Computer Science & Engineering Department
I. I. T. Kharagpur

Principles of Programming Languages: CS40032

Elective

Assignment – 2: λ-Calculus *Marks: 20*

Assign Date: *18th January, 2021* Submit Date: *23:55, 22nd January, 2021*

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1. Reduce the following λ-expressions. Show every step of α-, β-, η- and δ-reductions. **[2 * 7 = 14]**

   (a) $(\lambda z.\ z)\ (\lambda y.\ y\ y)\ (\lambda x.\ x\ a)$

   (b) $(\lambda z.\ z)\ (\lambda z.\ z\ z)\ (\lambda z.\ z\ y)$

   (c) $(\lambda x.\ \lambda y.\ x\ y\ y)\ (\lambda a.\ a)\ b$

   (d) $(\lambda x.\ \lambda y.\ x\ y\ y)\ (\lambda y.\ y)\ y$

   (e) $(\lambda x.\ x\ x)\ (\lambda y.\ y\ x)\ z$

   (f) $(\lambda x.\ (\lambda y.\ (x\ y))\ y)\ z$

   (g) $(((\lambda x.\ (\lambda y.\ (x\ y))\ (\lambda y.\ y))\ w)$

2. Solve the following using Y combinator **[2 + 4 = 6]**

   (a) Write the recursive definition for $TriProduct$ where $TriProduct(n)$ can be defined as

   $$\begin{array}{lcll} TriProduct(n) & = & n^* \ (TriProduct(n\ -\ 1) & if\ n > 3 \\ & & +\ TriProduct(n\ -\ 2) + & \\ & & TriProduct(n-3)), & \\ & = & 5, & if\ n = 3 \\ & = & 2, & if\ n = 2 \\ & = & 1 & if\ n = 1 \end{array}$$

   Using Y combinator, encode the above recursive definition of $TriProduct$ as λ-expressions

   (b) Reduce $TriProduct$ 4. Show every step of β- and δ- reductions. You may skip α-reduction steps with a mention of the step.