DS-288 Numerical Methods
UMC-202 Introduction to Numerical Methods
Due date: August 24, 2025 (11:59 PM)

Homework-1

Total 100 points | Weightage 10%

Read the following instructions carefully.

- Write your NAME and SR. NUMBER on the first page of the report(only one PDF for all questions in order). Start each question on a new page.
- In coding exercises, also give algorithm/background theory along with code and discuss attached plots briefly to get full credit for that question.
- Only LaTeX version of the report is acceptable (find the attached demo LaTeX file). Use Python/Matlab for coding. Give proper annotations and comments in code wherever required.
- Submit your report as a single PDF named **LastFiveDigitsSRNo_Name.pdf**.
- For each question, give a separate code file `.ipynb`, i.e., for Question 1 name **LastFiveDigitsSRNo_Name_q1.ipynb**.
- Upload both the **PDF** and **.ipynb** files to Teams **(without zipping)**.
- Don't use any inbuilt functions for solving problems (i.e., np.linalg.solve); use a proper algorithm to get credits. Marks will be deducted if plagiarism is found in the report or codes. Late submissions won't be accepted.
- Using LLM's for writing the report is strictly prohibited.

---

1. A spherical buoy is being designed for oceanographic data collection. The buoy has a radius of 0.5 meters and is made from a composite material with a uniform density of $\rho_{obj} = 600 \text{ kg/m}^3$. It will be deployed in seawater, which has a density of approximately $\rho_{water} = 1025 \text{ kg/m}^3$.

   According to Archimedes' principle, a floating object is in equilibrium when the weight of the object ($W_{obj} = \rho_{obj} \times V_{sphere} \times g$) is equal to the weight of the fluid it displaces ($W_{disp} = \rho_{water} \times V_{disp} \times g$). Your task is to determine the depth to which the buoy will sink in the water.

. . .

(a) Set up an equation $f(h) = 0$ that represents the equilibrium condition ($W_{obj} = W_{disp}$). Perform the Bisection method to approximate the root of $f(h)$, with error tolerance $\epsilon = 10^{-4}$. Report the number of iterations and the root found in each iteration in a table. Explain your choice of interval $[a, b]$. [15 marks]

(b) Explain why the bisection method is guaranteed to converge for this problem. [5 marks]

Note: $V_{disp} = \frac{\pi h^2}{3}(3R - h)$. The error at the i^{th} iteration is $|h_i - h_{i-1}|$.

2. Using Newton's and Modified Newton's methods, find the solution of $f(x) = 0$ for the given functions. Iterate until you reach a relative tolerance of 10^{-7} between successive iterates. For each case, report the root obtained and the number of iterations required for each method. Plot the rate of convergence (hint: use the theoretical concept discussed in class, use a log plot, and observe the slope) and provide your detailed observations on the convergence rate using the plot.

   (a) $f(x) = (x^2 - e^{-x}\cos x)^2$ with initial guess $x_0 = 5$. [15 marks]

   (b) $f(x) = \tan^{-1}(2x + 3) + 1$ with initial guess $x_0 = -1$. [15 marks]

   Note: Use relative error between successive iterates:

$$\text{Relative error} = \left| \frac{x_{n+1} - x_n}{x_{n+1}} \right|$$

.

3. An economist is studying the market for a new brand of wireless headphones. Based on market research, they have modeled the weekly supply and demand for the headphones as functions of the price 'p' in dollars.

   - The Demand Equation (how many units consumers will buy) is:

$$Q_d = 10,000 - 20p$$

   - The Supply Equation (how many units manufacturers will produce) is influenced by production costs and is non-linear:

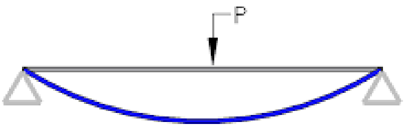
$$Q_s = 150\sqrt{p - 25}$$

   The market reaches equilibrium when the quantity demanded equals the quantity supplied ($Q_d = Q_s$). Algebraically rearrange this equation into the form $p = g(p)$. Now, using the fixed point iteration method, find this equilibrium price 'p'. Use the initial price $p_0 = \$150$ with tolerance error $\epsilon = 10^{-4}$. Report the number of iterations and the roots found during each iteration in a table. [25 marks]

4. An engineer is analyzing the deflection of a 10-meter-long steel I-beam that is simply supported at both ends and carries a heavy load P at its center. For the first half of the beam ($0 \leq x \leq 5$), the equation for the vertical deflection $y(x)$ (in meters) at a distance x from one end is given by:

$$y(x) = 10^{-4}(300x - 4x^3)$$

The engineer needs to find the exact location x along the first half of the beam where the deflection is precisely 0.06 meters.



Set up an equation that represents the condition where the deflection $y(x)$ is equal to -0.06 m. Choose the initial interval $[1, 5]$ and use Regula Falsi method to approximate the location x with error tolerance $\epsilon = 10^{-6}$. Report the number of iterations and the root found at each iteration in a tabular form. [25 marks]