19CS30048 SUMAS JAIN

3) Given

$A^T\eta = 0$ & $b^T\eta = 0$ [Taking transpose both sides]

Construct a new matrix C

$$C = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ b_1 & b_2 & b_3 \end{bmatrix} = \begin{bmatrix} A^T \\ b^T \end{bmatrix}$$

for $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$ & $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

we know that

$C\eta = 0$ since $A^T\eta = 0$ & $b^T\eta = 0$

$\Rightarrow$ So all the elements of $C\eta$ will become 0.

$\Rightarrow$ That implies if η is non-zero (given) then $Cx = 0$ has a solution.

$\Rightarrow$ That implies rows and colums of C are dependent.

$$\alpha_1 (a_{11}, a_{21}, a_{31}) + \alpha_2 (a_{12}, a_{22}, a_{32}) =$$

$$(b_1, b_2, b_3)$$

has a solution ~~with $\alpha_1 \neq 0$ or $\alpha_2 \neq 0$~~

hence. for $Ax = b$.

$x = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$ will always exist.