DS 288 (AUG) 3:0 Numerical Methods
Sample Midterm Exam [1]

Name:——————————————————————

Instructions:

1. `Duration: 60 Minutes (07:00 - 08:00 ).`
2. `Total number of points: 25 (25% of the final grade).`
3. `Solo effort only. Closed book exam, you are allowed to use one sheet of written notes.`
4. `Type in the answers in the space provided, rough work in your sheets/note book.`

Part I:Write either True/False and explain the answer for each question. **No credit will be given if the explanation of answer is missing**. *[Total points: 5x2 = 10 points].*

1. Least squares curve fitting using a polynomial of degree N requires that the number of data points used always be less than or equal to N+1.

   True/False.

   Explanation:

2. Cubic splines are especially accurate for functions with discontinuous first derivatives.

   True/False.

   Explanation:

3. Rounding errors behave similarly in both numerical differentiation and integration because both types of expressions are derived from the Taylor series of expansion.

   True/False.

[1]Conducted on: `August 30, 2021`.

Explanation:

4. Neville's Table is a method for generating the coefficients of successively higher degree polynomials.

   True/False.

   Explanation:

5. Adaptive quadrature attempts to adjust the spacing between function samples in order to uniformly distribute the error across the interval of integration by evaluating the error term through comparison of the numerical approximation to the exact answer.

   True/False.

   Explanation:

*Part II: Short answer questions [Total points: **3x5 = 15 points**].*

6. Investigate the possibility of developing a quadrtic spline for the following data set. Specifically how many constrains are needed and write them explicitly. Do they match the number of variables (unknowns) to be determined?. Do any problems arise that might cause you not to recommend such a spline?.

x	1.0	1.1	1.2	1.3	1.4
$f(x)$	1.684370	1.949477	2.199796	2.439189	2.670324

7. A colleague is interested in computing Hermite polynomials evaluated at specific values of x and has discovered the following recursion relation

$$H_{n+2}(x) = 2xH_{n+1}(x) - 2(n+1)H_n(x) \tag{1}$$

where $H_0(x) = 1$ and $H_1(x) = 2x$. Assuming that rounding error present in the Hermite polynomial evaluations (i.e., in $H_n(x)$ for all $n \geq 0$), would you recommend this process as a good way to evaluate higher order Hermite Polynomials?. Defend your answer.

8. The Secant method can be written in simpler form as

$$P_n = \frac{f(P_{n-1})P_{n-2} - f(P_{n-2})P_{n-1}}{f(P_{n-1}) - f(P_{n-2})} \tag{2}$$

Compare this with Secant method in terms of convergence and accuracy of P_n (solution at the n^{th} iteration).