

19CS30048

SUMAS TAIN

$$(Q_2) \quad A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad b_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

\Rightarrow For least sq. solⁿ, geometrically columns of A should form a combination such that it is

closest to b_1 ,
 \Rightarrow Same as projection of b_1 on colspace of A .

Linear combination \Rightarrow

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$x_1 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 \\ 0 \end{pmatrix}$$

This represents the xy plane.

Projection of b_1 on xy plane is $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

Solving $\begin{pmatrix} x_1 + x_2 \\ x_2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, we get,

$$x_1 = 0$$

$$x_2 = 0$$

$$\hat{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

For b_2

Projection on xy plane = $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

$$x_1 + x_2 = 1$$

$$x_2 = 1$$

$$x_1 = 0 \quad x_2 = 1$$

$$\text{So } \hat{x} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$