Linear Algebra for AI & ML

Swanand Khare
srkhare@maths.iitkgp.ac.in

Text books :

1) Introduction to Applied Linear Algebra: vectors, matrices, least squares
Boyd, Vandenberghe

2) Linear Algebra and learning from data
G. Strang

F4 Wed 10 to 11
Thur 9 to 10
Fri 11 to 1

Vectors & matrices

- LS problems
  - non-linear
  - constrained
  - multi-objective

Data fitting / regression

- Matrix decompositions
  - QR (LS problems)
  - SVD (PCA)
  - Low rank approximations (NN - reducing number of parameters in training)
  - Low rank matrix completion
    - → multilinear algebra (tensors)

Vectors:

Typically column vector.

$$x, y \in \mathbb{R}^n$$

n ← how many components/entries

$\mathbb{R}$ ↑ set of real numbers

(Field)

x is an n-vector.

$$\mathbb{R}^{m \times n}$$

$$x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n$$

rows

columns

Transpose:

$x \in \mathbb{R}^n$; $x \in \mathbb{R}^{n \times 1}$

$x^T \in \mathbb{R}^{1 \times n}$ $x^T = [x_1 \ \ x_2 \ \ \dots \ \ x_n] \in \mathbb{R}^{1 \times n}$

Examples:

i) colour

3-vector ; each entry between 0 & 1.

$x \in \mathbb{R}^3$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{matrix} \rightarrow \text{Red} \\ \rightarrow \text{Green} \\ \rightarrow \text{Blue} \end{matrix}$$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \rightarrow \text{red} \quad , \quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \rightarrow \text{green}$$

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \rightarrow \text{yellow}$$

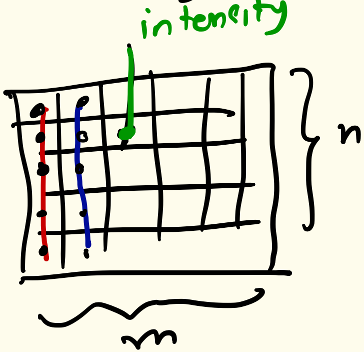
2) Time-series / Signal

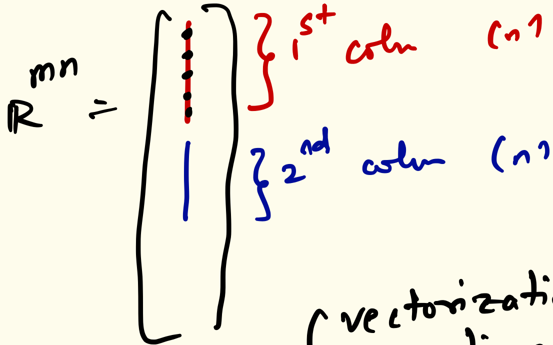
n - values of quantity at n-different time instances.

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n$$

$\underline{n = 31}$

3) Grey-Scale images





(vectorization operation

intensity values $\in [0,1]$

MNIST $\boxed{6}$ 27 × 27

4) Word count & histogram.

Dictionary on n-words

$$\mathbb{R}^n \qquad \begin{array}{r} 1^{st} \rightarrow \\ 2^{nd} \rightarrow \\ \rightarrow \\ \\ n^{th} \rightarrow \end{array} \begin{bmatrix} 5 \\ 0 \\ 1 \\ \vdots \\ 10 \end{bmatrix}$$

↑ word-count vector

5) Features/attributes vector.

$$\mathbb{R}^k \qquad \begin{bmatrix} * \\ \vdots \\ * \end{bmatrix} \begin{array}{l} \leftarrow 1^{st} \text{ feature value} \\ \\ \leftarrow k^{th} \text{ feature value} \end{array}$$

Operations on vectors.

i) vector addition

x & $y \in \mathbb{R}^n$

$x + y$ = component wise or entrywise addition

$$\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} + \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} x_1+y_1 \\ \vdots \\ x_n+y_n \end{pmatrix}$$

$x, y, z \in \mathbb{R}^n$

i) Commutative : $x+y = y+x$

ii) associative : $(x+y)+z = x+(y+z)$

iii) $\exists$ ~~a~~ the $0 \in \mathbb{R}^n$ $\quad 0 = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$ s.t. $x+0 = 0+x = x$ additive identity

iv) $\exists$ ~~a~~ the vector $b \in \mathbb{R}^n$ s.t. $x+b = b+x = 0$ (additive inverse)

Scalar - multiplication:

$x \in \mathbb{R}^n$ and $\alpha \in \mathbb{R}$

$$\alpha . x = \alpha x \in \mathbb{R}^n$$

$$\alpha x = \begin{bmatrix} \alpha x_1 \\ \alpha x_2 \\ \vdots \\ \alpha x_n \end{bmatrix} \in \mathbb{R}^n$$

Properties: $\alpha, \beta, \gamma \in \mathbb{R}$, $x, y \in \mathbb{R}^n$

i) $(\alpha+\beta) x = \alpha x + \beta x$

ii) $\alpha(x+y) = \alpha x + \alpha y$

iii) $(\alpha\beta) x = \alpha(\beta x)$

iv) $-1 . x = -x \leftarrow$ additive inverse of $x \in \mathbb{R}^n$

Ex:

$x \in \mathbb{R}^n$: audio signal

$\alpha \in \mathbb{R}_+$

αx

$\mathbb{R}^2, \mathbb{R}^3$