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4) i) $L = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \middle| \, x_1 + x_2 + x_3 = 0 \right\}$

Properties:-

① Commutative addition

$$\underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}}_{X} + \underbrace{\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}}_{Y} = \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \\ x_3 + y_3 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \\ y_1 \end{pmatrix} + \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

So, $X + Y = Y + X$

② Associativity addition.

$$(X+Y) + Z = \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \\ x_3 + y_3 \end{pmatrix} + \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} x_1 + y_1 + z_1 \\ x_2 + y_2 + z_2 \\ x_3 + y_3 + z_3 \end{pmatrix}$$

$$X + (Y + Z) = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} y_1 + z_1 \\ y_2 + z_2 \\ y_3 + z_3 \end{pmatrix} = \begin{pmatrix} x_1 + y_1 + z_1 \\ x_2 + y_2 + z_2 \\ x_3 + y_3 + z_3 \end{pmatrix}$$

So, $X + (Y + Z) = (X + Y) + Z$

③ Additive identity

$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \in L \quad [0 + 0 + 0 = 0]$

④ Additive inverse.

If $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \in L$ then $\begin{pmatrix} -x_1 \\ -x_2 \\ -x_3 \end{pmatrix} \in L \quad [\Sigma x_i = 0]$

(5) Cumulative scalar multiplication

$$\alpha(\beta x) = \alpha \begin{pmatrix} \beta x_1 \\ \beta x_2 \\ \beta x_3 \end{pmatrix} = \begin{pmatrix} \alpha\beta x_1 \\ \alpha\beta x_2 \\ \alpha\beta x_3 \end{pmatrix} = (\alpha\beta)x.$$

(6) Multiplication identity.

for $1 \in R$

$$1 \cdot X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = X$$

(7) Distribution ~~of~~ scalar multiplication

$$(\alpha+\beta) X = \begin{pmatrix} (\alpha+\beta) x_1 \\ (\alpha+\beta) x_2 \\ (\alpha+\beta) x_3 \end{pmatrix} = \begin{pmatrix} \alpha x_1 + \beta x_1 \\ \alpha x_2 + \beta x_2 \\ \alpha x_3 + \beta x_3 \end{pmatrix} = \alpha X + \beta X.$$

(8) $$\alpha(X+Y) = \alpha \begin{pmatrix} x_1+y_1 \\ x_2+y_2 \\ x_3+y_3 \end{pmatrix} = \begin{pmatrix} \alpha x_1 + \alpha y_1 \\ \alpha x_2 + \alpha y_2 \\ \alpha x_3 + \alpha y_3 \end{pmatrix} =$$

$$\begin{pmatrix} \alpha x_1 \\ \alpha x_2 \\ \alpha x_3 \end{pmatrix} + \begin{pmatrix} \alpha y_1 \\ \alpha y_2 \\ \alpha y_3 \end{pmatrix} = \alpha X + \alpha Y$$

Hence $\mathcal{L}$ is a vector space & since $\mathcal{L} \subseteq R^3$ then $\mathcal{L}$ is also a vector subspace of R^3.

(ii) $v_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$, $v_2 \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$

V_1 and V_2 form basis of L.

ie. $\text{span}\{V_1, V_2\} = L$

Finding a vector v such that it is ~~perpendicular to~~ orthogonal to L.

Let's take cross product of V_1 and V_2 and make the $\|v\|_2 = 1$.

$$v_1 \times v_2 = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$$

$$\| v_1 \times v_2 \|_2 = \sqrt{3}$$

to make norm 1

divide by $\sqrt{3}$

$$v = \begin{pmatrix} -1/\sqrt{3} \\ -1/\sqrt{3} \\ -1/\sqrt{3} \end{pmatrix}$$

$$Q = I - 2vv^T$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - 2 \begin{bmatrix} -\frac{1}{\sqrt{3}} \\ -1/\sqrt{3} \\ -1/\sqrt{3} \end{bmatrix} \begin{bmatrix} -1/\sqrt{3} & -1/\sqrt{3} & \frac{-1}{\sqrt{3}} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 2/3 & 2/3 & 2/3 \\ 2/3 & 2/3 & 2/3 \\ 2/3 & 2/3 & 2/3 \end{bmatrix}$$

$$\Theta = \begin{bmatrix} 1/3 & -2/3 & -2/3 \\ -2/3 & 1/3 & -2/3 \\ -2/3 & -2/3 & 1/3 \end{bmatrix}$$