DS-288 Numerical Methods
UMC-202 Introduction to Scientific Computing
Mid Sem Exam 2025

Time: 2 hr

Name:

SR. No.: Weight 30% (90 Points)

Part-I (Multiple Choice Questions) [Total: $4 \times 5 = 20$]

- *There is no penalty for incorrect answers.*
- ***More than one option may be correct. Credits will be given only if the correct option(s) and no wrong option(s) are marked. Partial credit(s) will be given.***

1. Richardson extrapolation improves accuracy by:
   (a) Reducing the effective step size
   (b) Cancelling the leading error term
   (c) Using higher derivatives
   (d) Combining results from different step sizes

2. Solving the BVP
$$-y'' = f(x), \quad y(0) = y(1) = 0$$
   with central differences yields a linear system that is:
   (a) Toeplitz and tridiagonal
   (b) Dense and symmetric
   (c) Upper triangular
   (d) Diagonal

3. The local truncation error of the explicit Euler method is:
   (a) $O(h)$

(b) $O(h^2)$

(c) $O(h^3)$

(d) $O(h^4)$

4. Consider the forward difference approximation for the second derivative:

$$f''(x_i) \approx \frac{af_i + bf_{i+1} + cf_{i+2}}{h^2}.$$

What are the values of a, b, and c that make this approximation *second-order accurate*?

(a) $a = 1, b = -2, c = 1$

(b) $a = 2, b = -5, c = 4$

(c) $a = -2, b = 5, c = -4$

(d) $a = 1, b = -3, c = 3$

Part-II (Assertion type question) [Total: $4 \times 5(1+4) = 20$]

- *Write True/False in the empty box and explain the reasoning for each question.*

1. An $n-$point Gaussian quadrature rule integrates exactly any polynomial of degree up to $2n-1$.

Explanation:

2. Is $f(x, y) = 2x + 3y$ is a Lipschitz function on the domain $R = \{(x, y) : 0 \leq x \leq 1, -\infty < y < \infty\}$.

Explanation:

3. The central difference second-derivative formula is exact for all polynomials of degree ≤ 3.

Explanation:

4. For the linear system $Ax \approx b$ (overdetermined), the normal equations $A^\top Ax = A^\top b$ yield a least-squares solution only if A has full column rank.

Explanation:

Part-III (Fill in the numerical values) [Total: $4 \times 5 = 20$]

- *Write your answers in the missing place. Credits will be awarded only for the final answer.*

1. Using Simpson's rule with $n = 2$ subintervals, the value of

$$\int_0^2 (1 + x^2)dx$$

is approximated as __________.

2. When solving $f(x) = e^{-x} - x$ using Newton-Raphson with $x_0 = 0$, the absolute error $|x_1 - x^*|$ after the first iteration (where x^* is the true root ≈ 0.567143) is __________ (round to two decimals).

3. Using the 3rd-order Taylor polynomial of e^x around $x = 1$, the estimated value of $e^{1.5}$ is __________.

4. For $f(x) = x(|x| + |x - 1|)$ the smallest Lipschitz constant L in the interval $[-2, 2]$ is __________.

Part-IV (Subjective type question) [Total: $2 \times 15 = 30$]

- *Write your answers in the space provided.*

1. (a) Since set of Legendre polynomials, $\{P_n(x)\}$, is orthogonal on $[-1, 1]$ with respect to the weight function $w(x) \equiv 1$. Use first three Legendre polynomials, $P_0(x) = 1, P_1(x) = x$, and $P_2(x) = \frac{3x^2-1}{2}$ to approximate $f(x) = x^3 + x^2 + x$ as $f(x) \approx \alpha P_0 + \beta P_1 + \gamma P_2$. [1.5+1.5+1.5]

   (b) Determine a, b, c such that the following formula

$$\int_0^h f(x)dx = h\left\{af(0) + bf(\frac{h}{3}) + cf(h)\right\}$$

   is exact for the polynomial of as high order as possible.

2. Approximate the integral below using Composite Simpson's rule with $n = m = 4$. In every intermediate arithmetic step, round to 3 decimal places.

$$\int_1^3 \frac{1}{x}\,dx$$

Compare with the exact value $\ln 3 = 1.099$.