Computer Science & Engineering Department
I. I. T. Kharagpur

Principles of Programming Languages: CS40032

Elective

Assignment – 1: λ-Calculus *Marks: 25*

Assign Date: *14^{th} January, 2021* Submit Date: *23:55, 21^{st} January, 2021*

Instructions: Please solve the questions using pen and paper and scan the images. Every image should contain your roll number and name.

1. Fully parenthesize the following λ-expressions: **[1.5 * 3 = 4.5]**

   (a) $\lambda x.\ x\ z\ \lambda y.\ x\ y$

   (b) $(\lambda x.\ x\ z)\ \lambda y.\ w\ \lambda w.\ w\ y\ z\ x$

   (c) $\lambda x.\ x\ y\ \lambda x.\ y\ x$

2. Mark the free variables in the following λ-expressions: **[1.5 * 3 = 4.5]**

   (a) $\lambda x.\ x\ z\ \lambda y.\ x\ y$

   (b) $(\lambda x.\ x\ z)\ \lambda y.\ w\ \lambda w.\ w\ y\ z\ x$

   (c) $\lambda x.\ x\ y\ \lambda x.\ y\ x$

3. Prove the following using encoding in λ-calculus: **[2 * 8 = 16]**

   (a) $NOT(NOT\ TRUE) = TRUE$
   Given:
   $$NOT = \lambda x.\ ((x\ FALSE)\ TRUE)$$
   $$TRUE = \lambda x.\ \lambda y.\ x$$
   $$FALSE = \lambda x.\ \lambda y.\ y$$

   (b) $OR\ FALSE\ TRUE = TRUE$
   Given:
   $$OR = \lambda x.\ \lambda y.\ ((x\ TRUE)\ y)$$
   $$TRUE = \lambda x.\ \lambda y.\ x$$
   $$FALSE = \lambda x.\ \lambda y.\ y$$

   (c) $SUCC\ 2 = 3$
   Given:
   $$2 = \lambda f.\ \lambda y.\ f\ (f\ y)$$
   $$3 = \lambda f.\ \lambda y.\ f\ (f\ (f\ y))$$
   $$SUCC = \lambda z.\ \lambda f.\ \lambda y.\ f\ (z\ f\ y)$$

   (d) $(Y\ FACT)\ 2 = 2$
   Given:
   $$Y = \lambda f.\ (\lambda x.\ f\ (x\ x))\ (\lambda x.\ f\ (x\ x))$$
   $$FACT = \lambda f.\ \lambda n.\ IF\ n = 0\ THEN\ 1\ ELSE\ n\ ^*\ (f\ (n\ -\ 1))$$

   (e) Given: $mul = \lambda n.\lambda m.\lambda x.\ (n\ (m\ x))$

   Solve: $mul\ \overline{3}\ \overline{3}$

   (f) Solve: $add\ \overline{8}\ \overline{1}$
   Given: $add = \lambda n.\lambda m.\lambda f.\lambda x.\ n\ f\ (m\ f\ x)$

   (g) $IF\ FALSE\ THEN\ x\ ELSE\ y = y$
   Given:
   $$IF\ a\ THEN\ b\ ELSE\ c = a\ b\ c$$
   $$TRUE = \lambda x.\ \lambda y.\ x$$
   $$FALSE = \lambda x.\ \lambda y.\ y$$

   (h) Prove: add and mul are commutative
   Given:
   $mul = \lambda n.\lambda m.\lambda x.\ (n\ (m\ x))$
   $add = \lambda n.\lambda m.\lambda f.\lambda x.\ n\ f\ (m\ f\ x)$