Topics:

- Forward/Backward/Centered Differences
- Numerical Quadrature
- Composite Integration
- Newton-Cotes Open/Closed Formulas
- Romberg Integration
- Adaptive Quadrature
- Gauss Quadrature

Sections in text: 4.1–4.7 (except 4.2)

Study Questions–III [1]

1. Describe how to derive $f'''(x)$ to $O(h^3)$. How many function evaluations would be involved?. Could a centered difference formula be derived?.

2. How many more sample points are needed to increase a central difference formula from $O(h^m)$ to $O(h^{m+2})$?. Is this the same number that would be required for a backward difference formula?.

3. How does one interpret the error term when using a *single* application of a given quadrature rule?. How about for a composite quadrature rule?.

4. What happens to the error term behavior when composite rule integration is used (i.e., relative to a single application of the quadrature rule)?

5. What problem arise when $h \to 0$ in difference formulas?. What is the best strategy to adopt to help to overcome these difficulties?.

6. What is Romberg integration? and why is it useful?.

7. What is the rationale for adaptive quadrature?. What assumptions were made to develop the adaptive quadrature scheme studied in class?.

8. What is Gauss quadrature? Why is it more accurate than a quadrature rule with uniform sampling of the integrand with the same number of function evaluations?

9. How are the sample points chosen in Gauss quadrature? Is it generally useful in any interval [a,b]?.

10. How many error removals can be achieved in Romberg integration after 6 panel doublings?.

11. What is a panel and how many are involved in Simpson's $\frac{3}{8}^{th}$ rule (single application)?

12. What are Newton-Cotes open formulas?. How do they differ from Newton-Cotes closed formulas?. In terms of the accuracy benchmark for a single application of these rules, is either more accurate than the other for same number of integrand evaluations?.

13. Why is an optimal h theoretically possible to find for numerical differentiation? Is the same true for numerical integration?

14. How do rounding errors behave in numerical differentiation relative to numerical integration?.

[1] Posted on: `September 15, 2021`.