Computer Science & Engineering Department
I. I. T. Kharagpur

Principles of Programming Languages: CS40032

Elective

Assignment – 2: λ-Calculus *Marks: 20*

Assign Date: *18th January, 2021* Submit Date: *23:55, 22nd January, 2021*

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1. Reduce the following λ-expressions. Show every step of α-, β-, η- and δ-reductions. **[2 * 7 = 14]**

   (a) $(\lambda z.\ z)\ (\lambda y.\ y\ y)\ (\lambda x.\ x\ a)$

   (b) $(\lambda z.\ z)\ (\lambda z.\ z\ z)\ (\lambda z.\ z\ y)$

   (c) $(\lambda x.\ \lambda y.\ x\ y\ y)\ (\lambda a.\ a)\ b$

   (d) $(\lambda x.\ \lambda y.\ x\ y\ y)\ (\lambda y.\ y)\ y$

   (e) $(\lambda x.\ x\ x)\ (\lambda y.\ y\ x)\ z$

   (f) $(\lambda x.\ (\lambda y.\ (x\ y))\ y)\ z$

   (g) $(((\lambda x.\ (\lambda y.\ (x\ y))\ (\lambda y.\ y))\ w)$

   BEGIN SOLUTION

(λz.z) (λy.y y) (λx.x a)
(λz.z) (λy.y y) (λx.x a) → // β-reduction = body[sym/replacemen
(λy.y y) (λx.x a) → // z[z/(λy.y y)] replace z with λy.y
(λx.x a) (λx.x a) → // y y[y/(λx.x a)] replace y with λx.x
(λx.x a) a → a a // x a[x/(λx.x a)] replace x with λx.x
// x a[x/a] replace x with a

(λz.z) (λz.z z) (λz.z y)
(λz.z) (λz.z z) (λz.z y) → // β-reduction: replace z with λz.z z
(λz.z z) (λz.z y) → // β-reduction: replace z with λz.z y
(λz.z y) (λz.z y) → // β-reduction: replace z with λz.z y
(λz.z y) y → y y // β-reduction: replace z with y

(λx.λy.x y y) (λa.a) b
(λx.λy.x y y) (λa.a) b → // β-reduction: replace x with λa.a
(λy.(λa.a) y y) b → // β-reduction: replace y with b
(λa.a) b b → b b // β-reduction: replace a with b

(λx.λy.x y y) (λy.y) y
(λx.λy.x y y) (λy.y) y → // α-conversion: rename y to a
(λx.λa.x a a) (λy.y) y → // β-reduction: replacing x with λy.y
(λa.(λy.y) a a) y → // β-reduction: replacing a with y
(λy.y) y y → y y // β-reduction: replacing y with y

(λx.x x) (λy.y x) z
(λx.x x) (λy.y x) z → // β-reduction: replacing x with λy.y x
(λy.y x) (λy.y x) z → // β-reduction: replacing y with λy.y x
(λy.y x) x z → // β-reduction: replacing y with x
x x z

(λx. (λy. (x y)) y) z
(λx. (λy. (x y)) y) z → // α-conversion: rename y to a
(λx. (λa. (x a)) y) z → // β-reduction: replacing x with z
(λa. (z a)) y → // β-reduction: replacing a with y
z y

((λx.x x) (λy.y)) (λy.y)
((λx.x x) (λy.y)) (λy.y) → // β-reduction: replacing x with λy.y
((λy.y) (λy.y)) (λy.y) → // β-reduction: replacing y with λy.y
(λy.y) (λy.y) → // β-reduction: replacing y with λy.y
λy.y

(((λx. λy.(x y))(λy.y)) w)
(((λx. λy.(x y))(λy.y)) w) → // α-conversion: rename y to a
(((λx. λa.(x a))(λy.y)) w) → // β-reduction: replacing x with λy.y
((λa.((λy.y) a)) w) → // β-reduction: replacing a with w
(λy.y) w → // β-reduction: replacing y with λy.y

   END SOLUTION

2. Solve the following using Y combinator **[2 + 4 = 6]**

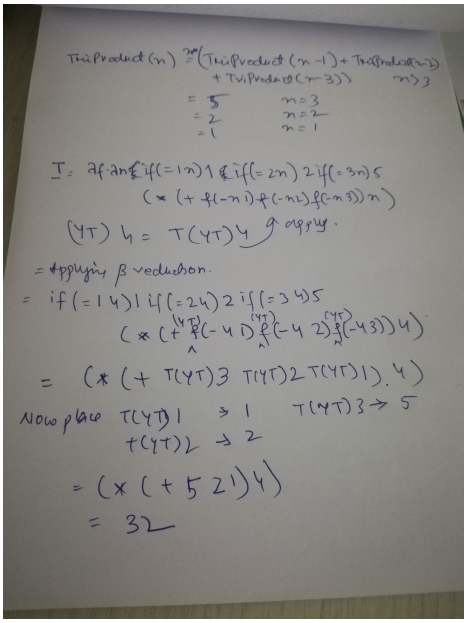
   (a) Write the recursive definition for $TriProduct$ where $TriProduct(n)$ can be defined as

$$
\begin{aligned}
TriProduct(n) &= n^* \ (TriProduct(n-1) && if\ n > 3 \\
&\quad + \ TriProduct(n-2) + \\
&\quad TriProduct(n-3)), \\
&= 5, && if\ n = 3 \\
&= 2, && if\ n = 2 \\
&= 1 && if\ n = 1
\end{aligned}
$$

Using Y combinator, encode the above recursive definition of $TriProduct$ as λ-expressions

(b) Reduce $TriProduct$ 4. Show every step of β- and δ- reductions. You may skip α-reduction steps with a mention of the step.

BEGIN SOLUTION



END SOLUTION