Indian Institute of Science, Banglore
Department of Computational and Data Sciences (CDS)

DS284: Numerical Linear Algebra

Quiz 1 [Posted Aug 6, 2021]

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Notations: (i) Vectors $\mathbf{v}$ and matrices $\mathbf{M}$ are denoted by bold faced lower case and upper case alphabets respectively. (ii) Set of all real numbers is denoted by $\mathbb{R}$ (iii) Set of all n dimensional vectors is denoted by $\mathbb{R}^n$ and set of all $m \times n$ matrices is denoted by $\mathbb{R}^{m\times n}$

1. For what values of k, the following set $\mathbb{S}$ will not form a basis of $\mathbb{R}^3$, $\mathbb{S} = \{$(k,1,1),(1,k,1),(1,1,k)$\}$
   (a) No such real k exist
   (b) -2
   (c) 1
   (d) -1

2. Which of the following property is/are true for square matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$?
   (a) $\mathbf{A(BC) = (AB)C}$
   (b) $\mathbf{ABC = CBA}$
   (c) $\mathbf{A}^n = 0$ implies $\mathbf{A}$=0
   (d) $\mathbf{A}^T\mathbf{A}$ is symmetric

3. For matrices $\mathbf{A}$ of size $m \times n$, $\mathbf{B}$ of size $n \times p$, $\mathbf{C}$ of size $p \times q$ and vector $\mathbf{v}$ in $\mathbb{R}^n$. Which of the following is/are true?
   (a) $\mathbf{Av}$ has mn multiplications and m(n-1) additions.
   (b) $\mathbf{Av}$ has mn multiplications and mn additions.
   (c) $\mathbf{AB}$ has mnp multiplications and m(n-1)p additions.
   (d) $\mathbf{AB}$ has mnp multiplications and mnp additions.
   (e) Number of Multiplications in $\mathbf{A(BC)}$ = Number of Multiplications in $\mathbf{(AB)C}$

4. Let $\mathbf{P}$ be 3×3 non-null real matrix, If there exist a 3×2 real matrix $\mathbf{A}$ and 2×3 real matrix $\mathbf{B}$ such that $\mathbf{P} = \mathbf{AB}$ then-

   (a) $\mathbf{Px} = \mathbf{0}$ has a unique solution where $\mathbf{0} = (0,0,0)$ and $\mathbf{x} \in \mathbb{R}^3$.

   (b) There exists a $\mathbf{b} \in \mathbb{R}^3$ such that $\mathbf{Px} = \mathbf{b}$ has no solution.

   (c) There exists a non-zero $\mathbf{b} \in \mathbb{R}^3$ such that $\mathbf{Px} = \mathbf{b}$ has a unique solution.

   (d) There exists a non-zero $\mathbf{b} \in \mathbb{R}^3$ such that $\mathbf{P}^T\mathbf{x} = \mathbf{b}$ has a unique solution.

5. Given that $4x + 2y + z = 31$ and $2x + 4y - z = 19$, then $9x + 7y + z = ?$

   (a) Can't be calculated uniquely.

   (b) $\frac{281}{3}$

   (c) $\frac{182}{3}$

   (d) $\frac{218}{3}$

6. Let, $\mathbf{M} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & -1 & -1 \end{pmatrix}$. If $\mathbb{V} = \left\{ \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} \in \mathbb{R}^3 \;\middle|\; \mathbf{M} \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$ and $\mathbb{W} = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \;\middle|\; \mathbf{M} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$, Then-

   (a) $\dim(\mathbb{W}) = 1$

   (b) $\dim(\mathbb{V}) = 1$

   (c) $\dim(\mathbb{W}) = 2$

   (d) $\mathbb{V} \cap \mathbb{W} = \{(0,0,0)\}$

7. Let, $\mathbb{V}$ be a vector space with a basis $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ [Note that $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are vectors], which means $\dim(\mathbb{V}) = 3$. Then which of the following is/are also a basis of $\mathbb{V}$?

   (a) $\{\mathbf{a} + \mathbf{b} + \mathbf{c},\ \mathbf{b} + \mathbf{c},\ \mathbf{c} - \mathbf{b}\}$

   (b) $\{\mathbf{a} + \mathbf{b} + \mathbf{c},\ \mathbf{b} + \mathbf{c},\ \mathbf{c}\}$

   (c) $\{\mathbf{a} - \mathbf{b},\ \mathbf{b} - \mathbf{c},\ \mathbf{c} - \mathbf{a}\}$

   (d) $\{\mathbf{a} - \mathbf{b} + \mathbf{c},\ -\mathbf{a} + \mathbf{b} + \mathbf{c},\ \mathbf{a} + \mathbf{b} - \mathbf{c}\}$

8. Is it possible to have 3 non-parallel vectors in a 2 dimensional subspace such that $\mathbf{u}^T\mathbf{v} < 0$, $\mathbf{v}^T\mathbf{w} < 0$ and $\mathbf{u}^T\mathbf{w} < 0$? Also is it possible to have 3 non-parallel vectors in a 2 dimensional subspace such that $\mathbf{u}^T\mathbf{v} = 0$, $\mathbf{v}^T\mathbf{w} = 0$ and $\mathbf{u}^T\mathbf{w} = 0$?

   (a) Yes, No

   (b) No, Yes

   (c) Yes, Yes

   (d) No, No

9. Which of the following is/are true about the matrix vector multiplication operation of $\mathbf{Q}$ and $\mathbf{v}$, where $\mathbf{Q}$ is an orthogonal matrix of size $m \times m$ and $\mathbf{v}$ is a vector in R^m?

   (a) Preserves the length of $\mathbf{v}$

   (b) Changes the length of $\mathbf{v}$

   (c) Corresponds to a rigid rotation of $\mathbf{v}$

   (d) Corresponds to a reflection of $\mathbf{v}$

   (e) Determinant of $\mathbf{Q}$ is ± 1

10. How do you test the "solvability" of a system of linear equations $\mathbf{Ax} = \mathbf{b}, \mathbf{A} \in \mathbb{R}^{m \times m}$?

    (a) Check if $\mathbf{b}$ lies in the column space of $\mathbf{A}$

    (b) Check if $\mathbf{b}$ lies in the row space of $\mathbf{A}$

    (c) Check if the column rank of $\mathbf{A} = m$

    (d) Check if determinant of $\mathbf{A}$ is 0

11. For a matrix $\mathbf{A} \in \mathbb{R}^{m \times n}$, not necessarily square $(m \geq n)$ if $\mathbf{Ax} = \mathbf{0}$ has infinite solutions ($\mathbf{x}$ are the solutions). Which of the following is/are false?

    (a) dimension of null space of $\mathbf{A}$ is always 0

    (b) $\mathbf{A}$ can never be a full rank matrix

    (c) Columns of $\mathbf{A}$ are always linearly dependent

    (d) Rows of $\mathbf{A}$ are always linearly independent