Indian Institute of Science, Banglore
Department of Computational and Data Sciences (CDS)

DS284: Numerical Linear Algebra

Quiz 1 [Posted Aug 25, 2022]

Faculty Instructor: Dr. Phani Motamarri
TAs: Ashish Rout, Dibya Nayak, Gourab Panigrahi, Nikhil Kodali

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Notations: (i) Vectors $\mathbf{v}$ and matrices $\mathbf{M}$ are denoted by bold faced lower case and upper case alphabets respectively. (ii) Set of all real numbers is denoted by $\mathbb{R}$ (iii) Set of all n dimensional vectors is denoted by $\mathbb{R}^n$ and set of all $m \times n$ matrices are denoted by $\mathbb{R}^{m\times n}$

1. For matrices $\mathbf{A}$ of size $m \times n$, $\mathbf{B}$ of size $n \times p$, $\mathbf{C}$ of size $p \times q$ and vector $\mathbf{v}$ in $\mathbb{R}^n$. Which of the following is/are true?
   (a) $\mathbf{Av}$ has mn multiplications and m(n-1) additions. [True]
   (b) $\mathbf{Av}$ has mn multiplications and mn additions. [False]
   (c) $\mathbf{AB}$ has mnp multiplications and m(n-1)p additions. [True]
   (d) $\mathbf{AB}$ has mnp multiplications and mnp additions. [False]
   (e) Number of Multiplications in $\mathbf{A(BC)}$ = Number of Multiplications in $\mathbf{(AB)C}$ [False]

2. Let $\mathbf{P}$ be 3×3 full-rank real matrix, If there exist a 3×4 real matrix $\mathbf{A}$ and 4×3 real matrix $\mathbf{B}$ such that $\mathbf{P} = \mathbf{AB}$ then
   (a) $\mathbf{Px} = \mathbf{0}$ has a unique solution where $\mathbf{0}$ = (0,0,0) and $\mathbf{x} \in \mathbb{R}^3$. [True]
   (b) There exists a $\mathbf{b} \in \mathbb{R}^3$ such that $\mathbf{Px} = \mathbf{b}$ has no solution. [False]
   (c) There exists a non-zero $\mathbf{b} \in \mathbb{R}^3$ such that $\mathbf{Px} = \mathbf{b}$ has a unique solution. [True]
   (d) There exists a non-zero $\mathbf{b} \in \mathbb{R}^3$ such that $\mathbf{P}^T\mathbf{x} = \mathbf{b}$ has a unique solution. [True]

3. Let, $\mathbf{M} = \begin{pmatrix} 10 & -2 & 6 \\ 5 & 1 & -4 \end{pmatrix}$. If $\mathbb{V} = \left\{ \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} \in \mathbb{R}^3 \;\middle|\; \mathbf{M}\begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$ and $\mathbb{W} = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \;\middle|\; \mathbf{M}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$, Then-
   (a) $\dim(\mathbb{W}) = 1$ [True]
   (b) $\dim(\mathbb{V}) = 1$ [False]
   (c) $\dim(\mathbb{W}) = 2$ [False]
   (d) $\mathbb{V} \cap \mathbb{W} = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$ [True]

4. For what values of k, the following set $\mathbb{S}$ will not form a basis of $\mathbb{R}^3$, $\mathbb{S} = \{$(k,2,2),(2,k,2),(6,6,k)$\}$
   (a) No such real k exists [False]
   (b) 6 [False]
   (c) 2 [True]
   (d) 4 [True]

5. Let, $\mathbb{V}$ be a vector space with a basis $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ [Note that $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are vectors], which means $\dim(\mathbb{V}) = 3$. Then which of the following is/are also a basis of $\mathbb{V}$?
   (a) $\{\mathbf{a}+\mathbf{b}+\mathbf{c},\ \mathbf{b}+\mathbf{c},\ \mathbf{c}-\mathbf{b}\}$ [True]
   (b) $\{\mathbf{a}+\mathbf{b}+\mathbf{c},\ \mathbf{b},\ \mathbf{c}\}$ [True]
   (c) $\{\mathbf{a}-\mathbf{b},\ \mathbf{b}-\mathbf{c},\ \mathbf{c}-\mathbf{a}\}$ [False]
   (d) $\{\mathbf{a}-\mathbf{b}+\mathbf{c},\ -\mathbf{a}+\mathbf{b}+\mathbf{c},\ \mathbf{c}\}$ [False]

6. Consider a matrix $\mathbf{A} \in \mathbb{R}^{n\times n}$, knowing that $\mathbf{A}^2$ is invertible and its inverse is $\mathbf{B}$. Which of the following is/are true?
   (a) Left inverse ($\mathbf{L} \in \mathbb{R}^{n\times n}$ such that $\mathbf{LA} = \mathbf{I}$) of $\mathbf{A}$ exists [True]
   (b) $\mathbf{A}$ and $\mathbf{B}$ commute i.e. $\mathbf{AB} = \mathbf{BA}$ [True]
   (c) $det(\mathbf{A}^T\mathbf{BA}) = 1$ [True]
   (d) $\forall \mathbf{b} \in \mathbb{R}^n\ \exists \mathbf{x} \in \mathbb{R}^n$ such that $\mathbf{A}^k\mathbf{x} = \mathbf{b}$, $\forall k \in \mathbb{N}$ [True]

7. Is it possible to have 3 non-parallel vectors in a 2 dimensional subspace such that $\mathbf{u}^T\mathbf{v} > 0$, $\mathbf{v}^T\mathbf{w} > 0$ and $\mathbf{u}^T\mathbf{w} < 0$? Also is it possible to have 3 non-parallel vectors in a 2 dimensional subspace such that $\mathbf{u}^T\mathbf{v} = 0$, $\mathbf{v}^T\mathbf{w} = 0$ and $\mathbf{u}^T\mathbf{w} = 0$?
   (a) Yes, No [True]
   (b) No, Yes [False]
   (c) Yes, Yes [False]
   (d) No, No [False]

8. Which of the following is/are true about the matrix vector multiplication operation of $\mathbf{Q}$ and $\mathbf{v}$, where $\mathbf{Q}$ is an orthogonal matrix of size $m \times m$ and $\mathbf{v}$ is a vector in $\mathbb{R}^m$?
   (a) Preserves the length of $\mathbf{v}$ [True]
   (b) Changes the length of $\mathbf{v}$ [False]
   (c) May correspond to a rigid rotation of $\mathbf{v}$ [True]
   (d) May correspond to a reflection of $\mathbf{v}$ [True]
   (e) Determinant of $\mathbf{Q}$ is ± 1 [True]

9. Suppose we know that for a matrix $\mathbf{A} \in \mathbb{R}^{m\times n}, \mathbf{A}^T\mathbf{A} = \mathbf{I}$, where $\mathbf{I}$ is the $n \times n$ identity matrix. Which of the following is/are true?
   (a) $\mathbf{A}\mathbf{A}^T$ is always an identity matrix [False]
   (b) $\mathbf{Ax}$=$\mathbf{0}$ has only a trivial solution $\mathbf{x}$=$\mathbf{0}$ for $\mathbf{x} \in \mathbb{R}^n$ [True]
   (c) Columns of $\mathbf{A}$ are always linearly independent [True]
   (d) Rows of $\mathbf{A}$ are always linearly independent [False]

10. Consider $\mathbf{A}$ and $\mathbf{B}$ as orthogonal matrices of same dimensions. Which of the following is/are always true?
    (a) $\mathbf{A} + \mathbf{B} = \mathbf{A}\mathbf{B}^T\mathbf{B} + \mathbf{A}\mathbf{A}^T\mathbf{B}$ [True]
    (b) $det(\mathbf{A}^T\mathbf{B} - \mathbf{B}^T\mathbf{A}) = det(\mathbf{A} + \mathbf{B}) \times det(\mathbf{A} - \mathbf{B})$ [True]
    (c) $(\mathbf{AB})^n$ is an orthogonal matrix $\forall\ n \in \mathbb{N}, n \geq 3$. [True]
    (d) If $det(\mathbf{A}^T) + det(\mathbf{B}^T) = 0$, then $det(\mathbf{A} + \mathbf{B}) = 0$ [True]