1-D Unsteady Diffusion Equation solution

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The 1D Unsteady Diffusion equation has been solved with the given initial condition and boundary conditions.The file oneD.cpp contains c++ program which solves the equation and the file PlotGraphs.m contains MATLAB program to plot data generated from solving the equation. The 1D unsteady diffusion equation as follows,

$$\frac{\partial u}{\partial t} = \alpha * \left(\frac{\partial^2 u}{\partial x^2}\right)$$

Finite Difference explicit scheme was used to discretize the equation. For time, forward difference scheme of first order was used and, for spatial discretization central diffrencing scheme of second order was used. The final discretized equation is as follows,

$$u_i^{n+1} = u_i^n + \left(\frac{\alpha * \Delta t}{\Delta x^2}\right) * (u_{i+1}^n - 2 * u_i^n + u_{i-1}^n)$$

$$d = \left(\frac{\alpha * \Delta t}{\Delta x^2}\right)$$

where α is 1, n is time index and i is space index with $\Delta t = 0.00001s$ and $\Delta x = 0.0078125$ units. Obtained d value was 0.176, means solution is stable (stability condition for FTCS method, d $<$ 0.5). The analytical solution is as follows,

Boundary conditions (periodic)

$$u(0,t) = u(1,t) = 0$$

Initial condition

$$u(x,0) = sin(2\pi x)$$

By method of separation of variables, PDE solution is

$$u(x,t) = \sum_{n=1}^{\infty} B_n e^{-n^2\pi^2 t} sin(n\pi x)$$

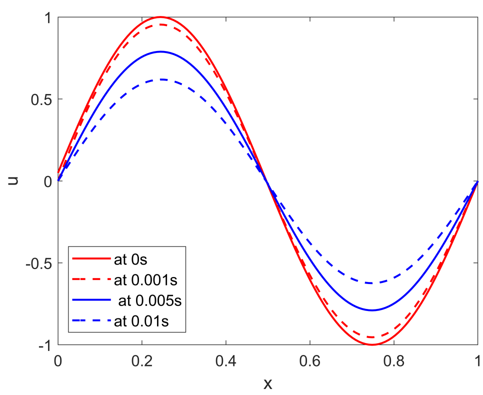
where

$$B_n = 2 * \int_0^1 sin(2\pi x) sin(n\pi x) dx$$

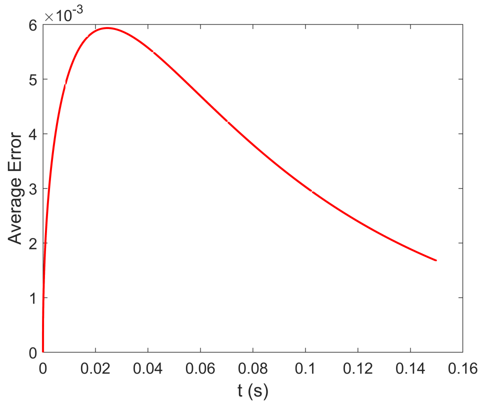
Final solution is ,

$$u(x,t) = \sum_{n=1}^{\infty} \left(\frac{sin(2\pi - n\pi)}{2\pi - n\pi} - \frac{sin(2\pi + n\pi)}{2\pi + n\pi}\right) sin(n\pi x) * e^{-n^2\pi^2 t}$$

The average error is calculated by subtracting numerical solution from analytical solution at each time step.The solution output was saved for t= 0, 0.001 (100 time-step) , 0.005 (500 time-step) and 0.01 (1000 time-step) seconds, plots are as follows,



(a) solution of $u(x,t)$



(b) Average domain error at each Δt