2)

$$\text{avg}(x) = \left(\frac{1_n}{n}\right)^T x$$

$$= \frac{1}{n}(1_n)^T x$$

$$\text{std}(x) = \frac{\| x - \text{avg}(x) 1_n \|_2}{\sqrt{n}}$$

(a) $$\text{avg}(\alpha x + \beta 1_n) = \left(\frac{1_n}{n}\right)^T (\alpha x + \beta 1_n)$$

$$= \frac{1}{n}(1_n)^T(\alpha x) + \frac{\beta}{n}\underbrace{(1_n)^T \cdot 1_n}_{n}$$

$$= \alpha \cdot \underbrace{\frac{1}{n}(1_n)^T(x)}_{\text{avg}(x)} + \beta$$

$$= \alpha \cdot \text{avg}(x) + \beta$$

(b) $$\text{std}(\alpha x + \beta 1_n) = \frac{\| \alpha x + \beta 1_n - \text{avg}(\alpha x + \beta 1_n) \cdot 1_n \|_2}{\sqrt{n}}$$

$$= \frac{\| \alpha x + \beta 1_n - (\alpha \, \text{avg}(x) + \beta) 1_n \|}{\sqrt{n}}$$

$$= \frac{\| \alpha x - \alpha \, \text{avg}(x) 1_n \|}{\sqrt{n}}$$

$$= \alpha \frac{\| x - \text{avg}(x) 1_n \|}{\sqrt{n}}$$

$$= \alpha \, \text{std}(x)$$