Feedback — Quiz 7: Sections 15-16

You submitted this quiz on **Sat 17 Nov 2012 8:04 PM PST**. You got a score of **9.00** out of **9.00**.

Section 15 covers Randomness and Random Walk Models.

Section 16 focuses on the Colonel Blotto Game.

Both Models can be used to model competition, and there is some math involved in both. The second half of the quiz deals with the specifics of Blotto, so be ready to allocate your troops and choose your fronts.

Good luck!

Question 1

Which one of the following examples best illustrates the "Paradox of Skill"?

Your Answer		Score	Explanation
◉ All the sprinters in the Olympic 100m dash are extremely skilled. However, since they are so close to each other in skill level, the winner is often determined by luck.	✓	1.00	
Total		1.00 / 1.00	

Question Explanation

The "Paradox of Skill" states that when the difference in skill between groups or individuals is very small, the winner is often determined by luck.

Consequently, even when all competitors are extremely skilled, the winner is often determined by luck.

[See 15.3, "Skill vs. Luck"]

Question 2

A law firm has favorable outcomes in 80% of cases that go to court. They have repeated this feat for many, many years. They recruit the top talent and have some of the top lawyers in the country as partners. If we were to use the Skill vs. Luck Model to determine how much of the law firm's success derives from skill, and how much derives from luck, would we expect the variable a to be closer to 0 or to 1?

Your Answer		Score	Explanation
◉ 0	✓	1.00	
Total		1.00 / 1.00	

Question Explanation

The model tells us that Outcome = $a * Luck + (1 - a) * Skill$.

If a is close to 1, it means that outcomes are heavily influenced by luck.

If a is close to 0, it means that luck does not play a great role, indicating the influence of skill.

In this case, we expect a to be close to 0, because the question tells us that the law firm's success derives mostly from skill. There are several hints that tell us this: the law firm is successful over a sustained period of time ("many, many years"), and they have some of the "top lawyers" in the country. [See 15.3, "Skill and Luck"]

Question 3

Brett runs a casino. The odds that a person wins on a single hand of blackjack are 50%. Brett has had millions of people come through his casino. One day, a pit boss tells Brett that someone has just won 20 hands of blackjack in a row. Brett concludes that due to the length of this streak, the wins cannot possibly be fair (or in other words, random). Is Brett's reasoning correct? That is to say, is Brett correct to conclude that a person cannot possibly win 20 hands of blackjack in a row without cheating?

Your Answer		Score	Explanation
◉ No. It is possible that 20 wins in a row is random. Brett's reasoning is not correct.	✓	1.00	
Total		1.00 / 1.00	

Question Explanation

The answer is "No," Brett's reasoning is not correct.

Brett's reasoning is flawed according to the Binary Random Walk model. This model tells us that for any number k, a random walk will pass both $-k$ and $+k$ an infinite number of times, and in terms of coin flips, there will be a streak of k heads and k tails an infinite number of times.

Since the odds here are 50%, we can compare the blackjack game to flipping coins. When flipping a coin, there will be streaks, and it is possible for there to be 20 wins in a row, even though the process is random. In fact, the odds of 20 wins in a row are about one in a million.

[See 15.4, "Random Walks"]

Question 4

If you're rolling a fair die, what is the probability that you will roll a one 10 times in a row?

Your Answer		Score	Explanation
◉ $\frac{1^{10}}{6}$	✓	1.00	
Total		1.00 / 1.00	

Question Explanation

The probability of rolling a one on any single toss equals $\frac{1}{6}$, so the probability of getting ten in a row is $\frac{1^{10}}{6}$

[See 15.4, "Random Walks"]

Question 5

Each year a small clothing company produces a new sweater design. The sales of each design have been shown to be independent random variables. Owing to changes in fashion, the company produces each design for three years. Assume that the value of the firm depends on its sales and that we can write the value at time t as follows: $V_t = S_t + S_{t-1} + S_{t-2}$ where S_t equals sales at time t. Which of the following will be true of the firm's value? More than one may be true.

Your Answer		Score	Explanation
☑ The firm's value one year will be correlated with it's value in the next year.	✓	0.33	
☐ The firm's value in 2015 will share values with its value in 2012.	✓	0.33	
☑ The firm's value in two years will be correlated with its value this year.	✓	0.33	
Total		1.00 / 1.00	

Question Explanation

Two of the three are true.

We can write the value at time t as follows: $V_t = S_t + S_{t-1} + S_{t-2}$ where S_t equals sales at time t.

Therefore, the value at time $t+1$ shares variables with the value at time t, and so does the value at time $t+2$.

So the firm's value in one and two years will be correlated with its value this year.

However, if we look at the values in 2012 and 2015, we get the following:

$V_{2012} = S_{2012} + S_{2011} + S_{2010}$ and $V_{2015} = S_{2015} + S_{2014} + S_{2013}$.

There is no overlap in the variables for 2012 and 2015.

[See 15.6, "Finite Memory Random Walks"]

Question 6

Imagine the following scenario: Rufus and Cornelius are running in an election for Prime Minister. Cornelius has $1,000 to spend in advertising, while Rufus only has $500.

There are 5 districts participating in this election. In order to win the election, a candidate must win 3 of 5 districts. Whichever candidate spends the most in advertising in a district will $always$ win that district.

Assume that Cornelius distributes his advertising budget equally among the 5 districts. Is it possible for Rufus to win the election?

Your Answer		Score	Explanation
◉ No, it is not possible for Rufus to win.	✓	1.00	
Total		1.00 / 1.00	

Question Explanation

This is a Colonel Blotto game.

We assume that Cornelius distributes his money evenly, meaning he spends $200 in each of the 5 districts.

This means that in order to win 3 of the 5 districts, Rufus must spend at least 201+201+201=$603.

Since Rufus has only $500, he cannot possibly win the election.

[See 16.1, "Colonel Blotto Game"]

Question 7

Sasha has 80 troops allocated on 3 fronts. She has 40 on the 1st front, 40 on the 2nd front, and 0 on the 3rd front. What is the $minimum$ number of troops you would need to beat Sasha in this Blotto game? Assume - given that we're looking for the $minimum$ - that you go second, having seen Sasha's allocation, and that there are only two turns: Sasha's first, then yours.

Your Answer	Score	Explanation

42	✓	1.00
Total		1.00 / 1.00

Question Explanation

You only need to place 1 troop on the 3rd front to win there.

You also need 41 troops to win either the 1st or 2nd front.

So 42 troops will be the minimum amount that allows you to beat Sasha on 2 of the 3 fronts.

[See 16.2, "Blotto: No Best Strategy"]

Question 8

A small group of revolutionaries are fighting against their current colonial government. The revolutionaries have 290 troops. The government has 500 troops. What is the $minimum$ number of fronts required for the revolutionaries to have any chance of winning the war? Winning the war is defined as winning more than 50% of fronts. Assume that the government always distributes its troops as evenly as possible across all fronts.

Your Answer		Score	Explanation
7	✓	1.00	
Total		1.00 / 1.00	

Question Explanation

Let's start with 3 fronts. The government would allocate Front 1=166, Front 2=167, Front 3=167. Therefore, the Revolutionaries would need $167 + 168 = 335$ troops to beat the government on 2 of 3 fronts and thus win the war. They only have 290, so it's not possible.

Let's try 5 fronts. The government's allocation would be 100 troops on each front. So the revolutionaries would need at least $101 + 101 + 101 = 303$ troops to beat the government on 3 of 5 fronts and thus win the war. They still don't have enough.

Let's try 7. The government's allocation will be: 71 troops on 4 fronts, and 72 troops on the other 3 fronts. So the revolutionaries need

$72 + 72 + 72 + 72 = 288$ to beat the government on 4 of 7 fronts and thus win the war. They have 290 troops; therefore, the revolutionaries could win the war if there were 7 fronts.

The revolutionaries could win on 9 fronts as well, but this question asks for the $minimum$ number of fronts that would allow the revolutionaries to win.

[See 16.4, "Blotto: Troop Advantages"]

Question 9

Country A is at war with Country B. Country B has far fewer resources than Country A. Which of the following is a good strategy for Country B?

Your Answer		Score	Explanation
◉ Country B should try to increase the number of fronts.	✓	1.00	
Total		1.00 / 1.00	

Question Explanation

Increasing the number of fronts increases the weaker country's chances of winning, where we define the weaker country as the one with fewer resources.

[See 16.4, "Blotto: Troop Advantages"]