

Parallel Programming - OpenMP Assignment

THEORY & METHODOLOGY

K-means clustering is an iterative unsupervised learning algorithm that segregates a set of data points into K clusters. [1]
[2] The diagram below shows how the algorithm iterates:

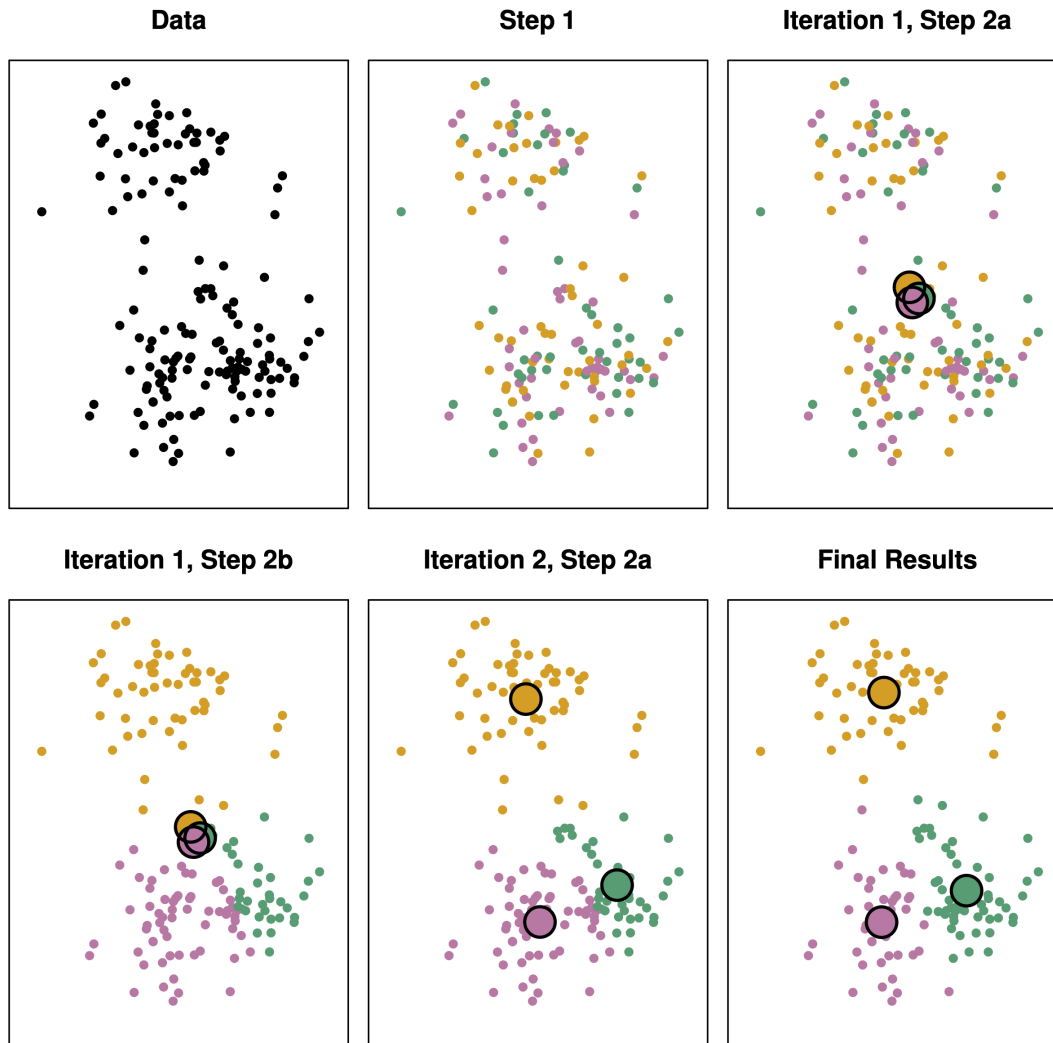


FIGURE 1. K-means iterative clustering algorithm. [3]

The basic (sequential) algorithm for K-means clustering is as follows:

1) Initialization:

- Choose the number of clusters, K . We use $K = 20$, as specified in the question.
- Initialize the K cluster centroids, C_1, C_2, \dots, C_K . We simply use the first K data points as the centroids for simplicity.¹

¹We can also use `kmeans++` algorithm, but such an algorithm will only increase the complexity. Our aim is check the time saved by using OpenMP, not obtain a perfect clustering algorithm. [4]

2) Assignment Step (Cluster Assignment):

- For each data point P_i in the dataset:
- Calculate its distance to each of the K centroids. We use Squared Euclidean distance:

$$d(P_i, C_j) = \|P_i - C_j\|^2$$

- Assign the data point P_i to the cluster of its nearest centroid.

3) Update Step (Centroid Recalculation):

- For each cluster j (from $j = 1$ to K):
- Recalculate the position of its centroid C_j to be the arithmetic mean of all data points P_i currently assigned to it.

4) Check for Convergence:

- After the update step, we check if the algorithm has converged. Convergence is reached if the cluster assignments (the sets S_j) did not change from the previous iteration.²
- If not converged, return to Step 3 (Assignment Step).

We can parallelise a few aspects of this algorithm:

- 1) The assignment step, wherein we assign each point to its nearest centroid, can be fully parallelised.
- 2) The accumulation step, where the per-thread local sums are merged into global, can be parallelised.
- 3) To check for convergence, we calculate the distance of each new centroid from its previous position. This can also be parallelised using OpenMP's `reduce` feature.

EXPERIMENTAL SETUP

Both the sequential and parallel programs were executed on the teaching cluster. The teaching cluster has the following specifications:

- **CPU:** Intel Xeon Gold 5318Y 24-core 2.1GHz
- **RAM:** 128 GB RAM
- **Storage (HDF5):** 2×12 TB SATA HDD, 1.92TB PCIe NVME M.2 Enterprise SSD
- **Operating System:** Cent OS
- **Job scheduling software:** SLURM

The program was executed parallelly using OpenMP. For testing the benefits of OpenMP 1, 4, 8, 16, 32 threads were used. When using 1 thread, the OpenMP program behaves exactly like a sequential program. To obtain more accurate readings, each experiment was repeated 5 times, and the execution time recorded.

The execution times (of each experiment) is then fed into Python, where the average corresponding to each thread count is calculated and output. The Python program also produces a graph using `matplotlib`.

²To accommodate for floating point errors, we assume convergence when the change in centroid positions between iterations is below a small threshold, ϵ . For the sake of this experiment, we take $\epsilon = 0.0001$.

RESULTS

As mentioned above, we calculated the average time over 5 experiments per number of threads. These averages are displayed in the below table:

TABLE 1. Table containing the average execution time against number of threads and whether static or dynamic mode was used.

Threads	Average Time with Dynamic (ms)	Average Time with Static (ms)
1	54.375	27.880
4	95.743	8.482
8	92.402	5.635
16	102.869	4.125
32	119.260	3.651

The graph obtained by plotting the averages is as follows:

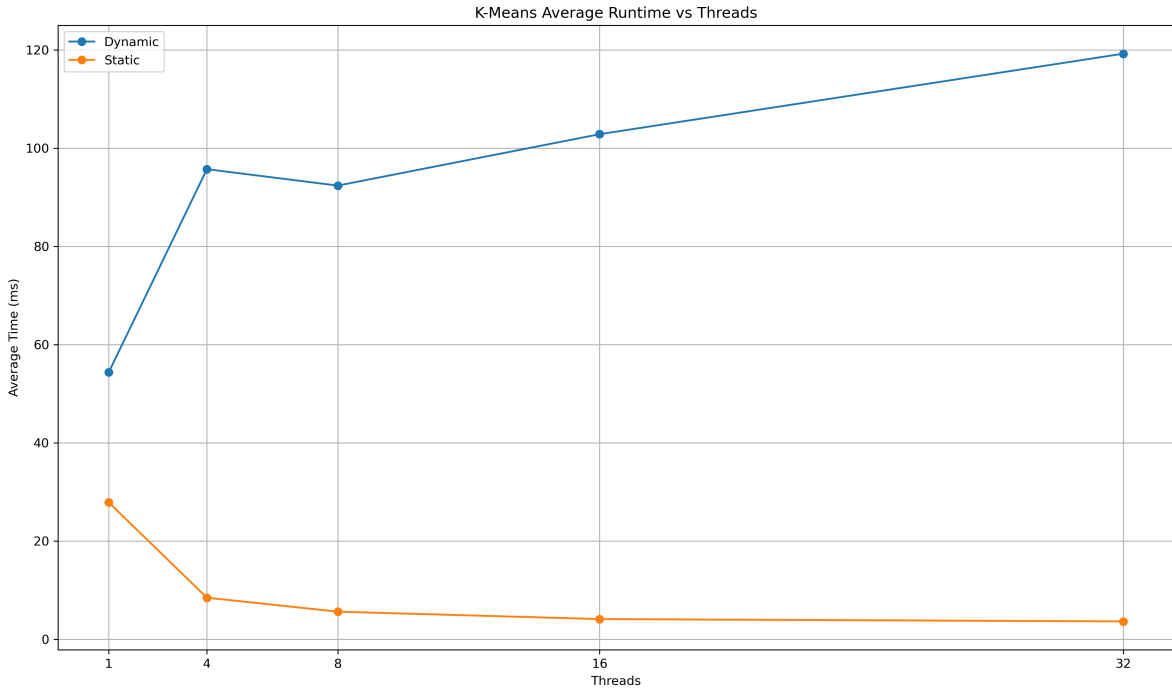


FIGURE 2. The graph depicting the average execution time against the number of threads.

OBSERVATIONS

Dynamic Scheduling. In the dynamic scheduling section, the opposite of what we had expected is happening. We expected the execution time to decrease as we provided the program with more and more threads. On the contrary, the execution time is increasing with increasing number of threads. The following explanation may provide an insight into this observation.

When we use dynamic scheduling, it maintains a central work queue of loop chunks. When a thread becomes idle, it must acquire a lock on this queue, take the next available chunk, and then release the lock. This strategy can be very efficient where

the loops are non-uniform, i.e., where some iterations take much longer than others. In other words, dynamic scheduling works best when a thread has a chance of executing faster than other threads.

In our case, the K-Means assignment loop is a highly uniform workload; every iteration takes a virtually identical amount of time, because each iteration essentially performs the same number of mathematical operations.

Static Scheduling. As can be seen in the graph, when we go from a sequential program to a parallel program with 4 threads, it takes just 30% of the time of the sequential program. This correlates to a speed-up of around 3.29.

However, as we increase the number of threads, the speed-up reduces. For example, we go from 4 threads to 8, the speedup is just 1.51 (which is less than half of the speed-up we obtained when going from a sequential program to 4 threads).

This is essentially the law of diminishing returns, wherein a higher "investment" need not guarantee the same "return on investment". In parallel programming, this is called as Amdahl's law. Amdahl's law can be stated as follows:

"The overall performance improvement gained by optimizing a single part of a system is limited by the fraction of time that the improved part is actually used." [5]

Amdahl's law can be visualised using the following graph:

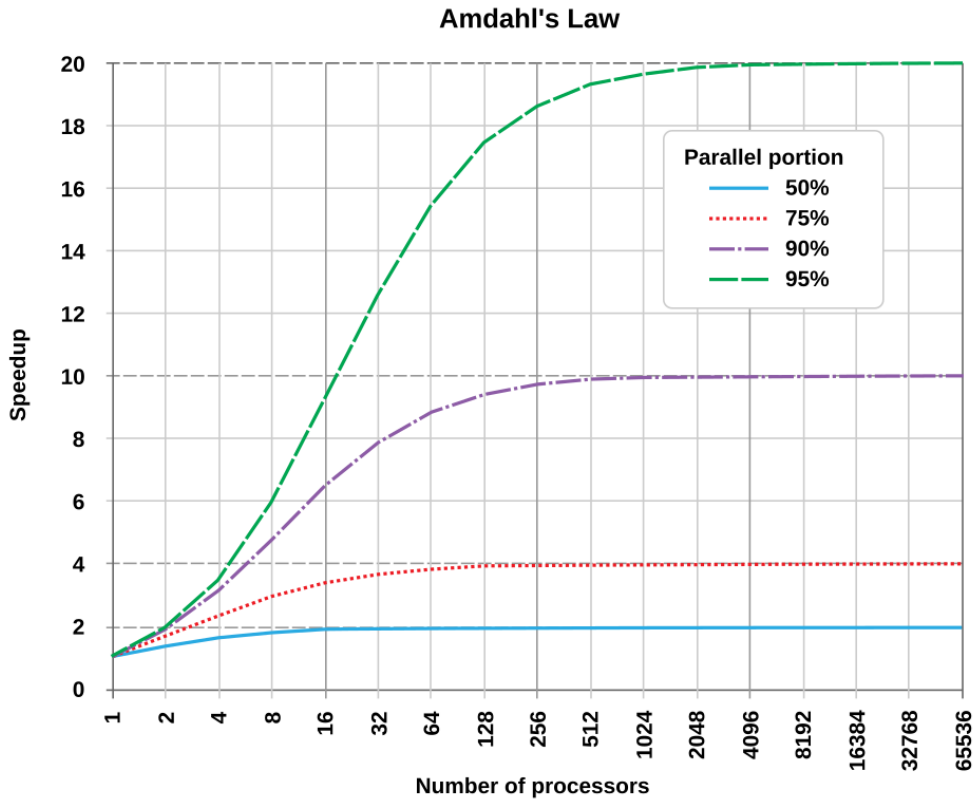


FIGURE 3. Visualisation of Amdahl's law. [6]

As can be seen in the graph, after a particular threshold, adding more processors (or threads) does not make any difference in the speed-up. What we have observed in our experiment corroborates Amdahl's law.

Dynamic vs. Static Scheduling. We can clearly see in the graph that the dynamic scheduling program required twice the execution of the static scheduling program, even for the program with a single thread. Ideally, both of these must have had the same execution time, due to them not being parallelised at all.

This is due to the unnecessary work of the scheduler’s lock acquiring/releasing mechanism for every chunk, even with no other threads to compete with. In other words, when we attempt to dynamically schedule a program, there is a lot of overhead associated with it. In fact, the overhead can be so high that the execution time is doubled, as happened in this case.

REFERENCES

- [1] E. Kavlakoglu and V. Winland, “What is k-means clustering?,”
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- [3] S. Bacallado and J. Taylor, “Stats202: K-means clustering,” 2022.
- [4] D. Arthur and S. Vassilvitskii, “k-means++: The advantages of careful seeding,” in *Proceedings of the Eighteenth Annual ACM-SIAM Symposium on Discrete Algorithms (SODA ’07)*, pp. 1027–1035, 2007.
- [5] M. D. Hill and M. R. Marty, “Amdahl’s law in the multicore era,” *Computer*, vol. 41, no. 7, pp. 33–38, 2008.
- [6] “Svg graph illustrating amdahl’s law.” Wikipedia, 2008.