

**Topics:**

- Interpolation
- Lagrange Polynomials
- Neville's Method
- Chebyshev Optimal Points
- Hermite Polynomials
- Piecewise Interpolation
- Splines
- Least Squares

Sections in text: 3.1–3.5, 8.1–8.3

**Study Questions–II <sup>1</sup>**

1. Given 8 samples of  $f(x)$  at distinct values of  $x$ , what is the order of the unique interpolating polynomial that exactly reproduces the data samples provided?.
2. How many roots (or zeros) does  $L_{N,k}(x)$  have? Sketch  $L_{5,0}(x)$ ,  $L_{5,2}(x)$ , and  $L_{5,4}(x)$ . What would  $L'_{5,0}(x)$  and  $L'_{5,2}(x)$  look like?.
3. What is Neville's method? What is Divided Differences? If the data in Neville Table is rearranged at the start, will all the subsequent tabular entries be the same? Which will remain the same and which will be different if starting data  $x_0$  through  $x_4$  is changed such that  $x_2$  and  $x_3$  are interchanged?.
4. What is the advantage of using a basis function expansion to represent  $P_N(x)$ , an  $n^{th}$  order polynomial? What is the disadvantage?
5. What is the value of using Chebyshev sampling of  $f(x)$  to construct an interpolant? In this situation where would the sample points be located? Is the resulting polynomial representable as a Lagrange polynomial basis function expansion?
6. What special feature does a Hermite polynomial possess? What order Hermite polynomial results if 5 data samples (i.e.,  $x_i$  with  $i = 0, 1, 2, 3$ , and 4) are used? What advantages and disadvantages result from using a Hermite interpolant?.
7. What is the motivation for using piecewise polynomial interpolation? Can such an interpolant be constructed from Lagrange polynomials? Hermite polynomials? What would be the advantages and disadvantages of each approach? Give an example of how this would be done in each case?
8. What is the concept behind a spline? Identify the constraints that could be imposed using a *quadratic* spline. Do any problems arise? Does this suggest why cubic splines are popular?
9. When would least squares curve fitting be appropriate? What is the difference between linear and nonlinear least squares? Give an example of a fit that results in a linear case and a nonlinear case.
10. What is the difference between discrete and continuous least squares? Why are orthogonal basis function expansions so useful in the continuous case?

---

<sup>1</sup>Posted on: September 1, 2021.