SUHAS NAGARAJ
119505303
suhas99

ENPM 667
PROBLEM SET - 5

PROBLEM - 1

Given :- $A \in R^{n \times n}$ and $B \in R^{m \times m}$.

To Show :- $X(t) = e^{At} X(0) e^{Bt}$ is the solution to the equation $\dot{X}(t) = AX(t) + X(t)B$

Solution :-

The matrix exponential of the form e^{At} is defined as

$$e^{At} = \sum_{n=0}^{\infty} \frac{A^n t^n}{n!}$$

At time $t=0$, $e^{At} = I + 0 + 0 \cdots$

$\therefore e^{At} = I$ @ time $t=0$

and $\frac{de^{At}}{dt} = Ae^{At} = e^{At} A$ — (1)

Now consider the equation $X(t) = e^{At} X(0) e^{Bt}$

Taking time derivative

$$\dot{X}(t) = \frac{d}{dt}\left(e^{At} X(0) e^{Bt}\right)$$

$$\dot{X}(t) = \left(\frac{d}{dt} e^{At}\right) X(0)\, e^{Bt} + e^{At} X(0) \frac{d}{dt} e^{Bt}$$

$$\dot{X}(t) = A e^{At} X(0)\, e^{Bt} + e^{At} X(0)\, B e^{Bt}$$

But from 1, $B e^{Bt} = e^{Bt} B \rightarrow$ replacing

$$\dot{X}(t) = A\left[e^{At} X(0)\, e^{Bt}\right] + \left[e^{At} X(0)\, e^{Bt}\right] B \quad \text{— } (2)$$

Hence, the term $e^{At} X(0)\, e^{Bt}$ can be the solution if at time $t=0$, $e^{At} X(0)\, e^{Bt} = X(0)$

Testing the above condition,

at $t=0$, $e^{A(0)} X(0)\, e^{B(0)} = X(0) \quad \text{— } (3)$

Hence, from comparing 2 with the given equation $\dot{X}(t) = A X(t) + X(t) B$ and from 3 we can conclude that $X(t) = e^{At} X(0)\, e^{Bt}$ is the solution for the equation $\dot{X}(t) = A X(t) + X(t) B$

Problem 2

Euclidean ball, $B(x_c, r)$ in R^n is given by

$$B(x_c, r) = \{x \in R^n \mid \|x - x_c\| \leq r\}$$

where $r > 0$ and $\|\cdot\|$ is the euclidean norm, $\|x\| = \sqrt{x^T x}$. The vector $x_c \in R^n$ is the center of the ball and r is the radius

To prove, $B(x_c, r)$ is convex set

Proof :- A set is convex if the line segment between any two points in the set, lies in the set.

Considering two points x_1 and x_2 in the euclidean ball, if $\theta_1 x_1 + (1-\theta) x_2 \in C$ for any θ, then the set is convex.

The interior of the euclidean ball is given by $\|x_1\| \leq r$ and $\|x_2\| \leq r$.

Considering the convex combination mentioned above

$\| \theta \bar{x}_1 + (1-\theta) \bar{x}_2 \|$

Using triangular inequality of norms,

$\|x\| + \|y\| \geq \|x+y\|$

$\therefore \| \theta \bar{x}_1 + (1-\theta) \bar{x}_2 \| \leq \| \theta \bar{x}_1 \| + \| (1-\theta) \bar{x}_2 \|$

$\Rightarrow \theta \|\bar{x}_1\| + (1-\theta) \|\bar{x}_2\| \geq \| \theta \bar{x}_1 + (1-\theta) \bar{x}_2 \|$

$\theta \in [0, 1]$ and $\theta, (1-\theta) \geq 0$

Here x_1 and x_2 are vectors inside euclidean ball such that thier norm is less than or equal to the radius of the ball

Here $\bar{x}_1 = x_1 - x_c$ where X_c is the centre of

and $\bar{x}_2 = x_2 - x_c$ the euclidean ball

$\therefore \|\bar{x}_1\| \leq r$

$\|\bar{x}_2\| \leq r$

$\therefore \theta \|\bar{x}_1\| + (1-\theta) \|\bar{x}_2\| \leq r.$

But $\| \theta \bar{x}_1 + (1-\theta) \bar{x}_2 \| \leq \theta \|\bar{x}_1\| + (1-\theta) \|\bar{x}_2\|$

From the two equations,

it can be deduced that

$$\|\theta_1 \bar{x} + (1-\theta)\bar{x}_2\| \leq r$$

Hence, as this norm is also less than the radius of the euclidean ball, it must lie inside the ball, which proves that the equation $(\theta_1 \bar{x} + (1-\theta)\bar{x})$ lies inside the ball

$$\|\theta x_1 + (1-\theta)x_2\| \in B(x_c, r)$$

$\therefore$ B is convex

PROBLEM-3

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$J = \int_0^\infty \left(x^T Q x + u^2\right) dt$$

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & \gamma \end{bmatrix}, \quad \gamma > 0$$

To solve :- Design an LQR and provide state feedback $u = kx$.

The given state space equation is of the form

$$\dot{x}(t) = Ax(t) + Bu(t)$$

Comparing, we get $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

Finding K, such that it minimizes the ~~lost~~ cost function $J(k, \vec{x}(0)) = \int_0^\infty x^T(t) Q x(t) + u_k^T(t) R u_k(t) dt$

⇒ Comparing the given cost function to the general form of cost function ~~g~~ mentioned above,

$$u_k^T(t) R u_k(t) = u^2$$

$$\therefore R = I$$

The gain matrix is represented by the formula $K = -R^{-1} B_K^T P$ where P is the solution to the Ricatti equation $A^T P + PA - PBR^{-1}B^T P = -Q$ where $R = I$ and A, B matrices are given.

Substituting,

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}^T \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} + \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} I$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}^T \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = -Q = \begin{bmatrix} -1 & 0 \\ 0 & -\gamma \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} + \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} P_{12} \\ P_{22} \end{bmatrix} I$$

$$\begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -\gamma \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 \\ P_{11} & P_{12} \end{bmatrix} + \begin{bmatrix} 0 & P_{11} \\ 0 & P_{21} \end{bmatrix} - \begin{bmatrix} P_{12} \\ P_{22} \end{bmatrix} \begin{bmatrix} P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -\gamma \end{bmatrix}$$

Here as P is symmetric matrix

$P_{21} = P_{12}$

$$\therefore \begin{bmatrix} 0 & 0 \\ P_{11} & P_{12} \end{bmatrix} + \begin{bmatrix} 0 & P_{11} \\ 0 & P_{12} \end{bmatrix} - \begin{bmatrix} P_{12} \\ P_{22} \end{bmatrix} \begin{bmatrix} P_{12} & P_{22} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -8 \end{bmatrix}$$

$$\begin{bmatrix} 0 & P_{11} \\ P_{11} & 2P_{12} \end{bmatrix} - \begin{bmatrix} P_{12}^2 & P_{12}P_{22} \\ P_{22}P_{12} & P_{22}^2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -8 \end{bmatrix}$$

$$\begin{bmatrix} -P_{12}^2 & P_{11} - P_{12}P_{22} \\ P_{11} - P_{22}P_{12} & 2P_{12} - P_{22}^2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -8 \end{bmatrix}$$

$P_{11} - P_{22}P_{12} = 0 \implies P_{11} = P_{22}P_{12}$

$-P_{12}^2 = -1 \implies P_{12}^2 = 1 \implies P_{12} = \pm 1$

Considering $P_{12} = 1$

$2P_{12} - P_{22}^2 = -8$

$+2 - P_{22}^2 = -8$.

Also $P_{11} = P_{22}$ $\therefore$ $P_{11} = P_{12} = \pm\sqrt{2+8}$

Now considering $P_{12} = -1$

$$-2 - P_{22}^2 = -\gamma$$

$$P_{22}^2 = \gamma - 2$$

$$P_{22} = \pm\sqrt{\gamma - 2}$$

Also $P_{11} = P_{22}(P_{12}) = P_{22}(-1)$

$$\therefore P_{11} = -P_{22}$$

In the above equation, P_{22} will be complex under some cases when γ is >0, hence we do not proceed with this.

$\Rightarrow$ Taking $P_{12} = 1$, we have

$$P_{22} = P_{11} = \pm\sqrt{2+\gamma}$$

$\rightarrow$ considering $P_{22} = +\sqrt{2+\gamma}$, P is

$$\begin{bmatrix} \sqrt{2+\gamma} & 1 \\ 1 & \sqrt{2+\gamma} \end{bmatrix}$$

The eigen values of P are given by :-

$$|P - \lambda I| = 0$$

$$\begin{vmatrix} \sqrt{2+\gamma} - \lambda & 1 \\ 1 & \sqrt{2+\gamma} - \lambda \end{vmatrix} = 0$$

$$\left(\sqrt{2+\gamma} - \lambda\right)^2 - 1 = 0.$$

$$\left(\sqrt{2+\gamma} - \lambda\right)^2 = 1$$

$$\left(\sqrt{2+\gamma} - \lambda\right) = \pm 1$$

$$\therefore \lambda = \sqrt{2+\gamma} \pm 1$$

Considering $P_{22} = -\sqrt{2+\gamma}$, P is

$$P = \begin{bmatrix} -\sqrt{2+\gamma} & 1 \\ 1 & -\sqrt{2+\gamma} \end{bmatrix}$$

The eigen values of P are given by

$$|P - I\lambda| = 0$$

$$\left|\begin{bmatrix} -\sqrt{2+\gamma} - \lambda & 1 \\ 1 & -\sqrt{2+\gamma} - \lambda \end{bmatrix}\right| = 0$$

$$\left(-\sqrt{2+\gamma}-\tau\right)^2 - 1 = 0$$

$$\left(-\sqrt{2+\gamma}-\tau\right)^2 = 1$$

$$-\sqrt{2+\gamma}-\tau = \pm 1$$

$$\tau = -\sqrt{2+\gamma} \pm 1$$

In the above case, the P matrix can become negative definate for certain values of γ, which is undesirable

$\therefore$ we consider $\tau = \sqrt{2+\gamma} \pm 1$

$$\therefore P = \begin{bmatrix} \sqrt{2+\gamma} & 1 \\ 1 & \sqrt{2+\gamma} \end{bmatrix}$$

This is the P matrix. Substituting in the equation

$$K = -R^{-1} B^T P$$

$$K = -I^{-1} \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{2+\gamma} & 1 \\ 1 & \sqrt{2+\gamma} \end{bmatrix}$$

$$K = -\begin{bmatrix} 1 & \sqrt{2+\gamma} \end{bmatrix}$$

The state feedback is given by

$$U = KX = -\begin{bmatrix} 1 & \sqrt{2+\gamma} \end{bmatrix} X$$

Substituting in the given state space equation

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot \left[-\begin{bmatrix} +1 & +\sqrt{2+\gamma} \end{bmatrix} X \right]$$

$$= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} X + \begin{bmatrix} 0 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & \sqrt{2+\gamma} \end{bmatrix} X$$

$$= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} X + \begin{bmatrix} 0 & 0 \\ -1 & -\sqrt{2+8} \end{bmatrix} X$$

$$\dot{X} = \begin{bmatrix} 0 & 1 \\ -1 & -\sqrt{2+8} \end{bmatrix} X$$

PROBLEM - 4

$$\dot{x}(t) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u(t)$$

$\Rightarrow$ The controllability matrix C is given by

$$C = [B \mid AB \mid A^2B]$$

Here matrix $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ is a $\underset{n \times n}{3 \times 3}$ matrix

and matrix $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ is a $\underset{n \times m}{3 \times 2}$ matrix

The controllability matrix C is a $n \times nm$ matrix

So it's 3×6 matrix

$$\therefore B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$AB = \underset{3 \times 3}{\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}} \underset{3 \times 2}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^2B = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 0 \\ 2 & 1 \end{bmatrix}$$

Substituting,

$$C = \begin{bmatrix} 0 & 1 & 1 & 1 & 2 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 2 & 1 \end{bmatrix}$$

The rank of the matrix is 2 which is less than $n = 3$. Therefore the system is uncontrollable

Defining $n \times n$ similarity transformation matrix

$$S = [V_1 \mid V_2 \mid \cdots \mid V_{n_r} \mid S_{n-n_r}]$$

Here S_{n-n_r} is selected as $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ so that

$$S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$ is invertible

Here $S^{-1} = \frac{1}{|S|} \text{Adj}(S) \Rightarrow$

$|S| = 1$

$$\text{Adj}(S) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\therefore \; S^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\hat{A} = S^{-1}AS = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\hat{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Here $\hat{A} = \left[\begin{array}{c|c} \underset{n_c \times n_1}{A_{11}} & A_{12} \\ \hline 0 & \underset{(n-n_c)(n-n_c)}{A_{22}} \end{array}\right]$

Comparing, we get, $A_{11} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

which is the controllable part.

$$\hat{B} = S^{-1}B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Here $\hat{B} = \left[\begin{array}{c} B_1 \\ \hline 0 \end{array}\right]$

Comparing, we get $B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

The standard form of the uncontrollable system

is $\dot{\vec{X}} = \hat{A}\vec{X} + \hat{B}\vec{U}$

$$\therefore \dot{\vec{X}} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \vec{X} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \vec{U}$$

and the controllable part is $A_{11} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and

$B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

The state space equation of the controllable part :-

$$\dot{\vec{X}} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \vec{x} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \vec{U}$$

Problem 5

$$\dot{X} = \begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix} X$$

Lyapunov Equation :-

$A^T P + PA = -Q \quad Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

An LTI system is stable if and only if for a positive definate symmetric matrix, there exists a symmetric positive definite matrix P such that the Lyapunov equation holds.

Taking P as $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}$

As $P_{12} = P_{21}$

The eigen ~~vector~~/value equation is

$$|P - \gamma I| = 0$$

$$\left| P - \begin{bmatrix} \gamma & 0 \\ 0 & \gamma \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} P_{11} - \gamma & P_{12} \\ P_{12} & P_{22} - \gamma \end{vmatrix} = 0$$

$$(P_{11} - \gamma)(P_{22} - \gamma) = P_{12}^2 \quad \text{—} (1)$$

From question, A is $A = \begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix}$

$$A^T = \begin{bmatrix} -3 & -1 \\ 2 & -1 \end{bmatrix}$$

Substituting in equation,

$$A^T P + PA = -Q$$

$$\begin{bmatrix} -3 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} + \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} \begin{bmatrix} -3 & 2 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -3P_{11} - P_{12} & -3P_{12} - P_{22} \\ 2P_{11} - P_{12} & 2P_{12} - P_{22} \end{bmatrix} + \begin{bmatrix} -3P_{11} - P_{12} & 2P_{11} - P_{12} \\ -3P_{12} - P_{22} & 2P_{12} - P_{22} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -6P_{11} - 2P_{12} & 2P_{11} - 4P_{12} - P_{22} \\ 2P_{11} - 4P_{12} - P_{22} & 4P_{12} - 2P_{22} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Equating

$$-6P_{11} - 2P_{12} = -1$$

$$2P_{11} - 4P_{12} - P_{22} = 0$$

$$4P_{12} - 2P_{22} = -1$$

Solving the 3 simultaneous linear equations, we get

$$P_{11} = \frac{7}{40} \qquad P_{12} = \frac{-1}{40} \qquad P_{22} = \frac{18}{40}$$

Substituting

$$P = \begin{bmatrix} \frac{7}{40} & -\frac{1}{40} \\ -\frac{1}{40} & \frac{18}{40} \end{bmatrix}$$

Substiting the values in 1

$$\left(\frac{7}{40} - \gamma\right)\left(\frac{18}{40} - \gamma\right) = \left(-\frac{1}{40}\right)^2$$

$$\frac{126}{40^2} - \frac{18}{40}\gamma - \frac{7}{40}\gamma + \gamma^2 = \frac{1}{40^2}$$

$$\gamma^2 - \frac{25}{40}\gamma + \frac{125}{40^2} = 0$$

Here $a = 1$ $b = -\frac{25}{40}$ $c = \frac{125}{40^2}$

$$\therefore \gamma = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{+\frac{25}{40} \pm \sqrt{\frac{25^2}{40^2} - \frac{4 \times 125}{40^2}}}{2}$$

$$= \frac{\frac{25}{40} \pm \sqrt{\frac{625 - 500}{40^2}}}{2}$$

$$\gamma = \frac{25}{80} \pm \frac{5\sqrt{5}}{80} = \frac{5(5 \pm \sqrt{5})}{80}$$

Here $5 \pm \sqrt{5}$ is always > 0, therefore the eigen values of P matrix is always positive making P positive definite.

Therefore the system is stable