(Q5) We have to find the vector $x \in R^n$ such that $\|x - a\|^2$, while also satisfying the given equation of average.

$$\boxed{1^T x = n\beta} \qquad \left[1 = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \right]$$

Using KKT equation as we have one equation we want to minimize while satisfying the other one.

$$\begin{bmatrix} 2AA^T & C^T \\ C & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} = \begin{bmatrix} 2A^T b \\ d \end{bmatrix}$$

Putting in the appropriate values.

$A = I$, $C = 1^T$, $b = a$, $d = n\beta$

$$\begin{bmatrix} 2I & 1 \\ 1^T & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} = \begin{bmatrix} 2a \\ n\beta \end{bmatrix}$$

$z \Rightarrow$ Lagrange multiplication vector

Using this equation.

$2x + z1 = 2a.$

$$\boxed{x = a - \left(\frac{z}{2}\right) 1}$$

Combining it with $1^T x = n\beta$

$$1^T \left(a - \left(\frac{z}{2}\right) 1\right) = n\beta$$

$$1^T a - \frac{nz}{2} = n\beta$$

$$\boxed{z = 2\left(\frac{1^T a}{n} - \beta\right)}$$

From this equation, substituting the value of z

$$x = a - \left(\frac{z}{2}\right) 1$$

$$x = a + \left(\beta - \frac{1^T a}{n}\right) 1$$

$$\boxed{x = a + \left(\beta - \text{avg}(a)\right) 1}$$

$$\left[\text{avg}(a) = \frac{1^T a}{n}\right]$$