

Computer Science & Engineering Department
I. I. T. Kharagpur

Principles of Programming Languages: CS40032

Elective

Assignment – 6: Denotaional Semantics

Marks: 25

Assign Date: 25th March, 2021

Submit Date: 23:55, 09th April, 2021

Instructions: Please solve the questions using pen and paper and scan the images. Every image should contain your roll number and name.

1. Given the algebras of natural numbers and truth values, simplify the following expressions. Show all the steps in your simplification. [6]

- (a) $((\text{six equals}(\text{two plus one})) \rightarrow \text{one}) \sqcap (\text{three minus one}) \text{ plus two}$
- (b) $(\text{two equals}(\text{true} \rightarrow \text{one}) \sqcap \text{two}) \text{ and true}$
- (c) $\text{not}(\text{false}) \rightarrow \text{not}(\text{true}) \sqcap \text{not}(\text{true})$

2. Modify the dynamic array algebra of Module so that arrays carry with them upper and lower bounds. The operations are altered so that: [6]

- (a) $\text{newarray} : \text{Nat} \times \text{Nat} \rightarrow \text{Array}$ establishes an empty array with lower and upper bounds set to the values of the two arguments.
- (b) access and update both compare their index argument against the lower and upper bounds of their array argument. (Hint: use $\text{Array} = (\text{Nat} \rightarrow A) \times \text{Nat} \times \text{Nat}$.)

3. Draw the abstract syntax tree that results from parsing the series of keystrokes $12 + 5 \pm M^+ M^+ - 55 =$.

Remember, keystrokes are entered and evaluated from left to right.

[5]

4. (a) Complete the definition of the algebra given below by defining these operations: [8]

- i. $\text{update-payrate} : \text{Rat} \times \text{Payroll-rec} \rightarrow \text{Payroll-rec}$
- ii. $\text{update-hours} : \text{Rat} \times \text{Payroll-rec} \rightarrow \text{Payroll-rec}$

Domain $\text{Payroll-rec} = \text{String} \times (\text{Day} + \text{Night}) \times \text{Rat}$

where $\text{Day} = \text{Rat}$ and $\text{Night} = \text{Rat}$

(The names Day and Night are aliases for two occurrences of Rat . We use $\text{dwage} \in \text{Day}$ and $\text{nwage} \in \text{Night}$ in the operations that follow.)

Operations

$\text{newemp} : \text{String} \rightarrow \text{Payroll-rec}$
 $\text{newemp}(\text{name}) = (\text{name}, \text{inDay}(\text{minimum-wage}), 0)$
 $\text{move-to-dayshift} : \text{Payroll-rec} \rightarrow \text{Payroll-rec}$
 $\text{move-to-dayshift}(\text{employee}) = (\text{employee} \downarrow 1, (\text{cases}(\text{employee} \downarrow 2)$
of $\text{isDay}(\text{dwage}) \rightarrow \text{inDay}(\text{dwage}) \sqcap \text{isNight}(\text{nwage}) \rightarrow \text{inDay}(\text{nwage})$
end), $\text{employee} \downarrow 3)$
 $\text{move-to-nightshift} : \text{Payroll-rec} \rightarrow \text{Payroll-rec}$
 $\text{move-to-nightshift}(\text{employee}) = (\text{employee} \downarrow 1, (\text{cases}(\text{employee} \downarrow 2)$
of $\text{isDay}(\text{dwage}) \rightarrow \text{inNight}(\text{dwage}) \sqcap \text{isNight}(\text{nwage}) \rightarrow \text{inNight}(\text{nwage})$
end), $\text{employee} \downarrow 3)$
 .
 .
 .
 $\text{compute-pay} : \text{Payroll-rec} \rightarrow \text{Rat}$
 $\text{compute-pay}(\text{employee}) = (\text{cases}(\text{employee} \downarrow 2)$ of $\text{isDay}(\text{dwage}) \rightarrow$
 $\text{dwage multrat}(\text{employee} \downarrow 3) \sqcap \text{isNight}(\text{nwage}) \rightarrow (\text{nwage multrat } 1.5)$
 $\text{multrat}(\text{employee} \downarrow 3)$ end)

- (b) Use the completed algebra to define a payroll record stating that
 - i. “Jane Doe” has been assigned a payroll record.

- ii. She is moved to the night shift.
- iii. She works 38 hours that week.
- iv. Her payrate goes to 9.00.

Next, write an expression denoting Jane Doe's pay for the week.