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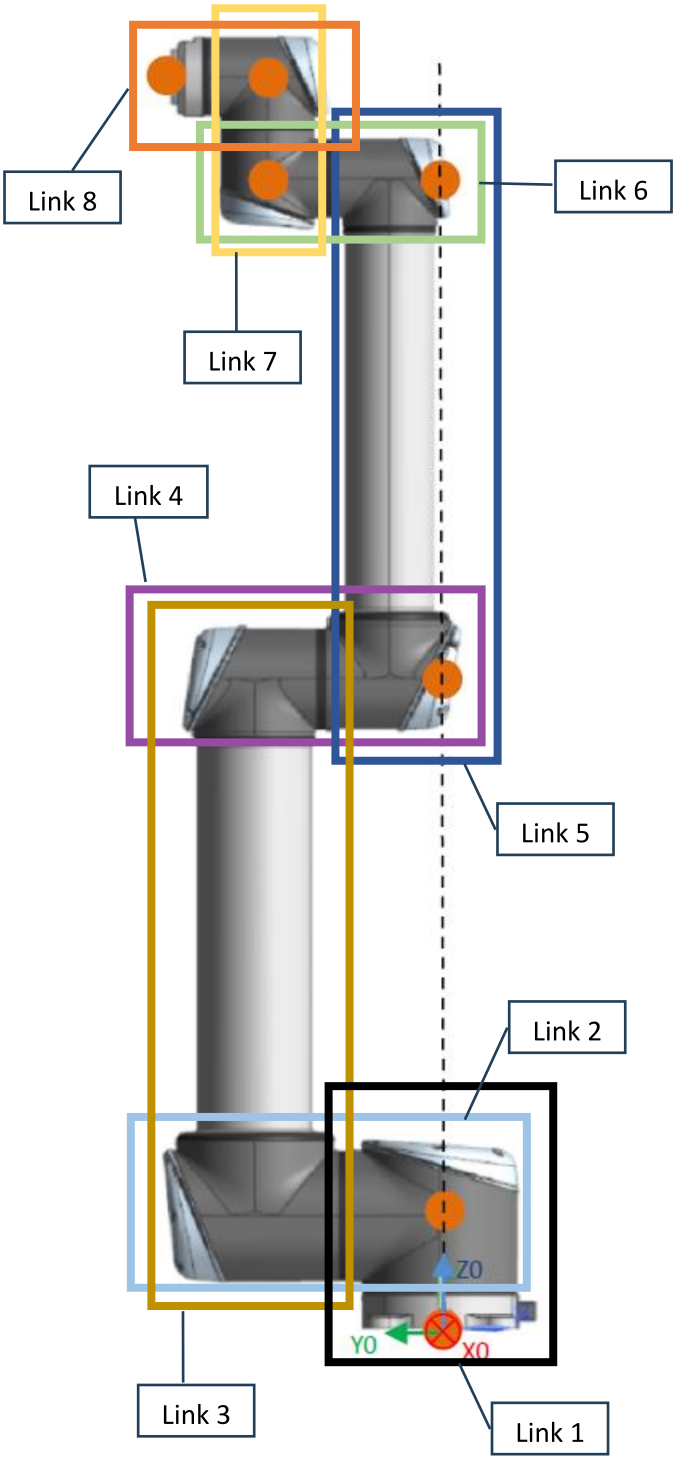
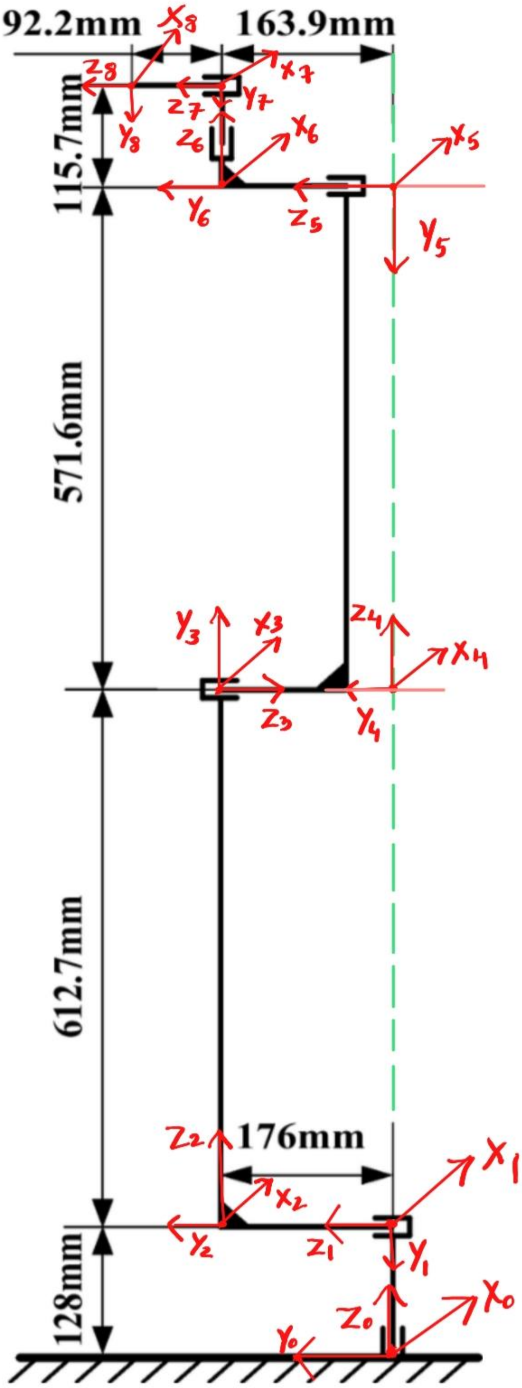
# ENPM 662 – Homework 3 Report

## 1. Position Kinematics – UR10

### D-H coordinate frames (Spong)



In the figures given above,

- Link 1 is attached to Revolute Joint 1 ($\theta_1$).
- Link 1 and Link 2 are interconnected by Revolute Joint 2 ($\theta_2$).
- Link 2 and Link 3 are interconnected by a Fixed Joint.
- Link 3 and Link 4 are interconnected by Revolute Joint 3 ($\theta_3$).
- Link 4 and Link 5 are interconnected by a Fixed Joint.
- Link 5 and Link 6 are interconnected by Revolute Joint 4 ($\theta_4$).
- Link 6 and Link 7 are interconnected by Revolute Joint 5 ($\theta_5$).
- Link 7 and Link 8 are interconnected by Revolute Joint 6 ($\theta_6$).
- The frames are attached as per Spong convention.
- The initial frame is given where x axis in into the plane; the same convention is followed for assigning the rest of the frames.

DH Table:

| Frames | Link | a (in mm) | $\alpha$ (in degree) | $\theta$ (in degree) | d (in mm) |
|---|---|---|---|---|---|
| Frame 0 – Frame 1 | 1 | 0 | $-90^0$ | $\theta_1$ | 128 |
| Frame 1 – Frame 2 | 2 | 0 | $90^0$ | $\theta_2$ | 176 |
| Frame 2 – Frame 3 | 3 | 0 | $90^0$ | 0 | 612.7 |
| Frame 3 – Frame 4 | 4 | 0 | $-90^0$ | $\theta_3$ | 176 |
| Frame 4 – Frame 5 | 5 | 0 | $-90^0$ | 0 | 571.6 |
| Frame 5 – Frame 6 | 6 | 0 | $90^0$ | $\theta_4$ | 163.9 |
| Frame 6 – Frame 7 | 7 | 0 | $-90^0$ | $\theta_5$ | 115.7 |
| Frame 7 – Frame 8 | 8 | 0 | $0^0$ | $\theta_6$ | 92.2 |

Link Transformation Matrices:

The general form of Link Transformation Matrix is given by:

$$T_i = \begin{bmatrix} \cos\theta_i & -(\sin\theta_i * \cos\alpha_i) & (\sin\theta_i * \sin\alpha_i) & (a_i * \cos\theta_i) \\ \sin\theta_i & (\cos\theta_i * \cos\alpha_i) & -(\cos\theta_i * \sin\alpha_i) & (a_i * \sin\theta_i) \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where "i" is the link number.

Therefore, referring to the DH table, the link transformation matrices for the links are given by:

i. For Link 1 (Frame 0 – Frame 1)

$$^{0}T_1 = \begin{bmatrix} \cos\theta_1 & -(\sin\theta_1 * \cos -90) & (\sin\theta_1 * \sin -90) & (0 * \cos\theta_1) \\ \sin\theta_1 & (\cos\theta_1 * \cos -90) & -(\cos\theta_1 * \sin -90) & (0 * \sin\theta_1) \\ 0 & \sin -90 & \cos -90 & 128 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_1 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & 128 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

ii. For Link 2 (Frame 1 – Frame 2)

$$^{1}T_2 = \begin{bmatrix} \cos\theta_2 & -(\sin\theta_2 * \cos 90) & (\sin\theta_2 * \sin 90) & (0 * \cos\theta_2) \\ \sin\theta_2 & (\cos\theta_2 * \cos 90) & -(\cos\theta_2 * \sin 90) & (0 * \sin\theta_2) \\ 0 & \sin 90 & \cos 90 & 176 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{1}T_2 = \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 & 0 \\ \sin\theta_2 & 0 & -\cos\theta_2 & 0 \\ 0 & 1 & 0 & 176 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

iii. For Link 3 (Frame 2 – Frame 3)

$$^{2}T_{3} = \begin{bmatrix} \cos 0 & -(\sin 0 * \cos 90) & (\sin 0 * \sin 90) & (0 * \cos 0) \\ \sin 0 & (\cos 0 * \cos 90) & -(\cos 0 * \sin 90) & (0 * \sin 0) \\ 0 & \sin 90 & \cos 90 & 612.7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{2}T_{3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 612.7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

iv. For Link 4 (Frame 3 – Frame 4)

$$^{3}T_{4} = \begin{bmatrix} \cos\theta_3 & -(\sin\theta_3 * \cos -90) & (\sin\theta_3 * \sin -90) & (0 * \cos\theta_3) \\ \sin\theta_3 & (\cos\theta_3 * \cos -90) & -(\cos\theta_3 * \sin -90) & (0 * \sin\theta_3) \\ 0 & \sin -90 & \cos -90 & 176 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{3}T_{4} = \begin{bmatrix} \cos\theta_3 & 0 & -\sin\theta_3 & 0 \\ \sin\theta_3 & 0 & \cos\theta_3 & 0 \\ 0 & -1 & 0 & 176 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

v. For Link 5 (Frame 4 – Frame 5)

$$^{4}T_{5} = \begin{bmatrix} \cos 0 & -(\sin 0 * \cos -90) & (\sin 0 * \sin -90) & (0 * \cos 0) \\ \sin 0 & (\cos 0 * \cos -90) & -(\cos 0 * \sin -90) & (0 * \sin 0) \\ 0 & \sin -90 & \cos -90 & 571.6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{4}T_{5} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 571.6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

vi. For Link 6 (Frame 5 – Frame 6)

$$^{5}T_{6} = \begin{bmatrix} \cos\theta_4 & -(\sin\theta_4 * \cos 90) & (\sin\theta_4 * \sin 90) & (0 * \cos\theta_4) \\ \sin\theta_4 & (\cos\theta_4 * \cos 90) & -(\cos\theta_4 * \sin 90) & (0 * \sin\theta_4) \\ 0 & \sin 90 & \cos 90 & 163.9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{5}T_{6} = \begin{bmatrix} \cos\theta_4 & 0 & \sin\theta_4 & 0 \\ \sin\theta_4 & 0 & -\cos\theta_4 & 0 \\ 0 & 1 & 0 & 163.9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

vii. For Link 7 (Frame 6 – Frame 7)

$$^{6}T_{7} = \begin{bmatrix} \cos\theta_5 & -(\sin\theta_5 * \cos -90) & (\sin\theta_5 * \sin -90) & (0 * \cos\theta_5) \\ \sin\theta_5 & (\cos\theta_5 * \cos -90) & -(\cos\theta_5 * \sin -90) & (0 * \sin\theta_5) \\ 0 & \sin -90 & \cos -90 & 115.7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{6}T_{7} = \begin{bmatrix} \cos\theta_5 & 0 & -\sin\theta_5 & 0 \\ \sin\theta_5 & 0 & \cos\theta_5 & 0 \\ 0 & -1 & 0 & 115.7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

viii. For Link 8 (Frame 7 – Frame 8)

$$^{7}T_{8} = \begin{bmatrix} \cos\theta_6 & -(\sin\theta_6 * \cos 0) & (\sin\theta_6 * \sin 0) & (0 * \cos\theta_6) \\ \sin\theta_6 & (\cos\theta_6 * \cos 0) & -(\cos\theta_6 * \sin 0) & (0 * \sin\theta_6) \\ 0 & \sin 0 & \cos 0 & 92.2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{7}T_{8} = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 92.2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The Final Transformation Matrix is given by post multiplying the matrices like:

$^{0}T_{8} = {}^{0}T_{1} * {}^{1}T_{2} * {}^{2}T_{3} * {}^{3}T_{4} * {}^{4}T_{5} * {}^{5}T_{6} * {}^{6}T_{7} * {}^{7}T_{8}$

$$^{0}T_{8} = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & 128 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 & 0 \\ \sin\theta_2 & 0 & -\cos\theta_2 & 0 \\ 0 & 1 & 0 & 176 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 612.7 \\ 0 & 0 & 0 & 1 \end{bmatrix} *$$

$$\begin{bmatrix} \cos\theta_3 & 0 & -\sin\theta_3 & 0 \\ \sin\theta_3 & 0 & \cos\theta_3 & 0 \\ 0 & -1 & 0 & 176 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 571.6 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \cos\theta_4 & 0 & \sin\theta_4 & 0 \\ \sin\theta_4 & 0 & -\cos\theta_4 & 0 \\ 0 & 1 & 0 & 163.9 \\ 0 & 0 & 0 & 1 \end{bmatrix} *$$

$$\begin{bmatrix} \cos\theta_5 & 0 & -\sin\theta_5 & 0 \\ \sin\theta_5 & 0 & \cos\theta_5 & 0 \\ 0 & -1 & 0 & 115.7 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 92.2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The Symbolic form of the transformation matrix is obtained through sympy (python) code)

Symbolic form of final transformation matrix

Column 1

$$\begin{bmatrix} -\sin(\theta_1)\sin(\theta_5)\cos(\theta_6) - \sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_6)\cos(\theta_1) + \sin(\theta_2)\sin(\theta_3)\cos(\theta_1)\cos(\theta_4)\cos(\theta_5)\cos(\theta_6) - \sin(\theta_2)\sin(\theta_4)\cos(\theta_1)\cos(\theta_3)\cos(\theta_5)\cos(\theta_6) - \sin(\theta_2)\sin(\theta_6)\cos(\theta_1)\cos(\theta_3)\cos(\theta_4) + \sin(\theta_3)\sin(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_5)\cos(\theta_6) + \sin(\theta_3)\sin(\theta_6)\cos(\theta_1)\cos(\theta_2)\cos(\theta_4) - \sin(\theta_4)\sin(\theta_6)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) + \cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5)\cos(\theta_6) \\ -\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_6) + \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4)\cos(\theta_5)\cos(\theta_6) - \sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\cos(\theta_3)\cos(\theta_5)\cos(\theta_6) - \sin(\theta_1)\sin(\theta_2)\sin(\theta_6)\cos(\theta_3)\cos(\theta_4) + \sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\cos(\theta_2)\cos(\theta_5)\cos(\theta_6) + \sin(\theta_1)\sin(\theta_3)\sin(\theta_6)\cos(\theta_2)\cos(\theta_4) - \sin(\theta_1)\sin(\theta_4)\sin(\theta_6)\cos(\theta_2)\cos(\theta_3) + \sin(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5)\cos(\theta_6) + \sin(\theta_5)\cos(\theta_1)\cos(\theta_6) \\ -\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\cos(\theta_5)\cos(\theta_6) - \sin(\theta_2)\sin(\theta_3)\sin(\theta_6)\cos(\theta_4) + \sin(\theta_2)\sin(\theta_4)\sin(\theta_6)\cos(\theta_3) - \sin(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5)\cos(\theta_6) - \sin(\theta_3)\sin(\theta_4)\sin(\theta_6)\cos(\theta_2) + \sin(\theta_3)\cos(\theta_2)\cos(\theta_4)\cos(\theta_5)\cos(\theta_6) - \sin(\theta_4)\cos(\theta_2)\cos(\theta_3)\cos(\theta_5)\cos(\theta_6) - \sin(\theta_6)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) \\ 0 \end{bmatrix}$$

Column 2

$$\begin{matrix} \sin(\theta_1)\sin(\theta_5)\sin(\theta_6) - \sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\cos(\theta_1)\cos(\theta_6) - \sin(\theta_2)\sin(\theta_3)\sin(\theta_6)\cos(\theta_1)\cos(\theta_4)\cos(\theta_5) + \sin(\theta_2)\sin(\theta_4)\sin(\theta_6)\cos(\theta_1)\cos(\theta_3)\cos(\theta_5) - \sin(\theta_2)\cos(\theta_1)\cos(\theta_3)\cos(\theta_4)\cos(\theta_6) - \sin(\theta_3)\sin(\theta_4)\sin(\theta_6)\cos(\theta_1)\cos(\theta_2)\cos(\theta_5) + \sin(\theta_3)\cos(\theta_1)\cos(\theta_2)\cos(\theta_4)\cos(\theta_6) - \sin(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_6) - \sin(\theta_6)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5) \\ -\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\cos(\theta_6) - \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_6)\cos(\theta_4)\cos(\theta_5) + \sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\sin(\theta_6)\cos(\theta_3)\cos(\theta_5) - \sin(\theta_1)\sin(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_6) - \sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\sin(\theta_6)\cos(\theta_2)\cos(\theta_5) + \sin(\theta_1)\sin(\theta_3)\cos(\theta_2)\cos(\theta_4)\cos(\theta_6) - \sin(\theta_1)\sin(\theta_4)\cos(\theta_2)\cos(\theta_3)\cos(\theta_6) - \sin(\theta_1)\sin(\theta_6)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5) - \sin(\theta_5)\sin(\theta_6)\cos(\theta_1) \\ \sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_6)\cos(\theta_5) - \sin(\theta_2)\sin(\theta_3)\cos(\theta_4)\cos(\theta_6) + \sin(\theta_2)\sin(\theta_4)\cos(\theta_3)\cos(\theta_6) + \sin(\theta_2)\sin(\theta_6)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5) - \sin(\theta_3)\sin(\theta_4)\cos(\theta_2)\cos(\theta_6) - \sin(\theta_3)\sin(\theta_6)\cos(\theta_2)\cos(\theta_4)\cos(\theta_5) + \sin(\theta_4)\sin(\theta_6)\cos(\theta_2)\cos(\theta_3)\cos(\theta_5) - \cos(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_6) \\ 0 \end{matrix}$$

Column 3

$$\begin{matrix} -\sin(\theta_1)\cos(\theta_5) - \sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_1)\cos(\theta_4) + \sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_3) - \sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2) - \sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) \\ -\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_4) + \sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_3) - \sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) + \cos(\theta_1)\cos(\theta_5) \\ \sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5) + \sin(\theta_2)\sin(\theta_5)\cos(\theta_3)\cos(\theta_4) - \sin(\theta_3)\sin(\theta_5)\cos(\theta_2)\cos(\theta_4) + \sin(\theta_4)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3) \\ 0 \end{matrix}$$

Column 4

$$\begin{bmatrix} -92.2\sin(\theta_1)\cos(\theta_5) - 163.9\sin(\theta_1) + 115.7\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\cos(\theta_1) - 92.2\sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_1)\cos(\theta_4) + 92.2\sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_3) + 115.7\sin(\theta_2)\cos(\theta_1)\cos(\theta_3)\cos(\theta_4) + 571.6\sin(\theta_2)\cos(\theta_1)\cos(\theta_3) + 612.7\sin(\theta_2)\cos(\theta_1) - 92.2\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2) - 115.7\sin(\theta_3)\cos(\theta_1)\cos(\theta_2)\cos(\theta_4) - 571.6\sin(\theta_3)\cos(\theta_1)\cos(\theta_2) + 115.7\sin(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) - 92.2\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) \\ 115.7\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4) - 92.2\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_4) + 92.2\sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_3) + 115.7\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)\cos(\theta_4) + 571.6\sin(\theta_1)\sin(\theta_2)\cos(\theta_3) + 612.7\sin(\theta_1)\sin(\theta_2) - 92.2\sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2) - 115.7\sin(\theta_1)\sin(\theta_3)\cos(\theta_2)\cos(\theta_4) - 571.6\sin(\theta_1)\sin(\theta_3)\cos(\theta_2) + 115.7\sin(\theta_1)\sin(\theta_4)\cos(\theta_2)\cos(\theta_3) - 92.2\sin(\theta_1)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) + 92.2\cos(\theta_1)\cos(\theta_5) + 163.9\cos(\theta_1) \\ 92.2\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5) + 115.7\sin(\theta_2)\sin(\theta_3)\cos(\theta_4) + 571.6\sin(\theta_2)\sin(\theta_3) - 115.7\sin(\theta_2)\sin(\theta_4)\cos(\theta_3) + 92.2\sin(\theta_2)\sin(\theta_5)\cos(\theta_3)\cos(\theta_4) + 115.7\sin(\theta_3)\sin(\theta_4)\cos(\theta_2) - 92.2\sin(\theta_3)\sin(\theta_5)\cos(\theta_2)\cos(\theta_4) + 92.2\sin(\theta_4)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3) + 115.7\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) + 571.6\cos(\theta_2)\cos(\theta_3) + 612.7\cos(\theta_2) + 128 \\ 1 \end{bmatrix}$$

**Home Position Orientation :** In the robot's home position, as given in the problem, it is considered that joint angles are 0.

i.e. $\theta_1 = \theta_2 = \theta_3 = \theta_4 = \theta_5 = \theta_6 = 0$ degrees

Hence, for the home position, ${}^0T_8$ is :

$$
{}^0T_8 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 128 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 176 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 612.7 \\ 0 & 0 & 0 & 1 \end{bmatrix} *
$$

$$
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 176 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 571.6 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 163.9 \\ 0 & 0 & 0 & 1 \end{bmatrix} *
$$

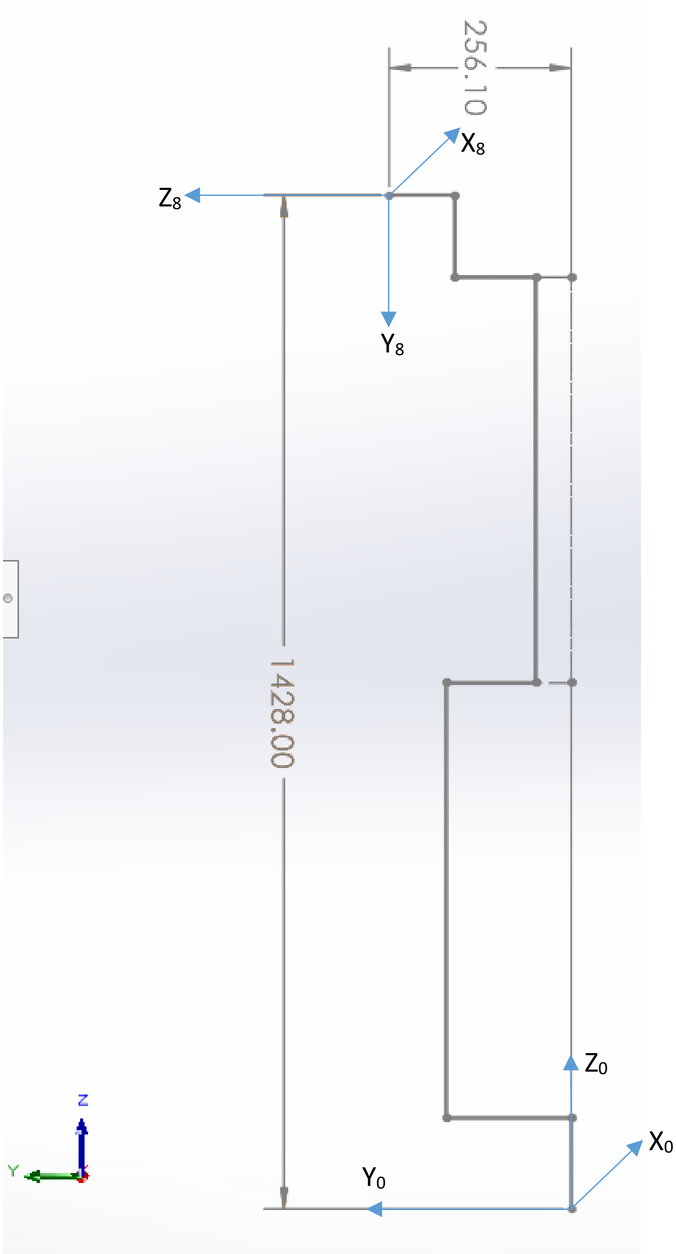
$$
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 115.7 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 92.2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0T_8 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 256.1 \\ 0 & -1 & 0 & 1428.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Geometric Validation

- The robot links are represented as straight lines.
- The robot is drawn according to the orientation considered and the link dimensions given in the problem.

Geometric validation for Home Position Orientation:




Home Orientation

$\theta_1 = \theta_2 = \theta_3 = \theta_4 = \theta_5 = \theta_6 = 0$ degrees

From the figure,

- $X_8 . X_0 = 1$ , $Y_8 . X_0 = 0$ , $Z_8 . X_0 = 0$
- $X_8 . Y_0 = 0$ , $Y_8 . Y_0 = 0$ , $Z_8 . Y_0 = 1$
- $X_8 . Z_0 = 0$ , $Y_8 . Z_0 = -1$ , $X_8 . Z_0 = 0$
- X Translation = 00 mm
- Y Translation = 256.1 mm
- Z Translation = 1428.00 mm

These are the exact values that we got from Transformation matrix, hence verified.

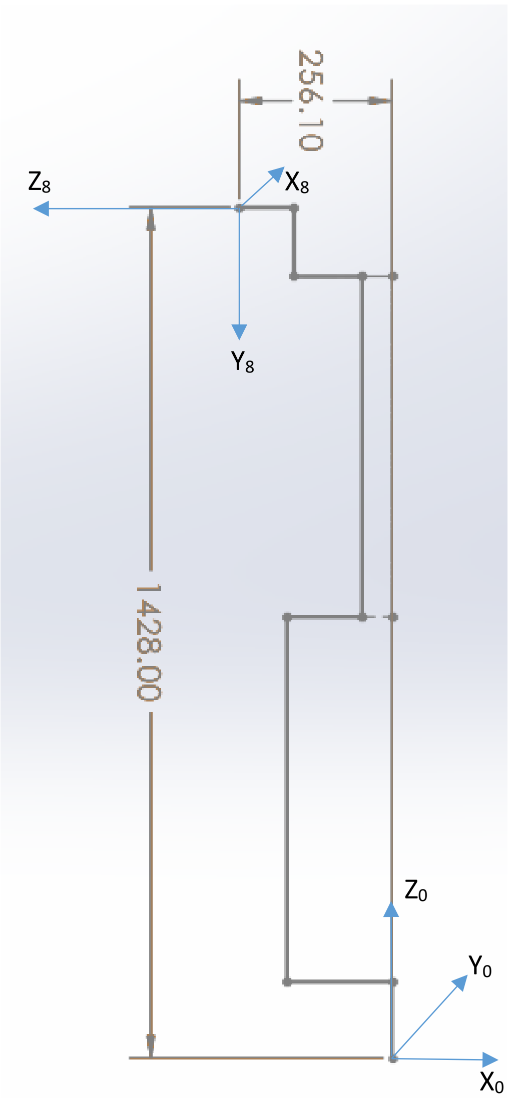
Validating the equations parametrically:

(the transformation matrices are obtained from python code)

i. Orientation 1:

$( \theta_1 , \theta_2 , \theta_3 , \theta_4 , \theta_5 , \theta_6 ) = ( 90 , 0 , 0 , 0 , 0 , 0 )$

$${}^{0}T_8 = \begin{bmatrix} 0 & 0 & -1 & -256.1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1428.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$




Orientation 1

$( \theta 1 , \theta 2 , \theta 3 , \theta 4 , \theta 5 , \theta 6 ) = ( 90 , 0 , 0 , 0 , 0 , 0 )$

From the figure,

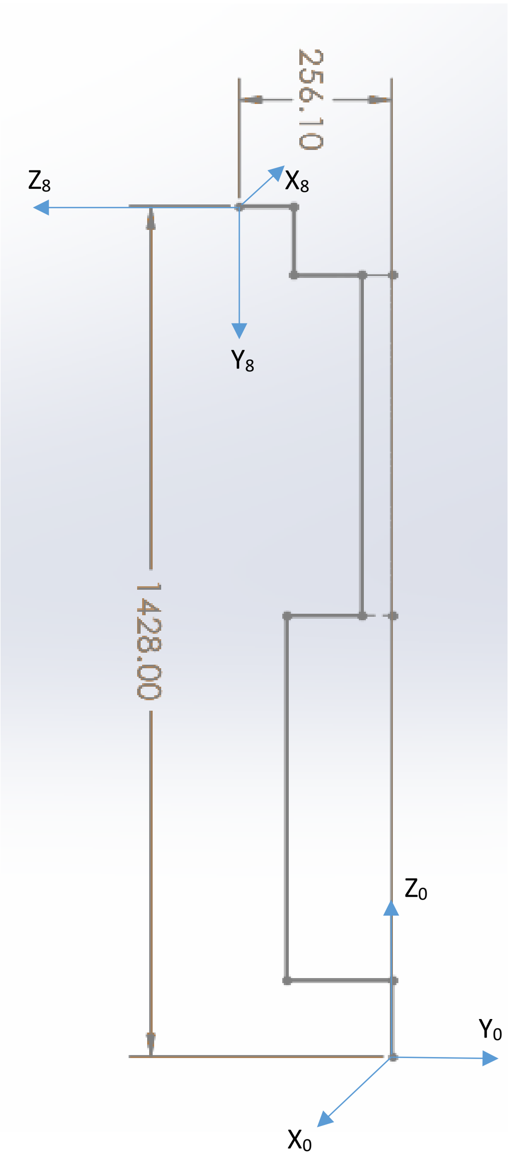
- $X_8 . X_0 = 0$ , $Y_8 . X_0 = 0$ , $Z_8 . X_0 = -1$
- $X_8 . Y_0 = 1$ , $Y_8 . Y_0 = 0$ , $Z_8 . Y_0 = 0$
- $X_8 . Z_0 = 0$ , $Y_8 . Z_0 = -1$ , $X_8 . Z_0 = 0$
- X Translation = - 256.1 mm
- Y Translation = 00 mm
- Z Translation = 1428.00 mm

These are the exact values that we got from Transformation matrix, hence verified.

ii. Orientation 2:

$( \theta_1 , \theta_2 , \theta_3 , \theta_4 , \theta_5 , \theta_6 ) = ( 180 , 0 , 0 , 0 , 0 , 0 )$

$$ {}^{0}T_8 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -256.1 \\ 0 & -1 & 0 & 1428.0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$




Orientation 2

$( \theta_1 , \theta_2 , \theta_3 , \theta_4 , \theta_5 , \theta_6 ) = ( 180 , 0 , 0 , 0 , 0 , 0 )$

From the figure,

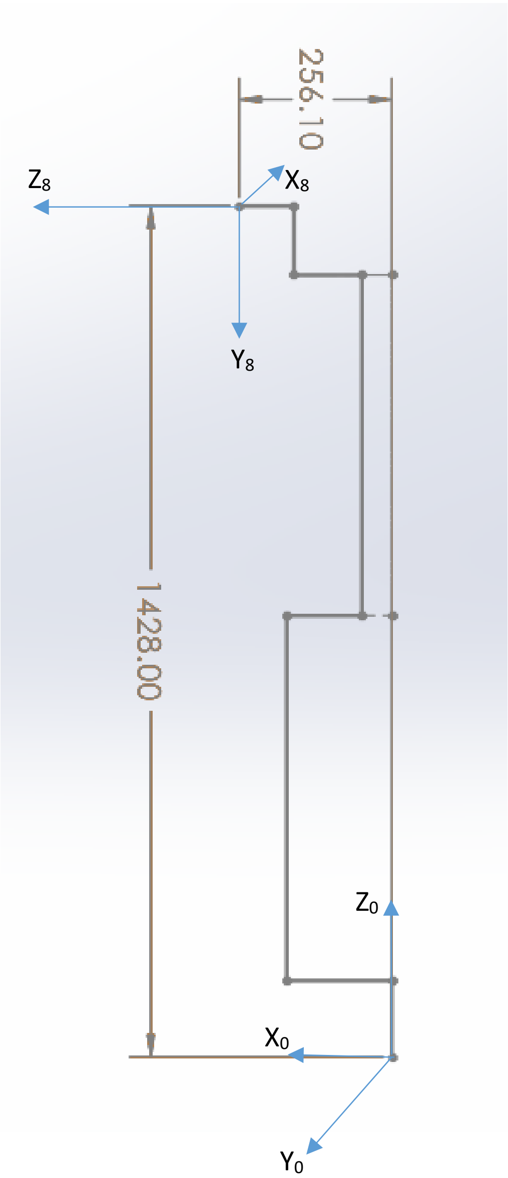
- $X_8 . X_0 = -1$ , $Y_8 . X_0 = 0$ , $Z_8 . X_0 = 0$
- $X_8 . Y_0 = 0$ , $Y_8 . Y_0 = 0$ , $Z_8 . Y_0 = -1$
- $X_8 . Z_0 = 0$ , $Y_8 . Z_0 = -1$ , $X_8 . Z_0 = 0$
- X Translation = 00 mm
- Y Translation = - 256.1 mm
- Z Translation = 1428.00 mm

These are the exact values that we got from Transformation matrix, hence verified.

iii. Orientation 3:

$(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6) = (270, 0, 0, 0, 0, 0)$

$$^{0}T_8 = \begin{bmatrix} 0 & 0 & 1 & 256.1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1428.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$




Orientation 3

$(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6) = (270, 0, 0, 0, 0, 0)$

From the figure,

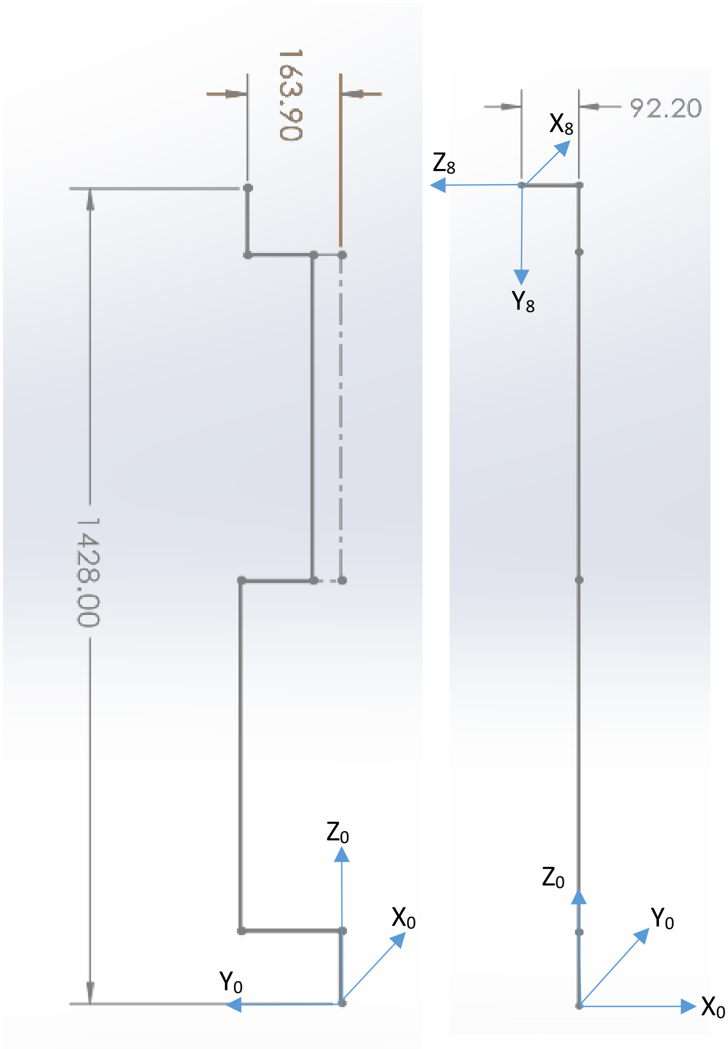
- $X_8 . X_0 = 0$ , $Y_8 . X_0 = 0$ , $Z_8 . X_0 = 1$
- $X_8 . Y_0 = -1$ , $Y_8 . Y_0 = 0$ , $Z_8 . Y_0 = 0$
- $X_8 . Z_0 = 0$ , $Y_8 . Z_0 = -1$ , $X_8 . Z_0 = 0$
- X Translation = 256.1 mm
- Y Translation = 00 mm
- Z Translation = 1428.00 mm

These are the exact values that we got from Transformation matrix, hence verified.

iv. Orientation 4:

$(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6) = (0, 0, 0, 0, 90, 0)$

$$
{}^{0}T_{8} = \begin{bmatrix} 0 & 0 & -1 & -92.2 \\ 1 & 0 & 0 & 163.9 \\ 0 & -1 & 0 & 1428.0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$




Orientation 4

$(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6) = (0, 0, 0, 0, 90, 0)$

From the figure,

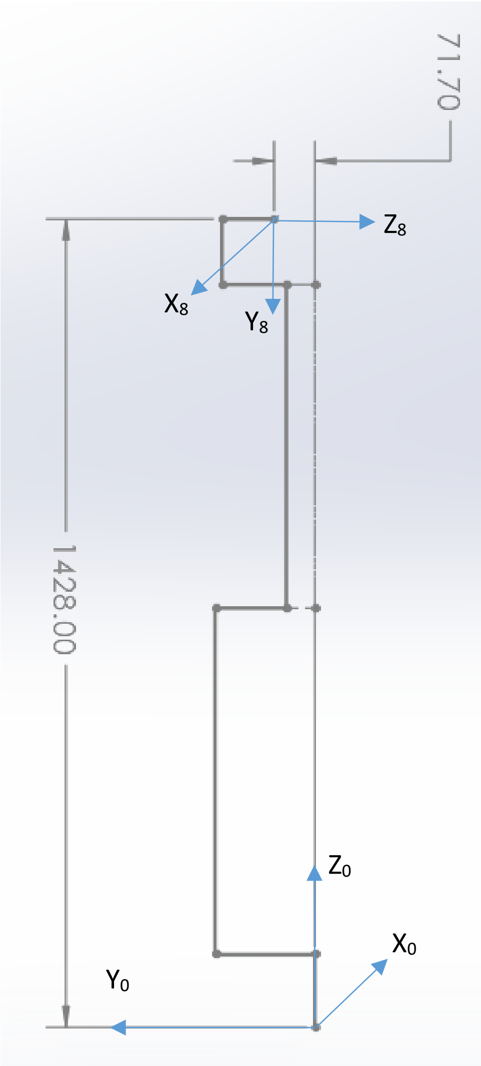
- $X_8 . X_0 = 0$ , $Y_8 . X_0 = 0$ , $Z_8 . X_0 = -1$
- $X_8 . Y_0 = 1$ , $Y_8 . Y_0 = 0$ , $Z_8 . Y_0 = 0$
- $X_8 . Z_0 = 0$ , $Y_8 . Z_0 = -1$ , $X_8 . Z_0 = 0$
- X Translation = -92.20 mm
- Y Translation = 163.90 mm
- Z Translation = 1428.00 mm

These are the exact values that we got from Transformation matrix, hence verified.

v. Orientation 5:

$(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6) = (0, 0, 0, 0, 180, 0)$

$$ {}^{0}T_8 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 71.7 \\ 0 & -1 & 0 & 1428.0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$




Orientation 5

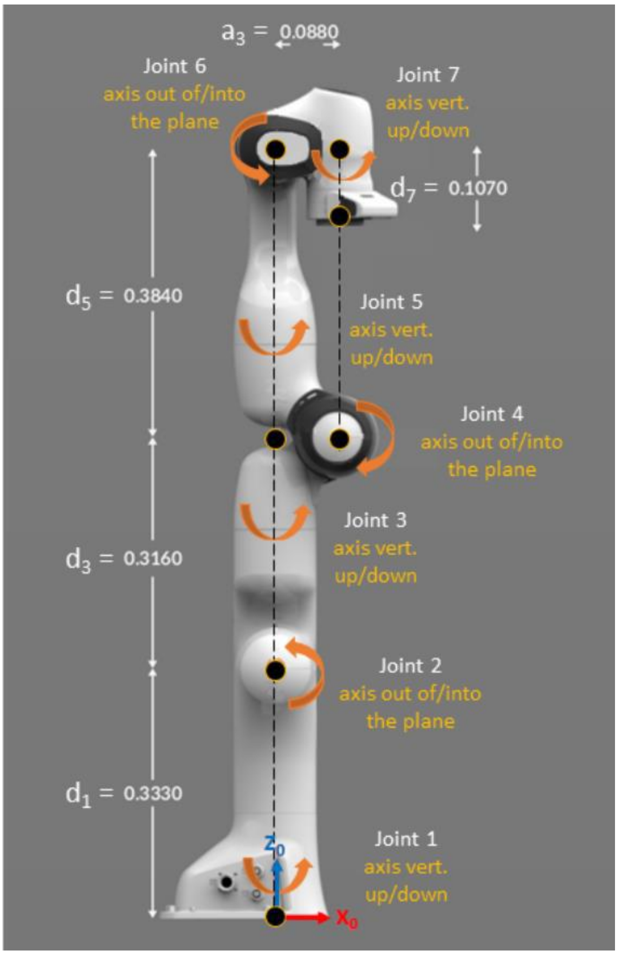
$(\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6) = (0, 0, 0, 0, 180, 0)$

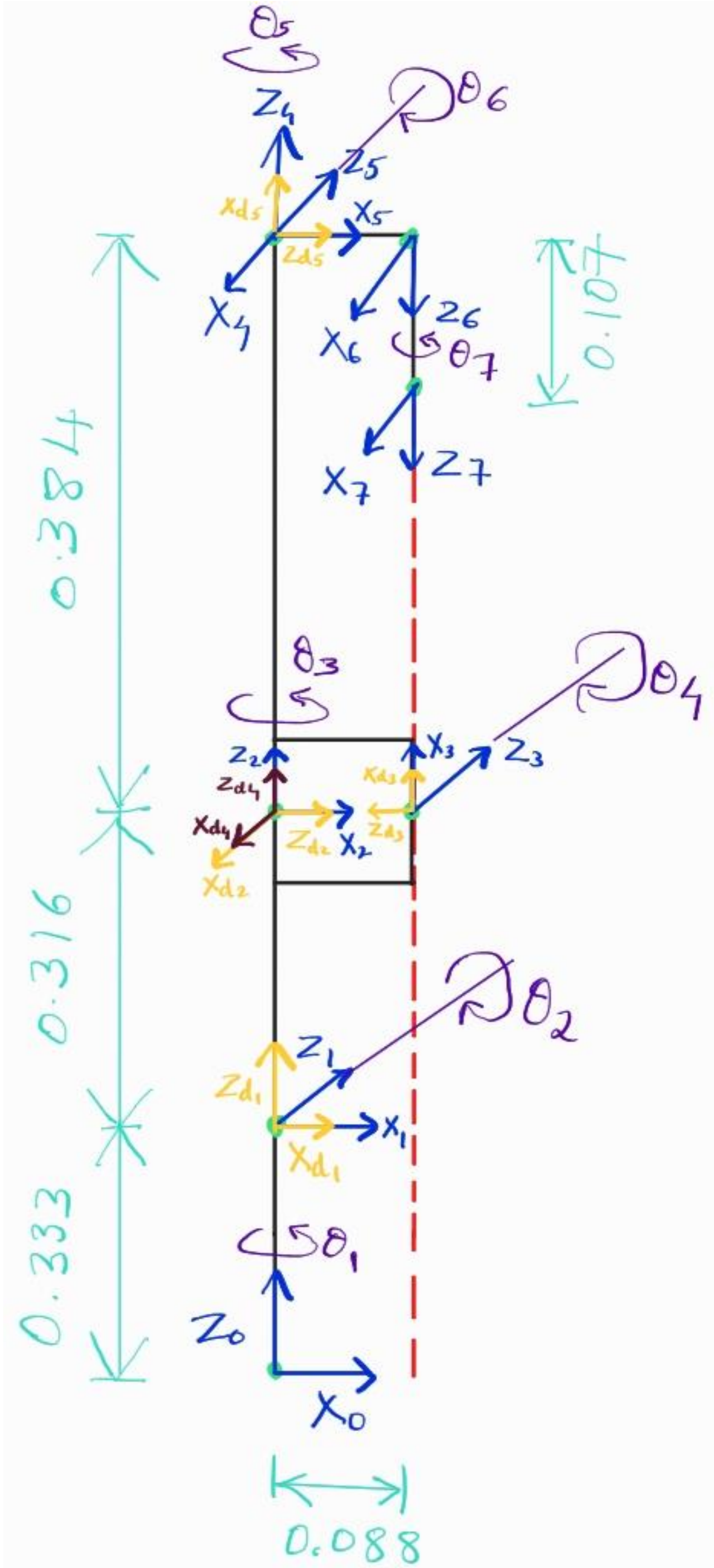
From the figure,

- $X_8 . X_0 = -1$ , $Y_8 . X_0 = 0$ , $Z_8 . X_0 = 0$
- $X_8 . Y_0 = 0$ , $Y_8 . Y_0 = 0$ , $Z_8 . Y_0 = -1$
- $X_8 . Z_0 = 0$ , $Y_8 . Z_0 = -1$ , $X_8 . Z_0 = 0$
- X Translation = 00 mm
- Y Translation = 71.70 mm
- Z Translation = 1428.00 mm

These are the exact values that we got from Transformation matrix, hence verified.

## 2. Position Kinematics - KUKA




D-H coordinate frames (Spong)

D-H Table (Spong Convention)

| Frames | a (in m) | $\alpha$ (in degree) | d (in m) | $\theta$ (in degree) |
|---|---|---|---|---|
| Frame 0 – Frame 1 | 0 | -90 | 0.333 | $\theta_1$ |
| Frame 1 – Frame $d_1$ | 0 | 90 | 0 | 0 |
| Frame $d_1$ – Frame 2 | 0 | 0 | 0.316 | $\theta_2$ |
| Frame 2 – Frame $d_2$ | 0 | -90 | 0 | $\theta_3$ - 90 |
| Frame $d_2$ – Frame 3 | 0 | 90 | 0.088 | -90 |
| Frame 3 – Frame $d_3$ | 0 | 90 | 0 | $\theta_4$ |
| Frame $d_3$ – Frame $d_4$ | 0 | -90 | 0.088 | -90 |
| Frame $d_4$ – Frame 4 | 0 | 0 | 0.384 | 0 |
| Frame 4 – Frame 5 | 0 | -90 | 0 | $\theta_5$ + 90 |
| Frame 5 – Frame $d_5$ | 0 | -90 | 0 | $\theta_6$ - 90 |
| Frame $d_5$ – Frame 6 | 0 | -90 | 0.088 | 90 |
| Frame 6 – Frame 7 | 0 | 0 | 0.107 | $\theta_7$ |

At the home position (given pose), the $\theta$ values are (from the figure) :

$\theta_1 = 0^o$

$\theta_2 = 0^o$

$\theta_3 = 0^o$

$\theta_4 = 0^o$

$\theta_5 = 0^o$

$\theta_6 = 0^o$

$\theta_7 = 0^o$

Hence at home position, the final transformation matrix is:

$${}^0T_n = \begin{bmatrix} 0 & -1 & 0 & 0.088 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0.926 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
from sympy import *
import math

t1=0
t2=0
t3=0
t4=-(math.pi/2)
t5=-(math.pi/2)
t6=0
t7=-(math.pi/2)
t8=0
t9=(math.pi/2)
t10=-(math.pi/2)
t11=(math.pi/2)
t12=0

a1=-(math.pi/2)
a2=(math.pi/2)
a3=0
a4=-(math.pi/2)
a5=(math.pi/2)
a6=(math.pi/2)
a7=-(math.pi/2)
a8=0
a9=-(math.pi/2)
a10=-(math.pi/2)
a11=-(math.pi/2)
a12=0
```

```
d1=0.333
d2=0
d3=0.316
d4=0
d5=0.088
d6=0
d7=0.088
d8=0.3840
d9=0
d10=0
d11=0.088
d12=0.107

m1 = Matrix([[cos(t1),-(sin(t1)*cos(a1)),(sin(t1)*sin(a1)),0],[sin(t1),(cos(t1)*cos(a1)),-(cos(t1)*sin(a1)),0],[0,sin(a1),cos(a1
m2 = Matrix([[cos(t2),-(sin(t2)*cos(a2)),(sin(t2)*sin(a2)),0],[sin(t2),(cos(t2)*cos(a2)),-(cos(t2)*sin(a2)),0],[0,sin(a2),cos(a2
m3 = Matrix([[cos(t3),-(sin(t3)*cos(a3)),(sin(t3)*sin(a3)),0],[sin(t3),(cos(t3)*cos(a3)),-(cos(t3)*sin(a3)),0],[0,sin(a3),cos(a3
m4 = Matrix([[cos(t4),-(sin(t4)*cos(a4)),(sin(t4)*sin(a4)),0],[sin(t4),(cos(t4)*cos(a4)),-(cos(t4)*sin(a4)),0],[0,sin(a4),cos(a4
m5 = Matrix([[cos(t5),-(sin(t5)*cos(a5)),(sin(t5)*sin(a5)),0],[sin(t5),(cos(t5)*cos(a5)),-(cos(t5)*sin(a5)),0],[0,sin(a5),cos(a5
m6 = Matrix([[cos(t6),-(sin(t6)*cos(a6)),(sin(t6)*sin(a6)),0],[sin(t6),(cos(t6)*cos(a6)),-(cos(t6)*sin(a6)),0],[0,sin(a6),cos(a6
m7 = Matrix([[cos(t7),-(sin(t7)*cos(a7)),(sin(t7)*sin(a7)),0],[sin(t7),(cos(t7)*cos(a7)),-(cos(t7)*sin(a7)),0],[0,sin(a7),cos(a7
m8 = Matrix([[cos(t8),-(sin(t8)*cos(a8)),(sin(t8)*sin(a8)),0],[sin(t8),(cos(t8)*cos(a8)),-(cos(t8)*sin(a8)),0],[0,sin(a8),cos(a8
m9 = Matrix([[cos(t9),-(sin(t9)*cos(a9)),(sin(t9)*sin(a9)),0],[sin(t9),(cos(t9)*cos(a9)),-(cos(t9)*sin(a9)),0],[0,sin(a9),cos(a9
m10 = Matrix([[cos(t10),-(sin(t10)*cos(a10)),(sin(t10)*sin(a10)),0],[sin(t10),(cos(t10)*cos(a10)),-(cos(t10)*sin(a10)),0],[0,sin
m11 = Matrix([[cos(t11),-(sin(t11)*cos(a11)),(sin(t11)*sin(a11)),0],[sin(t11),(cos(t11)*cos(a11)),-(cos(t11)*sin(a11)),0],[0,sin
m12 = Matrix([[cos(t12),-(sin(t12)*cos(a12)),(sin(t12)*sin(a12)),0],[sin(t12),(cos(t12)*cos(a12)),-(cos(t12)*sin(a12)),0],[0,sin

M=m1*m2*m3*m4*m5*m6*m7*m8*m9*m10*m11*m12

M
```

$$\begin{bmatrix} -6.12323399573677 \cdot 10^{-17} & -1.0 & 1.22464679914735 \cdot 10^{-16} & 0.088 \\ -1.0 & 6.12323399573677 \cdot 10^{-17} & 8.57655218742367 \cdot 10^{-33} & 5.38844591624836 \cdot 10^{-18} \\ -1.23259516440783 \cdot 10^{-32} & -1.22464679914735 \cdot 10^{-16} & -1.0 & 0.926 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Verifying the method through python code

(Consider very small values as 0)

APPENDIX

Problem 1 – Sympy code for symbolic transformation matrix and the matrices for parametric-geometric validation

```
from sympy import *
import math

## Code to get the symbolic final transformation matrix

theta_1 = symbols("theta_1")
theta_2 = symbols("theta_2")
theta_3 = symbols("theta_3")
theta_4 = symbols("theta_4")
theta_5 = symbols("theta_5")
theta_6 = symbols("theta_6")

## Defining transformation matrices

t1 = Matrix([[cos(theta_1),0,-sin(theta_1),0],[sin(theta_1),0,cos(theta_1),0],[0,-1,0,128],[0,0,0,1]])
t2 = Matrix([[cos(theta_2),0,sin(theta_2),0],[sin(theta_2),0,-cos(theta_2),0],[0,1,0,176],[0,0,0,1]])
t3 = Matrix([[1,0,0,0],[0,0,-1,0],[0,1,0,612.7],[0,0,0,1]])
t4 = Matrix([[cos(theta_3),0,-sin(theta_3),0],[sin(theta_3),0,cos(theta_3),0],[0,-1,0,176],[0,0,0,1]])
t5 = Matrix([[1,0,0,0],[0,0,1,0],[0,-1,0,571.6],[0,0,0,1]])
t6 = Matrix([[cos(theta_4),0,sin(theta_4),0],[sin(theta_4),0,-cos(theta_4),0],[0,1,0,163.9],[0,0,0,1]])
t7 = Matrix([[cos(theta_5),0,-sin(theta_5),0],[sin(theta_5),0,cos(theta_5),0],[0,-1,0,115.7],[0,0,0,1]])
t8 = Matrix([[cos(theta_6),-sin(theta_6),0,0],[sin(theta_6),cos(theta_6),0,0],[0,0,1,92.2],[0,0,0,1]])

T = expand(t1*t2*t3*t4*t5*t6*t7*t8)

print("ENPM662 Homework 3 Problem 1"'\n')

## Printing the successive transformation matrices

print("The successive link transformation matrices are as follows:"'\n')

print("The transformation matrix T_01 is:"'\n''\n')
pprint(t1)
print('\n''\n')

print("The transformation matrix T_12 is:"'\n''\n')
pprint(t2)
print('\n''\n')

print("The transformation matrix T_23 is:"'\n''\n')
pprint(t3)
print('\n''\n')


print("The transformation matrix T_34 is:"'\n''\n')
pprint(t4)
print('\n''\n')

print("The transformation matrix T_45 is:"'\n''\n')
pprint(t5)
print('\n''\n')

print("The transformation matrix T_56 is:"'\n''\n')
pprint(t6)
print('\n''\n')

print("The transformation matrix T_67 is:"'\n''\n')
pprint(t7)
print('\n''\n')

print("The transformation matrix T_78 is:"'\n''\n')
pprint(t8)
print('\n''\n')


print("The symbolic final transformation matrix (T_08) is:"'\n''\n')
pprint(T)                              ## Printing the final symbolic transformation matrix
print('\n''\n''\n')



## Code to get the parametric final transformation matrices for different geometrically known configurations

## Configurations considered
## Home Configuration - (theta-1 , theta-2 , theta-3 , theta-4 , theta-5 , theta-6) = (0,0,0,0,0,0) -
## Configuration 1 - (theta-1 , theta-2 , theta-3 , theta-4 , theta-5 , theta-6) = (90,0,0,0,0,0)
## Configuration 2 - (theta-1 , theta-2 , theta-3 , theta-4 , theta-5 , theta-6) = (180,0,0,0,0,0)
## Configuration 3 - (theta-1 , theta-2 , theta-3 , theta-4 , theta-5 , theta-6) = (270,0,0,0,0,0)
## Configuration 4 - (theta-1 , theta-2 , theta-3 , theta-4 , theta-5 , theta-6) = (0,0,0,0,90,0)
## Configuration 5 - (theta-1 , theta-2 , theta-3 , theta-4 , theta-5 , theta-6) = (0,0,0,0,180,0)
```

```python
print("Parametric final transformation matrices for validation (robot at different configurations): "'\n''\n')

vt1=[0,(0.5*math.pi),(math.pi),(1.5*math.pi),0,0]
vt2=[0,0,0,0,0,0]
vt3=[0,0,0,0,0,0]
vt4=[0,0,0,0,0,0]                                   ## Vectors to store theta values
vt5=[0,0,0,0,(0.5*math.pi),math.pi]
vt6=[0,0,0,0,0,0]
i=0

while i<len(vt1):
    th_1=vt1[i]
    th_2=vt2[i]
    th_3=vt3[i]
    th_4=vt4[i]                                 ## Defining theta values
    th_5=vt5[i]
    th_6=vt6[i]

    ## Defining the transformation matrices

    m1 = Matrix([[cos(th_1),0,-sin(th_1),0],[sin(th_1),0,cos(th_1),0],[0,-1,0,128],[0,0,0,1]])
    m2 = Matrix([[cos(th_2),0,sin(th_2),0],[sin(th_2),0,-cos(th_2),0],[0,1,0,176],[0,0,0,1]])
    m3 = Matrix([[1,0,0,0],[0,0,-1,0],[0,1,0,612.7],[0,0,0,1]])
    m4 = Matrix([[cos(th_3),0,-sin(th_3),0],[sin(th_3),0,cos(th_3),0],[0,-1,0,176],[0,0,0,1]])
    m5 = Matrix([[1,0,0,0],[0,0,1,0],[0,-1,0,571.6],[0,0,0,1]])
    m6 = Matrix([[cos(th_4),0,sin(th_4),0],[sin(th_4),0,-cos(th_4),0],[0,1,0,163.9],[0,0,0,1]])
    m7 = Matrix([[cos(th_5),0,-sin(th_5),0],[sin(th_5),0,cos(th_5),0],[0,-1,0,115.7],[0,0,0,1]])
    m8 = Matrix([[cos(th_6),-sin(th_6),0,0],[sin(th_6),cos(th_6),0,0],[0,0,1,92.2],[0,0,0,1]])

    H = (m1*m2*m3*m4*m5*m6*m7*m8)          ## Final transformation matrix

    ## For printing configuration number

    if i==0:
        print("Home Configuration - (0,0,0,0,0,0)"'\n''\n'"Transformation matrix:")
    if i==1:
        print("Configuration 1 - (90,0,0,0,0,0)"'\n''\n'"Transformation matrix:")
    if i==2:
        print("Configuration 2 - (180,0,0,0,0,0)"'\n''\n'"Transformation matrix:")
    if i==3:
        print("Configuration 3 - (270,0,0,0,0,0)"'\n''\n'"Transformation matrix:")
    if i==4:
        print("Configuration 4 - (0,0,0,0,90,0)"'\n''\n'"Transformation matrix:")
    if i==5:
        print("Configuration 5 - (0,0,0,0,180,0)"'\n''\n'"Transformation matrix:")


    print('\n')
    pprint(H)                ## Printing the final parametric transformation matrix
    print('\n''\n')

    i=i+1

print("Note - Please consider the values which are very small (e-5 and less) as 0")
```