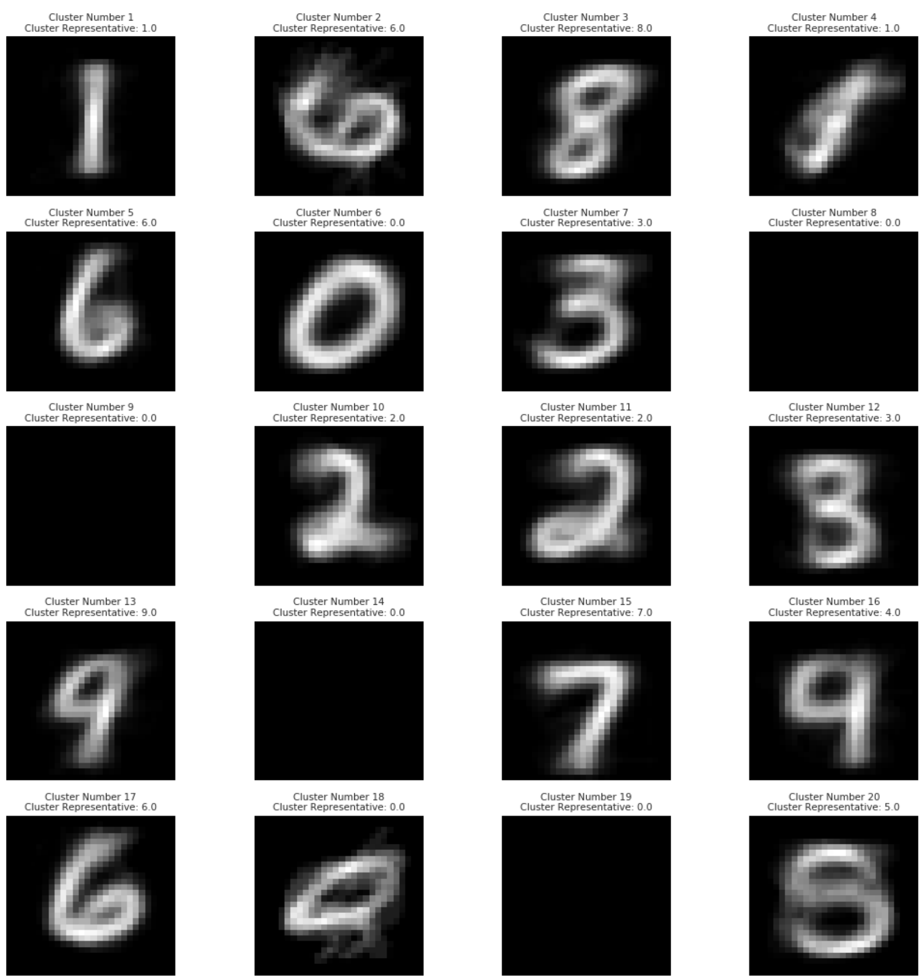
Image Clustering Using K-Means Algorithm

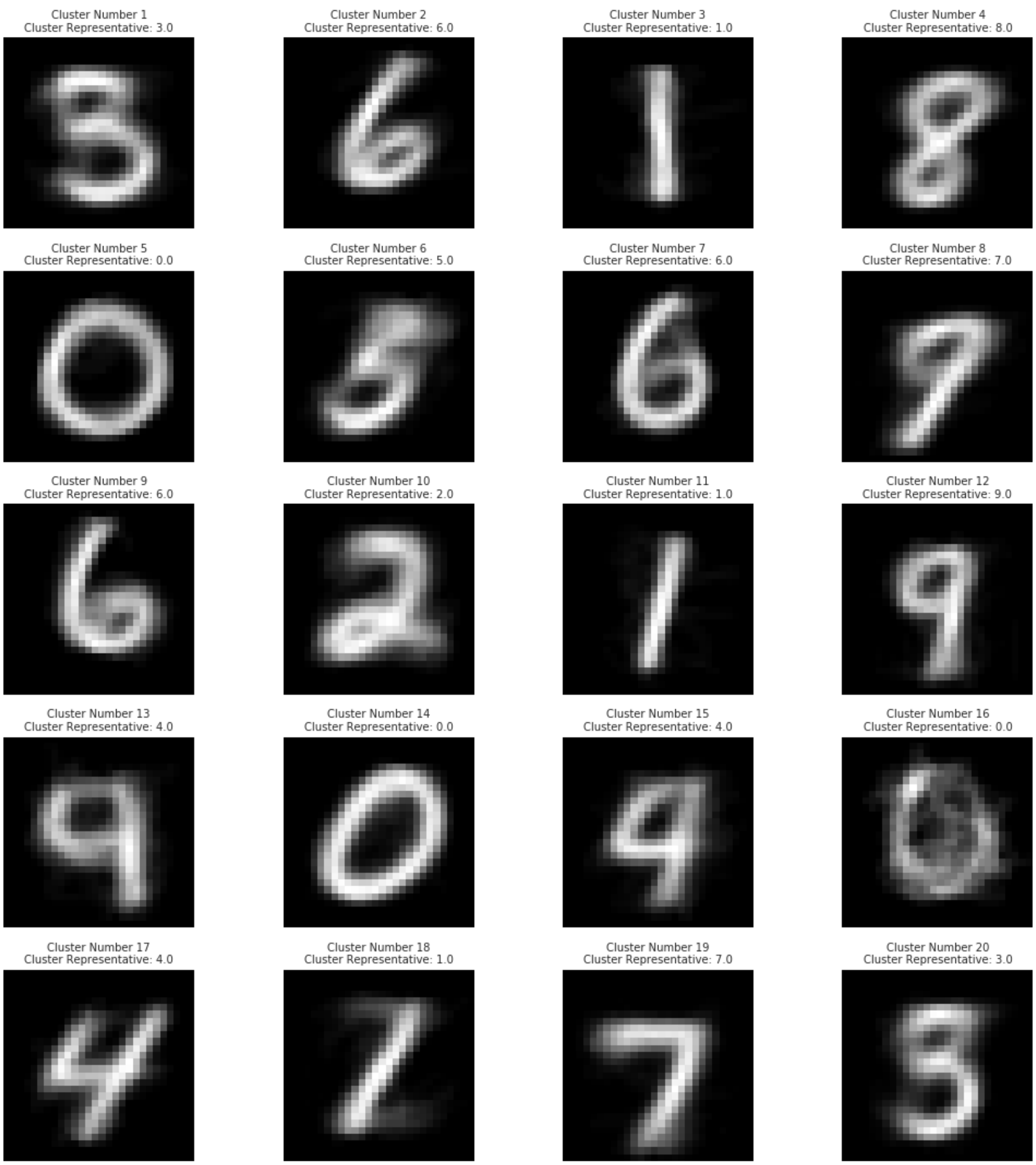
About the dataset
It is a dataset of 60,000 small square 28×28 pixel grayscale images of handwritten single digits between 0 and 9. The task is to classify a given image of a handwritten digit into one of 10 classes representing integer values from 0 to 9, inclusively.
We convert this into a vector of shape (784, 1000), 784 because we flatten the 28x28 matrix and 1000 because we have 100 images from all 10 digits.

a. For k = 20, we observed the following cluster representation. In each of the clusters, the images which were present in the highest number are also mentioned along with the picture. We did the whole process using 2 types of initialisations.
   i. **Random initialization of cluster representatives:** In this initialisation before the first iteration, clusters were initialised randomly and then clustering was performed.



Note: The following representatives are found by initialising the clusters such that their mean and standard deviation are the same as that of the training dataset. Also it might happen that certain clusters are empty during clustering assignment, which might lead to poor convergence and high J_{clust}.

ii. **Choose cluster representatives from the given data set:** In this initialisation at random 20 images were chosen and clusters were placed on those images then clustering was performed.



Number of iterations and accuracy on testing on random sample of 50 images which were not present in the training data for both cases:

K	Initialisation	Accuracy	Iterations	Final J_{clust}
20	Random	0.56	24	5.86
20	Training Data	0.72	19	5.83

LOSS		
K	**Random Initialisation**	**Training Data Initialisation**
5	6.57	6.50
6	6.45	6.42
7	6.38	6.39
8	6.32	6.28
9	6.24	6.23
10	6.21	6.17
11	6.16	6.13
12	6.11	6.08
13	6.07	6.01
14	6.05	6.01
15	6.02	5.95
16	5.97	5.92
17	5.90	5.90
18	5.92	5.87
19	5.864	5.837
20	5.867	5.839

J_{clust} vs Number of Clusters

Random Initialisation

Training Data Initialisation

For both the types of initialisations, best k is found to be k = 19. Ideally the best k should be 10 because there are 10 types of digits but because the images are written differently and are also rotated and blurred, this distortion occurs.

C)

Yes, the initial condition you choose has an influence on the k-means algorithm. Data-set initialization results in a lower J_{clust} loss, greater average accuracy on test datasets, and lower variance test accuracy as compared to random initialization. A random initialization is highly prone to identifying a cluster representative which is not in the distribution of any of the training set feature vectors. This results in the development of empty clusters, which must be addressed by reinitialization. This process takes extra iterations, that's the reason we observed 24 iterations with random initialisation and 19 with initialisation from the dataset. If we do not reinitialise the clusters, then at the end many clusters would just go empty and have no members.

BELOW ATTACHED IS THE FULL CODE OF EVERYTHING THAT IS IMPLEMENTED IN THIS QUESTION. EVERY GRAPH AND TABLE CAN BE CROSS-VERIFIED FROM THE RESULTS OF THE CODE BELOW.
THE CODE ALSO CONTAINS ADDITIONAL GRAPHS AND DATA (LIKE TEST ACCURACY FOR ALL K FROM 5 TO 20), WHICH CAN BE LOOKED INTO FOR MORE INSIGHT.

[1]:
```
import numpy as np
import random
import matplotlib.pyplot as plt
import seaborn as sns
from tensorflow.keras.datasets import mnist
from kmeans import KMeans
import argparse
sns.set(style='ticks', palette='Set2')

CONVERGENCE_DELTA = 1e-6
MAXIMUM_ITERATIONS = 100
```

[2]:
```
%%javascript
IPython.OutputArea.auto_scroll_threshold = 9999
```

```
<IPython.core.display.Javascript object>
```

[17]:
```
class KMeans():
    def __init__(
        self,
        x_train,
        y_train,
        num_clusters=3,
        seed: str = "random",
    ):
        self.dataset = x_train
        self.targets = y_train
        self.k = num_clusters
        self.num_features = x_train.shape[1]
        self.num_samples = x_train.shape[0]
        if seed == "random":
            self.centroids = self.random_initialise_centroids()
        elif seed == "custom":
            self.centroids = self.initialise_from_data()
        else:
            raise ValueError("Choose a seed between ['random', 'custom']")
        self.old_centroids = np.copy(self.centroids)
        self.cluster_labels = np.zeros(self.num_samples, dtype=int)
        for i in range(self.num_samples):
            self.cluster_labels[i] = np.argmin(
                np.linalg.norm(self.dataset[i]-self.centroids, ord=2, axis=1))


    def random_initialise_centroids(self):
        mean = np.mean(self.dataset, axis = 0)
        std = np.std(self.dataset, axis = 0)
        return np.random.randn(self.k, self.num_features)*std + mean
```

```
        return centroids

    def initialise_from_data(self):
        centroids = np.copy(self.dataset[np.random.choice(
                self.num_samples, self.k, replace=(False if self.k <= self.
↪num_samples else True))])
        return centroids

    def get_centroid_labels(self):
        centroid_labels = np.zeros(self.k)
        for i in range(self.k):
            count = np.bincount(self.targets[self.cluster_labels == i])
            if len(count) > 0:
                centroid_labels[i] = np.argmax(count)
        return centroid_labels

    def calculate_loss(self):
        loss = np.mean(np.linalg.norm(
            self.dataset - self.centroids[self.cluster_labels], ord=2, axis=1),␣
↪axis=0)
        return loss

    def fit(self):
        for i in range(MAXIMUM_ITERATIONS):
            for i in range(self.num_samples):
                self.cluster_labels[i] = np.argmin(
                    np.linalg.norm(self.dataset[i]-self.centroids, ord=2,␣
↪axis=1))
            prev_centers = np.copy(self.centroids)
            converged = True
            for i in range(self.k):
                alloted = self.dataset[self.cluster_labels == i]
                if len(alloted) > 0:
                    self.centroids[i] = np.mean(alloted, axis=0)
                else:
                    self.centroids[i] = np.zeros(self.num_features)
                if np.linalg.norm(prev_centers[i] - self.centroids[i]) >␣
↪CONVERGENCE_DELTA:
                    converged = False
            loss = self.calculate_loss()
            if converged is True:
                print(f"TOTAL ITERATIONS = {i}")
                break
            self.old_centroids = np.copy(self.centroids)

    def predict(self, x):
        labels = np.zeros(x.shape[0], dtype=int)
```

```
        for i in range(x.shape[0]):
            labels[i] = np.argmin(
                np.linalg.norm(x[i]-self.centroids, ord=2, axis=1))
        return self.get_centroid_labels()[labels]
```

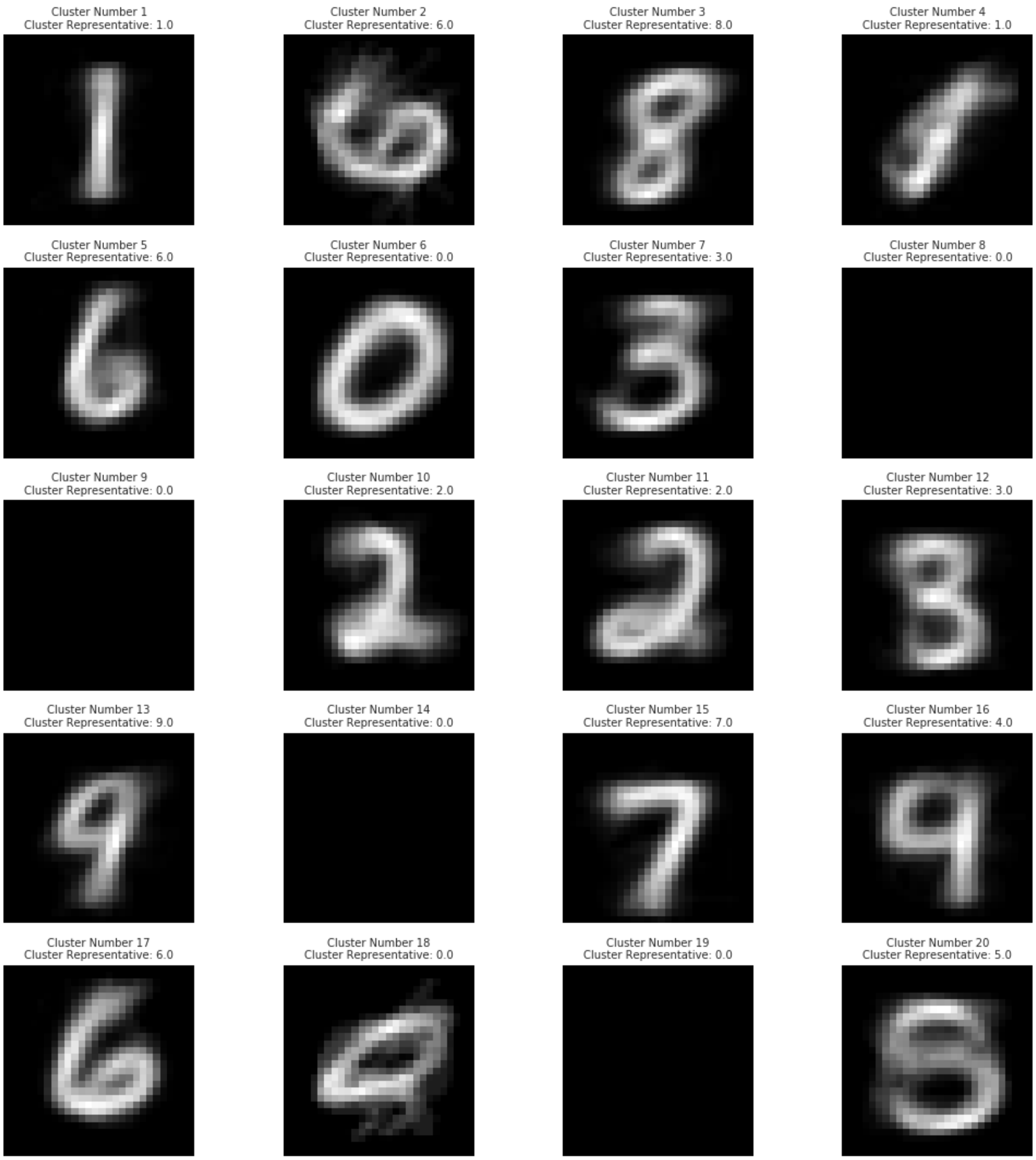
[18]:
```
def load_train_data(data_size=100):
    (x_train, y_train), (_, _) = mnist.load_data()
    x_train = x_train / 255
    x_train = x_train.reshape(x_train.shape[0], -1)
    digits = []
    targets = []
    for i in range(10):
        images = x_train[y_train == i]
        digits.append(images[np.random.choice(
            len(images), data_size, replace=False)])
        targets.append(np.full((data_size,), i))
    x_train = np.vstack(digits)
    y_train = np.hstack(targets)
    permutation = np.random.permutation(x_train.shape[0])
    x_train = x_train[permutation]
    y_train = y_train[permutation]
    return x_train, y_train

def load_test_data(data_size=50):
    (_, _), (x_test, y_test) = mnist.load_data()
    x_test = x_test / 255
    x_test = x_test.reshape(x_test.shape[0], -1)
    test_indices = np.random.choice(x_test.shape[0], data_size)
    x_test = x_test[test_indices]
    y_test = y_test[test_indices]
    return (x_test, y_test)

def plot_cluster_representatives(kmeans, centroids):
    centroid_images = np.copy(centroids.reshape(kmeans.k, 28, 28))
    centroid_images = centroid_images * 255
    centroid_labels = kmeans.get_centroid_labels()
    fig = plt.figure(figsize=(15, 15))
    nrows = 5
    ncols = 4
    for i in range(kmeans.k):
        fig.add_subplot(nrows, ncols, i+1)
        plt.imshow(centroid_images[i], cmap="gray")
        plt.title(f"Cluster Number {i+1}\nCluster Representative:␣
 ↪{centroid_labels[i]}", fontsize=10)
        plt.axis("off")
    fig.tight_layout()
    plt.show()
```

```python
def k_means_mnist(num_clusters=20, seed="random"):
    (x_train, y_train) = load_train_data()
    (x_test, y_test) = load_test_data()
    kmeans = KMeans(x_train, y_train,
                    num_clusters=num_clusters,
                    seed=seed)
    kmeans.fit()
    predictions = kmeans.predict(x_test)
    acc = np.mean(predictions == y_test)
    print(f"Accuracy: {acc}")
    plot_cluster_representatives(kmeans, kmeans.centroids)


def multiple_k_clusters(min_k = 0, max_k = 20, seed="random"):
    k = np.arange(start=min_k, stop=max_k+1, step=1, dtype=int)
    (x_train, y_train) = load_train_data()
    (x_test, y_test) = load_test_data()
    jclust = []
    accuracy = []
    for num_clusters in k:
        print("---------------------------")
        print(f"K = {num_clusters}")
        kmeans = KMeans(x_train, y_train,
                        num_clusters=num_clusters,
                        seed=seed)
        kmeans.fit()
        loss = kmeans.calculate_loss()
        print(f"TOTAL LOSS = {loss}")
        jclust.append(loss)
        predictions = kmeans.predict(x_test)
        acc = np.mean(predictions == y_test)
        print(f"Accuracy = {acc}\n")
        accuracy.append(acc)

    plt.plot(k, jclust)
    plt.title("J-clustering loss with number of clusters")
    plt.xlabel("Number of Clusters")
    plt.ylabel("J-Clustering Loss")
    plt.show()

    plt.plot(k, accuracy)
    plt.title("Test set accuracy with number of clusters")
    plt.xlabel("Number of Clusters")
    plt.ylabel("Accuracy on test set")
    plt.show()
```

[19]:
```
random.seed(70)
np.random.seed(70)
```

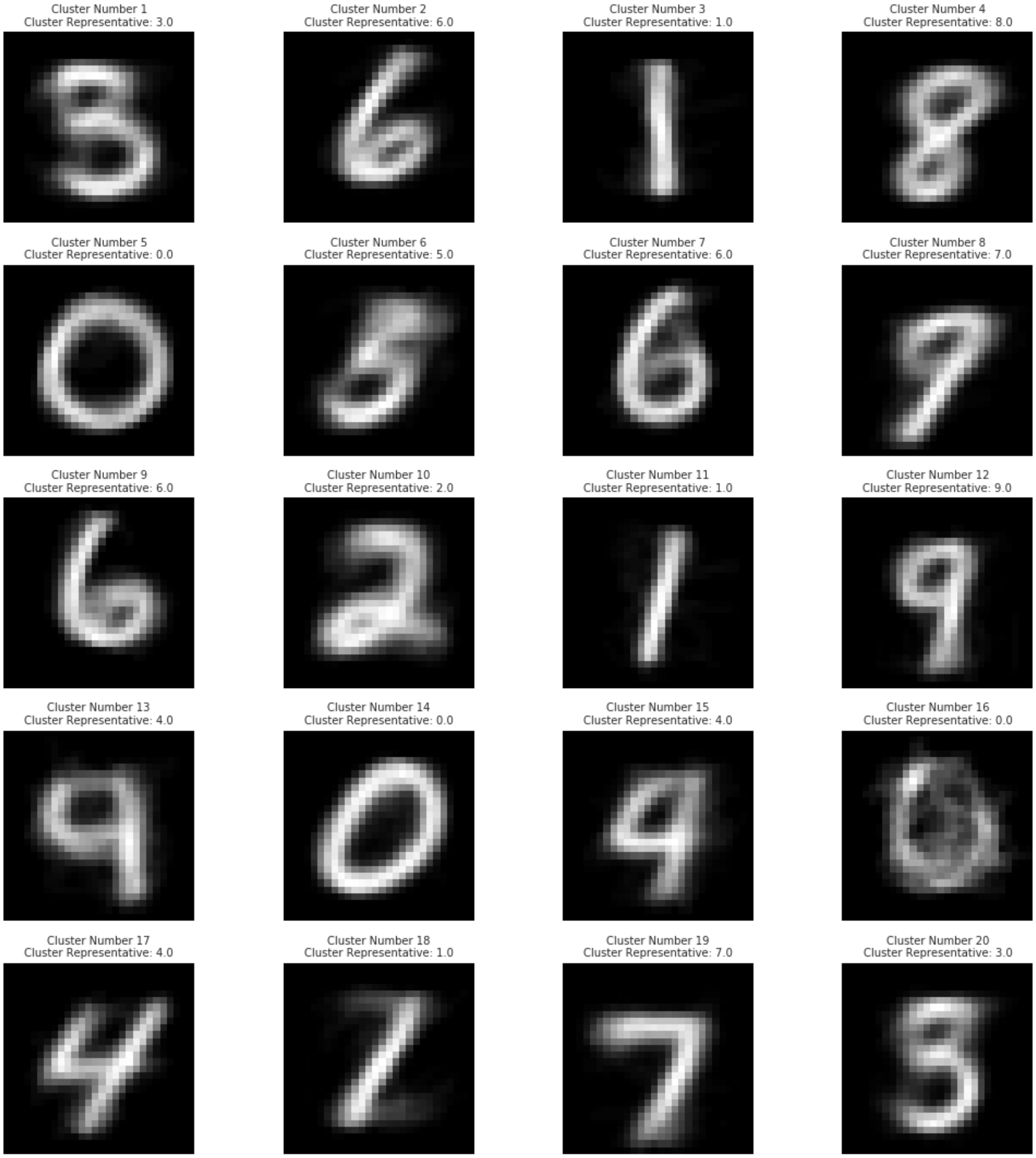
[16]:
```
k_means_mnist(seed = "random")
```

```
TOTAL ITERATIONS = 24
Accuracy: 0.56
```



[20]:
```
k_means_mnist(seed = "custom")
```

```
TOTAL ITERATIONS = 19
Accuracy: 0.72
```



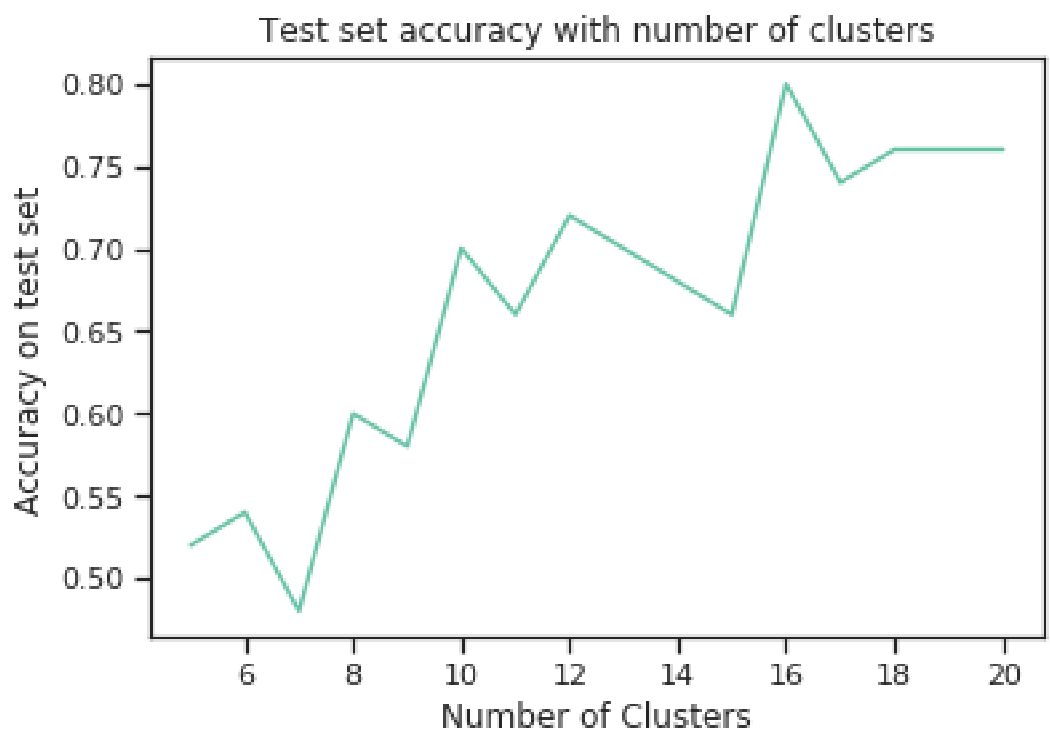
[21]:
```
multiple_k_clusters(min_k = 5, max_k = 20, seed = "custom")
```

```
---------------------------
K = 5
TOTAL ITERATIONS = 4
TOTAL LOSS = 6.50840838105185
Accuracy = 0.52
```

```
----------------------------
K = 6
TOTAL ITERATIONS = 5
TOTAL LOSS = 6.425665301537582
Accuracy = 0.54

----------------------------
K = 7
TOTAL ITERATIONS = 6
TOTAL LOSS = 6.396683941105112
Accuracy = 0.48

----------------------------
K = 8
TOTAL ITERATIONS = 7
TOTAL LOSS = 6.287688655285234
Accuracy = 0.6

----------------------------
K = 9
TOTAL ITERATIONS = 8
TOTAL LOSS = 6.236148643261253
Accuracy = 0.58

----------------------------
K = 10
TOTAL ITERATIONS = 9
TOTAL LOSS = 6.175016030542126
Accuracy = 0.7

----------------------------
K = 11
TOTAL ITERATIONS = 10
TOTAL LOSS = 6.139282865923727
Accuracy = 0.66

----------------------------
K = 12
TOTAL ITERATIONS = 11
TOTAL LOSS = 6.083365536633343
Accuracy = 0.72

----------------------------
K = 13
TOTAL ITERATIONS = 12
TOTAL LOSS = 6.016859778564274
Accuracy = 0.7
```

```
---------------------------
K = 14
TOTAL ITERATIONS = 13
TOTAL LOSS = 6.0168833418997565
Accuracy = 0.68

---------------------------
K = 15
TOTAL ITERATIONS = 14
TOTAL LOSS = 5.958255855521209
Accuracy = 0.66

---------------------------
K = 16
TOTAL ITERATIONS = 15
TOTAL LOSS = 5.929531560649528
Accuracy = 0.8

---------------------------
K = 17
TOTAL ITERATIONS = 16
TOTAL LOSS = 5.906489157842576
Accuracy = 0.74

---------------------------
K = 18
TOTAL ITERATIONS = 17
TOTAL LOSS = 5.874364455096953
Accuracy = 0.76

---------------------------
K = 19
TOTAL ITERATIONS = 18
TOTAL LOSS = 5.83799332391211
Accuracy = 0.76

---------------------------
K = 20
TOTAL ITERATIONS = 19
TOTAL LOSS = 5.839183432036803
Accuracy = 0.76
```


J-clustering loss with number of clusters
J-Clustering Loss
6.5
6.4
6.3
6.2
6.1
6.0
5.9
6
8
10
12
14
16
18
20
Number of Clusters


Test set accuracy with number of clusters
Accuracy on test set
0.80
0.75
0.70
0.65
0.60
0.55
0.50
6
8
10
12
14
16
18
20
Number of Clusters

[22]:
```
multiple_k_clusters(min_k = 5, max_k = 20, seed = "random")
```

```
---------------------------
K = 5
TOTAL ITERATIONS = 4
TOTAL LOSS = 6.5714835430122145
Accuracy = 0.4

---------------------------
K = 6
TOTAL ITERATIONS = 5
TOTAL LOSS = 6.454241270053107
Accuracy = 0.44

---------------------------
K = 7
TOTAL ITERATIONS = 6
TOTAL LOSS = 6.389698213233495
Accuracy = 0.46

---------------------------
K = 8
TOTAL ITERATIONS = 7
TOTAL LOSS = 6.325797242161661
Accuracy = 0.5

---------------------------
K = 9
TOTAL ITERATIONS = 8
TOTAL LOSS = 6.249483032309927
Accuracy = 0.56

---------------------------
K = 10
TOTAL ITERATIONS = 9
TOTAL LOSS = 6.211009299439668
Accuracy = 0.52

---------------------------
K = 11
TOTAL ITERATIONS = 10
TOTAL LOSS = 6.165129442672038
Accuracy = 0.54

---------------------------
K = 12
TOTAL ITERATIONS = 11
TOTAL LOSS = 6.112581756168993
```

```
Accuracy = 0.54

---------------------------
K = 13
TOTAL ITERATIONS = 12
TOTAL LOSS = 6.071516260660529
Accuracy = 0.68

---------------------------
K = 14
TOTAL ITERATIONS = 13
TOTAL LOSS = 6.053159375555776
Accuracy = 0.52

---------------------------
K = 15
TOTAL ITERATIONS = 14
TOTAL LOSS = 6.020901193092126
Accuracy = 0.68

---------------------------
K = 16
TOTAL ITERATIONS = 15
TOTAL LOSS = 5.977010115512925
Accuracy = 0.6

---------------------------
K = 17
TOTAL ITERATIONS = 16
TOTAL LOSS = 5.902915956134529
Accuracy = 0.66

---------------------------
K = 18
TOTAL ITERATIONS = 17
TOTAL LOSS = 5.921043077355468
Accuracy = 0.66

---------------------------
K = 19
TOTAL ITERATIONS = 18
TOTAL LOSS = 5.864230428946401
Accuracy = 0.66

---------------------------
K = 20
TOTAL ITERATIONS = 19
TOTAL LOSS = 5.86707713868915
```

```
Accuracy = 0.56
```