, column wise.

⇒ Similar to gaussian elimination this is done by subtracting multiples of each row from subsequent rows.

⇒ We can visualise this by seeing this as ~~ad~~ multiplication from a series of lower triangular matrices.

$$\underbrace{L_{n-1} \cdot L_{n-2} \cdots\cdots L_1}_{L^{-1}} \cdot A = U$$

PTO .

For example, consider a 4×4 matrix.

$$\underset{A}{\begin{bmatrix} x & x & x & x \\ x & x & x & x \\ x & x & x & x \\ x & x & x & x \end{bmatrix}} \longrightarrow \underset{L_1 A}{\begin{bmatrix} x & x & x & x \\ 0 & x & x & x \\ 0 & x & x & x \\ 0 & x & x & x \end{bmatrix}}$$

$$\downarrow$$

$$\underset{L_3 L_2 L_1 A}{\begin{bmatrix} x & x & x & x \\ 0 & x & x & x \\ 0 & 0 & x & x \\ 0 & 0 & 0 & x \end{bmatrix}} \longleftarrow \underset{L_1 L_2 A}{\begin{bmatrix} x & x & x & x \\ 0 & x & x & x \\ 0 & 0 & x & x \\ 0 & 0 & x & x \end{bmatrix}}$$

For a given L_k, it makes all the elements below the diagonal of column k 0. By subtracting multiples of rows $k+1, k+2 \dots n$ from row k.

In general, visualise as follows.

$$x_k = \begin{bmatrix} x_{1k} \\ x_{2k} \\ \vdots \\ x_{nk} \end{bmatrix} \xrightarrow{L_k} L_k x_k = \begin{bmatrix} x_{1k} \\ \vdots \\ x_{kk} \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

(At beginning of k^{th} step)

For this step we subtract l_{jk} times row k from row j.

$$l_{jk} = \frac{x_{jk}}{x_{kk}} \quad (k < j \leqslant n)$$

$$L_k = \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & -l_{k+1,k} & \ddots & & \\ & & -l_{k+2,k} & & \ddots & \\ & & \vdots & & & \\ & & -l_{n,k} & & & 1 \end{bmatrix}$$

Elements which are not shown are assumed to be 0.