DS-288 Numerical Methods
UMC-202 Introduction to Scientific Computing
Sample (Mid Sem 2025)

Time: 90 min

Name:

SR. No.: Weight 20% (100 Points)

Part-I (Multiple Choice Questions) [Total: $5 \times 6 = 30$]

- *There is no penalty for incorrect answers.*
- ***More than one option may be correct. Credits will be given only if all the correct and no wrong options are marked.***

1. Select the correct statements regarding root finding methods:

   (a) Bisection method always converges if $f(a)f(b) < 0$.

   (b) Newton-Raphson requires evaluation of both function and derivative.

   (c) Secant method requires only function evaluations.

   (d) False position method may stagnate if one endpoint remains fixed.

2. Select the correct statements comparing Newton-Raphson and Secant methods:

   (a) Newton-Raphson is usually faster than Secant.

   (b) Secant method requires two initial guesses.

   (c) Both methods require derivative evaluation.

   (d) Secant does not require derivative explicitly.

3. Select the correct properties of Lagrange interpolation polynomial:
   (a) It passes through all given data points.
   (b) Adding more points always change the polynomial completely.
   (c) The polynomial is unique for $n + 1$ distinct data points.
   (d) It always gives exact values outside the interpolation interval.

4. Select the correct statements regarding divided differences:
   (a) Zeroth-order divided difference is the function value at a point.
   (b) Higher-order divided differences are symmetric in their arguments.
   (c) For equally spaced nodes, divided differences reduce to finite differences.
   (d) Magnitude of higher-order divided differences always decreases.

5. Select the correct statements linking interpolation and root finding:
   (a) Lagrange interpolation can approximate a function before applying root-finding.
   (b) Divided differences are used to construct Newton's interpolating polynomial.
   (c) Bisection method can be applied to interpolating polynomials as well as original functions.
   (d) Newton-Raphson cannot be applied to interpolating polynomials.

Part-II (Assertion type question) [Total: $5 \times 6 = 30$]

- *Write True/False in the empty box and explain the reasoning for each question.*
- ***No credit will be given if the explanation of the answer is missing/incorrect.***

1. Talyor series expansion is valid for real-valued functions as well as complex-valued functions.

   Explanation:

2. The least squares curve fitting using a polynomial of degree N requires that the number of data points used always be less than or equal to N+1.

   Explanation:

3. The Regula Falsi method (false position method) always guarantees finding a root.

Explanation:

4. Piecewise cubic spline can produce an interpolating function that has continuous derivatives.

Explanation:

5. $f(x) = x + \sin x[1 + 2\cos 2x] - \sin 3x$, where $x \in \mathbb{R}$, then $f(x)$ is an algebraic function.

   Note: *An algebraic function is a function that can be expressed in the form of a polynomial with integer coefficients.*

   Explanation:

Part-III (Subjective type question) [Total: $10 + 15 + 15 = 40$]

- *Write your answers in the space provided.*

1. Find the root of $x^3 + x - 1 = 0$ using a fixed point iteration method that lies between 0 and 1.

2. Let $P_3(x)$ be the interpolating polynomial for the data $(0,0)$, $(\frac{1}{2}, y)$, $(1,3)$ and $(2,2)$. The coefficient of x^3 in $P_3(x)$ is 6. Find y.

3. Compute three iterations of the Newton's method to solve the system of equations with the initial approximation as $x_0 = 1.5$, and $y_0 = 0.5$.

$$x^2 + xy + y^2 = 7$$
$$x^3 + y^3 = 9$$