Computer Science & Engineering Department
I. I. T. Kharagpur

Principles of Programming Languages: CS40032

Elective

Assignment – 3: Typed λ-Calculus *Marks: 25*

Assign Date: *03rd February, 2021* Submit Date: *23:55, 10th February, 2021*

Instructions: Please solve the questions using pen and paper and scan the images. Every image should contain your roll number and name.

1. Derive the type of each lambda expression. Show all derivation steps. **[3 * 5 = 15]**

   (a) Given $\mathcal{E}_0 \cup y : bool$:

   $$y \ := \ true$$

   (b) Given type constants $func1 : A \rightarrow B$ and $func2 : (C \rightarrow B)$:

   $$\lambda(x \ : \ A).(func1 \ x); \lambda(q \ : \ C).(func2 \ q)$$

   (c) Given | be a constant of type $Bool \ \rightarrow \ Bool \ \rightarrow \ Bool$ and type of $true$ is $Bool$

   $$\lambda(\omega \ : \ Bool \ \rightarrow \ \pi). \ \lambda(x \ : \ Bool). \ (\omega \ (x \mid true))$$

   (d) Given + is type constant with the type $S \ \rightarrow \ S$.

   $$\lambda(f \ : \ S \ \rightarrow \ C). \ \lambda(x \ : \ S). \ f(+x)$$

   (e) Given $\mathcal{E}_0 = \{x \ : \ Ref \ Bool, \ y \ : \ Bool\}$ and the constants $succ \ : \ Int \ \rightarrow \ Int$, $true \ : \ Bool$, $4 \ : \ Int$:

   $$succ \ 4; \ x \ := \ true$$

2. Derive the type of each of the following expression. Any assumptions are welcome. **[5 * 2 = 10]**

   (a) Given the type of ϕ is $Float \ \rightarrow \ Integer$

   $$(\lambda(p : \ Float \ \rightarrow \ Integer). \ \lambda(f : \ Float \ \rightarrow \ Float). \ \lambda(y : \ Float). \ p \ (f \ (f \ y))) \ \phi$$

   (b) Given ϕ be the constant with the type $Bool \ \rightarrow \ Bool \ \rightarrow \ Bool$ and $true$ with the type $Bool$

   $$\lambda(func1 : \ Bool \ \rightarrow \ Char). \ \lambda(\tau : \ Bool). \ func1 \ (\tau \ \phi \ true)$$