ENPM 662 – Homework 5 Report

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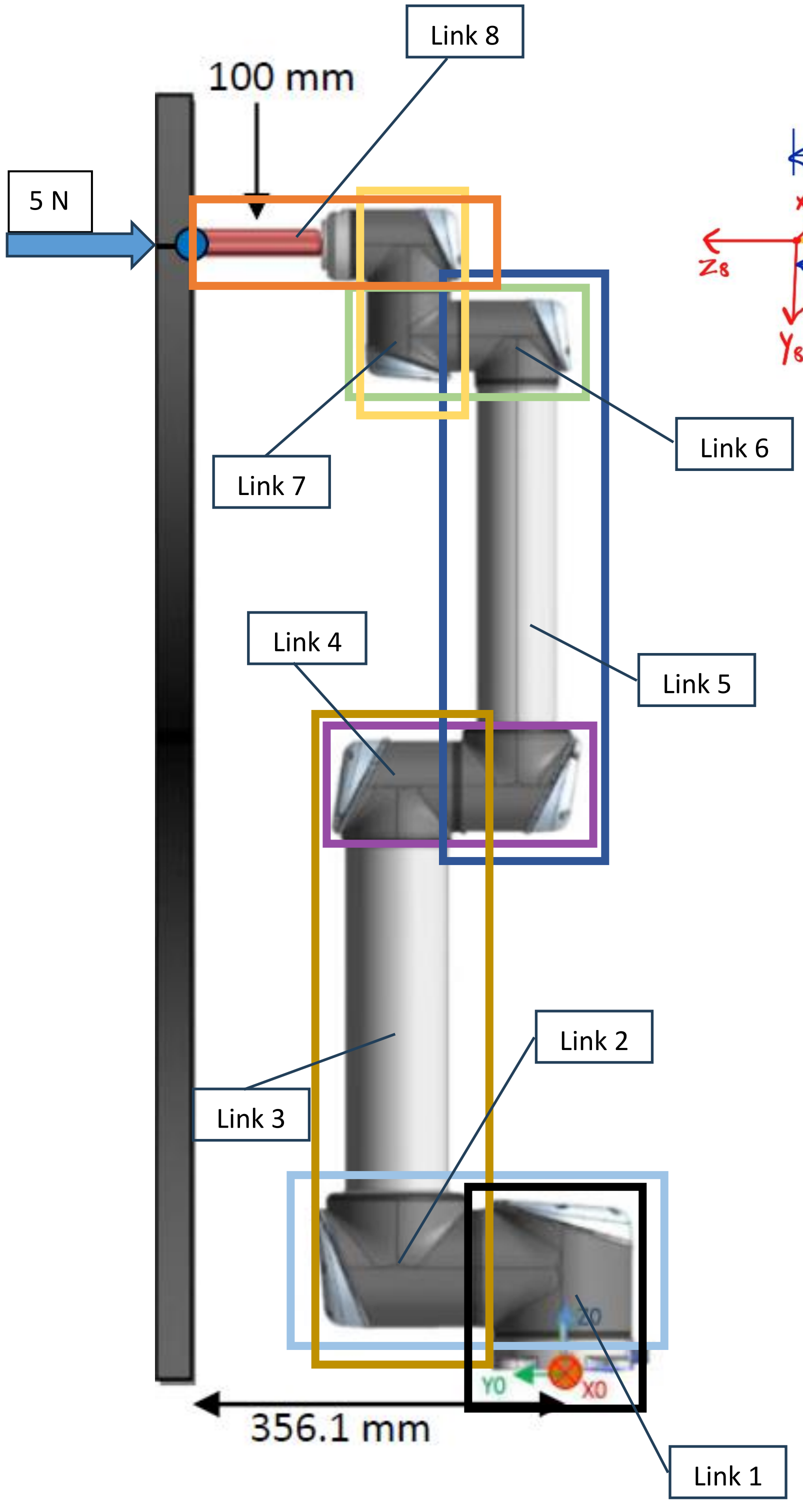
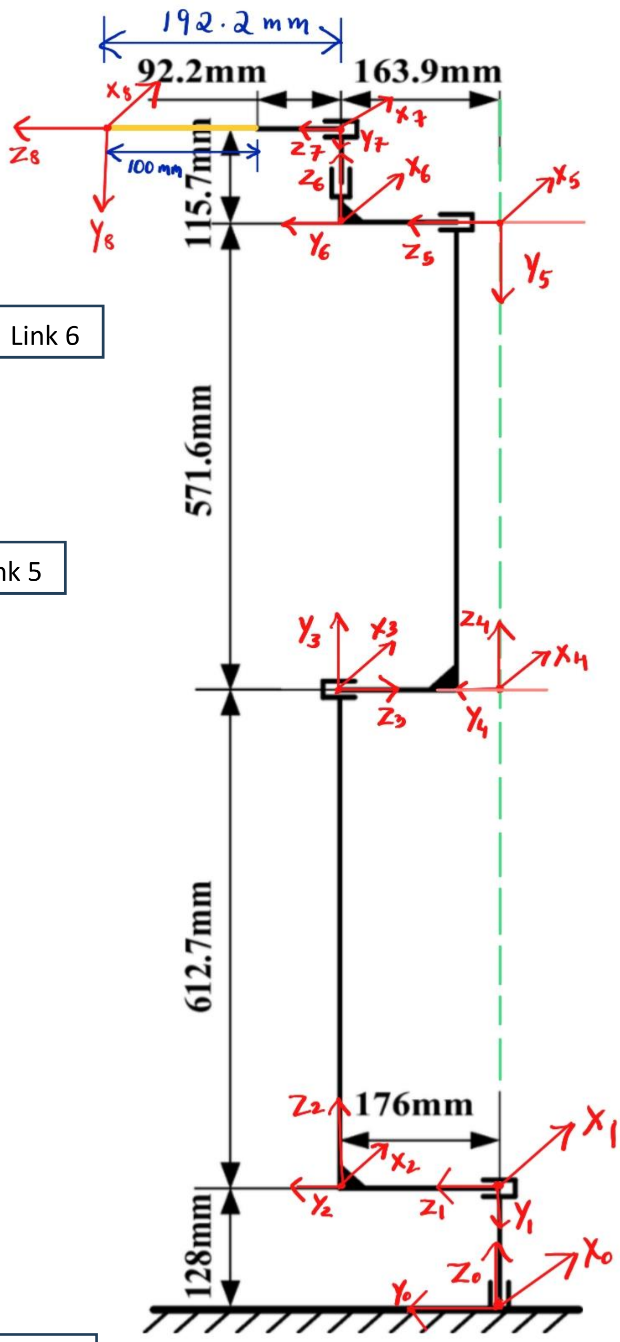




In the figures given above,

- Link 1 is attached to Revolute Joint 1 (θ_1).
- Link 1 and Link 2 are interconnected by Revolute Joint 2 (θ_2).
- Link 2 and Link 3 are interconnected by a Fixed Joint.
- Link 3 and Link 4 are interconnected by Revolute Joint 3 (θ_3).
- Link 4 and Link 5 are interconnected by a Fixed Joint.
- Link 5 and Link 6 are interconnected by Revolute Joint 4 (θ_4).
- Link 6 and Link 7 are interconnected by Revolute Joint 5 (θ_5).
- Link 7 and Link 8 are interconnected by Revolute Joint 6 (θ_6).
- Link 8 includes the pen, which is rigidly mounted to the end effector.
- The frames are attached as per Spong convention.
- The initial frame is given where x axis in into the plane; the same convention is followed for assigning the rest of the frames.
- The final frame is attached at the pen tip.

DH Table:

Frames	Link	a (in mm)	α (in degree)	θ (in degree)	d (in m)
Frame 0 – Frame 1	1	0	-90^0	θ_1	0.128
Frame 1 – Frame 2	2	0	90^0	θ_2	0.176
Frame 2 – Frame 3	3	0	90^0	0	0.6127
Frame 3 – Frame 4	4	0	-90^0	θ_3	0.176
Frame 4 – Frame 5	5	0	-90^0	0	0.5716
Frame 5 – Frame 6	6	0	90^0	θ_4	0.1639
Frame 6 – Frame 7	7	0	-90^0	θ_5	0.1157
Frame 7 – Frame 8	8	0	0^0	θ_6	0.1 + 0.0922

Link Transformation Matrices:

The general form of Link Transformation Matrix is given by:

$$T_i = \begin{bmatrix} \cos\theta_i & -(\sin\theta_i * \cos\alpha_i) & (\sin\theta_i * \ \sin\alpha_i) & (a_i * \cos\theta_i) \\ \sin\theta_i & (\cos\theta_i * \cos\alpha_i) & -(\cos\theta_i * \sin\alpha_i) & (a_i * \sin\theta_i) \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where “i” is the link number.

Therefore, referring to the DH table, the link transformation matrices for the links are given by:

i. For Link 1 (Frame 0 – Frame 1)

$${}^{0}T_1 = \begin{bmatrix} \cos\theta_1 & -(\sin\theta_1 * \cos -90) & (\sin\theta_1 * \ \sin -90) & (0 * \cos\theta_1) \\ \sin\theta_1 & (\cos\theta_1 * \cos -90) & -(\cos\theta_1 * \sin -90) & (0 * \sin\theta_1) \\ 0 & \sin -90 & \cos -90 & 0.128 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{0}T_1 = \begin{bmatrix} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & -1 & 0 & 0.128 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

ii. For Link 2 (Frame 1 – Frame 2)

$${}^{1}T_2 = \begin{bmatrix} \cos\theta_2 & -(\sin\theta_2 * \cos 90) & (\sin\theta_2 * \ \sin 90) & (0 * \cos\theta_2) \\ \sin\theta_2 & (\cos\theta_2 * \cos 90) & -(\cos\theta_2 * \sin 90) & (0 * \sin\theta_2) \\ 0 & \sin 90 & \cos 90 & 0.176 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{1}T_2 = \begin{bmatrix} \cos\theta_2 & 0 & \sin\theta_2 & 0 \\ \sin\theta_2 & 0 & -\cos\theta_2 & 0 \\ 0 & 1 & 0 & 0.176 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

iii. For Link 3 (Frame 2 – Frame 3)

$$^{2}T_{3} = \begin{bmatrix} \cos 0 & -(\sin 0 * \cos 90) & (\sin 0 * \ \sin 90) & (0 * \cos 0) \\ \sin 0 & (\cos 0 * \cos 90) & -(\cos 0 * \sin 90) & (0 * \sin 0) \\ 0 & \sin 90 & \cos 90 & 0.6127 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{2}T_{3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0.6127 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

iv. For Link 4 (Frame 3 – Frame 4)

$$^{3}T_{4} = \begin{bmatrix} \cos \theta_3 & -(\sin \theta_3 * \cos -90) & (\sin \theta_3 * \ \sin -90) & (0 * \cos \theta_3) \\ \sin \theta_3 & (\cos \theta_3 * \cos -90) & -(\cos \theta_3 * \sin -90) & (0 * \sin \theta_3) \\ 0 & \sin -90 & \cos -90 & 0.176 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{3}T_{4} = \begin{bmatrix} \cos \theta_3 & 0 & -\sin \theta_3 & 0 \\ \sin \theta_3 & 0 & \cos \theta_3 & 0 \\ 0 & -1 & 0 & 0.176 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

v. For Link 5 (Frame 4 – Frame 5)

$$^{4}T_{5} = \begin{bmatrix} \cos 0 & -(\sin 0 * \cos -90) & (\sin 0 * \ \sin -90) & (0 * \cos 0) \\ \sin 0 & (\cos 0 * \cos -90) & -(\cos 0 * \sin -90) & (0 * \sin 0) \\ 0 & \sin -90 & \cos -90 & 0.5716 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{4}T_{5} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0.5716 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

vi. For Link 6 (Frame 5 – Frame 6)

$$^{5}T_{6} = \begin{bmatrix} \cos\theta_4 & -(\sin\theta_4 * \cos 90) & (\sin\theta_4 * \ \sin 90) & (0 * \cos\theta_4) \\ \sin\theta_4 & (\cos\theta_4 * \cos 90) & -(\cos\theta_4 * \sin 90) & (0 * \sin\theta_4) \\ 0 & \sin 90 & \cos 90 & 0.1639 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{5}T_{6} = \begin{bmatrix} \cos\theta_4 & 0 & \sin\theta_4 & 0 \\ \sin\theta_4 & 0 & -\cos\theta_4 & 0 \\ 0 & 1 & 0 & 0.1639 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

vii. For Link 7 (Frame 6 – Frame 7)

$$^{6}T_{7} = \begin{bmatrix} \cos\theta_5 & -(\sin\theta_5 * \cos -90) & (\sin\theta_5 * \ \sin -90) & (0 * \cos\theta_5) \\ \sin\theta_5 & (\cos\theta_5 * \cos -90) & -(\cos\theta_5 * \sin -90) & (0 * \sin\theta_5) \\ 0 & \sin -90 & \cos -90 & 0.1157 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{6}T_{7} = \begin{bmatrix} \cos\theta_5 & 0 & -\sin\theta_5 & 0 \\ \sin\theta_5 & 0 & \cos\theta_5 & 0 \\ 0 & -1 & 0 & 0.1157 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

viii. For Link 8 (Frame 7 – Frame 8)

$$^{7}T_{8} = \begin{bmatrix} \cos\theta_6 & -(\sin\theta_6 * \cos 0) & (\sin\theta_6 * \ \sin 0) & (0 * \cos\theta_6) \\ \sin\theta_6 & (\cos\theta_6 * \cos 0) & -(\cos\theta_6 * \sin 0) & (0 * \sin\theta_6) \\ 0 & \sin 0 & \cos 0 & 0.1922 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{7}T_{8} = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0.1922 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The Final Transformation Matrix is given by post multiplying the matrices like:

$^{0}T_{8} = {}^{0}T_{1} * {}^{1}T_{2} * {}^{2}T_{3} * {}^{3}T_{4} * {}^{4}T_{5} * {}^{5}T_{6} * {}^{6}T_{7} * {}^{7}T_{8}$

Constructing the Jacobian Matrix

Method followed – Second Method

Steps:

1. Calculating the matrix T_i

$T_1 = T_1^0 = T_1^0$

$T_2 = T_2^0 = T_1^0 * T_2^1$

$T_3 = T_4^0 = T_1^0 * T_2^1 * T_3^2 * T_4^3$

$T_4 = T_6^0 = T_1^0 * T_2^1 * T_3^2 * T_4^3 * T_5^4 * T_6^5$

$T_5 = T_7^0 = T_1^0 * T_2^1 * T_3^2 * T_4^3 * T_5^4 * T_6^5 * T_7^6$

$T_6 = T_8^0 = T_1^0 * T_2^1 * T_3^2 * T_4^3 * T_5^4 * T_6^5 * T_7^6 * T_8^7$

2. Calculating Z_i (First 3 row values of the 3rd column of the T matrix)

Python Indexing Included

$$Z_1 = T_1[0:3,2] = \begin{bmatrix} -\sin(\theta_1) \\ \cos(\theta_1) \\ 0 \end{bmatrix}$$

$$Z_2 = T_2[0:3,2] = \begin{bmatrix} \sin(\theta_2)\cos(\theta_1) \\ \sin(\theta_1)\sin(\theta_2) \\ \cos(\theta_2) \end{bmatrix}$$

$$Z_3 = T_3[0:3,2] = \begin{bmatrix} \sin(\theta_2)\cos(\theta_1)\cos(\theta_3) - \sin(\theta_3)\cos(\theta_1)\cos(\theta_2) \\ \sin(\theta_1)\sin(\theta_2)\cos(\theta_3) - \sin(\theta_1)\sin(\theta_3)\cos(\theta_2) \\ \sin(\theta_2)\sin(\theta_3) + \cos(\theta_2)\cos(\theta_3) \end{bmatrix}$$

$$Z_4 = T_4[0:3,2] = \begin{bmatrix} (\sin(\theta_2)\sin(\theta_3)\cos(\theta_1) + \cos(\theta_1)\cos(\theta_2)\cos(\theta_3))\sin(\theta_4) - (-\sin(\theta_2)\cos(\theta_1)\cos(\theta_3) + \sin(\theta_3)\cos(\theta_1)\cos(\theta_2))\cos(\theta_4) \\ (\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + \sin(\theta_1)\cos(\theta_2)\cos(\theta_3))\sin(\theta_4) - (-\sin(\theta_1)\sin(\theta_2)\cos(\theta_3) + \sin(\theta_1)\sin(\theta_3)\cos(\theta_2))\cos(\theta_4) \\ -(-\sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3))\cos(\theta_4) + (-\sin(\theta_2)\cos(\theta_3) + \sin(\theta_3)\cos(\theta_2))\sin(\theta_4) \end{bmatrix}$$

$$Z_5 = T_5[0:3,2] = \begin{bmatrix} -((\sin(\theta_2)\sin(\theta_3)\cos(\theta_1) + \cos(\theta_1)\cos(\theta_2)\cos(\theta_3))\cos(\theta_4) + (-\sin(\theta_2)\cos(\theta_1)\cos(\theta_3) + \sin(\theta_3)\cos(\theta_1)\cos(\theta_2))\sin(\theta_4))\sin(\theta_5) - \sin(\theta_1)\cos(\theta_5) \\ -((\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + \sin(\theta_1)\cos(\theta_2)\cos(\theta_3))\cos(\theta_4) + (-\sin(\theta_1)\sin(\theta_2)\cos(\theta_3) + \sin(\theta_1)\sin(\theta_3)\cos(\theta_2))\sin(\theta_4))\sin(\theta_5) + \cos(\theta_1)\cos(\theta_5) \\ -((-\sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3))\sin(\theta_4) + (-\sin(\theta_2)\cos(\theta_3) + \sin(\theta_3)\cos(\theta_2))\cos(\theta_4))\sin(\theta_5) \end{bmatrix}$$

$$Z_6 = T_6[0{:}3,2] = \begin{bmatrix} -((\sin(\theta_2)\sin(\theta_3)\cos(\theta_1) + \cos(\theta_1)\cos(\theta_2)\cos(\theta_3))\cos(\theta_4) + (-\sin(\theta_2)\cos(\theta_1)\cos(\theta_3) + \sin(\theta_3)\cos(\theta_1)\cos(\theta_2))\sin(\theta_4))\sin(\theta_5) - \sin(\theta_1)\cos(\theta_5) \\ -((\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + \sin(\theta_1)\cos(\theta_2)\cos(\theta_3))\cos(\theta_4) + (-\sin(\theta_1)\sin(\theta_2)\cos(\theta_3) + \sin(\theta_1)\sin(\theta_3)\cos(\theta_2))\sin(\theta_4))\sin(\theta_5) + \cos(\theta_1)\cos(\theta_5) \\ -((-\sin(\theta_2)\sin(\theta_3) - \cos(\theta_2)\cos(\theta_3))\sin(\theta_4) + (-\sin(\theta_2)\cos(\theta_3) + \sin(\theta_3)\cos(\theta_2))\cos(\theta_4))\sin(\theta_5) \end{bmatrix}$$

3. Calculating h(q_1, q_2, ….. q_n)

h(q_1, q_2, ….. q_n) = P_8^0 = T_6 [0:3,3] =

$$\begin{bmatrix} -0.1922\sin(\theta_1)\cos(\theta_5) - 0.1639\sin(\theta_1) + 0.1157\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\cos(\theta_1) - 0.1922\sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_1)\cos(\theta_4) + 0.1922\sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_3) + 0.1157\sin(\theta_2)\cos(\theta_1)\cos(\theta_3)\cos(\theta_4) + 0.5716\sin(\theta_2)\cos(\theta_1)\cos(\theta_3) + 0.6127\sin(\theta_2)\cos(\theta_1) - 0.1922\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2) - 0.1157\sin(\theta_3)\cos(\theta_1)\cos(\theta_2)\cos(\theta_4) - 0.5716\sin(\theta_3)\cos(\theta_1)\cos(\theta_2) + 0.1157\sin(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) - 0.1922\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) \\ 0.1157\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4) - 0.1922\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_4) + 0.1922\sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_3) + 0.1157\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)\cos(\theta_4) + 0.5716\sin(\theta_1)\sin(\theta_2)\cos(\theta_3) + 0.6127\sin(\theta_1)\sin(\theta_2) - 0.1922\sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2) - 0.1157\sin(\theta_1)\sin(\theta_3)\cos(\theta_2)\cos(\theta_4) - 0.5716\sin(\theta_1)\sin(\theta_3)\cos(\theta_2) + 0.1157\sin(\theta_1)\sin(\theta_4)\cos(\theta_2)\cos(\theta_3) - 0.1922\sin(\theta_1)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) + 0.1922\cos(\theta_1)\cos(\theta_5) + 0.1639\cos(\theta_1) \\ 0.1922\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5) + 0.1157\sin(\theta_2)\sin(\theta_3)\cos(\theta_4) + 0.5716\sin(\theta_2)\sin(\theta_3) - 0.1157\sin(\theta_2)\sin(\theta_4)\cos(\theta_3) + 0.1922\sin(\theta_2)\sin(\theta_5)\cos(\theta_3)\cos(\theta_4) + 0.1157\sin(\theta_3)\sin(\theta_4)\cos(\theta_2) - 0.1922\sin(\theta_3)\sin(\theta_5)\cos(\theta_2)\cos(\theta_4) + 0.1922\sin(\theta_4)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3) + 0.1157\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) + 0.5716\cos(\theta_2)\cos(\theta_3) + 0.6127\cos(\theta_2) + 0.128 \end{bmatrix}$$

4. Calculating $\partial \mathbf{h}/\partial \mathbf{q}_i$

i. $\partial h / \partial q_1 = \partial h / \partial \theta_1$ =

$$\begin{bmatrix} -0.1157\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4) + 0.1922\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_4) - 0.1922\sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_3) - 0.1157\sin(\theta_1)\sin(\theta_2)\cos(\theta_3)\cos(\theta_4) - 0.5716\sin(\theta_1)\sin(\theta_2)\cos(\theta_3) - 0.6127\sin(\theta_1)\sin(\theta_2) + 0.1922\sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2) + 0.1157\sin(\theta_1)\sin(\theta_3)\cos(\theta_2)\cos(\theta_4) + 0.5716\sin(\theta_1)\sin(\theta_3)\cos(\theta_2) - 0.1157\sin(\theta_1)\sin(\theta_4)\cos(\theta_2)\cos(\theta_3) + 0.1922\sin(\theta_1)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) - 0.1922\cos(\theta_1)\cos(\theta_5) - 0.1639\cos(\theta_1) \\ -0.1922\sin(\theta_1)\cos(\theta_5) - 0.1639\sin(\theta_1) + 0.1157\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\cos(\theta_1) - 0.1922\sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_1)\cos(\theta_4) + 0.1922\sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_3) + 0.1157\sin(\theta_2)\cos(\theta_1)\cos(\theta_3)\cos(\theta_4) + 0.5716\sin(\theta_2)\cos(\theta_1)\cos(\theta_3) + 0.6127\sin(\theta_2)\cos(\theta_1) - 0.1922\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2) - 0.1157\sin(\theta_3)\cos(\theta_1)\cos(\theta_2)\cos(\theta_4) - 0.5716\sin(\theta_3)\cos(\theta_1)\cos(\theta_2) + 0.1157\sin(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) - 0.1922\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) \\ 0 \end{bmatrix}$$

ii. $\partial h / \partial q_2 = \partial h / \partial \theta_2$ =

$$\begin{bmatrix} 0.1922\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1) + 0.1157\sin(\theta_2)\sin(\theta_3)\cos(\theta_1)\cos(\theta_4) + 0.5716\sin(\theta_2)\sin(\theta_3)\cos(\theta_1) - 0.1157\sin(\theta_2)\sin(\theta_4)\cos(\theta_1)\cos(\theta_3) + 0.1922\sin(\theta_2)\sin(\theta_5)\cos(\theta_1)\cos(\theta_3)\cos(\theta_4) + 0.1157\sin(\theta_3)\sin(\theta_4)\cos(\theta_1)\cos(\theta_2) - 0.1922\sin(\theta_3)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_4) + 0.1922\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) + 0.1157\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) + 0.5716\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) + 0.6127\cos(\theta_1)\cos(\theta_2) \\ 0.1922\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5) + 0.1157\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4) + 0.5716\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) - 0.1157\sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\cos(\theta_3) + 0.1922\sin(\theta_1)\sin(\theta_2)\sin(\theta_5)\cos(\theta_3)\cos(\theta_4) + 0.1157\sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\cos(\theta_2) - 0.1922\sin(\theta_1)\sin(\theta_3)\sin(\theta_5)\cos(\theta_2)\cos(\theta_4) + 0.1922\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3) + 0.1157\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) + 0.5716\sin(\theta_1)\cos(\theta_2)\cos(\theta_3) + 0.6127\sin(\theta_1)\cos(\theta_2) \\ -0.1157\sin(\theta_2)\sin(\theta_3)\sin(\theta_4) + 0.1922\sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_4) - 0.1922\sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_3) - 0.1157\sin(\theta_2)\cos(\theta_3)\cos(\theta_4) - 0.5716\sin(\theta_2)\cos(\theta_3) - 0.6127\sin(\theta_2) + 0.1922\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2) + 0.1157\sin(\theta_3)\cos(\theta_2)\cos(\theta_4) + 0.5716\sin(\theta_3)\cos(\theta_2) - 0.1157\sin(\theta_4)\cos(\theta_2)\cos(\theta_3) + 0.1922\sin(\theta_5)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) \end{bmatrix}$$

iii. $\partial h / \partial q_3 = \partial h / \partial \theta_3 =$

$$\begin{bmatrix} -0.1922\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1) - 0.1157\sin(\theta_2)\sin(\theta_3)\cos(\theta_1)\cos(\theta_4) - 0.5716\sin(\theta_2)\sin(\theta_3)\cos(\theta_1) + 0.1157\sin(\theta_2)\sin(\theta_4)\cos(\theta_1)\cos(\theta_3) - 0.1922\sin(\theta_2)\sin(\theta_5)\cos(\theta_1)\cos(\theta_3)\cos(\theta_4) - 0.1157\sin(\theta_3)\sin(\theta_4)\cos(\theta_1)\cos(\theta_2) + 0.1922\sin(\theta_3)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_4) - 0.1922\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) - 0.1157\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) - 0.5716\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) \\ -0.1922\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5) - 0.1157\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4) - 0.5716\sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + 0.1157\sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\cos(\theta_3) - 0.1922\sin(\theta_1)\sin(\theta_2)\sin(\theta_5)\cos(\theta_3)\cos(\theta_4) - 0.1157\sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\cos(\theta_2) + 0.1922\sin(\theta_1)\sin(\theta_3)\sin(\theta_5)\cos(\theta_2)\cos(\theta_4) - 0.1922\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3) - 0.1157\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) - 0.5716\sin(\theta_1)\cos(\theta_2)\cos(\theta_3) \\ 0.1157\sin(\theta_2)\sin(\theta_3)\sin(\theta_4) - 0.1922\sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_4) + 0.1922\sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_3) + 0.1157\sin(\theta_2)\cos(\theta_3)\cos(\theta_4) + 0.5716\sin(\theta_2)\cos(\theta_3) - 0.1922\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2) - 0.1157\sin(\theta_3)\cos(\theta_2)\cos(\theta_4) - 0.5716\sin(\theta_3)\cos(\theta_2) + 0.1157\sin(\theta_4)\cos(\theta_2)\cos(\theta_3) - 0.1922\sin(\theta_5)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) \end{bmatrix}$$

iv. $\partial h / \partial q_4 = \partial h / \partial \theta_4 =$

$$\begin{bmatrix} 0.1922\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1) + 0.1157\sin(\theta_2)\sin(\theta_3)\cos(\theta_1)\cos(\theta_4) - 0.1157\sin(\theta_2)\sin(\theta_4)\cos(\theta_1)\cos(\theta_3) + 0.1922\sin(\theta_2)\sin(\theta_5)\cos(\theta_1)\cos(\theta_3)\cos(\theta_4) + 0.1157\sin(\theta_3)\sin(\theta_4)\cos(\theta_1)\cos(\theta_2) - 0.1922\sin(\theta_3)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_4) + 0.1922\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) + 0.1157\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) \\ 0.1922\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5) + 0.1157\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4) - 0.1157\sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\cos(\theta_3) + 0.1922\sin(\theta_1)\sin(\theta_2)\sin(\theta_5)\cos(\theta_3)\cos(\theta_4) + 0.1157\sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\cos(\theta_2) - 0.1922\sin(\theta_1)\sin(\theta_3)\sin(\theta_5)\cos(\theta_2)\cos(\theta_4) + 0.1922\sin(\theta_1)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3) + 0.1157\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) \\ -0.1157\sin(\theta_2)\sin(\theta_3)\sin(\theta_4) + 0.1922\sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_4) - 0.1922\sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_3) - 0.1157\sin(\theta_2)\cos(\theta_3)\cos(\theta_4) + 0.1922\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2) + 0.1157\sin(\theta_3)\cos(\theta_2)\cos(\theta_4) - 0.1157\sin(\theta_4)\cos(\theta_2)\cos(\theta_3) + 0.1922\sin(\theta_5)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) \end{bmatrix}$$

v. $\partial h / \partial q_5 = \partial h / \partial \theta_5 =$

$$\begin{bmatrix} 0.1922\sin(\theta_1)\sin(\theta_5) - 0.1922\sin(\theta_2)\sin(\theta_3)\cos(\theta_1)\cos(\theta_4)\cos(\theta_5) + 0.1922\sin(\theta_2)\sin(\theta_4)\cos(\theta_1)\cos(\theta_3)\cos(\theta_5) - 0.1922\sin(\theta_3)\sin(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_5) - 0.1922\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5) \\ -0.1922\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4)\cos(\theta_5) + 0.1922\sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\cos(\theta_3)\cos(\theta_5) - 0.1922\sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\cos(\theta_2)\cos(\theta_5) - 0.1922\sin(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5) - 0.1922\sin(\theta_5)\cos(\theta_1) \\ 0.1922\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\cos(\theta_5) + 0.1922\sin(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5) - 0.1922\sin(\theta_3)\cos(\theta_2)\cos(\theta_4)\cos(\theta_5) + 0.1922\sin(\theta_4)\cos(\theta_2)\cos(\theta_3)\cos(\theta_5) \end{bmatrix}$$

vi. $\partial h / \partial q_6 = \partial h / \partial \theta_6 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

5. Writing the Jacobian Matrix

$$J_8^0 = J = \begin{bmatrix} \partial h / \partial \theta_1 & \partial h / \partial \theta_2 & \partial h / \partial \theta_3 & \partial h / \partial \theta_4 & \partial h / \partial \theta_5 & \partial h / \partial \theta_6 \\ Z_1 & Z_2 & Z_3 & Z_4 & Z_5 & Z_6 \end{bmatrix} =$$

$$-0.1157 \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4) + 0.1922 \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_4) - 0.1922 \sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_3) - 0.1157 \sin(\theta_1)\sin(\theta_2)\cos(\theta_3)\cos(\theta_4) - 0.5716 \sin(\theta_1)\sin(\theta_2)\cos(\theta_3) - 0.6127 \sin(\theta_1)\sin(\theta_2) + 0.1922 \sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2) + 0.1157 \sin(\theta_1)\sin(\theta_3)\cos(\theta_2)\cos(\theta_4) + 0.5716 \sin(\theta_1)\sin(\theta_3)\cos(\theta_2) - 0.1157 \sin(\theta_1)\sin(\theta_4)\cos(\theta_2)\cos(\theta_3) + 0.1922 \sin(\theta_1)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) - 0.1922 \cos(\theta_1)\cos(\theta_5) - 0.1639 \cos(\theta_1)$$

$$-0.1922 \sin(\theta_1)\cos(\theta_5) - 0.1639 \sin(\theta_1) + 0.1157 \sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\cos(\theta_1) - 0.1922 \sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_1)\cos(\theta_4) + 0.1922 \sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_3) + 0.1157 \sin(\theta_2)\cos(\theta_1)\cos(\theta_3)\cos(\theta_4) + 0.5716 \sin(\theta_2)\cos(\theta_1)\cos(\theta_3) + 0.6127 \sin(\theta_2)\cos(\theta_1) - 0.1922 \sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2) - 0.1157 \sin(\theta_3)\cos(\theta_1)\cos(\theta_2)\cos(\theta_4) - 0.5716 \sin(\theta_3)\cos(\theta_1)\cos(\theta_2) + 0.1157 \sin(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) - 0.1922 \sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)$$

$$0$$

$$-\sin(\theta_1)$$

$$\cos(\theta_1)$$

$$0$$

Column 1

$$0.1922 \sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1) + 0.1157 \sin(\theta_2)\sin(\theta_3)\cos(\theta_1)\cos(\theta_4) + 0.5716 \sin(\theta_2)\sin(\theta_3)\cos(\theta_1) - 0.1157 \sin(\theta_2)\sin(\theta_4)\cos(\theta_1)\cos(\theta_3) + 0.1922 \sin(\theta_2)\sin(\theta_5)\cos(\theta_1)\cos(\theta_3)\cos(\theta_4) + 0.1157 \sin(\theta_3)\sin(\theta_4)\cos(\theta_1)\cos(\theta_2) - 0.1922 \sin(\theta_3)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_4) + 0.1922 \sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) + 0.1157 \cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) + 0.5716 \cos(\theta_1)\cos(\theta_2)\cos(\theta_3) + 0.6127 \cos(\theta_1)\cos(\theta_2)$$

$$0.1922 \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5) + 0.1157 \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4) + 0.5716 \sin(\theta_1)\sin(\theta_2)\sin(\theta_3) - 0.1157 \sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\cos(\theta_3) + 0.1922 \sin(\theta_1)\sin(\theta_2)\sin(\theta_5)\cos(\theta_3)\cos(\theta_4) + 0.1157 \sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\cos(\theta_2) - 0.1922 \sin(\theta_1)\sin(\theta_3)\sin(\theta_5)\cos(\theta_2)\cos(\theta_4) + 0.1922 \sin(\theta_1)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3) + 0.1157 \sin(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) + 0.5716 \sin(\theta_1)\cos(\theta_2)\cos(\theta_3) + 0.6127 \sin(\theta_1)\cos(\theta_2)$$

$$-0.1157 \sin(\theta_2)\sin(\theta_3)\sin(\theta_4) + 0.1922 \sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_4) - 0.1922 \sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_3) - 0.1157 \sin(\theta_2)\cos(\theta_3)\cos(\theta_4) - 0.5716 \sin(\theta_2)\cos(\theta_3) - 0.6127 \sin(\theta_2) + 0.1922 \sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2) + 0.1157 \sin(\theta_3)\cos(\theta_2)\cos(\theta_4) + 0.5716 \sin(\theta_3)\cos(\theta_2) - 0.1157 \sin(\theta_4)\cos(\theta_2)\cos(\theta_3) + 0.1922 \sin(\theta_5)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)$$

$$\sin(\theta_2)\cos(\theta_1)$$

$$\sin(\theta_1)\sin(\theta_2)$$

$$\cos(\theta_2)$$

Column 2

$$-0.1922 \sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1) - 0.1157 \sin(\theta_2)\sin(\theta_3)\cos(\theta_1)\cos(\theta_4) - 0.5716 \sin(\theta_2)\sin(\theta_3)\cos(\theta_1) + 0.1157 \sin(\theta_2)\sin(\theta_4)\cos(\theta_1)\cos(\theta_3) - 0.1922 \sin(\theta_2)\sin(\theta_5)\cos(\theta_1)\cos(\theta_3)\cos(\theta_4) - 0.1157 \sin(\theta_3)\sin(\theta_4)\cos(\theta_1)\cos(\theta_2) + 0.1922 \sin(\theta_3)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_4) - 0.1922 \sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) - 0.1157 \cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) - 0.5716 \cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$$

$$-0.1922 \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5) - 0.1157 \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4) - 0.5716 \sin(\theta_1)\sin(\theta_2)\sin(\theta_3) + 0.1157 \sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\cos(\theta_3) - 0.1922 \sin(\theta_1)\sin(\theta_2)\sin(\theta_5)\cos(\theta_3)\cos(\theta_4) - 0.1157 \sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\cos(\theta_2) + 0.1922 \sin(\theta_1)\sin(\theta_3)\sin(\theta_5)\cos(\theta_2)\cos(\theta_4) - 0.1922 \sin(\theta_1)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3) - 0.1157 \sin(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) - 0.5716 \sin(\theta_1)\cos(\theta_2)\cos(\theta_3)$$

$$0.1157 \sin(\theta_2)\sin(\theta_3)\sin(\theta_4) - 0.1922 \sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_4) + 0.1922 \sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_3) + 0.1157 \sin(\theta_2)\cos(\theta_3)\cos(\theta_4) + 0.5716 \sin(\theta_2)\cos(\theta_3) - 0.1922 \sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2) - 0.1157 \sin(\theta_3)\cos(\theta_2)\cos(\theta_4) - 0.5716 \sin(\theta_3)\cos(\theta_2) + 0.1157 \sin(\theta_4)\cos(\theta_2)\cos(\theta_3) - 0.1922 \sin(\theta_5)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)$$

$$\sin(\theta_2)\cos(\theta_1)\cos(\theta_3) - \sin(\theta_3)\cos(\theta_1)\cos(\theta_2)$$

$$\sin(\theta_1)\sin(\theta_2)\cos(\theta_3) - \sin(\theta_1)\sin(\theta_3)\cos(\theta_2)$$

$$\sin(\theta_2)\sin(\theta_3) + \cos(\theta_2)\cos(\theta_3)$$

Column 3

$$0.1922 \sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1) + 0.1157 \sin(\theta_2)\sin(\theta_3)\cos(\theta_1)\cos(\theta_4) - 0.1157 \sin(\theta_2)\sin(\theta_4)\cos(\theta_1)\cos(\theta_3) + 0.1922 \sin(\theta_2)\sin(\theta_5)\cos(\theta_1)\cos(\theta_3)\cos(\theta_4) + 0.1157 \sin(\theta_3)\sin(\theta_4)\cos(\theta_1)\cos(\theta_2) - 0.1922 \sin(\theta_3)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_4) + 0.1922 \sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3) + 0.1157 \cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)$$

$$0.1922 \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5) + 0.1157 \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4) - 0.1157 \sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\cos(\theta_3) + 0.1922 \sin(\theta_1)\sin(\theta_2)\sin(\theta_5)\cos(\theta_3)\cos(\theta_4) + 0.1157 \sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\cos(\theta_2) - 0.1922 \sin(\theta_1)\sin(\theta_3)\sin(\theta_5)\cos(\theta_2)\cos(\theta_4) + 0.1922 \sin(\theta_1)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3) + 0.1157 \sin(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)$$

$$-0.1157 \sin(\theta_2)\sin(\theta_3)\sin(\theta_4) + 0.1922 \sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_4) - 0.1922 \sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_3) - 0.1157 \sin(\theta_2)\cos(\theta_3)\cos(\theta_4) + 0.1922 \sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2) + 0.1157 \sin(\theta_3)\cos(\theta_2)\cos(\theta_4) - 0.1157 \sin(\theta_4)\cos(\theta_2)\cos(\theta_3) + 0.1922 \sin(\theta_5)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)$$

$$\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\cos(\theta_1) + \sin(\theta_2)\cos(\theta_1)\cos(\theta_3)\cos(\theta_4) - \sin(\theta_3)\cos(\theta_1)\cos(\theta_2)\cos(\theta_4) + \sin(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)$$

$$\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_4) + \sin(\theta_1)\sin(\theta_2)\cos(\theta_3)\cos(\theta_4) - \sin(\theta_1)\sin(\theta_3)\cos(\theta_2)\cos(\theta_4) + \sin(\theta_1)\sin(\theta_4)\cos(\theta_2)\cos(\theta_3)$$

$$\sin(\theta_2)\sin(\theta_3)\cos(\theta_4) - \sin(\theta_2)\sin(\theta_4)\cos(\theta_3) + \sin(\theta_3)\sin(\theta_4)\cos(\theta_2) + \cos(\theta_2)\cos(\theta_3)\cos(\theta_4)$$

Column 4

$$0.1922 \sin(\theta_1)\sin(\theta_5) - 0.1922 \sin(\theta_2)\sin(\theta_3)\cos(\theta_1)\cos(\theta_4)\cos(\theta_5) + 0.1922 \sin(\theta_2)\sin(\theta_4)\cos(\theta_1)\cos(\theta_3)\cos(\theta_5) - 0.1922 \sin(\theta_3)\sin(\theta_4)\cos(\theta_1)\cos(\theta_2)\cos(\theta_5) - 0.1922 \cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5)$$

$$-0.1922 \sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\cos(\theta_4)\cos(\theta_5) + 0.1922 \sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\cos(\theta_3)\cos(\theta_5) - 0.1922 \sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\cos(\theta_2)\cos(\theta_5) - 0.1922 \sin(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5) - 0.1922 \sin(\theta_5)\cos(\theta_1)$$

$$0.1922 \sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\cos(\theta_5) + 0.1922 \sin(\theta_2)\cos(\theta_3)\cos(\theta_4)\cos(\theta_5) - 0.1922 \sin(\theta_3)\cos(\theta_2)\cos(\theta_4)\cos(\theta_5) + 0.1922 \sin(\theta_4)\cos(\theta_2)\cos(\theta_3)\cos(\theta_5)$$

$$-\sin(\theta_1)\cos(\theta_5) - \sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_1)\cos(\theta_4) + \sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_3) - \sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2) - \sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4)$$

$$-\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_4) + \sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_3) - \sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) + \cos(\theta_1)\cos(\theta_5)$$

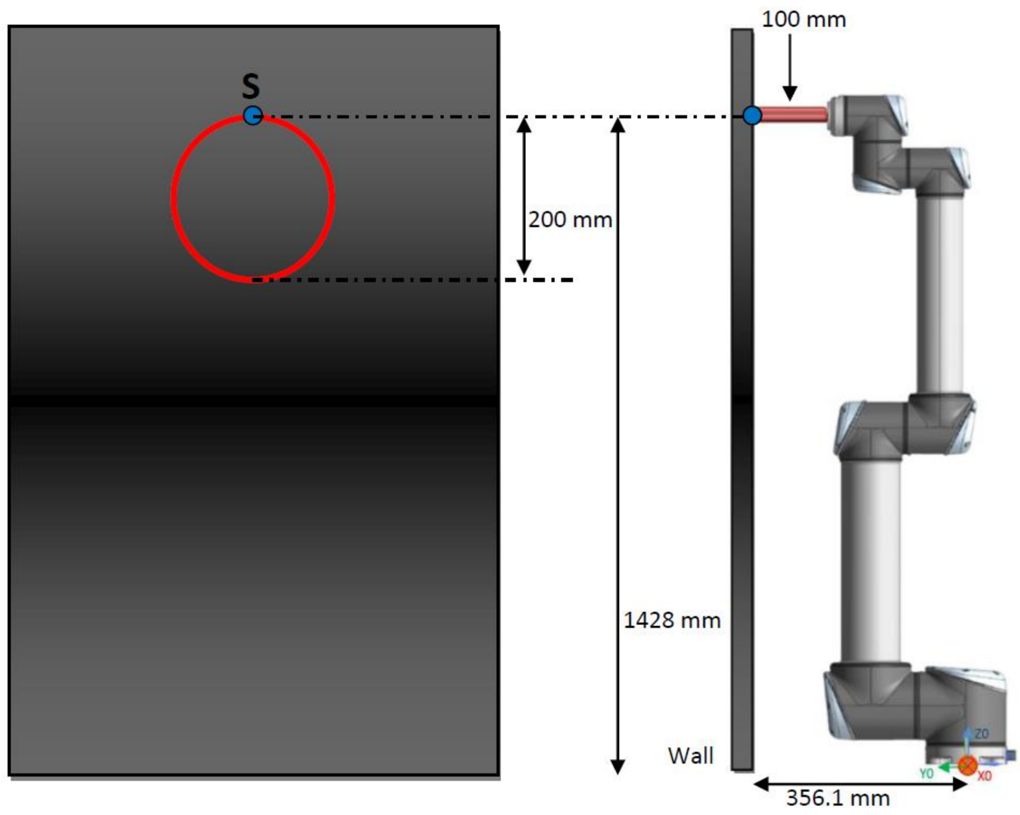
$$\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5) + \sin(\theta_2)\sin(\theta_5)\cos(\theta_3)\cos(\theta_4) - \sin(\theta_3)\sin(\theta_5)\cos(\theta_2)\cos(\theta_4) + \sin(\theta_4)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3)$$

Column 5

$$
\begin{array}{c}
0 \\
0 \\
0 \\
-\sin(\theta_1)\cos(\theta_5) - \sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_1)\cos(\theta_4) + \sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_3) - \sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_1)\cos(\theta_2) \\
- \sin(\theta_5)\cos(\theta_1)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) \\
-\sin(\theta_1)\sin(\theta_2)\sin(\theta_3)\sin(\theta_5)\cos(\theta_4) + \sin(\theta_1)\sin(\theta_2)\sin(\theta_4)\sin(\theta_5)\cos(\theta_3) - \sin(\theta_1)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5)\cos(\theta_2) - \sin(\theta_1)\sin \\
(\theta_5)\cos(\theta_2)\cos(\theta_3)\cos(\theta_4) + \cos(\theta_1)\cos(\theta_5) \\
\sin(\theta_2)\sin(\theta_3)\sin(\theta_4)\sin(\theta_5) + \sin(\theta_2)\sin(\theta_5)\cos(\theta_3)\cos(\theta_4) - \sin(\theta_3)\sin(\theta_5)\cos(\theta_2)\cos(\theta_4) + \sin(\theta_4)\sin(\theta_5)\cos(\theta_2)\cos(\theta_3)
\end{array}
$$

Column 6

Tracing the desired path (Updated)



Given:

- Circle Radius = 100mm = 0.1 m
- The robot is at home position when the tool (pen) is touching the wall. Hence,

  [q1 , q2 , q3 , q4 , q5 , q6] = [0 , 0 , 0 , 0 , 0 , 0] at (t = 0)

- Plotting starts from the top of the circle, from coordinates (0 , 0.3561 , 1.428)

Time considered for plotting the circle: T = 200 seconds

Hence the angular velocity is given by: $\omega = \frac{2*\pi}{T} = \frac{2*\pi}{200} = \frac{\pi}{100}$ rad/sec

The parametric equation of the circle is given by,

X = $r * \cos(\frac{\pi}{2} + \omega t)$

X = $100 * \cos(\frac{\pi}{2} + \frac{\pi}{100} * t)$ mm/sec

Z = A + ($r * \sin\left(\frac{\pi}{2} + \omega t\right)$) , where A = 100 mm (origin offset)

Z = 100 + ($100 * \sin(\frac{\pi}{2} + \frac{\pi}{100} * t)$)

Differentiate the parametric equation (wrt 't') to get velocity:

$$\dot{X} = \frac{d\,(100* \cos(\frac{\pi}{2}+\frac{\pi}{100}*t)}{dt} = \frac{-\,100 * \pi * \sin(\frac{\pi}{2}+\frac{\pi}{100}*t)}{100} = -\,\pi * \sin(\frac{\pi}{2} + \frac{\pi}{100} * t)$$

$$\dot{X} = -\pi * \cos\left(\frac{\pi}{100} * t\right) mm/s = -\frac{\pi}{1000} * \cos\left(\frac{\pi}{100} * t\right) \ m/s$$

$$\dot{Z} = \frac{d\,(100 + (\,100* \sin(\frac{\pi}{2}+\frac{\pi}{100}*t\,))}{dt} = \frac{100 * \pi * \cos(\frac{\pi}{2}+\frac{\pi}{100}*t)}{100} = \pi * \cos(\frac{\pi}{2} + \frac{\pi}{100} * t)$$

$$\dot{Z} = -\pi * \sin(\frac{\pi}{100} * t) \quad mm/s = -\frac{\pi}{1000} * \sin(\frac{\pi}{100} * t) \quad m/s$$

In our case, $\dot{Y}$ = 0 and the angular velocities (ω_x , ω_y , ω_z) = (0 , 0 , 0)

Therefore, the Velocity trajectory of the end-effector wrt base is:

$$V = \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} -\pi * \sin(\frac{\pi}{2} + \frac{\pi}{100} * t) \\ 0 \\ \pi * \cos(\frac{\pi}{2} + \frac{\pi}{100} * t) \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\pi * \cos(\frac{\pi}{100} * t) \\ 0 \\ -\pi * \sin(\frac{\pi}{100} * t) \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{\pi}{1000} * \cos(\frac{\pi}{100} * t) \\ 0 \\ -\frac{\pi}{1000} * \sin(\frac{\pi}{100} * t) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

In mm/s

In m/s

Now from t=0 to t=200, for a timestep of dt (dt is assumed as 0.5 seconds for this problem), perform the following calculations:

- Substitute current values of (q1 , q2 , q3 , q4 , q5 , q6) to the Jacobian equation J
- Substitute the current value of t to velocity trajectory equation V
- Use the Inverse Velocity Kinematics equation to calculate the Joint angular velocities

  $\dot{q} = J^{-1} * V$

  $$\begin{bmatrix} \dot{q1} \\ \dot{q2} \\ \dot{q3} \\ \dot{q4} \\ \dot{q5} \\ \dot{q6} \end{bmatrix} = J^{-1} * \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

- Use the joint angular velocities obtained from above equation to perform numeric integration and get the new joint angle values.

  i.e. $q_{new} = q_{current} + \dot{q} * dt$

- Plug in the new q values obtained after numeric integration into the forward position kinematics equation and extract the position of the end effector with respect to the base. This is done by extracting the last column of the final transformation matrix after substitution.
- Store the X and Z values and plot.

Computing Joint Torques

The robot dynamic equation is given by:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau + J^T(q)F$$

Here, as the system is assumed to be quasi-static, $\dot{q}$ *and* $\ddot{q}$ are assumed to be 0

Therefore, the dynamic equation changes to:

$$g(q) = \tau + J^T(q)F$$

Hence, the joint torques is given by:

$$\tau = g(q) - J^T(q)F$$

Where,

$g(q)$ is the gravity matrix

J^T is the transpose of the Jacobian

F is the force matrix

Computing gravity matrix $g(q)$:

$$g(q) = \frac{\partial P(q)}{\partial q}$$

$$\begin{bmatrix} g(q_1) \\ g(q_2) \\ g(q_3) \\ g(q_4) \\ g(q_5) \\ g(q_6) \end{bmatrix} = \begin{bmatrix} \frac{\partial P(q)}{\partial q_1} \\ \frac{\partial P(q)}{\partial q_2} \\ \frac{\partial P(q)}{\partial q_3} \\ \frac{\partial P(q)}{\partial q_4} \\ \frac{\partial P(q)}{\partial q_5} \\ \frac{\partial P(q)}{\partial q_6} \end{bmatrix}$$

Where P(q) is the total potential energy of the system

Computing Total Potential Energy P(q):

We know that Potential Energy,

PE = m*g*h

Where m is the mass of the body

g is the acceleration due to gravity

h is the height of the centre of mass of the body

In a serial robot, total potential energy is equal to the sum of the potential energy of its links.

Therefore, P = P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + P_8

(as I have considered 8 links: Refer the figure on page 1)

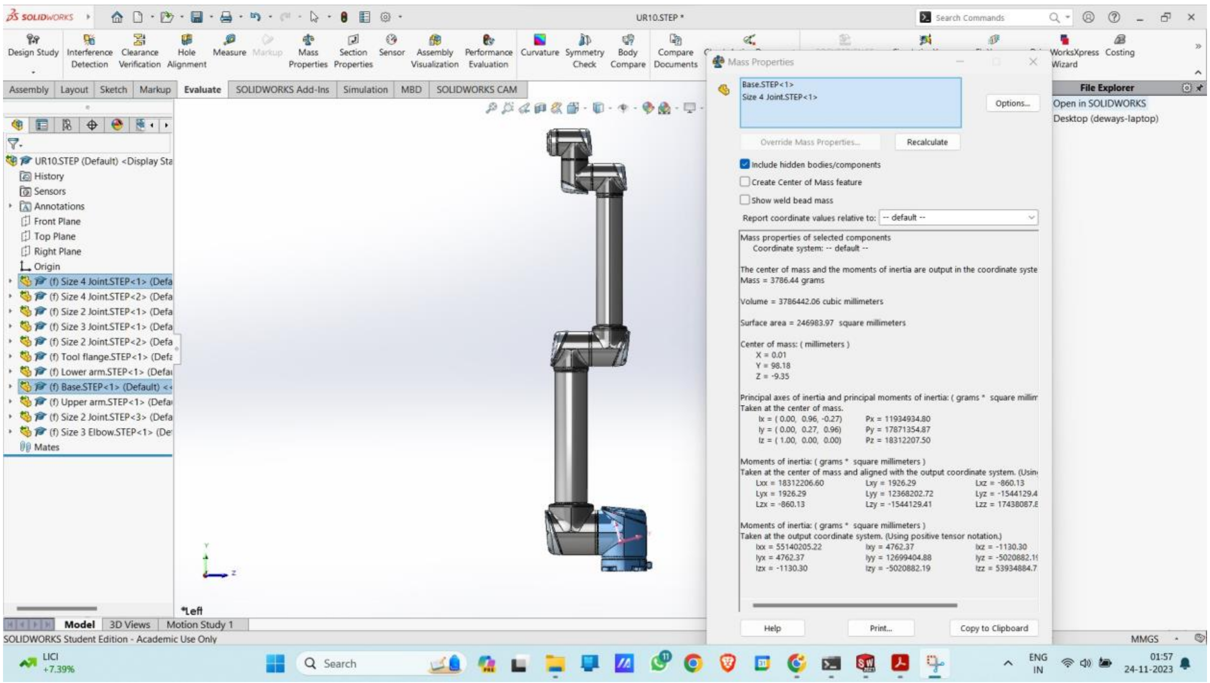
Note - The links of the robot are assumed to be cylindrical. Therefore, the centre of mass of the link is assumed to be at h/2, where h is the height of the cylinder (or link length).

$$P_i = m_i * g * h_i$$

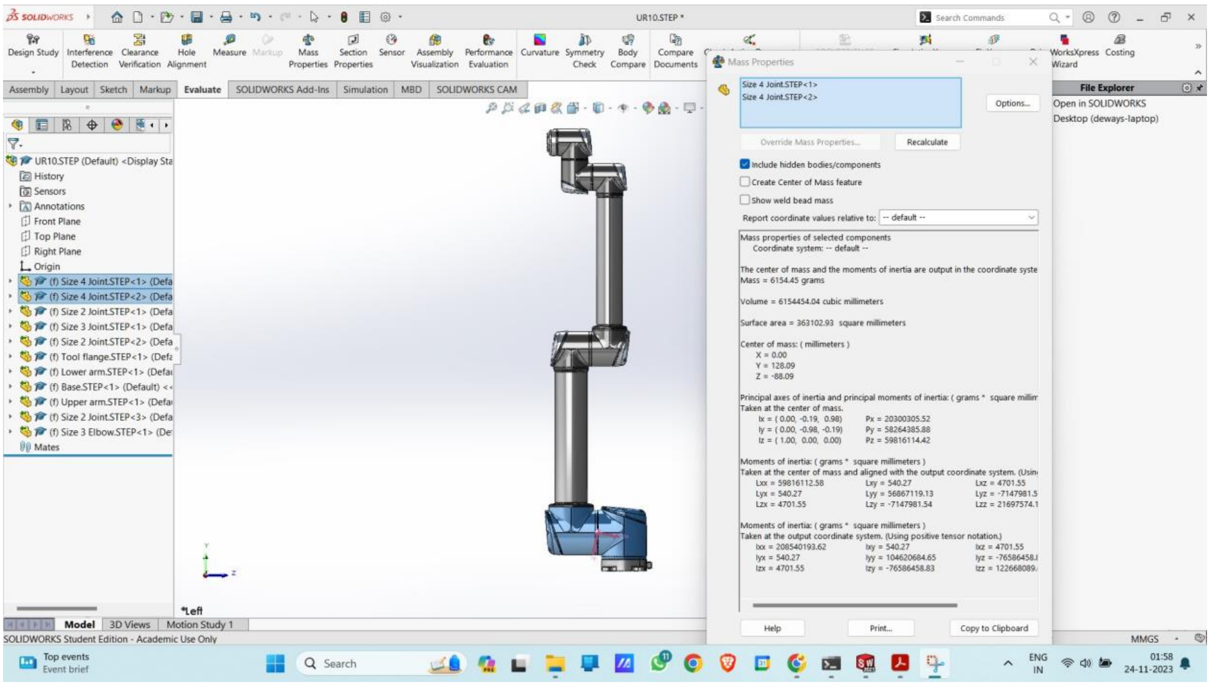
Considering link mass m_i

Note: Mass values are taken from STEP file of the robot.

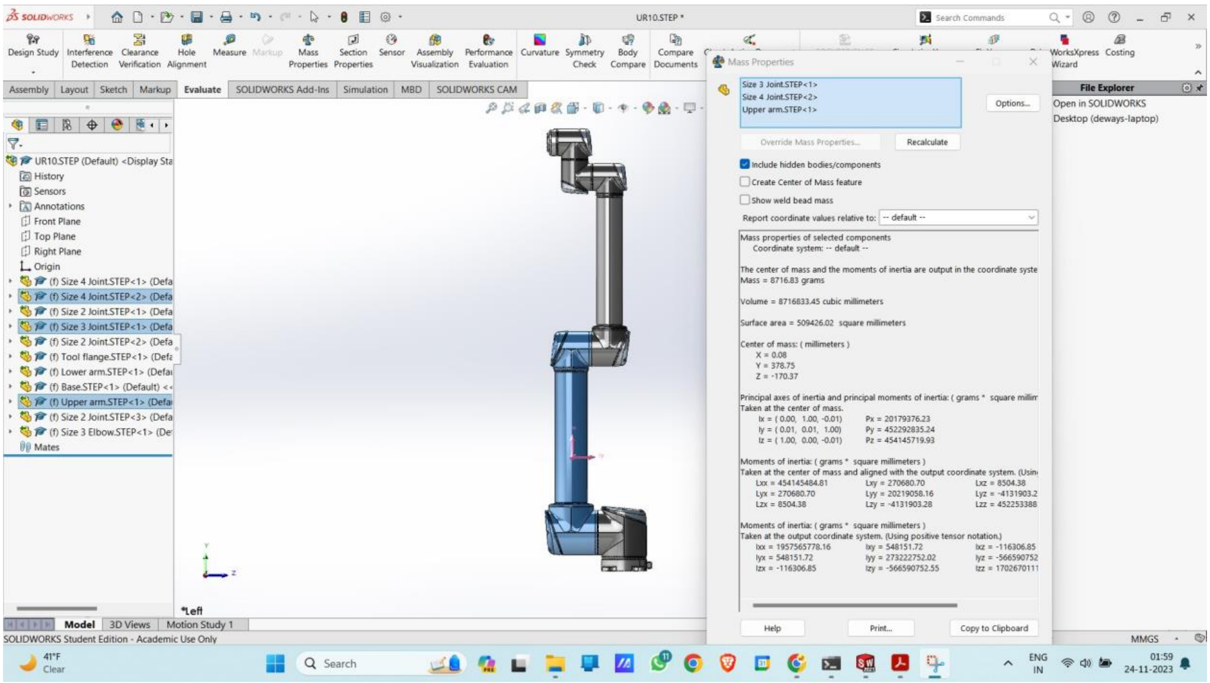
i. Mass m1 = 3.786 kg



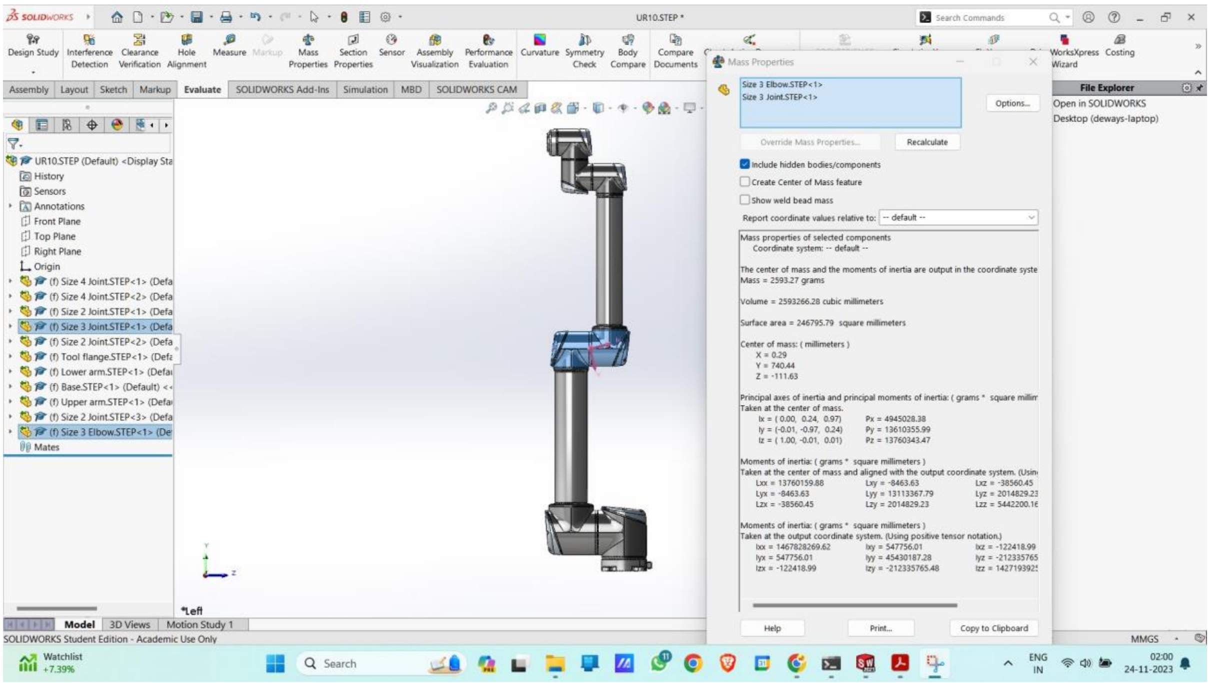
ii. Mass m2 = 6.154 kg



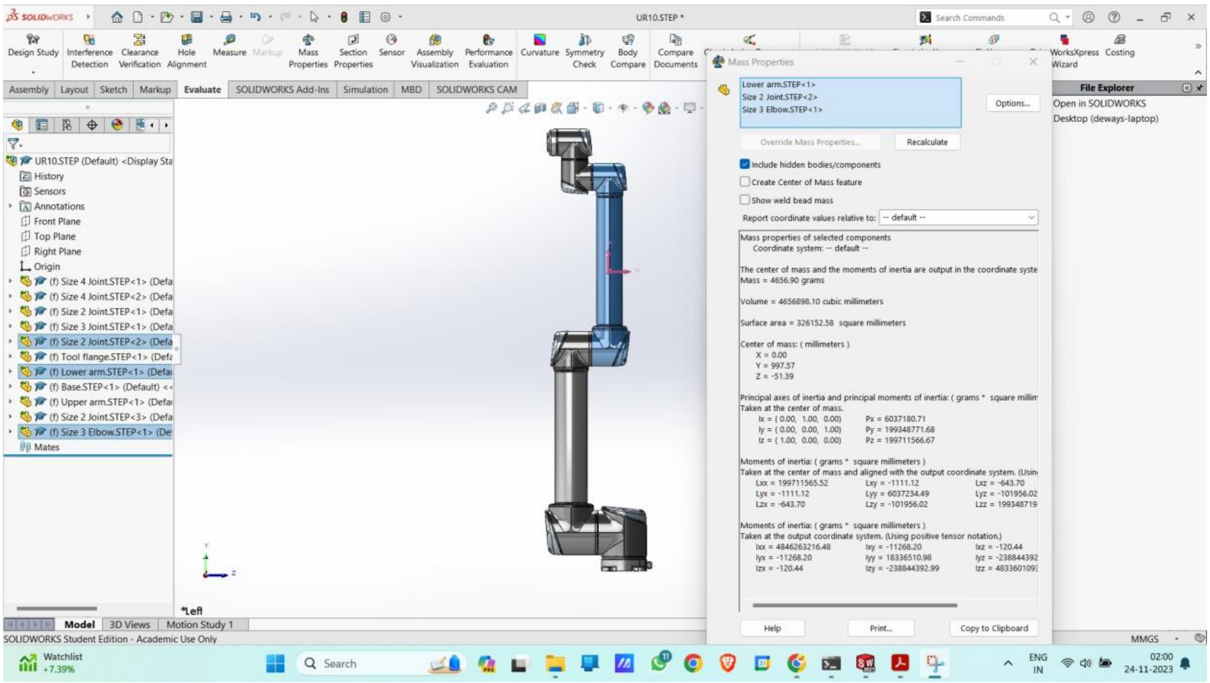
iii. Mass m3 = 8.716 kg



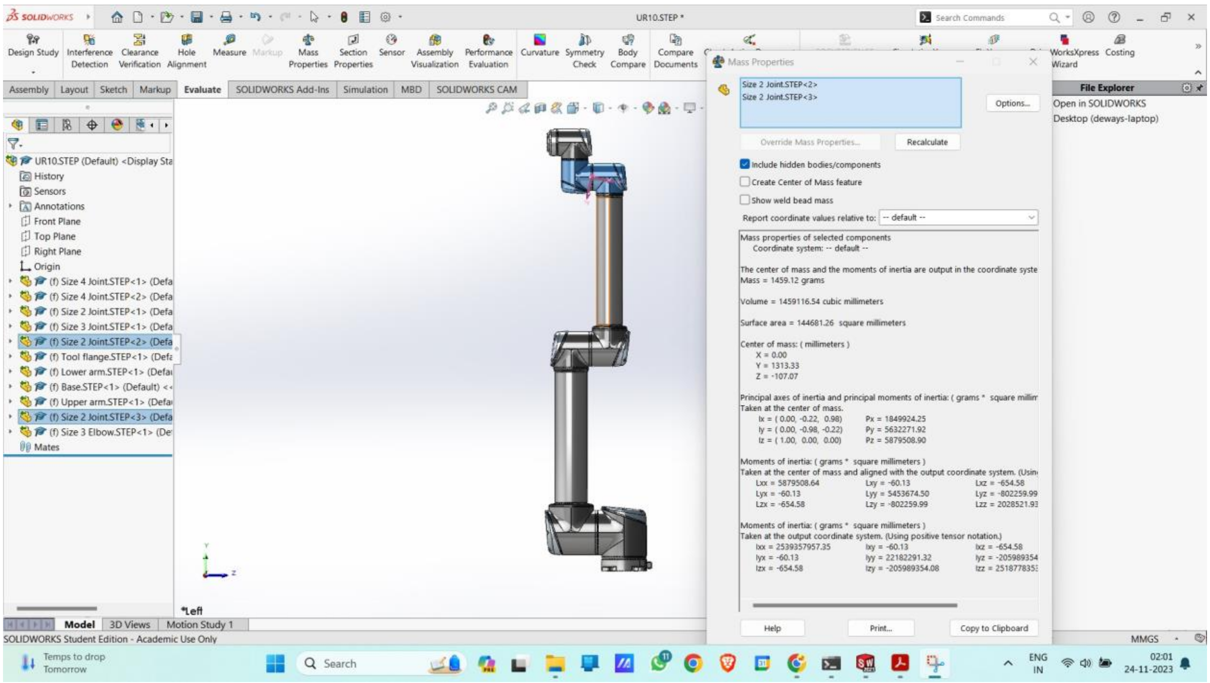
iv. Mass m4 = 2.593 kg



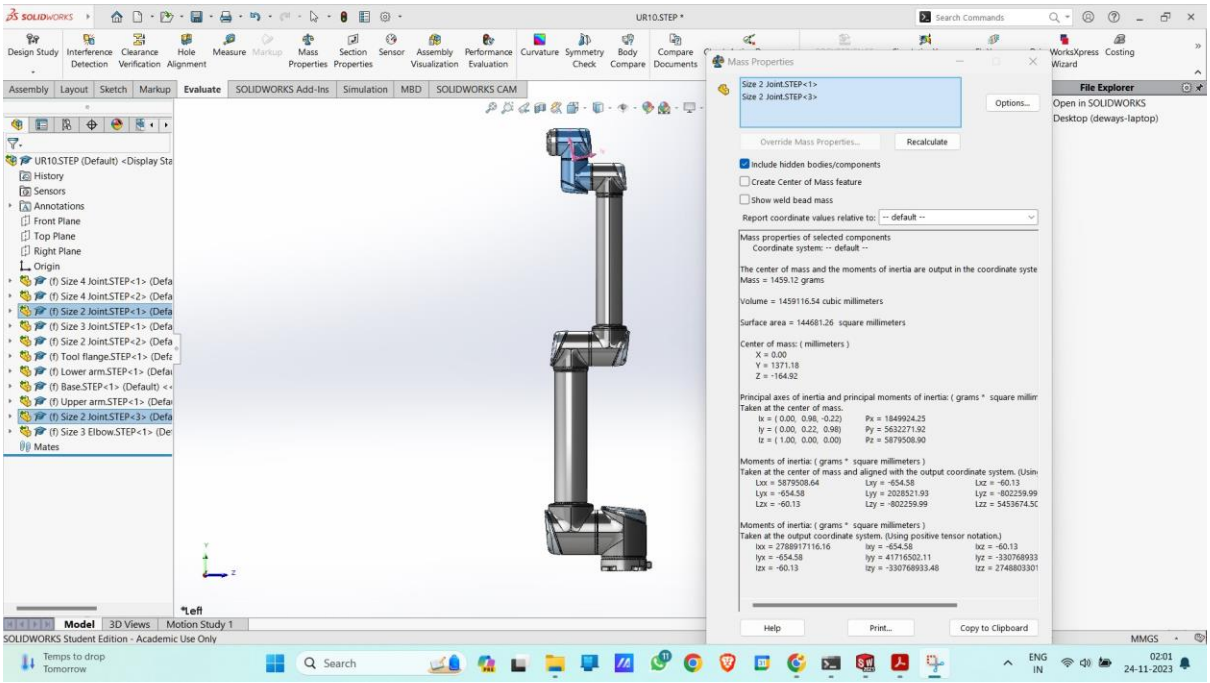
v. Mass m5 = 4.656 kg



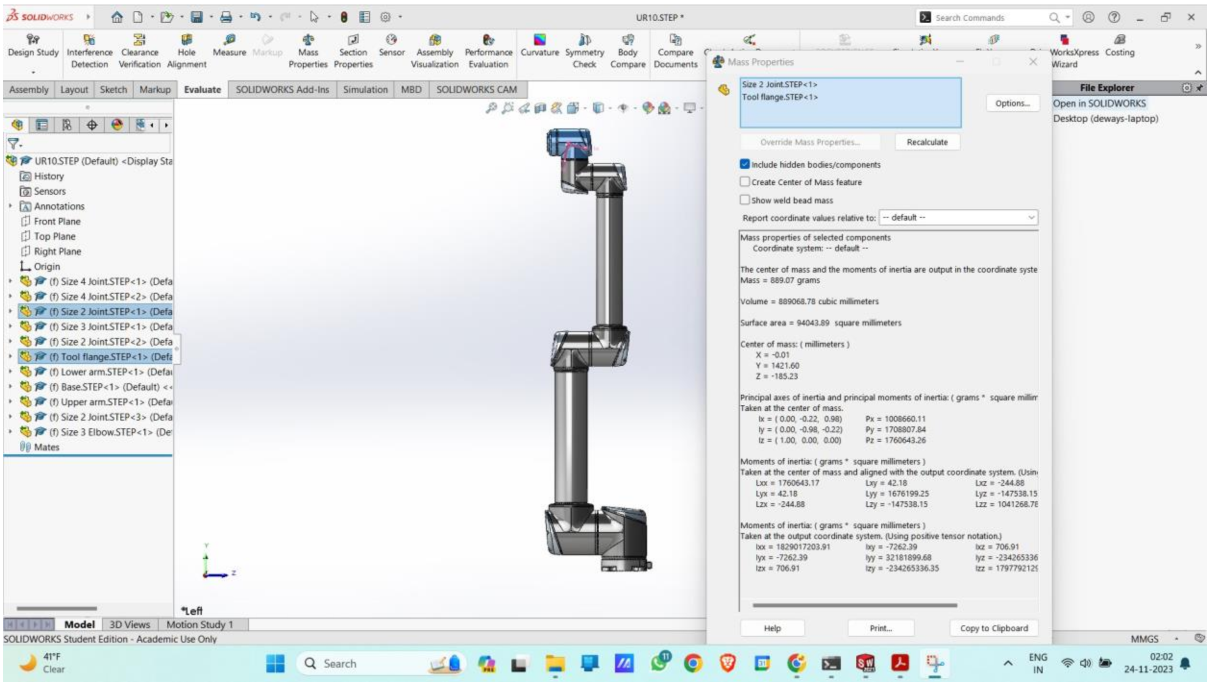
vi. Mass m6 = 1.459 kg



vii. Mass m7 = 1.459 kg



viii. Mass m8 = 0.889 kg



Note: Link heights are computed from the link lengths included in the data

Computing link height h_i (q)

The height of the centre of mass of individual links is a function of joint angles as it changes when the joint angle changes. To obtain this height as a function of joint angles, we can follow the steps:

i. Assume that the centre of mass of the cylindrical link is at the half of the height of the cylindrical link. (i.e. if the link length is 'd', then the centre of mass is assumed to be at 'd/2')

ii. Compute the transformation matrices, which gives us the transformation from the centre of the link to the base frame. This is done by considering the link length as half its true length, while substituting in the transformation matrix.

```
R1=t01.subs([(d1,0.128/2)])
R2=t01.subs([(d1,0.128)])*t12.subs([(d2,0.176/2)])
R3=t01.subs([(d1,0.128)])*t12.subs([(d2,0.176)])*t23.subs([(d3,0.6127/2)])
R4=t01.subs([(d1,0.128)])*t12.subs([(d2,0.176)])*t23.subs([(d3,0.6127)])*t34.subs([(d4,0.176/2)])
R5=t01.subs([(d1,0.128)])*t12.subs([(d2,0.176)])*t23.subs([(d3,0.6127)])*t34.subs([(d4,0.176)])*t45.subs([(d5,0.5716/2)])
R6=t01.subs([(d1,0.128)])*t12.subs([(d2,0.176)])*t23.subs([(d3,0.6127)])*t34.subs([(d4,0.176)])*t45.subs([(d5,0.5716)])*t56.subs([(d6,0.1639/2)])
R7=t01.subs([(d1,0.128)])*t12.subs([(d2,0.176)])*t23.subs([(d3,0.6127)])*t34.subs([(d4,0.176)])*t45.subs([(d5,0.5716)])*t56.subs([(d6,0.1639)])*t67.subs([(d7,0.1157/2)])
R8=t01.subs([(d1,0.128)])*t12.subs([(d2,0.176)])*t23.subs([(d3,0.6127)])*t34.subs([(d4,0.176)])*t45.subs([(d5,0.5716)])*t56.subs([(d6,0.1639)])*t67.subs([(d7,0.1157)])*t78.subs([(d8,0.922/2)])
```

iii. Extract the height of the link from the computed transformation matrices (4th column and 3rd row element)

```
h1=R1[2,3]
h2=R2[2,3]
h3=R3[2,3]
h4=R4[2,3]
h5=R5[2,3]
h6=R6[2,3]
h7=R7[2,3]
h8=R8[2,3]
```

This gives us the height of the links as a function of joint angles 'q'.

Substituting the values to get gravity matrix g(t)

- $P_i = m_i * g * h_i$

```
PE1=3.786*9.81*h1
PE2=6.154*9.81*h2
PE3=8.716*9.81*h3
PE4=2.593*9.81*h4
PE5=4.656*9.81*h5
PE6=1.459*9.81*h6
PE7=1.459*9.81*h7
PE8=0.889*9.81*h8
```

- P = $P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + P_8$

```
PE=PE1+PE2+PE3+PE4+PE5+PE6+PE7+PE8
```

- $g(q) \; = \begin{bmatrix} g(q_1) \\ g(q_2) \\ g(q_3) \\ g(q_4) \\ g(q_5) \\ g(q_6) \end{bmatrix} = \begin{bmatrix} \frac{\partial P(q)}{\partial q_1} \\ \frac{\partial P(q)}{\partial q_2} \\ \frac{\partial P(q)}{\partial q_3} \\ \frac{\partial P(q)}{\partial q_4} \\ \frac{\partial P(q)}{\partial q_5} \\ \frac{\partial P(q)}{\partial q_6} \end{bmatrix}$

```
G1=diff(PE,theta_1)
G2=diff(PE,theta_2)
G3=diff(PE,theta_3)
G4=diff(PE,theta_4)
G5=diff(PE,theta_5)
G6=diff(PE,theta_6)

G=Matrix([[G1],[G2],[G3],[G4],[G5],[G6]])
```

Computing the Joint Torques

$$\tau = g(q) - J^T(q)F$$

```
F = Matrix([[0],[5],[0],[0],[0],[0]])

JT=Transpose(JS)

GS=G.subs([(theta_1,the1),(theta_2,the2),(theta_3,the3),(theta_4,the4),(theta_5,the5),(theta_6,the6)])

tou_s=GS-(JT*F)
```

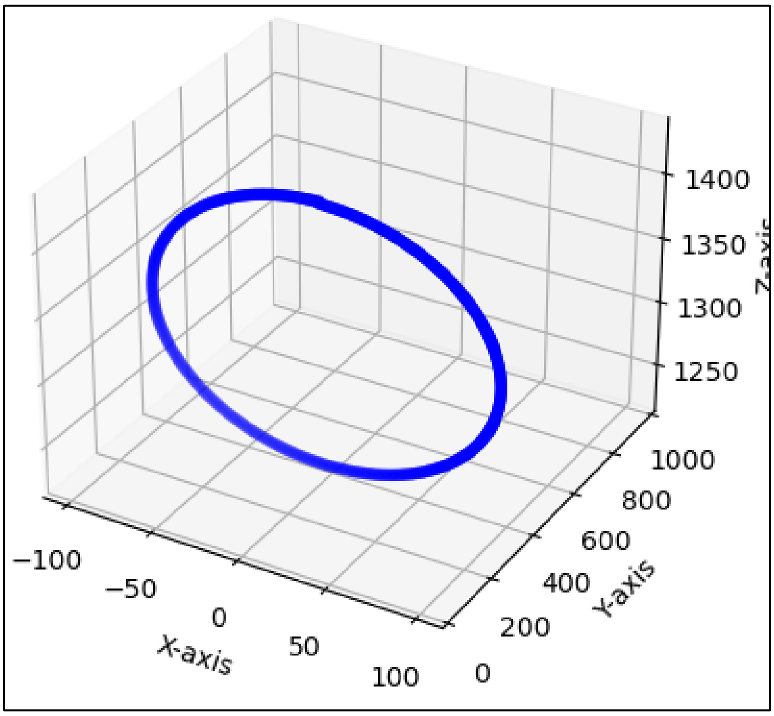
```
torque1.append((tou_s[0,0]))
torque2.append((tou_s[1,0]))
torque3.append((tou_s[2,0]))
torque4.append((tou_s[3,0]))
torque5.append((tou_s[4,0]))
torque6.append((tou_s[5,0]))
```

Results

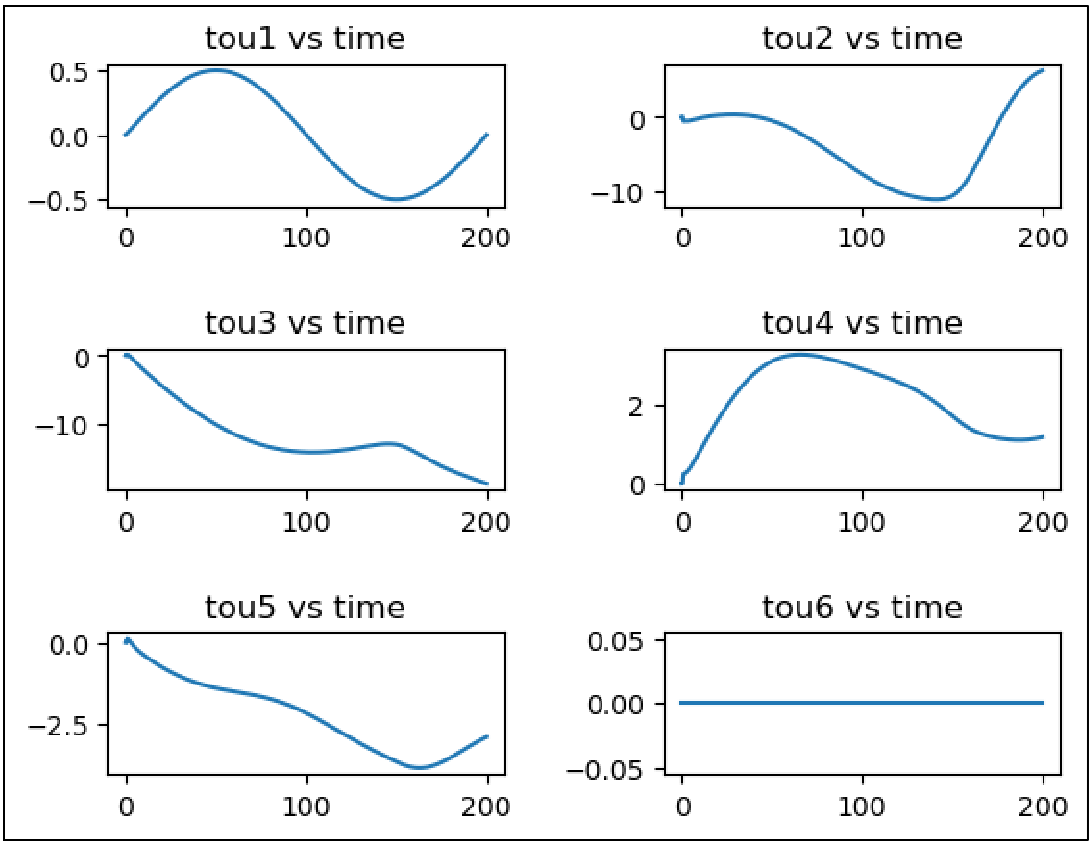
Symbolic Gravity matrix

$$\left[\begin{array}{c} 0 \\ -4.02042249\left(\left(-\sin(\theta_2)\sin(\theta_3)-\cos(\theta_2)\cos(\theta_3)\right)\cos(\theta_4)+\left(\sin(\theta_2)\cos(\theta_3)-\sin(\theta_3)\cos(\theta_2)\right)\sin(\theta_4)\right)\sin(\theta_5) \\ +1.8370250145\left(-\sin(\theta_2)\sin(\theta_3)-\cos(\theta_2)\cos(\theta_3)\right)\sin(\theta_4)-1.8370250145\left(\sin(\theta_2)\cos(\theta_3)-\sin(\theta_3)\cos(\theta_2)\right)\cos(\theta_4)-34.40137446\sin \\ (\theta_2)\cos(\theta_3)-92.647188018\sin(\theta_2)+34.40137446\sin(\theta_3)\cos(\theta_2) \\ -4.02042249\left(\left(\sin(\theta_2)\sin(\theta_3)+\cos(\theta_2)\cos(\theta_3)\right)\cos(\theta_4)+\left(-\sin(\theta_2)\cos(\theta_3)+\sin(\theta_3)\cos(\theta_2)\right)\sin(\theta_4)\right)\sin(\theta_5) \\ +1.8370250145\left(\sin(\theta_2)\sin(\theta_3)+\cos(\theta_2)\cos(\theta_3)\right)\sin(\theta_4)-1.8370250145\left(-\sin(\theta_2)\cos(\theta_3)+\sin(\theta_3)\cos(\theta_2)\right)\cos(\theta_4)+34.40137446\sin \\ (\theta_2)\cos(\theta_3)-34.40137446\sin(\theta_3)\cos(\theta_2) \\ -4.02042249\left(\left(-\sin(\theta_2)\sin(\theta_3)-\cos(\theta_2)\cos(\theta_3)\right)\cos(\theta_4)-\left(-\sin(\theta_2)\cos(\theta_3)+\sin(\theta_3)\cos(\theta_2)\right)\sin(\theta_4)\right)\sin(\theta_5) \\ +1.8370250145\left(-\sin(\theta_2)\sin(\theta_3)-\cos(\theta_2)\cos(\theta_3)\right)\sin(\theta_4)+1.8370250145\left(-\sin(\theta_2)\cos(\theta_3)+\sin(\theta_3)\cos(\theta_2)\right)\cos(\theta_4) \\ -4.02042249\left(\left(-\sin(\theta_2)\sin(\theta_3)-\cos(\theta_2)\cos(\theta_3)\right)\sin(\theta_4)+\left(-\sin(\theta_2)\cos(\theta_3)+\sin(\theta_3)\cos(\theta_2)\right)\cos(\theta_4)\right)\cos(\theta_5) \\ 0 \end{array}\right]$$

3D Plot of end effector position/trajectory obtained:



Joint Torque Plots:



Note: Y axis represent the torques in Kg m^2/ s^2 (Nm) and the X axis represent the time in seconds