19CS30048 SUHAS JAIN

2)

$$\theta_1^T = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 & 0 \\ -\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\theta_2^T = \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 \\ 0 & 1 & 0 \\ -\sin\theta_2 & 0 & \cos\theta_2 \end{bmatrix}$$

$$\theta_1^T a = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 & 0 \\ -\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} x \\ 0 \\ 2 \end{bmatrix}$$

$$x = \sqrt{1+2^2} = \sqrt{5}$$

$$-\sin\theta_1 + 2\cos\theta_1 = 0$$

$$\cos\theta_1 = \frac{1}{\sqrt{5}}$$

$$\sin\theta_1 = \frac{2}{\sqrt{5}}$$

$$\theta_2^T \theta_1^T a = \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 \\ 0 & 1 & 0 \\ -\sin\theta_2 & 0 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} \sqrt{5} \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} y \\ 0 \\ 0 \end{bmatrix}$$

$$y = \sqrt{5+4} = 3$$

(e)

$$-\sqrt{5}\sin\theta_2 + 2\cos\theta_2 = 0$$

$$\sin\theta_2 = \frac{2}{3}$$

$$\cos\theta_2 = \frac{\sqrt{5}}{3}$$

$$\theta_1^T = \begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 0 \\ \frac{-2}{\sqrt{5}} & \frac{+1}{\sqrt{5}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\theta_2^T = \begin{bmatrix} \frac{\sqrt{5}}{3} & 0 & \frac{2}{3} \\ 0 & 1 & 0 \\ -\frac{2}{3} & 0 & \frac{\sqrt{5}}{3} \end{bmatrix}$$

$$\theta_2^T \theta_1^T a = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

$$\theta = \theta_2^T \theta_1^T = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{-2}{\sqrt{5}} & \frac{+1}{\sqrt{5}} & 0 \\ \frac{-2}{3\sqrt{5}} & \frac{-4}{3\sqrt{5}} & \frac{\sqrt{5}}{3} \end{bmatrix}$$

$$L_{31} L_{21} a = \begin{bmatrix} \star \\ 0 \\ 0 \end{bmatrix}$$

$$L = L_{31} L_{21} = \begin{Bmatrix} R_2 - 2R_1 \\ R_3 - 2R_1 \end{Bmatrix}$$

$$a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad \Rightarrow \quad l_{21} = l_{31} = 2 = \frac{a_2}{a_1} = \frac{a_3}{a_1}$$

$$L_{21} = \{R_2 - 2R_1\}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$L_{31} = \{R_3 - 2R_1\}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

$$L = L_{31} L_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$