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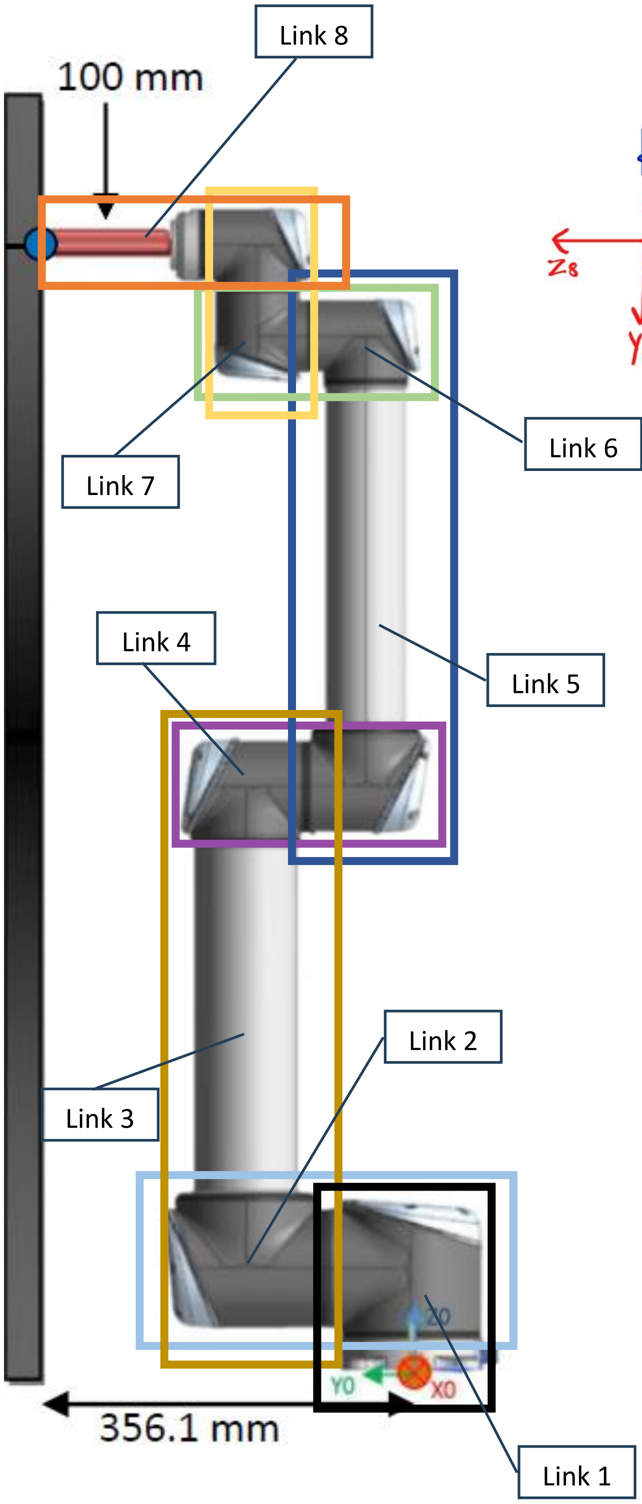
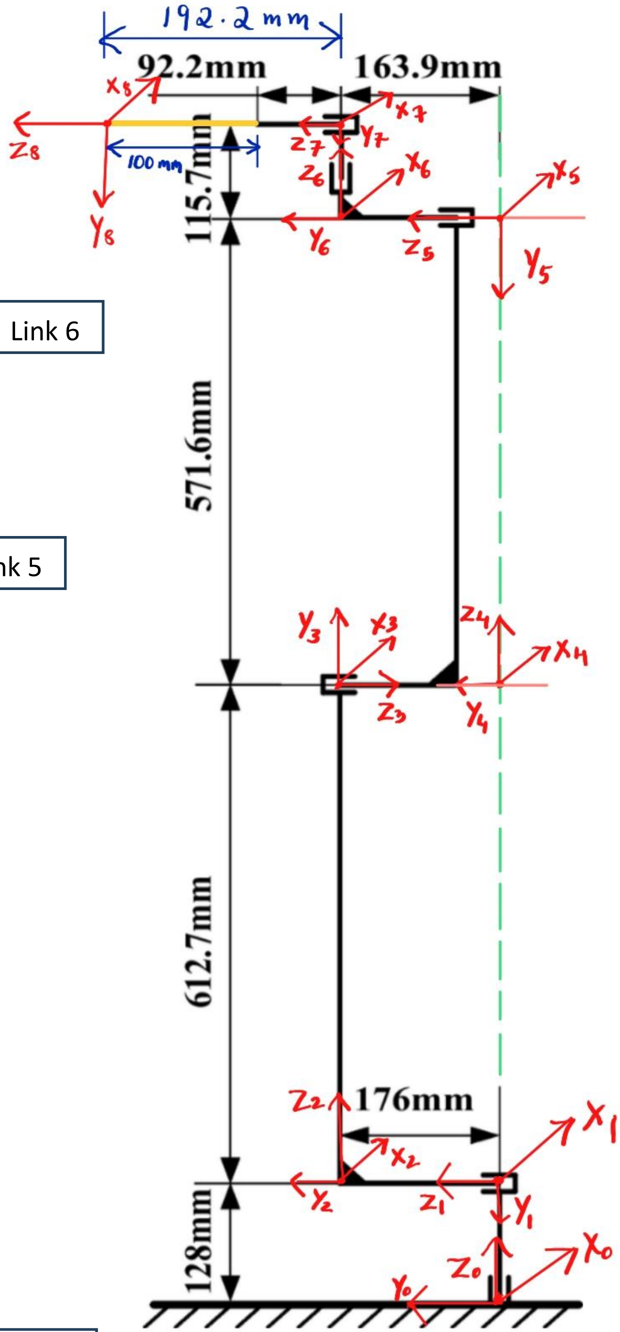
ENPM 662 – Homework 4 Report





In the figures given above,

- Link 1 is attached to Revolute Joint 1 (θ_1).
- Link 1 and Link 2 are interconnected by Revolute Joint 2 (θ_2).
- Link 2 and Link 3 are interconnected by a Fixed Joint.
- Link 3 and Link 4 are interconnected by Revolute Joint 3 (θ_3).
- Link 4 and Link 5 are interconnected by a Fixed Joint.
- Link 5 and Link 6 are interconnected by Revolute Joint 4 (θ_4).
- Link 6 and Link 7 are interconnected by Revolute Joint 5 (θ_5).
- Link 7 and Link 8 are interconnected by Revolute Joint 6 (θ_6).
- Link 8 includes the pen, which is rigidly mounted to the end effector.
- The frames are attached as per Spong convention.
- The initial frame is given where x axis in into the plane; the same convention is followed for assigning the rest of the frames.
- The final frame is attached at the pen tip.

DH Table:

Frames	Link	a (in mm)	α (in degree)	θ (in degree)	d (in mm)
Frame 0 – Frame 1	1	0	-90^0	θ_1	128
Frame 1 – Frame 2	2	0	90^0	θ_2	176
Frame 2 – Frame 3	3	0	90^0	0	612.7
Frame 3 – Frame 4	4	0	-90^0	θ_3	176
Frame 4 – Frame 5	5	0	-90^0	0	571.6
Frame 5 – Frame 6	6	0	90^0	θ_4	163.9
Frame 6 – Frame 7	7	0	-90^0	θ_5	115.7
Frame 7 – Frame 8	8	0	0^0	θ_6	100 + 92.2

Link Transformation Matrices:

The general form of Link Transformation Matrix is given by:

$$T_i = \begin{bmatrix} \cos\theta_i & -(\sin\theta_i * \cos\alpha_i) & (\sin\theta_i * \sin\alpha_i) & (a_i * \cos\theta_i) \\ \sin\theta_i & (\cos\theta_i * \cos\alpha_i) & -(\cos\theta_i * \sin\alpha_i) & (a_i * \sin\theta_i) \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where "i" is the link number.

Therefore, referring to the DH table, the link transformation matrices for the links are given by:

i. For Link 1 (Frame 0 – Frame 1)

$${}^0T_1 = \begin{bmatrix} \cos\theta_1 & -(\sin\theta_1 * \cos -90) & (\sin\theta_1 * \sin -90) & (0 * \cos\theta_1) \\ \sin\theta_1 & (\cos\theta_1 * \cos -90) & -(\cos\theta_1 * \sin -90) & (0 * \sin\theta_1) \\ 0 & \sin -90 & \cos -90 & 128 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0T_1 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & 128 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

ii. For Link 2 (Frame 1 – Frame 2)

$${}^1T_2 = \begin{bmatrix} \cos\theta_2 & -(\sin\theta_2 * \cos 90) & (\sin\theta_2 * \sin 90) & (0 * \cos\theta_2) \\ \sin\theta_2 & (\cos\theta_2 * \cos 90) & -(\cos\theta_2 * \sin 90) & (0 * \sin\theta_2) \\ 0 & \sin 90 & \cos 90 & 176 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^1T_2 = \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 & 0 \\ \sin\theta_2 & 0 & -\cos\theta_2 & 0 \\ 0 & 1 & 0 & 176 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

iii. For Link 3 (Frame 2 – Frame 3)

$$^{2}T_{3} = \begin{bmatrix} \cos 0 & -(\sin 0 * \cos 90) & (\sin 0 * \ \sin 90) & (0 * \cos 0) \\ \sin 0 & (\cos 0 * \cos 90) & -(\cos 0 * \sin 90) & (0 * \sin 0) \\ 0 & \sin 90 & \cos 90 & 612.7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{2}T_{3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 612.7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

iv. For Link 4 (Frame 3 – Frame 4)

$$^{3}T_{4} = \begin{bmatrix} \cos\theta_3 & -(\sin\theta_3 * \cos -90) & (\sin\theta_3 * \ \sin -90) & (0 * \cos\theta_3) \\ \sin\theta_3 & (\cos\theta_3 * \cos -90) & -(\cos\theta_3 * \sin -90) & (0 * \sin\theta_3) \\ 0 & \sin -90 & \cos -90 & 176 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{3}T_{4} = \begin{bmatrix} \cos\theta_3 & 0 & -\sin\theta_3 & 0 \\ \sin\theta_3 & 0 & \cos\theta_3 & 0 \\ 0 & -1 & 0 & 176 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

v. For Link 5 (Frame 4 – Frame 5)

$$^{4}T_{5} = \begin{bmatrix} \cos 0 & -(\sin 0 * \cos -90) & (\sin 0 * \ \sin -90) & (0 * \cos 0) \\ \sin 0 & (\cos 0 * \cos -90) & -(\cos 0 * \sin -90) & (0 * \sin 0) \\ 0 & \sin -90 & \cos -90 & 571.6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{4}T_{5} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 571.6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

vi. For Link 6 (Frame 5 – Frame 6)

$$ {}^{5}T_{6} = \begin{bmatrix} \cos\theta_4 & -(\sin\theta_4 * \cos 90) & (\sin\theta_4 * \sin 90) & (0 * \cos\theta_4) \\ \sin\theta_4 & (\cos\theta_4 * \cos 90) & -(\cos\theta_4 * \sin 90) & (0 * \sin\theta_4) \\ 0 & \sin 90 & \cos 90 & 163.9 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{5}T_{6} = \begin{bmatrix} \cos\theta_4 & 0 & \sin\theta_4 & 0 \\ \sin\theta_4 & 0 & -\cos\theta_4 & 0 \\ 0 & 1 & 0 & 163.9 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

vii. For Link 7 (Frame 6 – Frame 7)

$$ {}^{6}T_{7} = \begin{bmatrix} \cos\theta_5 & -(\sin\theta_5 * \cos -90) & (\sin\theta_5 * \sin -90) & (0 * \cos\theta_5) \\ \sin\theta_5 & (\cos\theta_5 * \cos -90) & -(\cos\theta_5 * \sin -90) & (0 * \sin\theta_5) \\ 0 & \sin -90 & \cos -90 & 115.7 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{6}T_{7} = \begin{bmatrix} \cos\theta_5 & 0 & -\sin\theta_5 & 0 \\ \sin\theta_5 & 0 & \cos\theta_5 & 0 \\ 0 & -1 & 0 & 115.7 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

viii. For Link 8 (Frame 7 – Frame 8)

$$ {}^{7}T_{8} = \begin{bmatrix} \cos\theta_6 & -(\sin\theta_6 * \cos 0) & (\sin\theta_6 * \sin 0) & (0 * \cos\theta_6) \\ \sin\theta_6 & (\cos\theta_6 * \cos 0) & -(\cos\theta_6 * \sin 0) & (0 * \sin\theta_6) \\ 0 & \sin 0 & \cos 0 & 192.2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{7}T_{8} = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 192.2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

The Final Transformation Matrix is given by post multiplying the matrices like:

$$^{0}T_8 = {}^{0}T_1 * {}^{1}T_2 * {}^{2}T_3 * {}^{3}T_4 * {}^{4}T_5 * {}^{5}T_6 * {}^{6}T_7 * {}^{7}T_8$$

$$^{0}T_8 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & 128 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 & 0 \\ \sin\theta_2 & 0 & -\cos\theta_2 & 0 \\ 0 & 1 & 0 & 176 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 612.7 \\ 0 & 0 & 0 & 1 \end{bmatrix} *$$

$$\begin{bmatrix} \cos\theta_3 & 0 & -\sin\theta_3 & 0 \\ \sin\theta_3 & 0 & \cos\theta_3 & 0 \\ 0 & -1 & 0 & 176 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 571.6 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \cos\theta_4 & 0 & \sin\theta_4 & 0 \\ \sin\theta_4 & 0 & -\cos\theta_4 & 0 \\ 0 & 1 & 0 & 163.9 \\ 0 & 0 & 0 & 1 \end{bmatrix} *$$

$$\begin{bmatrix} \cos\theta_5 & 0 & -\sin\theta_5 & 0 \\ \sin\theta_5 & 0 & \cos\theta_5 & 0 \\ 0 & -1 & 0 & 115.7 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 192.2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The Symbolic form of the transformation matrix is obtained through sympy (python) code)

Symbolic form of final transformation matrix

Column 1

$$\begin{bmatrix} -\sin(\theta_1)\sin(\theta_5)\cos(\theta_6) - \sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_6)\cos(\theta_1) + \sin(\theta_2)\sin(\theta_3)\cos(\theta_1)\cos(\theta_4)\cos(\theta_5)\cos(\theta_6) - \sin(\theta_2)\sin(\theta_4)\cos(\theta_1)\cos(\theta_3)\cos(\theta_5)\cos(\theta_6) - \sin(\theta_2)\sin(\theta_6)\cos(\theta_1)\cos(\theta_3)\cos(\theta_4) + \sin(\theta_3)\sin(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_5)\cos(\theta_6) + \sin(\theta_3)\sin(\theta_6)\cos(\theta_1)\cos(\theta_2)\cos(\theta_4) - \sin(\theta_4)\sin(\theta_6)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) + \cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5)\cos(\theta_6) \\ -\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_6) + \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4)\cos(\theta_5)\cos(\theta_6) - \sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\cos(\theta_3)\cos(\theta_5)\cos(\theta_6) - \sin(\theta_1)\sin(\theta_2)\sin(\theta_6)\cos(\theta_3)\cos(\theta_4) + \sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\cos(\theta_2)\cos(\theta_5)\cos(\theta_6) + \sin(\theta_1)\sin(\theta_3)\sin(\theta_6)\cos(\theta_2)\cos(\theta_4) - \sin(\theta_1)\sin(\theta_4)\sin(\theta_6)\cos(\theta_2)\cos(\theta_3) + \sin(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5)\cos(\theta_6) + \sin(\theta_5)\cos(\theta_1)\cos(\theta_6) \\ -\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\cos(\theta_5)\cos(\theta_6) - \sin(\theta_2)\sin(\theta_3)\sin(\theta_6)\cos(\theta_4) + \sin(\theta_2)\sin(\theta_4)\sin(\theta_6)\cos(\theta_3) - \sin(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5)\cos(\theta_6) - \sin(\theta_3)\sin(\theta_4)\sin(\theta_6)\cos(\theta_2) + \sin(\theta_3)\cos(\theta_2)\cos(\theta_4)\cos(\theta_5)\cos(\theta_6) - \sin(\theta_4)\cos(\theta_2)\cos(\theta_3)\cos(\theta_5)\cos(\theta_6) - \sin(\theta_6)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) \\ 0 \end{bmatrix}$$

Column 2

$$\begin{matrix} \sin(\theta_1)\sin(\theta_5)\sin(\theta_6) - \sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\cos(\theta_1)\cos(\theta_6) - \sin(\theta_2)\sin(\theta_3)\sin(\theta_6)\cos(\theta_1)\cos(\theta_4)\cos(\theta_5) + \sin(\theta_2)\sin(\theta_4)\sin(\theta_6)\cos(\theta_1)\cos(\theta_3)\cos(\theta_5) - \sin(\theta_2)\cos(\theta_1)\cos(\theta_3)\cos(\theta_4)\cos(\theta_6) - \sin(\theta_3)\sin(\theta_4)\sin(\theta_6)\cos(\theta_1)\cos(\theta_2)\cos(\theta_5) + \sin(\theta_3)\cos(\theta_1)\cos(\theta_2)\cos(\theta_4)\cos(\theta_6) - \sin(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_6) - \sin(\theta_6)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5) \\ -\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\cos(\theta_6) - \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_6)\cos(\theta_4)\cos(\theta_5) + \sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\sin(\theta_6)\cos(\theta_3)\cos(\theta_5) - \sin(\theta_1)\sin(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_6) - \sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\sin(\theta_6)\cos(\theta_2)\cos(\theta_5) + \sin(\theta_1)\sin(\theta_3)\cos(\theta_2)\cos(\theta_4)\cos(\theta_6) - \sin(\theta_1)\sin(\theta_4)\cos(\theta_2)\cos(\theta_3)\cos(\theta_6) - \sin(\theta_1)\sin(\theta_6)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5) - \sin(\theta_5)\sin(\theta_6)\cos(\theta_1) \\ \sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_6)\cos(\theta_5) - \sin(\theta_2)\sin(\theta_3)\cos(\theta_4)\cos(\theta_6) + \sin(\theta_2)\sin(\theta_4)\cos(\theta_3)\cos(\theta_6) + \sin(\theta_2)\sin(\theta_6)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5) - \sin(\theta_3)\sin(\theta_4)\cos(\theta_2)\cos(\theta_6) - \sin(\theta_3)\sin(\theta_6)\cos(\theta_2)\cos(\theta_4)\cos(\theta_5) + \sin(\theta_4)\sin(\theta_6)\cos(\theta_2)\cos(\theta_3)\cos(\theta_5) - \cos(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_6) \\ 0 \end{matrix}$$

Column 3

$$\begin{matrix} -\sin(\theta_1)\cos(\theta_5) - \sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_1)\cos(\theta_4) + \sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_3) - \sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2) - \sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) \\ -\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_4) + \sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_3) - \sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) + \cos(\theta_1)\cos(\theta_5) \\ \sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5) + \sin(\theta_2)\sin(\theta_5)\cos(\theta_3)\cos(\theta_4) - \sin(\theta_3)\sin(\theta_5)\cos(\theta_2)\cos(\theta_4) + \sin(\theta_4)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3) \\ 0 \end{matrix}$$

Column 4

$$\begin{bmatrix} -192.2\sin(\theta_1)\cos(\theta_5) - 163.9\sin(\theta_1) + 115.7\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\cos(\theta_1) - 192.2\sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_1)\cos(\theta_4) + 192.2\sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_3) + 115.7\sin(\theta_2)\cos(\theta_1)\cos(\theta_3)\cos(\theta_4) + 571.6\sin(\theta_2)\cos(\theta_1)\cos(\theta_3) + 612.7\sin(\theta_2)\cos(\theta_1) - 192.2\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2) - 115.7\sin(\theta_3)\cos(\theta_1)\cos(\theta_2)\cos(\theta_4) - 571.6\sin(\theta_3)\cos(\theta_1)\cos(\theta_2) + 115.7\sin(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) - 192.2\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) \\ 115.7\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4) - 192.2\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_4) + 192.2\sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_3) + 115.7\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)\cos(\theta_4) + 571.6\sin(\theta_1)\sin(\theta_2)\cos(\theta_3) + 612.7\sin(\theta_1)\sin(\theta_2) - 192.2\sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2) - 115.7\sin(\theta_1)\sin(\theta_3)\cos(\theta_2)\cos(\theta_4) - 571.6\sin(\theta_1)\sin(\theta_3)\cos(\theta_2) + 115.7\sin(\theta_1)\sin(\theta_4)\cos(\theta_2)\cos(\theta_3) - 192.2\sin(\theta_1)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) + 192.2\cos(\theta_1)\cos(\theta_5) + 163.9\cos(\theta_1) \\ 192.2\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5) + 115.7\sin(\theta_2)\sin(\theta_3)\cos(\theta_4) + 571.6\sin(\theta_2)\sin(\theta_3) - 115.7\sin(\theta_2)\sin(\theta_4)\cos(\theta_3) + 192.2\sin(\theta_2)\sin(\theta_5)\cos(\theta_3)\cos(\theta_4) + 115.7\sin(\theta_3)\sin(\theta_4)\cos(\theta_2) - 192.2\sin(\theta_3)\sin(\theta_5)\cos(\theta_2)\cos(\theta_4) + 192.2\sin(\theta_4)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3) + 115.7\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) + 571.6\cos(\theta_2)\cos(\theta_3) + 612.7\cos(\theta_2) + 128 \\ 1 \end{bmatrix}$$

Constructing the Jacobian Matrix

Method followed – Second Method

Steps:

1. Calculating the matrix T_i

$T_1 = T_1^0 = T_1^0$

$T_2 = T_2^0 = T_1^0 * T_2^1$

$T_3 = T_4^0 = T_1^0 * T_2^1 * T_3^2 * T_4^3$

$T_4 = T_6^0 = T_1^0 * T_2^1 * T_3^2 * T_4^3 * T_5^4 * T_6^5$

$T_5 = T_7^0 = T_1^0 * T_2^1 * T_3^2 * T_4^3 * T_5^4 * T_6^5 * T_7^6$

$T_6 = T_8^0 = T_1^0 * T_2^1 * T_3^2 * T_4^3 * T_5^4 * T_6^5 * T_7^6 * T_8^7$

2. Calculating Z_i (First 3 row values of the 3rd column of the T matrix)

Python Indexing Included

$$Z_1 = T_1[0:3,2] = \begin{bmatrix} -\sin(\theta_1) \\ \cos(\theta_1) \\ 0 \end{bmatrix}$$

$$Z_2 = T_2[0:3,2] = \begin{bmatrix} \sin(\theta_2)\cos(\theta_1) \\ \sin(\theta_1)\sin(\theta_2) \\ \cos(\theta_2) \end{bmatrix}$$

$$Z_3 = T_3[0:3,2] = \begin{bmatrix} \sin(\theta_2)\cos(\theta_1)\cos(\theta_3) - \sin(\theta_3)\cos(\theta_1)\cos(\theta_2) \\ \sin(\theta_1)\sin(\theta_2)\cos(\theta_3) - \sin(\theta_1)\sin(\theta_3)\cos(\theta_2) \\ \sin(\theta_2)\sin(\theta_3) + \cos(\theta_2)\cos(\theta_3) \end{bmatrix}$$

$$Z_4 = T_4[0:3,2] = \begin{bmatrix} (\sin(\theta_2)\sin(\theta_3)\cos(\theta_1) + \cos(\theta_1)\cos(\theta_2)\cos(\theta_3))\sin(\theta_4) - (-\sin(\theta_2)\cos(\theta_1)\cos(\theta_3) + \sin(\theta_3)\cos(\theta_1)\cos(\theta_2))\cos(\theta_4) \\ (\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + \sin(\theta_1)\cos(\theta_2)\cos(\theta_3))\sin(\theta_4) - (-\sin(\theta_1)\sin(\theta_2)\cos(\theta_3) + \sin(\theta_1)\sin(\theta_3)\cos(\theta_2))\cos(\theta_4) \\ -(-\sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3))\cos(\theta_4) + (-\sin(\theta_2)\cos(\theta_3) + \sin(\theta_3)\cos(\theta_2))\sin(\theta_4) \end{bmatrix}$$

$$Z_5 = T_5[0:3,2] = \begin{bmatrix} -((\sin(\theta_2)\sin(\theta_3)\cos(\theta_1) + \cos(\theta_1)\cos(\theta_2)\cos(\theta_3))\cos(\theta_4) + (-\sin(\theta_2)\cos(\theta_1)\cos(\theta_3) + \sin(\theta_3)\cos(\theta_1)\cos(\theta_2))\sin(\theta_4))\sin(\theta_5) - \sin(\theta_1)\cos(\theta_5) \\ -((\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + \sin(\theta_1)\cos(\theta_2)\cos(\theta_3))\cos(\theta_4) + (-\sin(\theta_1)\sin(\theta_2)\cos(\theta_3) + \sin(\theta_1)\sin(\theta_3)\cos(\theta_2))\sin(\theta_4))\sin(\theta_5) + \cos(\theta_1)\cos(\theta_5) \\ -((-\sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3))\sin(\theta_4) + (-\sin(\theta_2)\cos(\theta_3) + \sin(\theta_3)\cos(\theta_2))\cos(\theta_4))\sin(\theta_5) \end{bmatrix}$$

$$Z_6 = T_6[0:3,2] = \begin{bmatrix} -((\sin(\theta_2)\sin(\theta_3)\cos(\theta_1) + \cos(\theta_1)\cos(\theta_2)\cos(\theta_3))\cos(\theta_4) + (-\sin(\theta_2)\cos(\theta_1)\cos(\theta_3) + \sin(\theta_3)\cos(\theta_1)\cos(\theta_2))\sin(\theta_4))\sin(\theta_5) - \sin(\theta_1)\cos(\theta_5) \\ -((\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + \sin(\theta_1)\cos(\theta_2)\cos(\theta_3))\cos(\theta_4) + (-\sin(\theta_1)\sin(\theta_2)\cos(\theta_3) + \sin(\theta_1)\sin(\theta_3)\cos(\theta_2))\sin(\theta_4))\sin(\theta_5) + \cos(\theta_1)\cos(\theta_5) \\ -((-\sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3))\sin(\theta_4) + (-\sin(\theta_2)\cos(\theta_3) + \sin(\theta_3)\cos(\theta_2))\cos(\theta_4))\sin(\theta_5) \end{bmatrix}$$

3. Calculating h(q_1, q_2, ….. q_n)

h(q_1, q_2, ….. q_n) = P_8^0 = T_6 [0:3,3] =

$$\begin{bmatrix} -192.2\sin(\theta_1)\cos(\theta_5) - 163.9\sin(\theta_1) + 115.7\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\cos(\theta_1) - 192.2\sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_1)\cos(\theta_4) + 192.2\sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_3) + 115.7\sin(\theta_2)\cos(\theta_1)\cos(\theta_3)\cos(\theta_4) + 571.6\sin(\theta_2)\cos(\theta_1)\cos(\theta_3) + 612.7\sin(\theta_2)\cos(\theta_1) - 192.2\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2) - 115.7\sin(\theta_3)\cos(\theta_1)\cos(\theta_2)\cos(\theta_4) - 571.6\sin(\theta_3)\cos(\theta_1)\cos(\theta_2) + 115.7\sin(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) - 192.2\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) \\ 115.7\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4) - 192.2\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_4) + 192.2\sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_3) + 115.7\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)\cos(\theta_4) + 571.6\sin(\theta_1)\sin(\theta_2)\cos(\theta_3) + 612.7\sin(\theta_1)\sin(\theta_2) - 192.2\sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2) - 115.7\sin(\theta_1)\sin(\theta_3)\cos(\theta_2)\cos(\theta_4) - 571.6\sin(\theta_1)\sin(\theta_3)\cos(\theta_2) + 115.7\sin(\theta_1)\sin(\theta_4)\cos(\theta_2)\cos(\theta_3) - 192.2\sin(\theta_1)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) + 192.2\cos(\theta_1)\cos(\theta_5) + 163.9\cos(\theta_1) \\ 192.2\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5) + 115.7\sin(\theta_2)\sin(\theta_3)\cos(\theta_4) + 571.6\sin(\theta_2)\sin(\theta_3) - 115.7\sin(\theta_2)\sin(\theta_4)\cos(\theta_3) + 192.2\sin(\theta_2)\sin(\theta_5)\cos(\theta_3)\cos(\theta_4) + 115.7\sin(\theta_3)\sin(\theta_4)\cos(\theta_2) - 192.2\sin(\theta_3)\sin(\theta_5)\cos(\theta_2)\cos(\theta_4) + 192.2\sin(\theta_4)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3) + 115.7\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) + 571.6\cos(\theta_2)\cos(\theta_3) + 612.7\cos(\theta_2) + 128 \end{bmatrix}$$

4. Calculating $\partial \mathbf{h}/\partial \mathbf{q}_i$

i. $\partial \mathrm{h}/\partial \mathrm{q}_1 = \partial \mathrm{h}/\partial \theta_1$ =

$$\begin{bmatrix} -115.7\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4) + 192.2\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_4) - 192.2\sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_3) - 115.7\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)\cos(\theta_4) - 571.6\sin(\theta_1)\sin(\theta_2)\cos(\theta_3) - 612.7\sin(\theta_1)\sin(\theta_2) + 192.2\sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2) + 115.7\sin(\theta_1)\sin(\theta_3)\cos(\theta_2)\cos(\theta_4) + 571.6\sin(\theta_1)\sin(\theta_3)\cos(\theta_2) - 115.7\sin(\theta_1)\sin(\theta_4)\cos(\theta_2)\cos(\theta_3) + 192.2\sin(\theta_1)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) - 192.2\cos(\theta_1)\cos(\theta_5) - 163.9\cos(\theta_1) \\ -192.2\sin(\theta_1)\cos(\theta_5) - 163.9\sin(\theta_1) + 115.7\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\cos(\theta_1) - 192.2\sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_1)\cos(\theta_4) + 192.2\sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_3) + 115.7\sin(\theta_2)\cos(\theta_1)\cos(\theta_3)\cos(\theta_4) + 571.6\sin(\theta_2)\cos(\theta_1)\cos(\theta_3) + 612.7\sin(\theta_2)\cos(\theta_1) - 192.2\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2) - 115.7\sin(\theta_3)\cos(\theta_1)\cos(\theta_2)\cos(\theta_4) - 571.6\sin(\theta_3)\cos(\theta_1)\cos(\theta_2) + 115.7\sin(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) - 192.2\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) \\ 0 \end{bmatrix}$$

ii. $\partial \mathrm{h}/\partial \mathrm{q}_2 = \partial \mathrm{h}/\partial \theta_2$ =

$$\begin{bmatrix} 192.2\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1) + 115.7\sin(\theta_2)\sin(\theta_3)\cos(\theta_1)\cos(\theta_4) + 571.6\sin(\theta_2)\sin(\theta_3)\cos(\theta_1) - 115.7\sin(\theta_2)\sin(\theta_4)\cos(\theta_1)\cos(\theta_3) + 192.2\sin(\theta_2)\sin(\theta_5)\cos(\theta_1)\cos(\theta_3)\cos(\theta_4) + 115.7\sin(\theta_3)\sin(\theta_4)\cos(\theta_1)\cos(\theta_2) - 192.2\sin(\theta_3)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_4) + 192.2\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) + 115.7\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) + 571.6\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) + 612.7\cos(\theta_1)\cos(\theta_2) \\ 192.2\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5) + 115.7\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4) + 571.6\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) - 115.7\sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\cos(\theta_3) + 192.2\sin(\theta_1)\sin(\theta_2)\sin(\theta_5)\cos(\theta_3)\cos(\theta_4) + 115.7\sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\cos(\theta_2) - 192.2\sin(\theta_1)\sin(\theta_3)\sin(\theta_5)\cos(\theta_2)\cos(\theta_4) + 192.2\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3) + 115.7\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) + 571.6\sin(\theta_1)\cos(\theta_2)\cos(\theta_3) + 612.7\sin(\theta_1)\cos(\theta_2) \\ -115.7\sin(\theta_2)\sin(\theta_3)\sin(\theta_4) + 192.2\sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_4) - 192.2\sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_3) - 115.7\sin(\theta_2)\cos(\theta_3)\cos(\theta_4) - 571.6\sin(\theta_2)\cos(\theta_3) - 612.7\sin(\theta_2) + 192.2\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2) + 115.7\sin(\theta_3)\cos(\theta_2)\cos(\theta_4) + 571.6\sin(\theta_3)\cos(\theta_2) - 115.7\sin(\theta_4)\cos(\theta_2)\cos(\theta_3) + 192.2\sin(\theta_5)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) \end{bmatrix}$$

iii. $\partial \mathrm{h}/\partial \mathrm{q}_3 = \partial \mathrm{h}/\partial \theta_3$ =

$$\begin{bmatrix} -192.2\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1) - 115.7\sin(\theta_2)\sin(\theta_3)\cos(\theta_1)\cos(\theta_4) - 571.6\sin(\theta_2)\sin(\theta_3)\cos(\theta_1) + 115.7\sin(\theta_2)\sin(\theta_4)\cos(\theta_1)\cos(\theta_3) - 192.2\sin(\theta_2)\sin(\theta_5)\cos(\theta_1)\cos(\theta_3)\cos(\theta_4) - 115.7\sin(\theta_3)\sin(\theta_4)\cos(\theta_1)\cos(\theta_2) + 192.2\sin(\theta_3)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_4) - 192.2\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) - 115.7\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) - 571.6\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) \\ -192.2\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5) - 115.7\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4) - 571.6\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + 115.7\sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\cos(\theta_3) - 192.2\sin(\theta_1)\sin(\theta_2)\sin(\theta_5)\cos(\theta_3)\cos(\theta_4) - 115.7\sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\cos(\theta_2) + 192.2\sin(\theta_1)\sin(\theta_3)\sin(\theta_5)\cos(\theta_2)\cos(\theta_4) - 192.2\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3) - 115.7\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) - 571.6\sin(\theta_1)\cos(\theta_2)\cos(\theta_3) \\ 115.7\sin(\theta_2)\sin(\theta_3)\sin(\theta_4) - 192.2\sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_4) + 192.2\sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_3) + 115.7\sin(\theta_2)\cos(\theta_3)\cos(\theta_4) + 571.6\sin(\theta_2)\cos(\theta_3) - 192.2\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2) - 115.7\sin(\theta_3)\cos(\theta_2)\cos(\theta_4) - 571.6\sin(\theta_3)\cos(\theta_2) + 115.7\sin(\theta_4)\cos(\theta_2)\cos(\theta_3) - 192.2\sin(\theta_5)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) \end{bmatrix}$$

iv. $\partial h/\partial q_4 = \partial h/\partial \theta_4 =$

$$\begin{bmatrix} 192.2 \sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1) + 115.7 \sin(\theta_2)\sin(\theta_3)\cos(\theta_1)\cos(\theta_4) - 115.7 \sin(\theta_2)\sin(\theta_4)\cos(\theta_1)\cos(\theta_3) + 192.2 \sin(\theta_2)\sin(\theta_5)\cos(\theta_1)\cos(\theta_3)\cos(\theta_4) + 115.7 \sin(\theta_3)\sin(\theta_4)\cos(\theta_1)\cos(\theta_2) - 192.2 \sin(\theta_3)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_4) + 192.2 \sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) + 115.7 \cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) \\ 192.2 \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5) + 115.7 \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4) - 115.7 \sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\cos(\theta_3) + 192.2 \sin(\theta_1)\sin(\theta_2)\sin(\theta_5)\cos(\theta_3)\cos(\theta_4) + 115.7 \sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\cos(\theta_2) - 192.2 \sin(\theta_1)\sin(\theta_3)\sin(\theta_5)\cos(\theta_2)\cos(\theta_4) + 192.2 \sin(\theta_1)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3) + 115.7 \sin(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) \\ -115.7 \sin(\theta_2)\sin(\theta_3)\sin(\theta_4) + 192.2 \sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_4) - 192.2 \sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_3) - 115.7 \sin(\theta_2)\cos(\theta_3)\cos(\theta_4) + 192.2 \sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2) + 115.7 \sin(\theta_3)\cos(\theta_2)\cos(\theta_4) - 115.7 \sin(\theta_4)\cos(\theta_2)\cos(\theta_3) + 192.2 \sin(\theta_5)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) \end{bmatrix}$$

v. $\partial h/\partial q_5 = \partial h/\partial \theta_5 =$

$$\begin{bmatrix} 192.2 \sin(\theta_1)\sin(\theta_5) - 192.2 \sin(\theta_2)\sin(\theta_3)\cos(\theta_1)\cos(\theta_4)\cos(\theta_5) + 192.2 \sin(\theta_2)\sin(\theta_4)\cos(\theta_1)\cos(\theta_3)\cos(\theta_5) - 192.2 \sin(\theta_3)\sin(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_5) - 192.2 \cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5) \\ -192.2 \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4)\cos(\theta_5) + 192.2 \sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\cos(\theta_3)\cos(\theta_5) - 192.2 \sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\cos(\theta_2)\cos(\theta_5) - 192.2 \sin(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5) - 192.2 \sin(\theta_5)\cos(\theta_1) \\ 192.2 \sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\cos(\theta_5) + 192.2 \sin(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5) - 192.2 \sin(\theta_3)\cos(\theta_2)\cos(\theta_4)\cos(\theta_5) + 192.2 \sin(\theta_4)\cos(\theta_2)\cos(\theta_3)\cos(\theta_5) \end{bmatrix}$$

vi. $\partial h/\partial q_6 = \partial h/\partial \theta_6 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

5. Writing the Jacobian Matrix

$$J_8^0 = J = \begin{bmatrix} \partial h/\partial \theta_1 & \partial h/\partial \theta_2 & \partial h/\partial \theta_3 & \partial h/\partial \theta_4 & \partial h/\partial \theta_5 & \partial h/\partial \theta_6 \\ Z_1 & Z_2 & Z_3 & Z_4 & Z_5 & Z_6 \end{bmatrix} =$$

$$\begin{matrix} -115.7 \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4) + 192.2 \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_4) - 192.2 \sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_3) - 115.7 \sin(\theta_1)\sin(\theta_2)\cos(\theta_3)\cos(\theta_4) - 571.6 \sin(\theta_1)\sin(\theta_2)\cos(\theta_3) - 612.7 \sin(\theta_1)\sin(\theta_2) + 192.2 \sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2) + 115.7 \sin(\theta_1)\sin(\theta_3)\cos(\theta_2)\cos(\theta_4) + 571.6 \sin(\theta_1)\sin(\theta_3)\cos(\theta_2) - 115.7 \sin(\theta_1)\sin(\theta_4)\cos(\theta_2)\cos(\theta_3) + 192.2 \sin(\theta_1)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) - 192.2 \cos(\theta_1)\cos(\theta_5) - 163.9 \cos(\theta_1) \\ -192.2 \sin(\theta_1)\cos(\theta_5) - 163.9 \sin(\theta_1) + 115.7 \sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\cos(\theta_1) - 192.2 \sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_1)\cos(\theta_4) + 192.2 \sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_3) + 115.7 \sin(\theta_2)\cos(\theta_1)\cos(\theta_3)\cos(\theta_4) + 571.6 \sin(\theta_2)\cos(\theta_1)\cos(\theta_3) + 612.7 \sin(\theta_2)\cos(\theta_1) - 192.2 \sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2) - 115.7 \sin(\theta_3)\cos(\theta_1)\cos(\theta_2)\cos(\theta_4) - 571.6 \sin(\theta_3)\cos(\theta_1)\cos(\theta_2) + 115.7 \sin(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) - 192.2 \sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) \\ 0 \\ \\ -\sin(\theta_1) \\ \cos(\theta_1) \\ 0 \end{matrix}$$

Column 1

$$192.2 \sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1) + 115.7 \sin(\theta_2)\sin(\theta_3)\cos(\theta_1)\cos(\theta_4) + 571.6 \sin(\theta_2)\sin(\theta_3)\cos(\theta_1) - 115.7 \sin(\theta_2)\sin(\theta_4)\cos(\theta_1)\cos(\theta_3) + 192.2 \sin(\theta_2)\sin(\theta_5)\cos(\theta_1)\cos(\theta_3)\cos(\theta_4) + 115.7 \sin(\theta_3)\sin(\theta_4)\cos(\theta_1)\cos(\theta_2) - 192.2 \sin(\theta_3)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_4) + 192.2 \sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) + 115.7 \cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) + 571.6 \cos(\theta_1)\cos(\theta_2)\cos(\theta_3) + 612.7 \cos(\theta_1)\cos(\theta_2)$$

$$192.2 \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5) + 115.7 \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4) + 571.6 \sin(\theta_1)\sin(\theta_2)\sin(\theta_3) - 115.7 \sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\cos(\theta_3) + 192.2 \sin(\theta_1)\sin(\theta_2)\sin(\theta_5)\cos(\theta_3)\cos(\theta_4) + 115.7 \sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\cos(\theta_2) - 192.2 \sin(\theta_1)\sin(\theta_3)\sin(\theta_5)\cos(\theta_2)\cos(\theta_4) + 192.2 \sin(\theta_1)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3) + 115.7 \sin(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) + 571.6 \sin(\theta_1)\cos(\theta_2)\cos(\theta_3) + 612.7 \sin(\theta_1)\cos(\theta_2)$$

$$-115.7 \sin(\theta_2)\sin(\theta_3)\sin(\theta_4) + 192.2 \sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_4) - 192.2 \sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_3) - 115.7 \sin(\theta_2)\cos(\theta_3)\cos(\theta_4) - 571.6 \sin(\theta_2)\cos(\theta_3) - 612.7 \sin(\theta_2) + 192.2 \sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2) + 115.7 \sin(\theta_3)\cos(\theta_2)\cos(\theta_4) + 571.6 \sin(\theta_3)\cos(\theta_2) - 115.7 \sin(\theta_4)\cos(\theta_2)\cos(\theta_3) + 192.2 \sin(\theta_5)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)$$

$$\sin(\theta_2)\cos(\theta_1)$$

$$\sin(\theta_1)\sin(\theta_2)$$

$$\cos(\theta_2)$$

Column 2

$$-192.2 \sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1) - 115.7 \sin(\theta_2)\sin(\theta_3)\cos(\theta_1)\cos(\theta_4) - 571.6 \sin(\theta_2)\sin(\theta_3)\cos(\theta_1) + 115.7 \sin(\theta_2)\sin(\theta_4)\cos(\theta_1)\cos(\theta_3) - 192.2 \sin(\theta_2)\sin(\theta_5)\cos(\theta_1)\cos(\theta_3)\cos(\theta_4) - 115.7 \sin(\theta_3)\sin(\theta_4)\cos(\theta_1)\cos(\theta_2) + 192.2 \sin(\theta_3)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_4) - 192.2 \sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) - 115.7 \cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) - 571.6 \cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$$

$$-192.2 \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5) - 115.7 \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4) - 571.6 \sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + 115.7 \sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\cos(\theta_3) - 192.2 \sin(\theta_1)\sin(\theta_2)\sin(\theta_5)\cos(\theta_3)\cos(\theta_4) - 115.7 \sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\cos(\theta_2) + 192.2 \sin(\theta_1)\sin(\theta_3)\sin(\theta_5)\cos(\theta_2)\cos(\theta_4) - 192.2 \sin(\theta_1)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3) - 115.7 \sin(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) - 571.6 \sin(\theta_1)\cos(\theta_2)\cos(\theta_3)$$

$$115.7 \sin(\theta_2)\sin(\theta_3)\sin(\theta_4) - 192.2 \sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_4) + 192.2 \sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_3) + 115.7 \sin(\theta_2)\cos(\theta_3)\cos(\theta_4) + 571.6 \sin(\theta_2)\cos(\theta_3) - 192.2 \sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2) - 115.7 \sin(\theta_3)\cos(\theta_2)\cos(\theta_4) - 571.6 \sin(\theta_3)\cos(\theta_2) + 115.7 \sin(\theta_4)\cos(\theta_2)\cos(\theta_3) - 192.2 \sin(\theta_5)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)$$

$$\sin(\theta_2)\cos(\theta_1)\cos(\theta_3) - \sin(\theta_3)\cos(\theta_1)\cos(\theta_2)$$

$$\sin(\theta_1)\sin(\theta_2)\cos(\theta_3) - \sin(\theta_1)\sin(\theta_3)\cos(\theta_2)$$

$$\sin(\theta_2)\sin(\theta_3) + \cos(\theta_2)\cos(\theta_3)$$

Column 3

$$192.2 \sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1) + 115.7 \sin(\theta_2)\sin(\theta_3)\cos(\theta_1)\cos(\theta_4) - 115.7 \sin(\theta_2)\sin(\theta_4)\cos(\theta_1)\cos(\theta_3) + 192.2 \sin(\theta_2)\sin(\theta_5)\cos(\theta_1)\cos(\theta_3)\cos(\theta_4) + 115.7 \sin(\theta_3)\sin(\theta_4)\cos(\theta_1)\cos(\theta_2) - 192.2 \sin(\theta_3)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_4) + 192.2 \sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) + 115.7 \cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)$$

$$192.2 \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5) + 115.7 \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4) - 115.7 \sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\cos(\theta_3) + 192.2 \sin(\theta_1)\sin(\theta_2)\sin(\theta_5)\cos(\theta_3)\cos(\theta_4) + 115.7 \sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\cos(\theta_2) - 192.2 \sin(\theta_1)\sin(\theta_3)\sin(\theta_5)\cos(\theta_2)\cos(\theta_4) + 192.2 \sin(\theta_1)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3) + 115.7 \sin(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)$$

$$-115.7 \sin(\theta_2)\sin(\theta_3)\sin(\theta_4) + 192.2 \sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_4) - 192.2 \sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_3) - 115.7 \sin(\theta_2)\cos(\theta_3)\cos(\theta_4) + 192.2 \sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2) + 115.7 \sin(\theta_3)\cos(\theta_2)\cos(\theta_4) - 115.7 \sin(\theta_4)\cos(\theta_2)\cos(\theta_3) + 192.2 \sin(\theta_5)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)$$

$$(\sin(\theta_2)\sin(\theta_3)\cos(\theta_1) + \cos(\theta_1)\cos(\theta_2)\cos(\theta_3))\sin(\theta_4) - (-\sin(\theta_2)\cos(\theta_1)\cos(\theta_3) + \sin(\theta_3)\cos(\theta_1)\cos(\theta_2))\cos(\theta_4)$$

$$(\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + \sin(\theta_1)\cos(\theta_2)\cos(\theta_3))\sin(\theta_4) - (-\sin(\theta_1)\sin(\theta_2)\cos(\theta_3) + \sin(\theta_1)\sin(\theta_3)\cos(\theta_2))\cos(\theta_4)$$

$$-(-\sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3))\cos(\theta_4) + (-\sin(\theta_2)\cos(\theta_3) + \sin(\theta_3)\cos(\theta_2))\sin(\theta_4)$$

Column 4

$$192.2 \sin(\theta_1)\sin(\theta_5) - 192.2 \sin(\theta_2)\sin(\theta_3)\cos(\theta_1)\cos(\theta_4)\cos(\theta_5) + 192.2 \sin(\theta_2)\sin(\theta_4)\cos(\theta_1)\cos(\theta_3)\cos(\theta_5) - 192.2 \sin(\theta_3)\sin(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_5) - 192.2 \cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5)$$

$$-192.2 \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4)\cos(\theta_5) + 192.2 \sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\cos(\theta_3)\cos(\theta_5) - 192.2 \sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\cos(\theta_2)\cos(\theta_5) - 192.2 \sin(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5) - 192.2 \sin(\theta_5)\cos(\theta_1)$$

$$192.2 \sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\cos(\theta_5) + 192.2 \sin(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5) - 192.2 \sin(\theta_3)\cos(\theta_2)\cos(\theta_4)\cos(\theta_5) + 192.2 \sin(\theta_4)\cos(\theta_2)\cos(\theta_3)\cos(\theta_5)$$

$$-((\sin(\theta_2)\sin(\theta_3)\cos(\theta_1) + \cos(\theta_1)\cos(\theta_2)\cos(\theta_3))\cos(\theta_4) + (-\sin(\theta_2)\cos(\theta_1)\cos(\theta_3) + \sin(\theta_3)\cos(\theta_1)\cos(\theta_2))\sin(\theta_4))\sin(\theta_5) - \sin(\theta_1)\cos(\theta_5)$$

$$-((\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + \sin(\theta_1)\cos(\theta_2)\cos(\theta_3))\cos(\theta_4) + (-\sin(\theta_1)\sin(\theta_2)\cos(\theta_3) + \sin(\theta_1)\sin(\theta_3)\cos(\theta_2))\sin(\theta_4))\sin(\theta_5) + \cos(\theta_1)\cos(\theta_5)$$

$$-((-\sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3))\sin(\theta_4) + (-\sin(\theta_2)\cos(\theta_3) + \sin(\theta_3)\cos(\theta_2))\cos(\theta_4))\sin(\theta_5)$$

Column 5

$$0$$

$$0$$

$$0$$

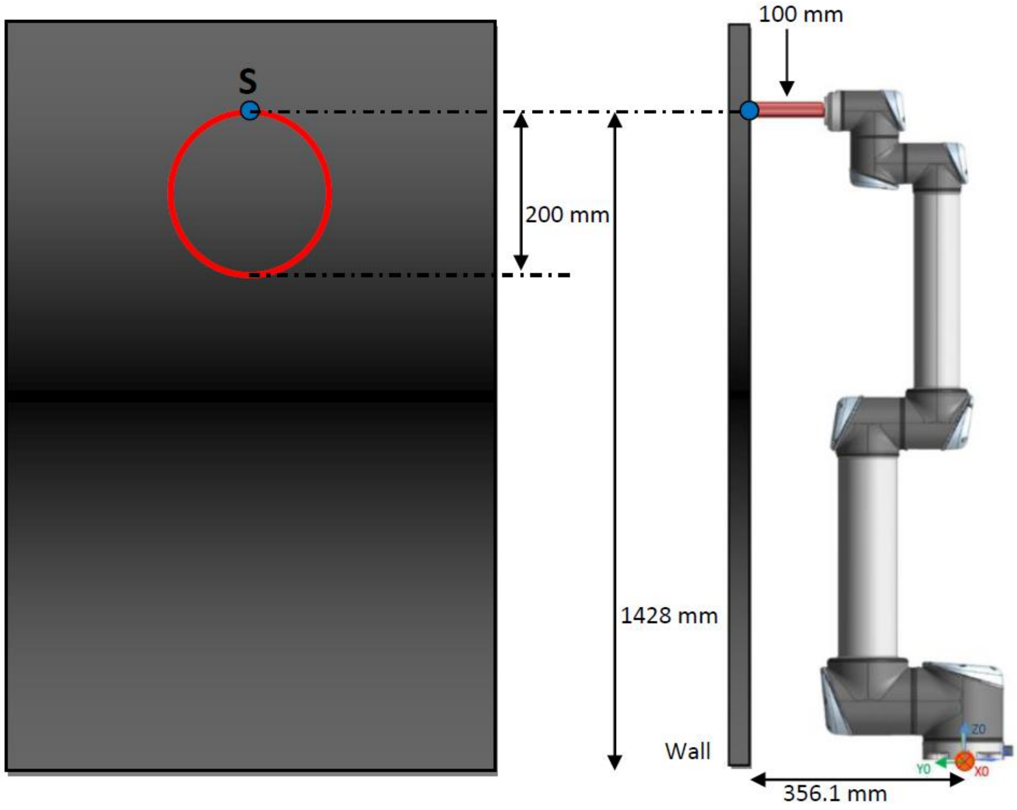
$$-((\sin(\theta_2)\sin(\theta_3)\cos(\theta_1) + \cos(\theta_1)\cos(\theta_2)\cos(\theta_3))\cos(\theta_4) + (-\sin(\theta_2)\cos(\theta_1)\cos(\theta_3) + \sin(\theta_3)\cos(\theta_1)\cos(\theta_2))\sin(\theta_4))\sin(\theta_5) - \sin(\theta_1)\cos(\theta_5)$$

$$-((\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + \sin(\theta_1)\cos(\theta_2)\cos(\theta_3))\cos(\theta_4) + (-\sin(\theta_1)\sin(\theta_2)\cos(\theta_3) + \sin(\theta_1)\sin(\theta_3)\cos(\theta_2))\sin(\theta_4))\sin(\theta_5) + \cos(\theta_1)\cos(\theta_5)$$

$$-((-\sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3))\sin(\theta_4) + (-\sin(\theta_2)\cos(\theta_3) + \sin(\theta_3)\cos(\theta_2))\cos(\theta_4))\sin(\theta_5)$$

Column 6

Tracing the desired path



Given:

- Circle Radius = 100mm
- The robot is at home position when the tool (pen) is touching the wall. Hence,

  [q1 , q2 , q3 , q4 , q5 , q6] = [0 , 0 , 0 , 0 , 0 , 0] at (t = 0)

- Plotting starts from the top of the circle, from coordinates (0 , 356.1 , 1428)

Time considered for plotting the circle: T = 15 seconds

Hence the angular velocity is given by: $\omega = \frac{2*\pi}{T} = \frac{2*\pi}{15}$ rad/sec

The parametric equation of the circle is given by,

X = $r * \cos(\frac{\pi}{2} + \omega t)$

X = $100 * \cos(\frac{\pi}{2} + \frac{2*\pi}{15} * t)$

Z = A + ($r * \sin\left(\frac{\pi}{2} + \omega t\right)$) , where A = 100 mm (origin offset)

Z = 100 + ($100 * \sin(\frac{\pi}{2} + \frac{2*\pi}{15} * t)$)

Differentiate the parametric equation (wrt 't') to get velocity:

$$\dot{X} = \frac{d\,(100* \cos(\frac{\pi}{2}+\frac{2*\pi}{15}*t)}{dt} = \frac{-\,100*2*\pi*\sin(\frac{\pi}{2}+\frac{2*\pi}{15}*t)}{15} = \frac{-\,40*\pi*\sin(\frac{\pi}{2}+\frac{2*\pi}{15}*t)}{3}$$

$$\dot{Z} = \frac{d\,(100 + (100* \sin(\frac{\pi}{2}+\frac{2*\pi}{15}*t))}{dt} = \frac{100*2*\pi*\cos(\frac{\pi}{2}+\frac{2*\pi}{15}*t)}{15} = \frac{40*\pi*\cos(\frac{\pi}{2}+\frac{2*\pi}{15}*t)}{3}$$

In our case, $\dot{Y}$ = 0 and the angular velocities (ω_x , ω_y , ω_z) = (0 , 0 , 0)

Therefore, the Velocity trajectory of the end-effector wrt base is:

$$V = \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} \frac{-\,40*\pi*\sin\left(\frac{\pi}{2}+\frac{2*\pi}{15}*t\right)}{3} \\ 0 \\ \frac{40*\pi*\cos(\frac{\pi}{2}+\frac{2*\pi}{15}*t)}{3} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Now from t-0 to t=15, for a timestep of t (t is assumed as 0.01 seconds for this problem), perform the following calculations:

- Substitute current values of (q1 , q2 , q3 , q4 , q5 , q6) to the Jacobian equation J
- Substitute the current value of t to velocity trajectory equation V
- Use the Inverse Velocity Kinematics equation to calculate the Joint angular velocities

  $$\dot{q} = J^{-1} * V$$

  $$\begin{bmatrix} \dot{q1} \\ \dot{q2} \\ \dot{q3} \\ \dot{q4} \\ \dot{q5} \\ \dot{q6} \end{bmatrix} = J^{-1} * \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

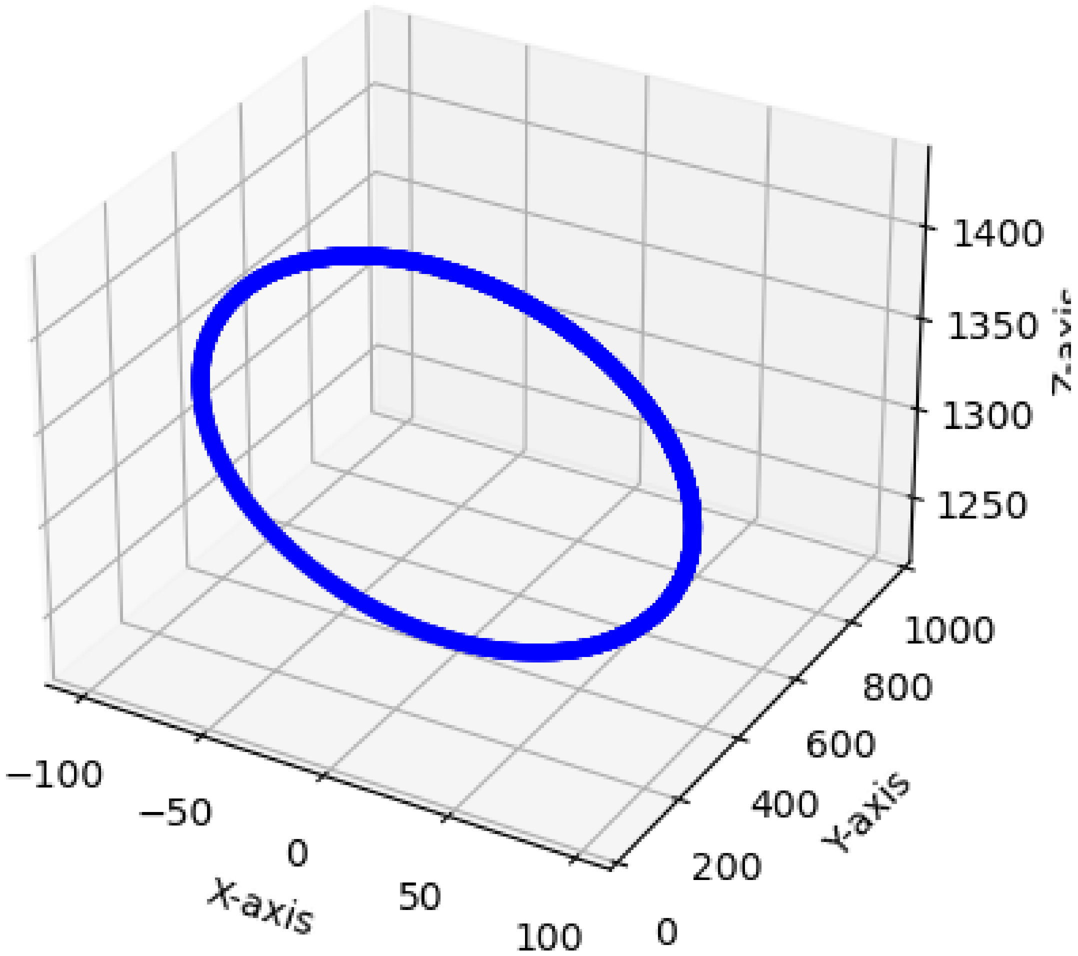
- Use the joint angular velocities obtained from above equation to perform numeric integration and get the new joint angle values.

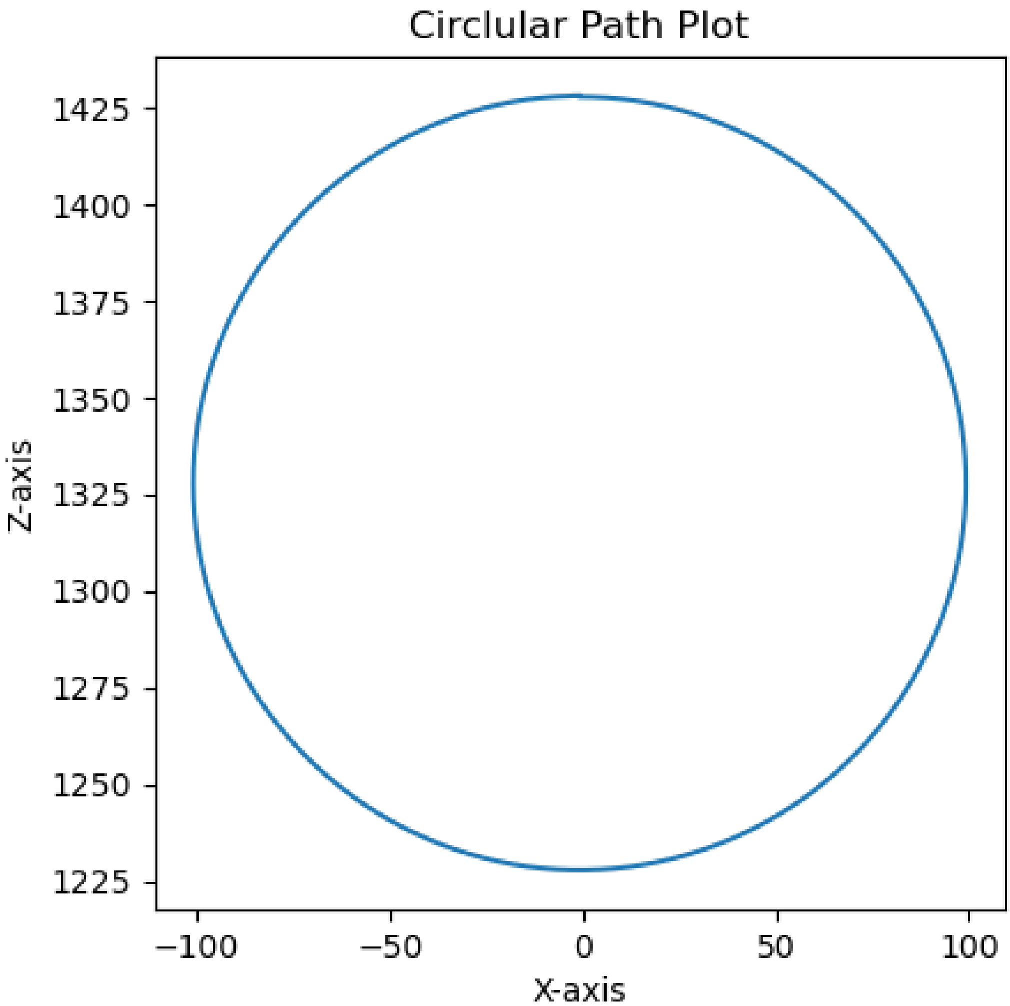
  i.e. q_{new} = $q_{current}$ + $\dot{q}$ * t

- Plug in the new q values obtained after numeric integration into the forward position kinematics equation and extract the position of the end effector with respect to the base. This is done by extracting the last column of the final transformation matrix after substitution.
- Store the X and Z values and plot them by taking X on the x-axis and Z on the y-axis (2D).

Result:

The plot obtained from Forward Kinematics Equation of the robot:





The circle is drawn between 1428 mm and 1228 mm about the Z axis and between -100 mm and 100 mm about the X axis.

APPENDIX

Code for the HW4

```python
from sympy import *
import math
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import axes3d,Axes3D

## Code to get the symbolic final transformation matrix

theta_1 = symbols("theta_1")
theta_2 = symbols("theta_2")
theta_3 = symbols("theta_3")
theta_4 = symbols("theta_4")
theta_5 = symbols("theta_5")
theta_6 = symbols("theta_6")
t=symbols("t")

## Defining transformation matrices

t1 = Matrix([[cos(theta_1),0,-sin(theta_1),0],[sin(theta_1),0,cos(theta_1),0],[0,-1,0,128],[0,0,0,1]])
t2 = Matrix([[cos(theta_2),0,sin(theta_2),0],[sin(theta_2),0,-cos(theta_2),0],[0,1,0,176],[0,0,0,1]])
t3 = Matrix([[1,0,0,0],[0,0,-1,0],[0,1,0,612.7],[0,0,0,1]])
t4 = Matrix([[cos(theta_3),0,-sin(theta_3),0],[sin(theta_3),0,cos(theta_3),0],[0,-1,0,176],[0,0,0,1]])
t5 = Matrix([[1,0,0,0],[0,0,1,0],[0,-1,0,571.6],[0,0,0,1]])
t6 = Matrix([[cos(theta_4),0,sin(theta_4),0],[sin(theta_4),0,-cos(theta_4),0],[0,1,0,163.9],[0,0,0,1]])
t7 = Matrix([[cos(theta_5),0,-sin(theta_5),0],[sin(theta_5),0,cos(theta_5),0],[0,-1,0,115.7],[0,0,0,1]])
t8 = Matrix([[cos(theta_6),-sin(theta_6),0,0],[sin(theta_6),cos(theta_6),0,0],[0,0,1,192.2],[0,0,0,1]])
```

```python
## Obtaining Matrix T

T1=t1
T2=t1*t2
T3=t1*t2*t3*t4
T4=t1*t2*t3*t4*t5*t6
T5=t1*t2*t3*t4*t5*t6*t7
T6=t1*t2*t3*t4*t5*t6*t7*t8*t9

## Obtaining Column Matrices Z

Z1=T1[0:3,2]
Z2=T2[0:3,2]
Z3=T3[0:3,2]
Z4=T4[0:3,2]
Z5=T5[0:3,2]
Z6=T6[0:3,2]

## Obtaining Row Matrix P

P=expand(T6[0:3,3])

## Obtaining partial derivate of P wrt different joint angles

p1=diff(P,theta_1)
p2=diff(P,theta_2)
p3=diff(P,theta_3)
p4=diff(P,theta_4)
p5=diff(P,theta_5)
p6=diff(P,theta_6)
```

```python
## Obtaining Jacobian Matrix J

J1=Matrix.hstack(p1,p2,p3,p4,p5,p6)
J2=Matrix.hstack(Z1,Z2,Z3,Z4,Z5,Z6)
J=Matrix.vstack(J1,J2)

print("The Jacobian is: ",'\n')
pprint(J)

t1=0.0000001
t2=0.0000001
t3=0.0000001
t4=0.0000001
t5=0.0000001
t6=0.0000001
t=0
xxx=[]
yyy=[]
zzz=[]

# Loop to get trajectory values

while t<=15:

    #Substituting the angle values into the Jacobian

    JS=J.subs([(theta_1,t1),(theta_2,t2),(theta_3,t3),(theta_4,t4),(theta_5,t5),(theta_6,t6)])

    # Computing velocity trajectory

    X_dot = Matrix([[-(40/3)*math.pi*sin((math.pi/2)+(((2*math.pi)/15)*t))],[0],[(40/3)*math.pi*cos((math.pi/2)+(((2*math.pi)/15)*t))],[0],[0],[0]])

```

```python
    #Computing Joint Velocities

    Q_dot = JS.pinv() * X_dot
    q1_dot=Q_dot[0,0]
    q2_dot=Q_dot[1,0]
    q3_dot=Q_dot[2,0]
    q4_dot=Q_dot[3,0]
    q5_dot=Q_dot[4,0]
    q6_dot=Q_dot[5,0]

    #Computing Joint Angle

    q1=t1+(q1_dot*0.01)
    q2=t2+(q2_dot*0.01)
    q3=t3+(q3_dot*0.01)
    q4=t4+(q4_dot*0.01)
    q5=t5+(q5_dot*0.01)
    q6=t6+(q6_dot*0.01)

```

```python
    ## Substituting the new joint angle values to Final transformation matrix

    TU=T6.subs([(theta_1,q1),(theta_2,q2),(theta_3,q3),(theta_4,q4),(theta_5,q5),(theta_6,q6)])

    ## Extracting Position values of the end effector wrt base

    xx=TU[0,3]
    yy=TU[1,3]
    zz=TU[2,3]
    xxx.append(xx)
    yyy.append(yy)
    zzz.append(zz)
    t1=q1
    t2=q2
    t3=q3
    t4=q4
    t5=q5
    t6=q6
    t=t+0.01


## 2D plot of the circle

fig, ax = plt.subplots()
plt.plot(xxx,zzz)
ax.set_aspect('equal')
plt.title('Circlular Path Plot')
plt.xlabel('X-axis')
plt.ylabel('Z-axis')
plt.show()

```

```
## 3D plot of the circle

fig = plt.figure()

ax1 = fig.add_subplot(111, projection='3d')
ax1.scatter(xxx, yyy, zzz,color='b',linewidths=0.1)
ax1.set_xlabel('X-axis')
ax1.set_ylabel('Y-axis')
ax1.set_zlabel('Z-axis')
ax1.set_ylim3d(0, 1000)

```