

# Simulation of Hawkes Process

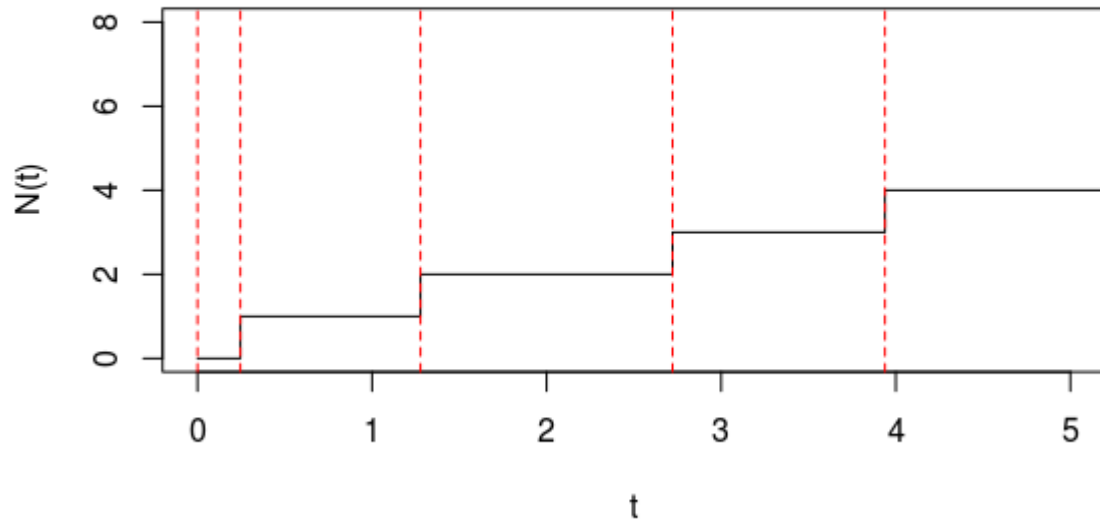
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# Poisson Process

- Homogeneous

$$P\{X(t) = n\} = e^{-\lambda t} \frac{(\lambda t)^n}{n!} \quad n = 0, 1, 2, \dots$$

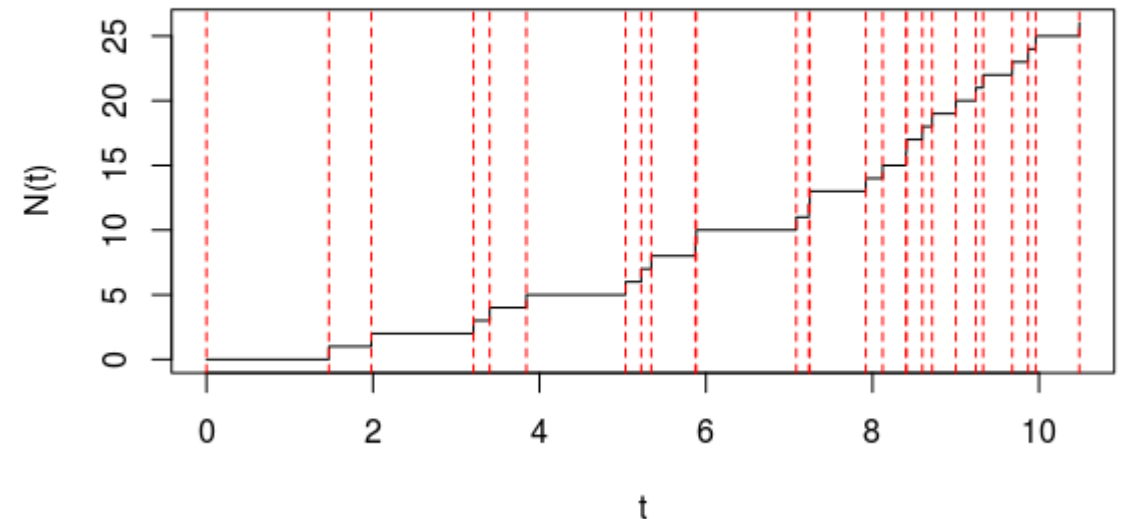
**Homogeneous Poisson process with rate=1**



- Inhomogeneous

$$P\{X(t) = n\} = e^{-\Lambda(t)} \frac{\{\Lambda(t)\}^n}{n!} \quad n = 0, 1, 2, \dots$$

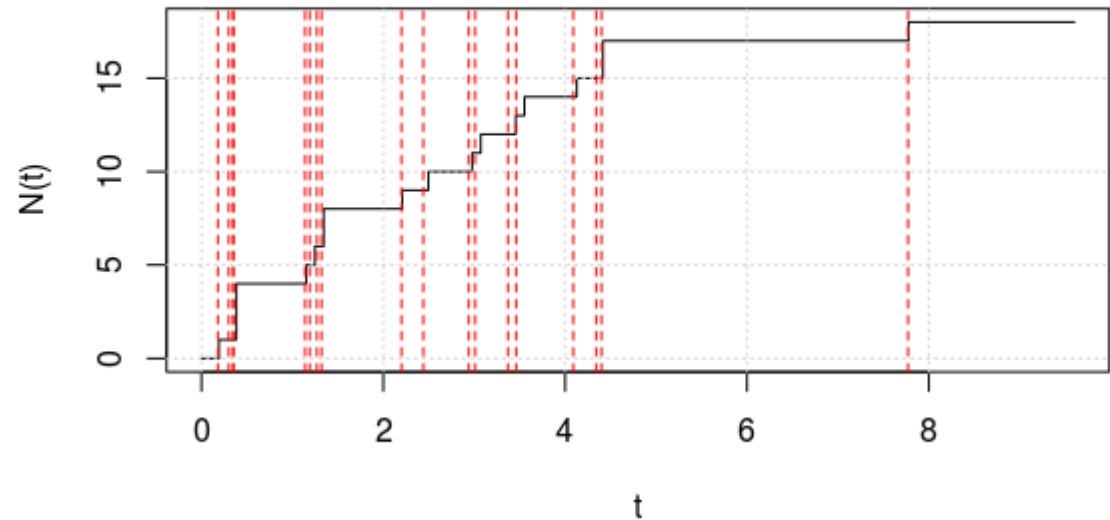
**Inhomogeneous Poisson Process with rate= $t/2$**



# Hawkes Process

$$\Lambda(t) = \lambda_0 + \sum_{t_i < t} \alpha e^{-\beta(t-t_i)}$$

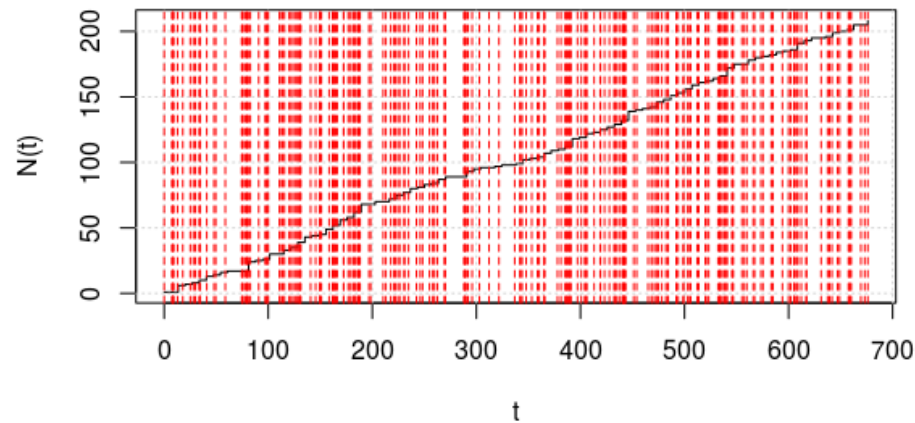
Hawkes Process, lambda = 1, alpha = 2, beta = 3



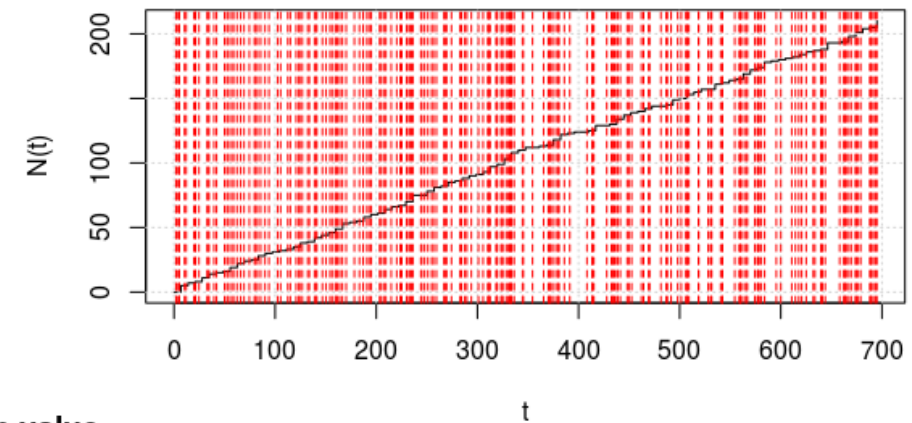
$$l = \sum_{i=1}^k \log \left[ \lambda + \alpha \sum_{j=1}^{i-1} e^{-\beta(t_i - t_j)} \right] - \lambda t_k + \frac{\alpha}{\beta} \sum_{i=1}^k \left[ e^{-\beta(t_k - t_i)} - 1 \right]$$

# Chicago Burglary data in Beat 423 (2017-18)

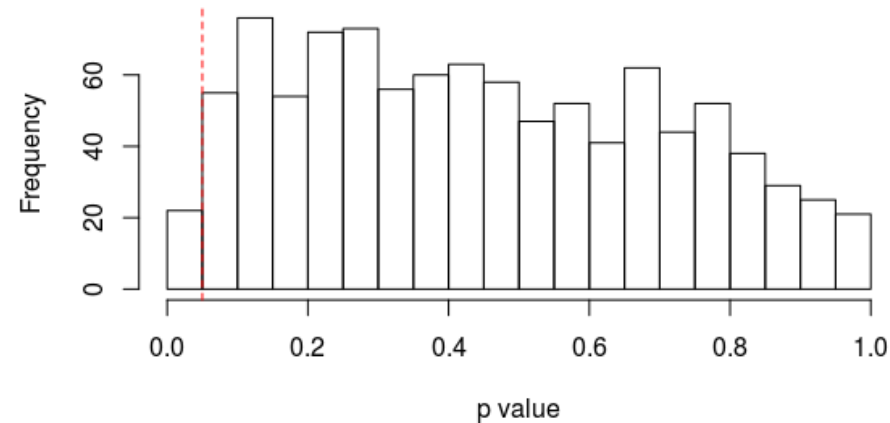
Chicago Burglary data in Beat 423 from 2017 to present



Simulated Hawkes data with  $\lambda = 0.29$ ,  $\alpha = 0.04$ ,  $\beta = 0.79$



Histogram of p value



Thank you

Simulation of Hawkes Process

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<https://rstudio.cloud/spaces/4116/project/113564>