19CS30048 SUHAS JAIN

Q5)

(i)

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \longrightarrow \begin{bmatrix} x_1 \\ \frac{x_1+x_2}{2} \\ \frac{x_2+x_3}{2} \\ \vdots \\ x_n \end{bmatrix}$$

$f : R^n \to R^{n+1}$

$$f(x+y) = \begin{bmatrix} x_1+y_1 \\ \frac{(x_1+y_1)+(x_2+y_2)}{2} \\ \vdots \\ \frac{(x_n+y_n)+(x_{n-1}+y_{n-1})}{2} \\ (x_n+y_n) \end{bmatrix}$$

$$f(x+y) = \begin{bmatrix} x_1 \\ \frac{(x_1+x_2)}{2} \\ \vdots \\ \frac{(x_n+x_{n-1})}{2} \\ x_n \end{bmatrix} + \begin{bmatrix} y_1 \\ \frac{(y_1+y_2)}{2} \\ \vdots \\ \frac{(y_n+y_{n-1})}{2} \\ y_n \end{bmatrix}$$

$$f(x+y) = f(x) + f(y) \quad — \quad (1)$$

$$f(ax) = \begin{bmatrix} ax_1 \\ \frac{(ax_1+x_2)}{2} \\ \vdots \\ \frac{(ax_n+ax_{n-1})}{2} \\ ax_n \end{bmatrix} = a\begin{bmatrix} x_1 \\ \frac{(x_1+x_2)}{2} \\ \vdots \\ \frac{(x_n+x_{n-1})}{2} \\ x_n \end{bmatrix} = af(x)$$

$$f(ax) = a \cdot f(x) \quad - (2)$$

From (1) & (2) f is a linear transformation.

(ii) $$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \longmapsto \sum_{i=1}^{n} |x_i|$$

$$f : R^n \to R$$

$$f(x+y) = f\left(\begin{bmatrix} x_1+y_1 \\ x_2+y_2 \\ \vdots \\ x_n+y_n \end{bmatrix}\right) = \sum_{i=1}^{n} |x_i+y_i|$$

$$f(x) + f(y) = \sum_{i=1}^{n} |x_i| + \sum_{i=1}^{n} |y_i|$$

in general $|x| + |y| \neq |x+y|$

so $f(x) + f(y) \neq f(x+y)$

Counter - example

$$x = \begin{bmatrix} 2 \\ 2 \\ \vdots \\ 2 \end{bmatrix}_{n \times 1} \qquad y = \begin{bmatrix} -1 \\ -1 \\ \vdots \\ -1 \end{bmatrix}_{n \times 1}$$

$f(x+y) = n \qquad f(x) + f(y) = 3n.$

f is NOT a linear transformation

(iii) $f : R^n \longrightarrow R^n$

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \longrightarrow \begin{bmatrix} \max(x_1, 0) \\ \max(x_2, 0) \\ \vdots \\ \max(x_n, 0) \end{bmatrix}$$

$$f(ax) = \begin{bmatrix} \max(ax_1, 0) \\ \max(ax_2, 0) \\ \vdots \end{bmatrix}$$

$f(ax) \neq a.f(x)$ (because a can change the sign of some x_i)

Counter Example

$$x = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \qquad f(x) = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \vdots \end{bmatrix}$$

if $a = -1$

$$f(ax) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \neq a.f(x)$$

f is NOT a linear transformation