

(Q8) For least square problem

- Optimal residue $\hat{y}^d = Ax - b$ should be perpendicular to the column space of A .

- For any vector $z \in R^n$, $Az \perp \hat{y}^d$

$$(Az)^T \hat{y}^d = 0$$

(Inner product of orthogonal vectors)

- Since $\phi_1(x) = 1$, first column of matrix A will be filled with 1's.

- Thus vector $1 \in R^N$ lies in the column space of A .

- $A = \begin{bmatrix} 1 & & \\ 1 & & \\ 1 & & \\ \vdots & & \\ 1 & & \end{bmatrix}$

- From this we get
 $Ae_1 \perp \hat{y}^d$

$$Ae_1 = 1$$

$$\boxed{1 \perp \hat{y}^d}$$

$$\text{avg}(\hat{\delta}^d) = \frac{\mathbf{1}^T \hat{\delta}^d}{N}$$

As $\mathbf{1}$ and $\hat{\delta}^d$ are perpendicular

$$\text{avg}(\hat{\delta}^d) = 0$$

(Inner product of
2 perpendicular
vectors)