DS 288: Numerical Methods

Aug- 19- 2021

- Fill Google form
- Information files
  - * GRADING
  - * GENERAL INFORMATION
  - * CLASS SCHEDULE
- PROGRAMMING
  - MATLAB (NEW)

TWO RELATED ASPECTS

- THEORY : LECTURES, READING, EXAMS, STUDY QUESTIONS
- APPLICATION (ANALYSIS): HOMEWORKS (COMPUTER PROGRAMMING)

* EXAMS: TEST UNDERSTANDING OF THEORY (NO PROGRAMMING).

HOMEWORK:

- PRACTICAL EXPERIENCE TO DEMONSTRATE PRINCIPALS
- MOST METHODS ARE ALREADY "CANNED"
  - EXISTING & WELL PROVEN

* UNDERSTANDING ADVANTAGE & LIMITATION OF EACH NUMERICAL METHOD.

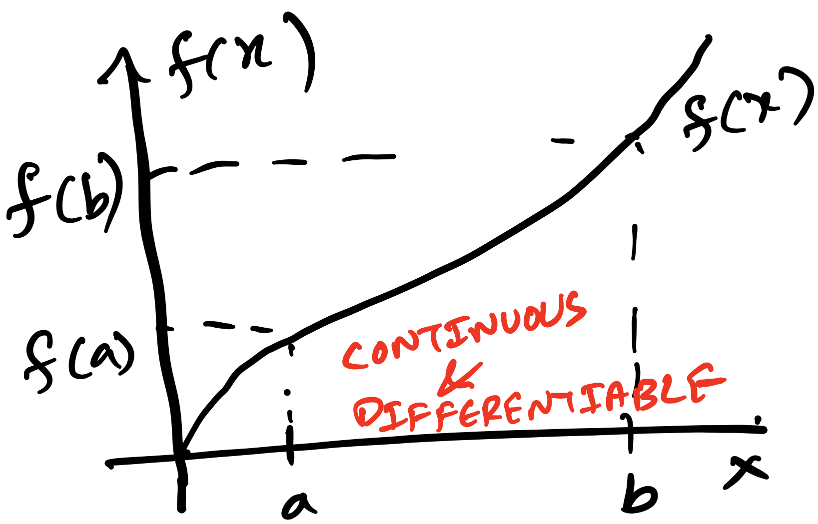
* NUMERICAL METHODS
  - NOT ANALYTICAL SOLUTION
  - MULTIPLE NUMERICAL METHODS MIGHT EXIST
  - APPROXIMATE SOLUTION (LIMITED PRECISION OF COMPUTING)
    * LESS ACCURATE COMPARED TO ANALYTICAL SOLUTION

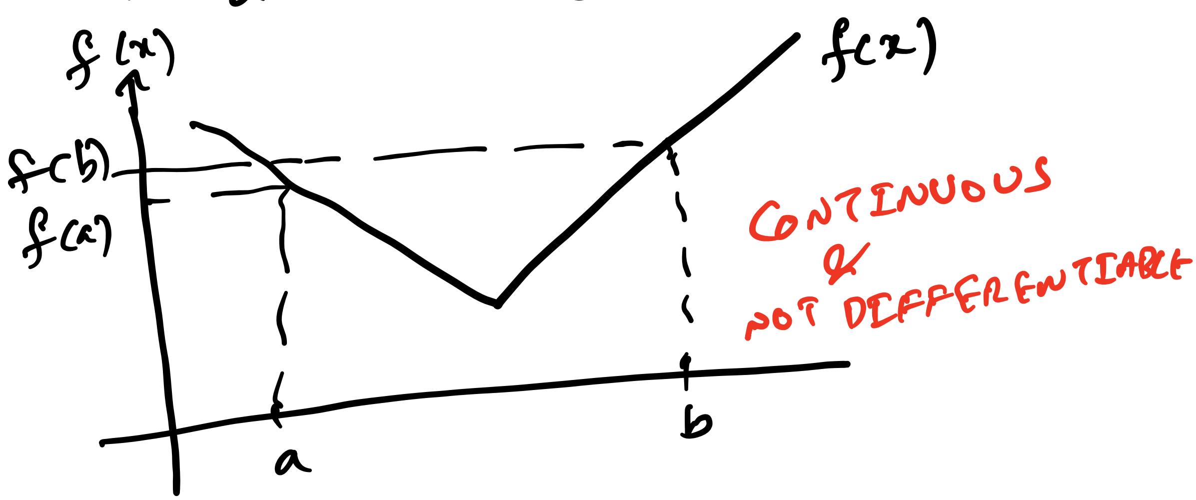
CHAPTER 1 - MATHEMATICAL PRELIMINARIES

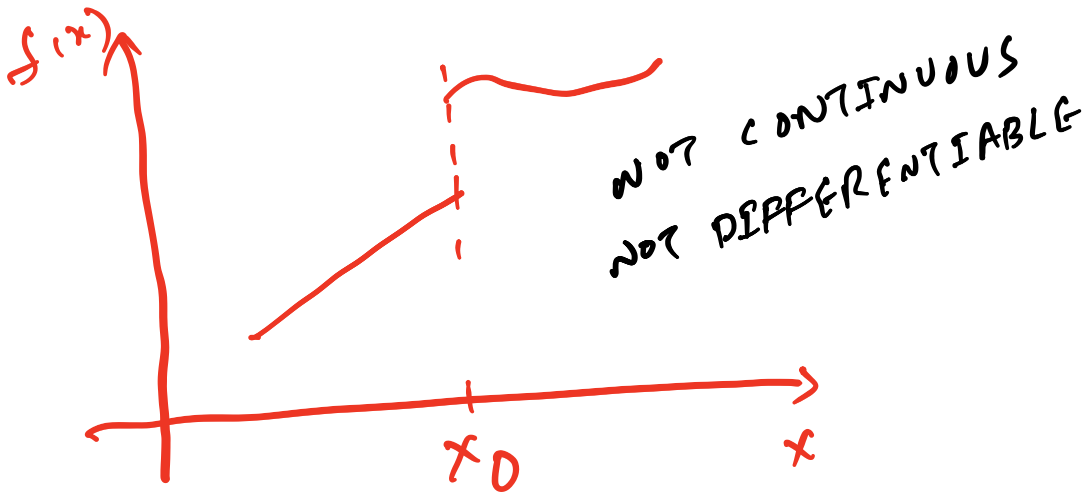
[[READING : §1.1 - 1.3]]

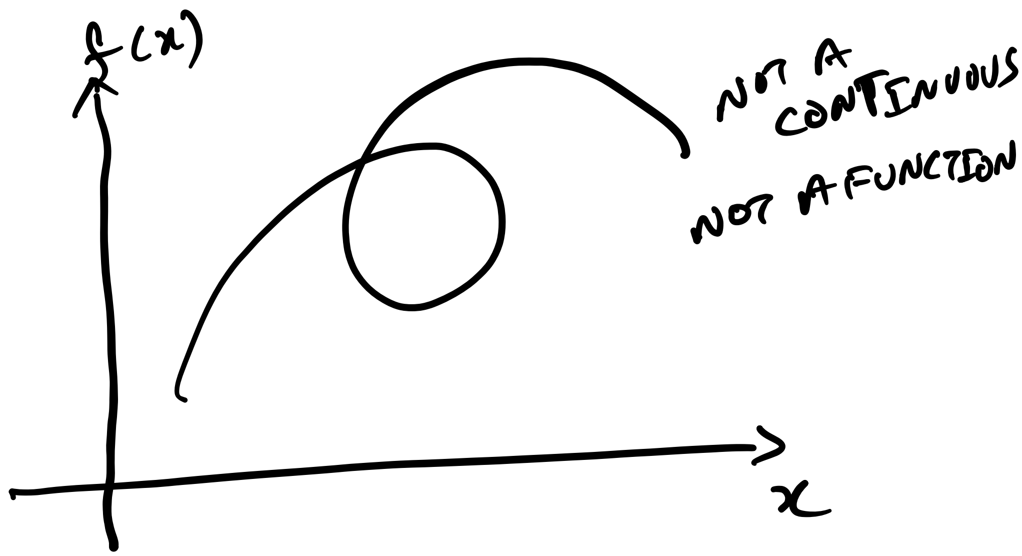
CALCULUS REVIEW

- EXAMPLES OF FUNCTIONS.









<u>CONTINUITY:</u>

$\lim_{x \to x_0} f(x)$ EXISTS AND IS <u>SINGLE-VALUED</u>

for all $x_0 \in [a,b]$

(REGARDLESS OF DIRECTION OF APPROACH)

DIFFERENTIABLITY

$$\underbrace{\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}}$$ EXISTS & IS SINGLE-VALUED

$$= \left. \frac{df(x)}{dx} \right|_{x = x_0} = f'(x_0) \rightarrow \text{SLOPE}$$

for all $x_0 \in [a, b]$

* EXAMPLE:

$$f(x) = \begin{cases} f_1(x) & x \in [a, x_0] \\ f_2(x) & x \in (x_0, b] \end{cases}$$

OPEN

SLOPE at x_0 for $f(x)$

$$\lim_{x \to x_0^-} \frac{f(x) - f(x_0)}{x - x_0} = \frac{f_1(x) - f_1(x_0)}{x - x_0}$$

$$\lim_{x \to x_0^+} \frac{f(x) - f(x_0)}{x - x_0} = \frac{f_2(x) - f_1(x_0)}{x - x_0}$$

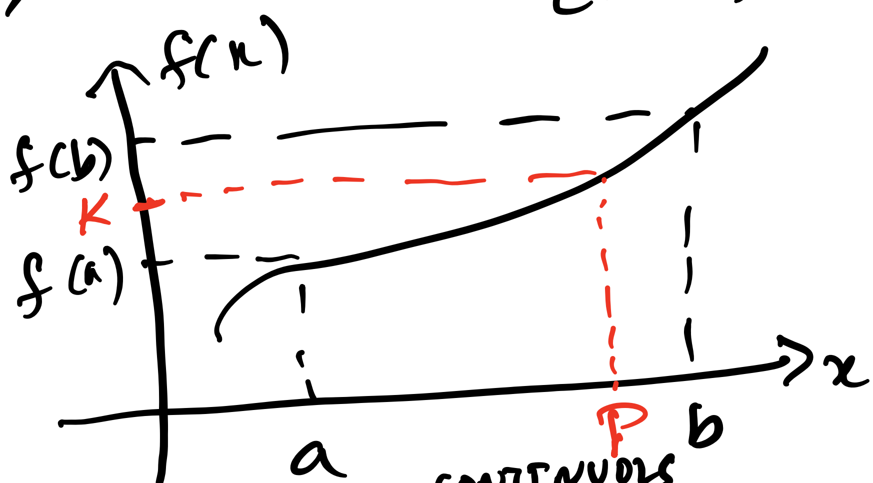
CSTN

NOT DIFFERENTIABLE

NOT A SINGLE VALUED

A FEW IMPORTANT THEOREMS

1) INTERMEDIATE VALUE THEOREM (IVT)

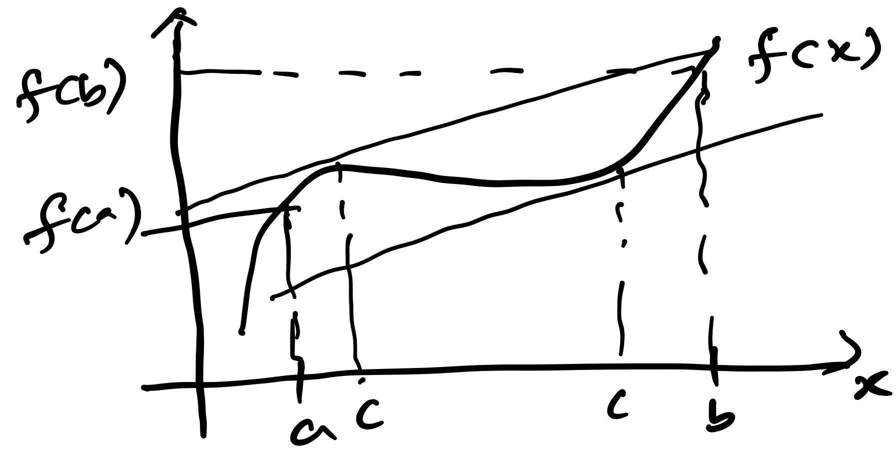


continuous

If $f \in C[a,b]$ & K is between $f(a)$ & $f(b)$ then $\exists$ a number $P \in (a,b)$ for which $f(P) = K$

(2) MEAN VALUE THEOREM (MVT)

If $f \in C[a,b]$ & differentiable on $[a,b]$ then a number $c \in (a,b)$ exists which $f'(c) = \dfrac{f(b)-f(a)}{b-a}$



(3) TAYLOR THEOREM [[IMP]]

IF f HAS N+1 DERIVATIES WHICH EXIST ON $[a,b]$ THEN

$$f(x) = P_N(x) + R_N(x)$$

WHERE

$P_N(x) \equiv$ POLYNOMIAL OF DEGREE 'N'

$R_N(x) \equiv$ REMAINDER OF DEGREE 'N+1'

N^{TH} ORDER TAYLOR POLYNOMIAL.

$$P_N(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)(x-x_0)^2}{2!} + \frac{f'''(x_0)}{3!}(x-x_0)^3 + \cdots + \frac{f^{(N)}(x_0)}{N!}(x-x_0)^N$$

$$= \sum_{K=0}^{N} \frac{f^{(K)}(x_0)}{K!}(x-x_0)^K$$

$\forall \underbrace{x \in [a,b]}_{\text{VALID OVER } [a,b]}$ & also $\underline{x_0} \in [a,b]$ → EXPAND ABOUT x_0.

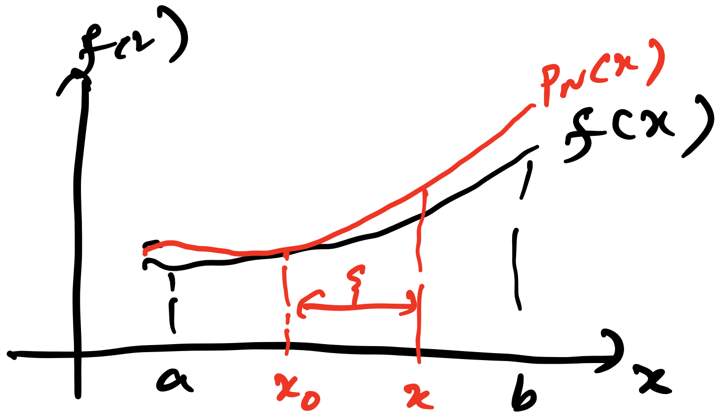
$$\lim_{N\to\infty} P_N(x) = f(x)$$

REMAINDER $R_N(x)$ → TRUNCATION ERROR

$$R_N(x) = \frac{f^{(N+1)}(\xi)\,(x-x_0)^{N+1}}{(N+1)!}$$

WHERE $\xi \in [x, x_0]$

[NOT SURE WHERE ξ IS THOUGH]



TRUNCATION ERROR → APPROXIMATE

UPPER BOUND CAN BE ESTIMATED
(worst case scenario)

MATH IN COMPUTERS (§1.2)

$(A+B)+C \neq A+(B+C)$

$$A*(B*C) \neq (A*B)*C$$

$$A*(B+C) \neq (A*B)+(A*C)$$

ORDER OF OPERATIONS MATTERS

EXAMPLE

$$\underbrace{\overbrace{dx+dx+dx+\cdots dx}^{A}}_{N\text{-TIMES}} = \overbrace{N*dx}^{B}$$

$A = B$ (MATHEMATICALLY)

(SINGLE PRECISION)

dx	N	A	B	ERROR
10^{-6}	10^{6}	1.00904	1.0000	~1%
10^{-7}	10^{7}	1.06477	1.0000	~6%
10^{-8}	10^{8}	0.25000	1.0000	~75%

$x = x + dx$ → NEVER DO THIS IN LOOP

$x = j * dx$ ✓

- CAUSE FOR THESE ERRORS "ROUND OFF"
- UNAVOIDABLE
- AS # OPERATIONS ↑ ROUND OFF ERROR ↑

QUESTION: IS IT MANAGEABLE??

CLASSES OF ERROR GROWTH

[§1.3]

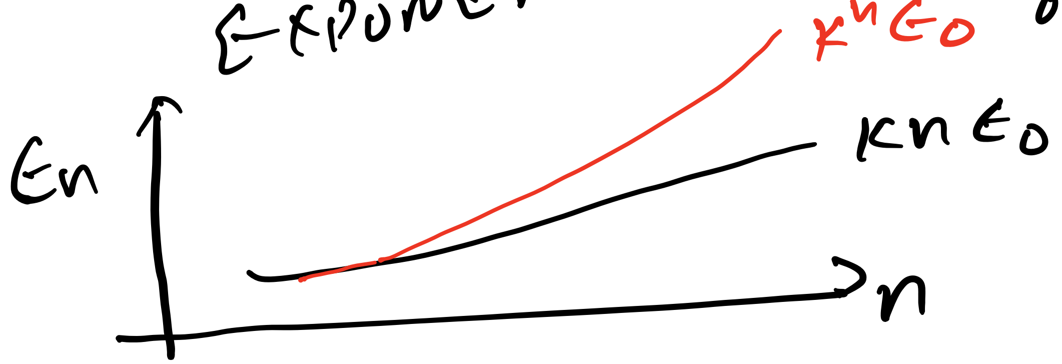
MANY ALGORITHMS REQUIRE STEPPING IN TIME OR SPACE OR ITERATING TO DETERMINE A SOLUTION.

1) $\epsilon_n \propto k n \epsilon_0 \rightarrow$ LINEAR GROWTH [OK]

2) $\epsilon_n \propto (k)^n \epsilon_0 \qquad k > 1.$

$\downarrow$

EXPONENTIAL GROWTH [NOT OK]



$\epsilon_0 \rightarrow$ ERROR AT INITIAL STEP

* RECURSIVE EQUATIONS
(OR DIFFERENE EQUATIONS)

[§1.3]

(a) $P_n = \frac{1}{3} P_{n-1}$ (b) $P_n = \frac{10}{3} P_{n-1} - P_{n-2}$

CAN WE EXPLAIN THE BEHAVIOR OF ERRORS FOR COMPUTATION OF P_n?

END OF CLASS