DS 288: Numerical Methods

Sep-28-2021

* MIDTERM:

  OCTOBER-5 (TUESDAY) @ 7AM

* PREPARE ONE PAGE OF WRITTEN NOTES (A4 SIZE)

* ROUGH PAGES ARE ALLOWED TO BE USED.

* SUBSCRIPT: a_i → a_i

  SUPERSCRIPT: a^i → a^i

  $R_{3,2}$ → R_(3,2)

  DIVIDE: $\frac{a}{b}$ → a/b

Topics:

- Error Growth
- Difference Equations
- Convergence Rates
- Root Multiplicity
- Fixed Points (Single and Multiple Variable)
- Fixed Point Methods: Newton, Secant, Modified Newton, False Position

Study Questions–I [1]

1. If a computational process can be described by a linear constant coefficient difference equation, under what conditions will exponential error growth result?.

IF λ IS ROOT OF CHARACTERSTIC EQN; $|\lambda| > 1$

2. What is a fixed point and can the existence of all fixed points be guaranteed? Why or why not?.

P IS A FIXED PT IF $g(P)=P$; NO, IF $|g'(x)|>1$ THEN MULTIPLE FIXED POINTS POSSIBLE

3. Can a fixed point iteration be guaranteed to converge? What conditions are required to prove convergence for *some* starting value? For *all* starting values on an interval [a, b]?.

YES (i) & (ii)*, (i), (ii) & (iii) ← FROM NEXT PAGES

4. Define convergence rate in a asymptotic sense. Under what conditions is a fixed point iteration quadratically convergent?

$\lim_{n\to\infty} E_{n+1}/E_n^{\alpha} = \lambda$ $\alpha \to$ ASYMPTOTIC CONV. RATE, $E_n \to$ ERROR AT STEP n; $\alpha = 2$, $g'(P)=0$, $g''(P)\neq 0$

5. When is Newtons method quadratically convergent? Linearly convergent? .

SIMPLE ROOT ($m=1$) → QUADRATIC
$m \geq 2$ → LINEAR

6. What is the rationale for the development of modified Newtons method? Why does it work?.

$m \geq 2$ → QUADRATIC NEW ITERATES $g(x)$ IN MOD. NEWTON'S METHOD

7. What are the advantages and disadvantages of the Secant method? How fast does it converge?

+ DO NOT HAVE TO KNOW $f'(x)$ − STARTING 2 VALUES ARE REQUIRED. CONVERGENCE: 1.62

8. What is a root multiplicity and how does it effect the convergence rate of various methods?

$m \geq 2$ SECANT METHOD: $\alpha = 1$ $\lim_{n\to\infty}$ $\alpha = 1 \Rightarrow \lambda \to 1$

9. Show graphically the difference between the Secant method and the method of False Position.

10. What conditions are necessary for convergence of a fixed point iteration involving multiple variables (and equations)? What conditions are need for such a process to be quadratically convergent?

[1] Posted on: `August 26, 2021`.

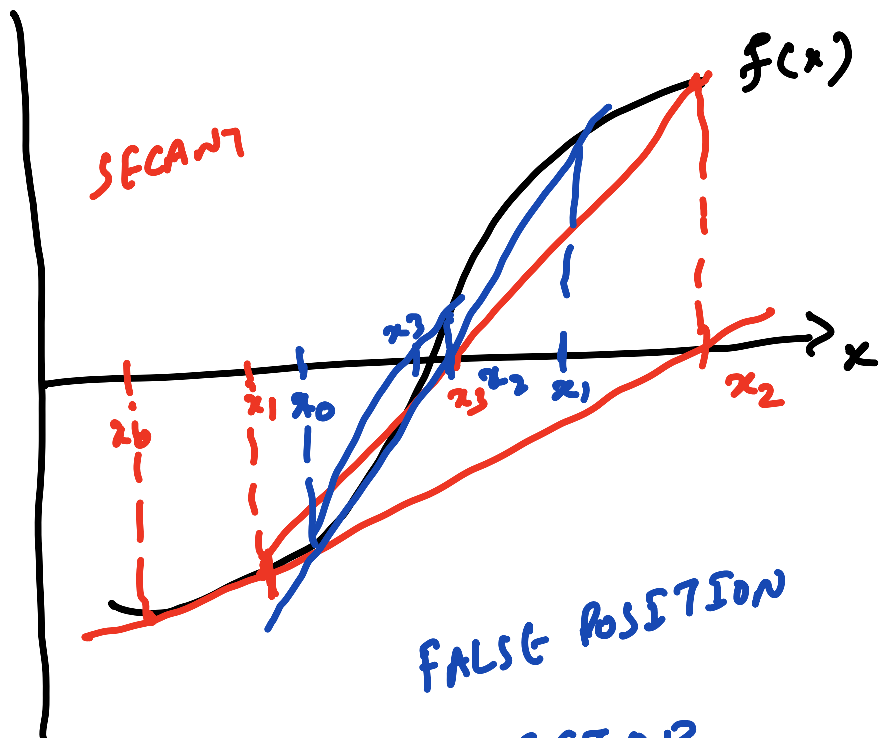
ANS:
(3) (i) $g(x)$ $C \in [a,b]$

(ii) $g(x)$ BOUNDED BY $[a,b]$ ON $[a,b]$

(iii) $|g'(x)| < 1$ FOR ALL $x \in [a,b]$ FOR ALL STARTING VALUES

$* \; |g'(p)| < 1$ FOR SOME STARTING VALUE

ANS
(9)



(ANS 10) $G(\underline{p}) = \underline{p}$ $\quad \underline{p} \to$ VECTOR

(i) $g_i(\underline{x})$ $C \in [\underline{a},\underline{b}]$ $\quad i = 1, 2, \ldots n$

(ii) $g_i(\underline{x})$ BOUNDED BY $[\underline{a},\underline{b}]$ ON $[\underline{a},\underline{b}]$

(iii) $\sum_{j=1}^{N} \left| \frac{\partial g_i(\underline{x})}{\partial x_j} \right| < 1 \quad i = 1, 2, \cdots N$

FOR $\alpha = 2$; $\frac{\partial g_i(\underline{x})}{\partial x_j} = 0 \quad i, j = 1, 2, \cdots N$

& $\frac{\partial^2 g_i(\underline{x})}{\partial x_j \partial x_k} \neq 0 \quad$ FOR $i, j, k = 1, 2, \cdots N$