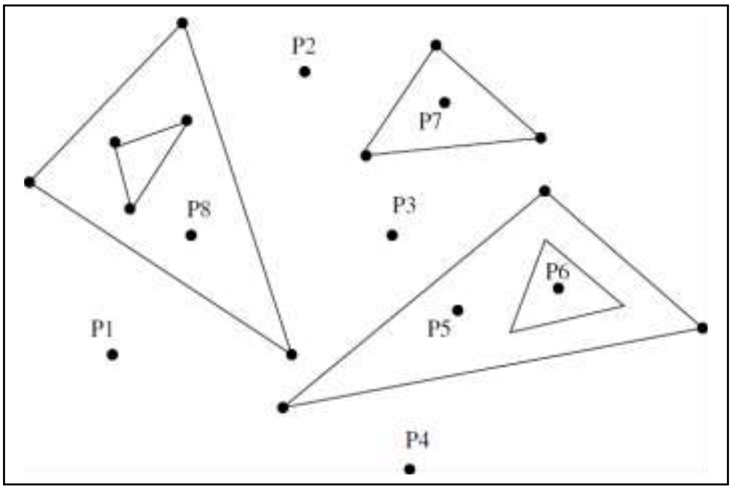
CS60064: Computational Geometry, Spring 2022

Homework Set – 04: Total points: 100; Credit: 10%;

Issued on 03 March 2022, **Due:** 13 March 2022, 11:55 pm; Please submit on Moodle by the due date.

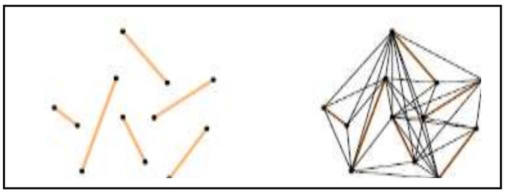
1. (25 points) Let S be a set of n triangles in the plane as shown in the figure below. The boundaries of the triangles are disjoint, but one may completely enclose another. Let P be a set of n points in the plane. Outline an $O(n \log n)$ algorithm that reports all points in P that lie outside all triangles in S. For example, in this figure, points P1, P2, P3, and P4 will only be reported.



2. (25 points) Let $A(L)$ denote a simple arrangement of a set L of n lines. Describe an $O(n \log n)$-time algorithm for constructing the convex hull of all intersection points.

3. (25 points) Prove that the test for collinearity of three points in a set of n points on the plane, is as hard as checking whether there exist three distinct integers a, b, c among a set of n integers, such that $(a + b + c) = 0$.

4. (25 points) Consider the visibility graph $G(V, E)$ of a set of n disjoint line-segments on the plane. Assume for simplicity that no three end-points of line-segments are collinear, and no line segment is horizontal or vertical. Each end-point defines a vertex in V. An edge (v_1, v_2) appears in E when the two end-points corresponding to v_1 and v_2 either belong to the same segment, or are visible on the plane. An example of visibility graph for a set of six line-segments is shown below.



(a) Show that G admits a cycle through all vertices. (25 points)

(b) Show that G can be constructed in $O(n^2)$-time using arrangement and duality.

Submission of the solution for Part (b) is **not required**; this may be treated as reading assignment only (David Mount's Lecture Notes).