19CS30048
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4) $Ax = b$, $A \in R^{m \times n}$, $b \in R^m$

We need to find $x \in R^n$

Let $A = [a_1 \; a_2 \; \ldots \; a_n]$

a_i = i^{th} column of the matrix

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$Ax = x_1 \begin{bmatrix} | \\ a_1 \\ | \end{bmatrix} + x_2 \begin{bmatrix} | \\ a_2 \\ | \end{bmatrix} \cdots + x_n \begin{bmatrix} | \\ a_n \\ | \end{bmatrix}.$$

We can observe the following :-

- Ax is written as linear combination of columns of A.
- Thus Ax will lie in the column space of ~~columns of~~ A.
- If b does not lie in the column space of ~~columns of~~ A then no solution will exist.
- If $b \in$ colspace (A), then atleast one solution exists.

In conclusion we can say that:-

- If colspace (A) ~~...~~ spans R^m, then we will definitely have a solution because we can represent b in terms of columns of A.
- For this solution to be unique we can observe that colums of A must form a basis of R^m.
- ~~For~~ If that happens then $m = n$.

Thus for uniqueness colums of A must form a basis of colspace (A).

To Summarise,

(i) If $b \in$ colspace(A), then there exists a solution.

(ii) If columns of ~~A~~ a basis of colspace (A) then this solution is unique.

(iii) If none of these conditions are satisfied then no-solution exists.