Indian Institute of Science, Banglore
Department of Computational and Data Sciences (CDS)

DS284: Numerical Linear Algebra

Quiz 1

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Notations: (i) Vectors $\mathbf{v}$ and matrices $\mathbf{M}$ are denoted by bold faced lower case and upper case alphabets respectively. (ii) Set of all real numbers is denoted by $\mathbb{R}$ (iii) Set of all n dimensional vectors is denoted by $\mathbb{R}^n$ and set of all $m \times n$ matrices is denoted by $\mathbb{R}^{m\times n}$

1. For what values of k, the following set $\mathbb{S}$ will not form a basis of $\mathbb{R}^3$, $\mathbb{S} = \{(k,1,1),(1,k,1),(1,1,k)\}$ (1 Points)
   (a) No such real k exist
   (b) -2
   (c) 1
   (d) -1

2. Which of the following property is/are true for square matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$? (2 Points)
   (a) $\mathbf{A(BC) = (AB)C}$
   (b) $\mathbf{ABC = CBA}$
   (c) $\mathbf{A}^n = 0$ implies $\mathbf{A}$=0
   (d) $\mathbf{A}^{-1}$ always exists.

3. For a matrix $\mathbf{A} \in \mathbb{R}^{m\times n}$, not necessarily square $(m \geq n)$ if $\mathbf{Ax} = \mathbf{0}$ has infinite solutions ($\mathbf{x}$ are the solutions). Which of the following is/are false? (2 Points)
   (a) dimension of null space of $\mathbf{A}$ is always 0
   (b) $\mathbf{A}$ can never be a full rank matrix
   (c) Columns of $\mathbf{A}$ are always linearly dependent
   (d) Rows of $\mathbf{A}$ are always linearly independent

4. Let, $\mathbf{M} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -1 & -1 \end{pmatrix}$. If $\mathbb{V} = \left\{ \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} \in \mathbb{R}^3 \;\middle|\; \mathbf{M} \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$ and

$\mathbb{W} = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \;\middle|\; \mathbf{M} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$, Then- (2 Points)

(a) $\dim(\mathbb{W}) = 1$

(b) $\dim(\mathbb{V}) = 1$

(c) $\dim(\mathbb{W}) = 2$

(d) $\mathbb{V} \cap \mathbb{W} = \{(0,0,0)\}$

5. Let, $\mathbb{V}$ be a vector space with a basis $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ [Note that $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are vectors], which means $\dim(\mathbb{V}) = 3$. Then which of the following is/are also a basis of $\mathbb{V}$? (2 Points)

(a) $\{\mathbf{a}+\mathbf{b}+\mathbf{c},\ \mathbf{b}+\mathbf{c},\ \mathbf{c}-\mathbf{b}\}$

(b) $\{\mathbf{a}+\mathbf{b}+\mathbf{c},\ \mathbf{b}+\mathbf{c},\ \mathbf{c}\}$

(c) $\{\mathbf{a}-\mathbf{b},\ \mathbf{b}-\mathbf{c},\ \mathbf{c}-\mathbf{a}\}$

(d) $\{\mathbf{a}-\mathbf{b}+\mathbf{c},\ -\mathbf{a}+\mathbf{b}+\mathbf{c},\ \mathbf{a}+\mathbf{b}-\mathbf{c}\}$

6. Is it possible to have 3 non-parallel vectors in a 2 dimensional subspace such that $\mathbf{u}^T\mathbf{v} < 0$, $\mathbf{v}^T\mathbf{w} < 0$ and $\mathbf{u}^T\mathbf{w} < 0$? Also is it possible to have 3 non-parallel vectors in a 2 dimensional subspace such that $\mathbf{u}^T\mathbf{v} = 0$, $\mathbf{v}^T\mathbf{w} = 0$ and $\mathbf{u}^T\mathbf{w} = 0$? (1 Points)

(a) Yes, No

(b) No, Yes

(c) Yes, Yes

(d) No, No