(Q4)

$$\underset{x \in R}{\operatorname{argmin}} \sum_{i=1}^{N} \lambda_i (x - b_i)^2$$

$$\frac{\partial J}{\partial x} = \sum_{i=1}^{N} 2\lambda_i (x - b_i) = 0$$

$$\sum_{i=1}^{N} \lambda_i \hat{x} = \sum_{i=1}^{N} \lambda_i b_i$$

$$\hat{x} = \frac{\sum_{i=1}^{N} \lambda_i b_i}{\sum_{i=1}^{N} \lambda_i}$$

let. $\sum_{i=1}^{N} \lambda_i = S$

$$\hat{x} = \sum_{i=1}^{N} \left(\frac{\lambda_i}{S}\right) b_i$$

$$\omega_i = \frac{\lambda_i}{S}$$

$\omega_i > 0$ as $\lambda_i > 0$

$$\sum_{i=1}^{N} \omega_i = \frac{1}{S}\left(\sum_{i=1}^{N} \lambda_i\right) = \frac{S}{S} = 1$$

$\therefore$ $\boxed{\sum_{i=1}^{N} \omega_i = 1}$

$$\boxed{\omega_i = \frac{\lambda_i}{\sum \lambda_i}}$$