Computer Science & Engineering Department
I. I. T. Kharagpur

Principles of Programming Languages: CS40032

Elective

Assignment – 3: Typed λ-Calculus *Marks: 25*

Assign Date: *02nd February, 2021* Submit Date: *23:55, 09th February, 2021*

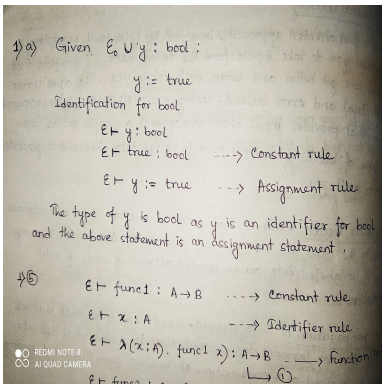
Instructions: Please solve the questions using pen and paper and scan the images. Every image should contain your roll number and name.

1. Derive the type of each lambda expression. Show all derivation steps. **[3 * 5 = 15]**

   (a) Given $\mathcal{E}_0 \cup y : bool$:

   $$y := true$$

   Solution:



   (b) Given type constants $func1 : A \rightarrow B$ and $func2 : (C \rightarrow B)$:

   $$\lambda(x \ : \ A).(func1 \ x); \lambda(q \ : \ C).(func2 \ q)$$

   Solution: $C \ \rightarrow \ B$

   (c) Given | be a constant of type $Bool \ \rightarrow \ Bool \ \rightarrow \ Bool$ and type of *true* is *Bool*

   $$\lambda(\omega \ : \ Bool \ \rightarrow \ \pi). \ \lambda(x \ : \ Bool). \ (\omega \ (x \mid true))$$

   Solution: $(Bool \ \rightarrow \ \pi) \ \rightarrow \ (Bool \ \rightarrow \ \pi)$

   (d) Given + is type constant with the type $S \ \rightarrow \ S$.

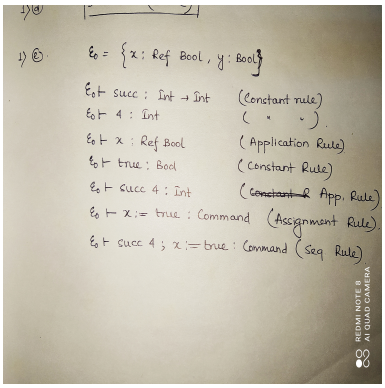
   $$\lambda(f \ : \ S \ \rightarrow \ C). \ \lambda(x \ : \ S). \ f(+x)$$

   Solution: $S \ \rightarrow \ C \ \rightarrow \ (S \ \rightarrow \ C)$

   (e) Given $\mathcal{E}_0 = \{x \ : \ Ref \ Bool, \ y \ : \ Bool\}$ and the constants $succ \ : \ Int \ \rightarrow \ Int$, $true \ : \ Bool$, $4 \ : \ Int$:

   $$succ \ 4; \ x \ := \ true$$

   Solution:



2. Derive the type of each of the following expression. Any assumptions are welcome. **[5 * 2 = 10]**

   (a) Given the type of ϕ is $Float \rightarrow Integer$

   $$(\lambda(p : Float \rightarrow Integer).\ \lambda(f : Float \rightarrow Float).\ \lambda(y : Float).\ p\ (f\ (f\ y)))\ \phi$$

   Solution: $(Float \rightarrow Float) \rightarrow (Float \rightarrow Integer)$

   (b) Given ϕ be the constant with the type $Bool \rightarrow Bool \rightarrow Bool$ and $true$ with the type $Bool$

   $$\lambda(func1 : Bool \rightarrow Char).\ \lambda(\tau : Bool).\ func1\ (\tau\ \phi\ true)$$

   Solution: $Bool \rightarrow Char \rightarrow (Bool \rightarrow Char)$