NuminaMath: The largest public dataset in AI4Maths with 860k pairs of competition math problems and solutions

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Abstract

Numina is an open AI4Maths initiative dedicated to advancing both artificial and human intelligence in the field of mathematics. In this paper, we present the NuminaMath dataset, a comprehensive collection of 860,000 pairs of competition math problems and solutions. This dataset is designed to enhance the mathematical reasoning capabilities of large language models (LLMs) and stands as the largest math dataset ever released in the field. The NuminaMath dataset includes problems ranging from high school level to advanced competition-level, all meticulously annotated in a chain-of-thought manner. We detail the construction, composition, and potential applications of the dataset, and provide experimental results that demonstrate its effectiveness. The power of this dataset is underscored by its role in fine-tuning a model to win the 1st AIMO Progress Prize, showcasing its significant impact on the development of state-of-the-art mathematical reasoning models.

1 Introduction

Mathematics is a universal language, fundamental to various domains including science, engineering, and economics. The complexity and beauty of mathematical problems make them an excellent testbed for developing and evaluating artificial intelligence (AI) models. Despite the significance of competition-level mathematics, there are very few publicly available datasets. Notable datasets include MATH, with a training set of 7.5k problems, and OrcaMath, focused on grade-school mathematics with a subset of problems similar to GSM8K. High-performing models like DeepSeekMath and others often achieve state-of-the-art performance without publishing their datasets, creating a gap in the availability of resources for the broader AI research community.

The NuminaMath dataset, an open-source initiative, addresses this gap by providing a comprehensive resource for training AI models to solve complex mathematical problems. This dataset consists of one million pairs of math problems and solutions, annotated in a chain-of-thought (CoT) format to facilitate reasoning and problem-solving by AI. This dataset is the biggest Math dataset ever released, and has allowed the Numina team to win the 1st AIMO progress prize.

The primary motivation for focusing on competition-level mathematics is its potential to serve as a precursor for advanced reasoning tasks in AI. Achieving a gold medal in the International Mathematical Olympiad (IMO) is a benchmark for exceptional mathematical achievement and a strong predictor of future success. IMO gold medalists are 50 times more likely to win a Fields Medal compared to a typical Cambridge PhD graduate. Furthermore, half of all Fields medalists participated in the IMO during their youth. Thus, by developing AI models capable of solving IMO-level problems, we can build intuition for what works in high-level mathematical reasoning.

To push the limits of open large language models (LLMs), it is crucial to combine chain-of-thought reasoning with tool use, such as code interpreters. Our research utilized this dataset to win the Progress Prize, demonstrating the practical application and success of the Numina dataset in advancing AI capabilities.

This paper also explores the performance of various models, including 7B, 16B, and 70B parameter models, and evaluates how well GPT-4 and Claude 3.5 perform on recent competitions like AMC 2023 and AIME 2024 using code interpreters. Additionally, we investigate whether our models can outperform these large models.

We provide a detailed analysis of the dataset, breaking it down by source and visualizing the distribution of problems per country for Olympiad-level problems. We also include token statistics (input/output), the number of Python blocks in the tool-integrated reasoning (TIR) format, and the distribution of problem types, particularly those with integer outputs.

In summary, the Numina dataset represents a significant step forward in developing AI models capable of tackling complex mathematical problems. By providing detailed, step-by-step solutions, the dataset facilitates deeper understanding and reasoning. We invite contributions from the community to further enhance the dataset and its applications.

2 Related Works

Several datasets have been developed to aid in the training of AI models for mathematical problem solving. The Mathematics Dataset by DeepMind, the MATH [Hendrycks et al., 2021] dataset, and the GSM8K [Cobbe et al., 2021] Dataset are notable examples. These datasets provide a range of problems from elementary arithmetic to advanced mathematics, annotated in various formats.

- **Math LLMs:** Numerous excellent models have emerged in the world of LLMs for math. In this report, we will draw a distinction between closed models (hidden weight and undisclosed data), open weights models (public weights but private data), and open data models (both weights and data are public). In the world of closed models, Minerva [Lewkowycz et al., 2022] was among the first to pursue mathematics training at scale. Recently, GPT-4 [OpenAI, 2024], Gemini 1.5 [GeminiTeam, 2024], and Claude 3.5 [Anthropic, 2024] have been among the strongest contenders. Other impressive models in this space have been open-weight but trained at least partially on private data, including Llemma [Azerbayev et al., 2024], MetaMath [Yu et al., 2023], WizardMath [Luo et al., 2023], and DeepSeekMath [Shao et al., 2024].

- **Chain of thought (CoT) / Scratchpads:** CoT [Wei et al., 2023] and scratchpads [Nye et al., 2021] were first introduced as prompting strategies to augment the problem-solving capacities of LLMs by incrementally decomposing problems into smaller steps. While transformers are tightly bound in the complexity of problems solvable in a single forward pass [Merrill and Sabharwal, 2024], looped transformers can solve a much more expressive set of problems [Merrill and Sabharwal, 2023, Giannou et al., 2023].

- **Finetuning with bootstrapped CoT trajectories:** One successful approach for distilling the performance gains of CoT has been to leverage in-context-learning (ICL) to sample well-formed CoT trajectories for existing problem solution pairs, filter based on correctness, and then utilize these intermediate samples for fine-tuning. Such an approach can push models to sample from increasingly effective problem-solving strategies. In the math domain, successful works including ToRA [Gou et al., 2024], WizardMath [Luo et al., 2023], and DeepSeekMath [Shao et al., 2024] all utilized variants of this strategy to bootstrap synthetic training data for fine-tuning. In addition, Chen et al. [2024] have characterized the boundaries of this type of approach, as well as presenting strategies for determining optimal fine-tuning data mixtures.
- **Use of synthetic problems:** Even with the amazing collection of existing math datasets, public data availability still remains a limiting factor in scaling up training. As such, there has been a recent focus on harnessing LLMs to generate novel problems, which can then be used to bootstrap training data. Li et al. [2024], as well as MetaMath [Yu et al., 2023] and MuMath [You et al., 2024] have explored this approach, fine-tuning on synthetic data, revealing strong performance gains. Microsoft's Phi family of models [Gunasekar et al., 2023, Li et al., 2023, Javaheripi et al., 2023, Abdin et al., 2024] have also presented substantial evidence for the effectiveness of synthetic data.
- **Leveraging external tools:** In approaching problems with underlying symbolic structure including mathematical and logical reasoning, augmenting LLMs with external "tools", such as access to a Python interpreter [Chen et al., 2022, Gao et al., 2023, He-Yueya et al., 2023] or SAT-solver [Olausson et al., 2023, Pan et al., 2023, Ye et al., 2024], has proved a powerful tactic. In such settings, an LLM operates as a declarative programmer, generating a formal specification of a given problem in code, that is then offloaded to an external solver, which can return an exact solution.
- **Tool-integrated reasoning:** In drawing from the complementary strengths of reasoning over text and programs, recent works have adopted combined strategies that train on both CoT and program trajectories [Yue et al., 2023]. These works, especially ToRA [Gou et al., 2024] and MuMath-Code [Yin et al., 2024], which interleave planning over free-form text with program generation and execution in joint trajectories, formed the basis of the most successful strategies explored and adopted in the development of this dataset and corresponding models.
- **Decoding algorithms:** While the base sampling distributions of LLMs may contain correct solutions to problems, it is often the case that greedy generation is a sub-optimal decoding strategy. Instead, it is often preferable to sample, steer, and filter multiple candidate trajectories, and use a scoring function and decision rule to rerank or reconcile the diverse responses. Self-consistency [Wang et al., 2022], self-refine [Madaan et al., 2024], and CoRe [Zhu et al., 2022], among other works, can be construed as variants of such a general strategy. The outputs of such online inference can also be used to bootstrap fine-tuning data for distillation, as in Yu et al. [2024] and STaR [Zelikman et al., 2022].

3 Datasets

A significant challenge in the development of open math LLMs has been the lack of substantial high-quality data. The training splits of the popular MATH [Hendrycks et al., 2021] and GSM8K [Cobbe et al., 2021] datasets have been frequently used, but their scale is limited, with less than 20k samples combined. The Microsoft orca-math-word-problems-200k dataset [Mitra et al., 2024] has been a valuable contribution to this space, adding 200K samples of grade level synthetic math problems.

With this report, we contribute the NuminaMath-CoT dataset, which consists of 860K problem-solution pairs covering a wide range of mathematical topics, from high school exercises to competition-level problems. The data sources include Chinese high school math exercises, US and international mathematics olympiad problems, and problems collected from online forums. It is the largest math problems dataset ever released.

Dataset Name	**Number of Samples for Training**	**Number of Competition Level Samples**
MATH	7500	7500
GSM8K	7470	0
orca-math	200K	0
NuminaMath-CoT	860K	400K

Table 1: Datasets and Sample Sizes

3.1 Data Collection and Processing

The data collection process involved acquiring data from the internet, extracting problems from PDFs using OCR, segmenting them into problem-solution pairs, translating them into English, and realigning them to produce a chain-of-thought reasoning format. The final answers were formatted to ensure consistency and accuracy. Here we describe the data collection and processing method by data source. The specific processing method will be detailed in the next subsection.

- **MATH and GSM8K:** We follow the recommendation of deepseek-math [Shao et al., 2024] and realign [Fan et al., 2024], use gpt4 to reformat the reference solution of the original dataset in the CoT format.
- **Orca-Math:** orca-math is a very nice large scale grade school level synthetic dataset. We apply a regex to simplify identify the answer of the provided solution in the original orca-math dataset, and use *boxed* to enclose the final answer. If you already have a copy of the orca-math dataset in your training data, or a better formatted version, it is not necessary to include this part of the NuminaMath dataset. For example, https://huggingface.co/datasets/PawanK gpt-4o-200k could be a better alternative, where the final solution is written by gpt4-o.
- **AMC and AMIE:** AMC and AMIE are the one of the most popular competition problem set. It covers different levels of difficulties, from grade school competition to almost IMO level; it includes various topics in math, from arithmetic, geometry, combinatorics, to algebra; Finally, problem from AMC or AIME has clear value output, either being a multi choice (AMC) or integer (AIME). We collected AMC and AMIE problem statements from the aops wiki website (https://artofproblemsolving.com/wiki/). The problem is already in latex format. Several good solutions are proposed by the aops community for each problem. We then select the first one with a *boxed* symbol in the solution.

  This dataset accounts for around 6500 problems, however, not all of them can be used in the training set. Part of this data has been used to create the MATH dataset [Hendrycks et al., 2021]. We have to perform a particular decontamination process using LLM embedding (see section decontamination), which results in around 4300 problems in the training set.

  Finally, we apply the same method to realign the reference solution of the training data into CoT format using gpt4.

- **Aops Forum:** The aops forum alos has a large collection of competition problems across the world. We have crawled all the data from the Contest Collection page of the website.

  However, the reference solution is not collected in the forum. Instead, replies might contain one good solution or at least hint at the solution. We pick all replies with a *boxed* or *blacksqure* symbol, then choose the one with the most latex symbols.

  Once we pick the best reply, we consider it as a reference solution and feed it into the realignment prompt for gpt4 to rewrite the solution as before.

- **Chinese k12 Exam:** We follow [Shao et al., 2024]'s fine tuning method, to collect chinese k12 education math exercises. These exercises are not competition level but will provide a solid base knowledge and good arithmetic skills for the model to learn.

  We retrieve all our exercises from public exam papers. Most of them can be found and downloaded from https://www.shijuan1.com/ or [He et al., 2023].

  When the exam paper comes as a PDF format, we apply OCR and regex segmentation to extract problem solution pairs. Then use gpt4 to translate and realign the reference solution.

- **Synthetic Data:** We follow [Li et al., 2024] to create synthetic math data using the MATH dataset and the training split of AMC-AMIE dataset.

  In [Li et al., 2024] the synthetic problem is first sampled using a seed problem by gpt4, with a high temperature (0.8). In the second stage, a new solution is created with greedy decoding. In our approach, we directly use the solution of the first stage for cost saving reasons.

- **World Olympiads Data:** We collect 152k problem-solution pairs from the following sources

  - International contests and their respective shortlists (IMO, APMO, BMO, etc.)
  - National and regional contests with a breakdown per country shown at Figure 2
  - Problem-solving forums, puzzle and olympiad books, summer school materials

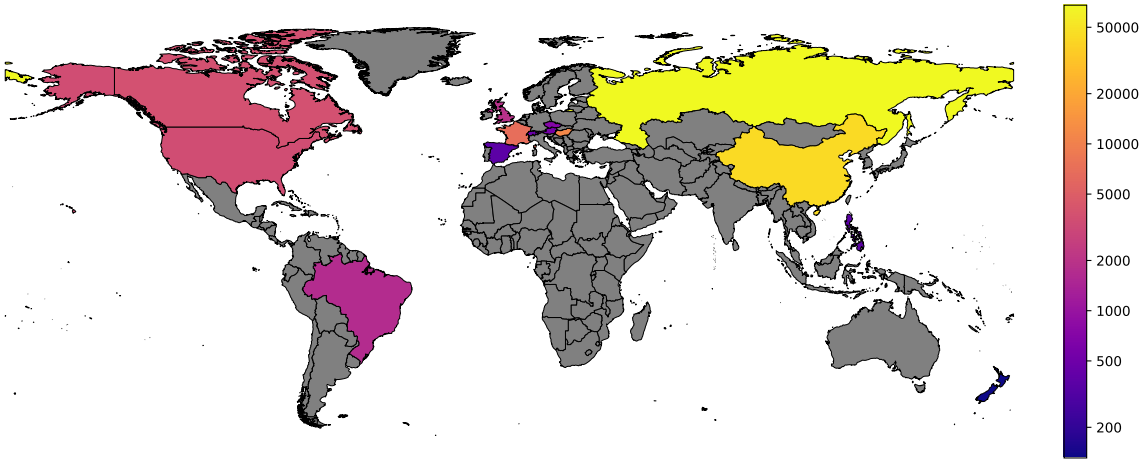

Figure 1: Number of collected national contests problems per country

Typically, files containing problems were found in PDF or scraped from sites in HTML format. In the latter case, they were converted to PDF for the correct formula representation. Then the full pipeline described in the next section was employed.

Combining all these data together, we then get the NuminaMath-CoT dataset. Here is the breakdown by source:

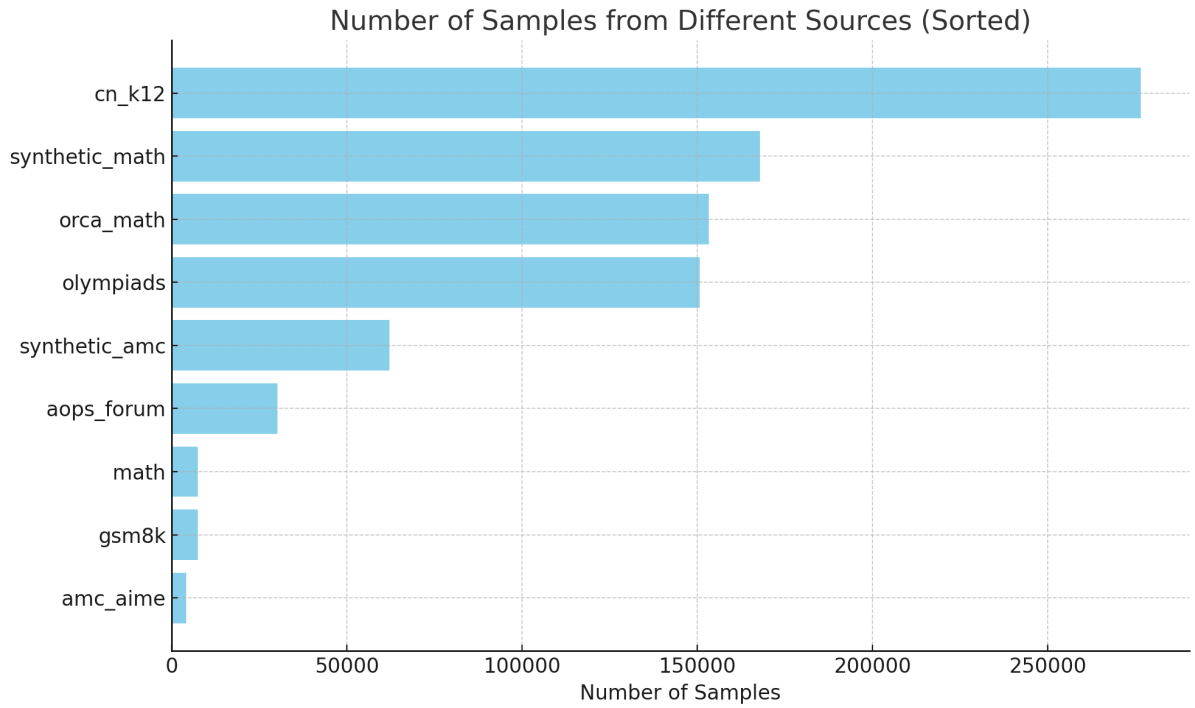

Figure 2: Number of samples per data source

3.2 Processing Pipeline

The purpose of the processing pipeline is to convert the original solution to a uniform format: English, CoT, and a conclusion either with *boxed* or with ■.

The data processing pipeline contains multiple steps.

- **OCR from PDFs:** Most of the raw data comes in PDF format. We use the API of https://mathpix.com/ to convert to pdf to markdown format. Depending on the segmentation strategy, we either convert the whole pdf (a sequence of problems and solutions) into text or convert a well segmented problem (single problem) into markdown text.

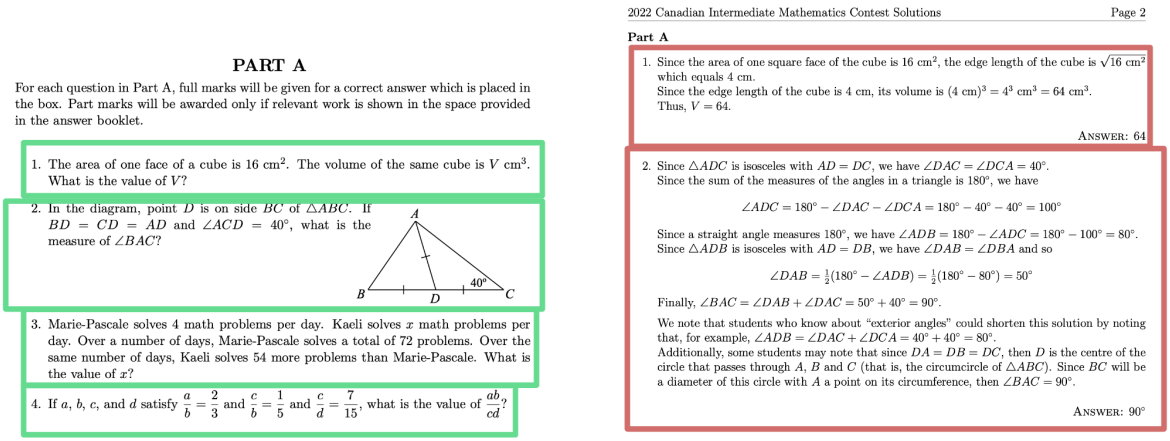

Figure 3: Illustration of manual segmentation

- **Problem Solution Segmentation:** Part of World Olympiads Data was segmented by matching the starts of problems and solutions with a corresponding regex that was engineered manually for the chosen contests. Another part, that was hard to match with regex, was segmented manually at the PDF level and then converted to text with OCR.
- **Reformatting:** We use gpt-4o to reformat the extracted text solution. We use the batch API of gpt-4o to perform the following steps:
    1. Translate the solution to English
    2. CoT Realignment
    3. Box the final answer

3.3 NuminaMath-TIR

We have first created the NuminaMath-CoT dataset, so to

Once we have created NuminaMath-TIR, it is straightforward to create the TIR (tool integrated reasoning). We follow the same approach of [Gou et al., 2024] and especially their prompt to sample TIR data from gpt-4o. Here is the process to create this TIR dataset:

1. Extract a subset of around 100K problems with value output from the NuminaMath-CoT dataset.
2. We sample a solution using the gpt-4o assistant API for each problem with a temperature of 0.8.
3. We filter the negative sample where the model generated answer did not match the reference answer. For integer output problem we use exact match, for other expressions, we use gpt as a judge to determine the match.
4. Repeat the same process on the negative problems.

3.4 Decontamination

A good internal validation set is one of the keys to our success in winning the AIMO Progress Prize. To reflect the true performance of the models, we need to perform decontamination with a lot of caution. We have chosen the well known benchmarks, MATH and GSM8K, as well as AMIE 2024 and AMC12 2023 to evaluate our model. We propose the following two steps decontamination strategies.

- **Exact string match:** we follow [Shao et al., 2024] to remove 10-gram exact match from all datasets, except the synthetic dataset, the MATH train set. Because the synthetic dataset are created from seed datasets where the decontamination is already conducted. We notice that there are a fraction of problems in the MATH test set, just the paraphrasing of the MATH train set with different numbers. We do consider this not as contamination (so do the synthetic data generated using MATH).
- **Nearest neighbor search using embedding:** We have seen increasing use of the AIME and AMC problems as part of training or finetuning set for math LLM [Zhou et al., 2024]. However, we need to take extra caution to decontaminate this dataset. The MATH test set is made using an important part of the AIME-AMC exercises Hendrycks et al. [2021], where some problems are modifications of the original problem to facilitate the parsing. The MATH test set will add statements such as $express the final result as \frac{m}{n}, compute m+n$ which makes exact string matches unreliable.

Worse, in our case, as we have collected the olympiad dataset all over the world, some of the problems that we have collected, are the translated version of AMC-AIME problems.

To better decontaminate, we compute the mistral embedding for each of our problems except for MATH train and the synthetic datasets, then we remove all the problems with an embedding distance smaller than 0.15. This is an empirical value, above which we did not observe contamination.

Here is an example:

Problem from AIME:	**Problem from MATH test:**
A sphere is inscribed in the tetrahedron whose vertices are $A = (6,0,0), B = (0,4,0), C = (0,0,2)$, and $D = (0,0,0)$. The radius of the sphere is $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.	A sphere is inscribed in the tetrahedron whose vertices are $A = (6,0,0)$, $B = (0,4,0)$, $C = (0,0,2)$, and $D = (0,0,0)$. Find the radius of the sphere.
The original problem in 2001 AIME I Problems/Problem 12	The modified version in the MATH test set

4 Experiments

We conducted several experiments to evaluate the effectiveness of the Numina dataset in training AI models. Our models were fine-tuned using a two-stage process inspired by the MuMath-Code paper.

4.1 Fine-Tuning Stages

- **Stage 1:** Fine-tuning on a large, diverse dataset of natural language math problems and solutions, with Chain of Thought (CoT) annotations to facilitate reasoning.
- **Stage 2:** Fine-tuning on a synthetic dataset of tool-integrated reasoning, where problems are decomposed into rationales, Python programs, and outputs.

We trained models at two scales:

- 7B, based on DeepSeekMath-Base 7B
- 72B, based on Qwen2-72B

All models were trained with a global batch size of 32, a learning rate of 2×10^{-5}, and a cosine scheduler. Training was run on 8-32 H100 GPUs, depending on the model size, and took less than 24 hours for both the stages. Due to time constraints, the NuminaMath 72B model was only trained for 2 epochs in Stage 1. Further details of hyper-parameters are shown in table 2.

4.2 Tool-integrated reasoning (TIR)

At inference time, TIR models have been trained to leverage interleaved CoT rationales alongside Python code generation, execution, and repair, as outlined in Algorithm 1, and described below. The complete implementation can be found in the linked public GitHub repository.

Each run of TIR begins with a problem, x, from a dataset, $\mathcal{X}$. The goal of TIR is to sample a candidate solution, y, where the valid solution space may vary between datasets, e.g., $\mathcal{Y}_{\texttt{AIMO}} = \{i \in \mathbb{N} \mid 0 \leq i \leq 999\}$. TIR begins by initializing a context, $\boldsymbol{c}$, with an initial prompt $\boldsymbol{c}_0$

Table 2: Hyper-parameters used in the experiments

	NuminaMath-7B		**NuminaMath-15B**		**NuminaMath-70B**	
stage	CoT	TIR	CoT	TIR	CoT	TIR
block size	2048	1024	2048	2048	2048	2048
epochs	3	4	4	5	1	3
warmup ratio	0	0.1	0.1	0.1	0.1	0.1

containing only x. This context is then extended through up to k rounds of interaction. On each iteration, i, TIR uses a sampler, $\texttt{S}$, and an LLM, θ, to sample text containing CoT and Python source code, $\boldsymbol{z}_i$, until reaching the stop keyword $w_{\texttt{stop}} = \texttt{```output}$. After sampling $\boldsymbol{z}_i$, TIR first checks if a candidate solution has been generated, which would be wrapped in the keyword $w_{\texttt{answer}} = \texttt{\boxed\{\}}$. If an answer is present, TIR applies a response parser, $\texttt{R}$, to the output, which acts to sanitize the text and return only the final numerical response with any units and other formatting removed. If no valid response is present, TIR assesses whether any code has been generated by matching a regular expression with the python region keyword $w_{\texttt{python}} = \texttt{```python(.*)```}$. If no such region is available, $\boldsymbol{z}_i$ is discarded, and TIR proceeds to the next iteration, resampling a fresh block of text. If such a region is available, the Python source code is passed to the Python interpreter, $\texttt{I}$, which parses and executes the source code. The result, $\boldsymbol{r}_i$ from running $I(\boldsymbol{z}_i)$ may include the output of print statements, or a truncated `Traceback` if an exception was raised. The running context is then extended, proceeding to the next round of interaction, via setting $\boldsymbol{c}_i$ to $\boldsymbol{c}_{i-1} \oplus \boldsymbol{z}_i \oplus \boldsymbol{r}_i$, where $\oplus$ denotes concatenation. Thus, by the end of interaction, $\boldsymbol{c} = \boldsymbol{c}_0 \boldsymbol{z}_1 \boldsymbol{r}_1 \boldsymbol{z}_2 \boldsymbol{r}_2 \ldots \boldsymbol{z}_{\leq k}$, where either a candidate answer, y, is successfully extracted from $\boldsymbol{z}_{\leq k}$ else an error keyword $w_{\texttt{error}}$, e.g., -1 for $\mathcal{Y}_{\texttt{AIMO}}$, is returned.

Algorithm 1 Tool-integrated reasoning (TIR)

Require: LLM θ, sampler $\texttt{S}$, interpreter $\texttt{I}$, response parser $\texttt{R}$, problem x, blocks k, keywords w

$\boldsymbol{c}_0 \leftarrow x$ ▷ Initialize context to problem text
for $i \leftarrow 1 : k$ **do**
  $\boldsymbol{z}_i \sim \texttt{S}\left(p_\theta\left(\cdot \mid \boldsymbol{c}_{i-1}\right), w_{\texttt{stop}}\right)$ ▷ Sample CoT and code until stop token
  if $w_{\texttt{answer}}$ in $\boldsymbol{z}_i$ **then** ▷ Parse response and return answer if generated
    $y \leftarrow \texttt{R}\left(\boldsymbol{z}_i\right)$
    return y
  else if $w_{\texttt{python}}$ not in $\boldsymbol{z}_i$ **then** ▷ Regenerate block if code missing
    continue
  end if
  $\boldsymbol{r}_i \leftarrow \texttt{I}\left(\boldsymbol{z}_i\right)$ ▷ Strip code, run interpreter, and return output or traceback
  $\boldsymbol{c}_i \leftarrow \boldsymbol{c}_{i-1} \oplus \boldsymbol{z}_i \oplus \boldsymbol{r}_i$ ▷ Update context with generated text and interpreter result
end for
return $w_{\texttt{error}}$ ▷ If can't generate an answer in k blocks

4.2.1 Incorporating self-consistency (SC-TIR)

TIR (Algorithm 1) describes the process for sampling a single candidate solution y for an initial problem x. However, TIR does not proceed through direct samples from the conditional distribution $p_\theta\left(y \mid x\right)$, which would correspond to a direct prediction of the solution. Rather, TIR jointly draws a sequence of samples from an auxiliary latent variable, the trace of generated CoT planning and Python source code. This sequence is denoted $\boldsymbol{z}$. Generations drawn from TIR thus correspond to samples from $p_\theta\left(y, \boldsymbol{z} \mid x\right)$. In order to effectively calculate $p_\theta\left(y \mid x\right)$,

it is thus necessary to marginalize out the generated traces, which can be achieved via the following summation:

$$p_\theta(y \mid x) = \sum_{z \in \mathcal{Z}} p_\theta(y, z \mid x)$$

In the context of latent variable modeling, this equation is typically referred to as the marginal likelihood, and is often computationally infeasible when $\mathcal{Z}$ is large, as is the case here. This type of scenario occurs frequently in practice, and various approximation strategies exist. Most notably in the context of marginalizing LLM reasoning traces, Wang et al. [2022] propose self-consistency (SC), which approximates the marginalization and application of a maximum a-posteriori (MAP) decision rule by drawing a finite number n of samples $\boldsymbol{y}$ from $p_\theta(y, z \mid x)$ and then applying a majority voting procedure:

$$y_{SC} = \operatorname*{argmax}_y \sum_{i=1}^{n} \mathbb{1}(\boldsymbol{y}_i = y)$$

In the case of SC-TIR, we generate n samples from TIR, then apply a filter, `F`, which removes ill-formed responses outside the support of $\mathcal{Y}$, and finally apply self-consistency majority voting, as outlined in Algorithm 2.

Algorithm 2 Self-consistent TIR (SC-TIR)

Require: subroutine `TIR`, response filter `F` , problem x, number of samples n

$\boldsymbol{y} \leftarrow \texttt{empty}(n)$
for $i \leftarrow 1 : n$ **do** ▷ Sample n independent TIR trajectories from problem
  $\boldsymbol{y}_i \sim \texttt{TIR}(x)$
end for
$y_{SC} \leftarrow \texttt{maj}(\texttt{F}(\boldsymbol{y}))$ ▷ Filter invalid answers and apply majority vote decision rule
return y_{SC}

4.3 Evaluation Metrics

We evaluated our models using various metrics, including accuracy on the MATH benchmark and performance on internal validation sets consisting of AMC and AIME problems. The results demonstrated significant improvements in problem-solving capabilities, particularly in handling complex reasoning tasks.

		NuminaMath 7B		Qwen2 7B	Llama3 8B	DeepSeekMath 7B		DART-Math 7B
		CoT	TIR	Instruct	Instruct	Instruct	RL	CoT
GSM8k Grade school math	0-shot	76.3%	84.6%	82.3%	79.6%	82.8%	88.2%	86.6%
MATH Math problem solving	0-shot	55.8%	68.1%	49.6%	30.0%	46.8%	51.7%	53.6%
AMC 2023 Competition-level math	0-shot maj@64	11/40 18/40	20/40 31/40	10/40 13/40	2/40 9/40	7/40 13/40	9/40 14/40	11/40 16/40
AIME 2024 Competition-level math	0-shot maj@64	0/30 1/30	5/30 10/30	1/30 4/30	0/30 2/30	1/30 1/30	1/30 1/30	1/30 1/30

Table 3: Comparison of various 7B and 8B parameter language models on different math benchmarks. All scores except those for NuminaMath 7B TIR are reported without tool-integrated reasoning.

		NuminaMath 72B CoT	NuminaMath 72B TIR	Qwen2 72B Instruct	Llama3 70B Instruct	Claude 3.5 Sonnet	GPT-4o 0513
GSM8k Grade school math	0-shot	91.4%	91.5%	91.1%	93.0%	96.4%	95.8%
MATH Math problem solving	0-shot	68.0%	75.8%	59.7%	50.4%	71.1%	76.6%
AMC 2023 Competition-level math	0-shot maj@64	21/40 24/40	24/40 34/40	19/40 21/40	13/40 13/40	- -	20/40 -
AIME 2024 Competition-level math	0-shot maj@64	1/30 3/30	5/30 12/30	3/30 4/30	0/30 2/30	- -	2/30 -

Table 4: Comparison of various open weight and proprietary language models on different math benchmarks. All scores except those for NuminaMath 72B TIR are reported without tool-integrated reasoning.

5 Conclusion and Future Works

The NuminaMath dataset is the largest math dataset even released. It represents a significant step forward in the development of AI models capable of solving complex mathematical problems. By providing detailed, step-by-step solutions, the dataset facilitates deeper understanding and reasoning. The successful use of this dataset in winning the Progress Prize underscores its potential in advancing the capabilities of AI in mathematical problem-solving.

Future work will focus on several key areas to enhance the dataset and its applications:

- **Expanding the Dataset:** We aim to increase the diversity and quantity of problems in the dataset, particularly by incorporating more problems from various international mathematics competitions and higher education levels. This expansion will help in creating a more robust training resource for AI models.
- **Refining the Annotation Process:** Improving the quality and accuracy of annotations, especially for complex problems, is crucial. Further refinement of the chain-of-thought (CoT) annotations will enhance the reasoning capabilities of the models trained on this dataset.
- **Integrating Tool-Based Reasoning:** Exploring new methods for integrating tool-based reasoning, such as incorporating more advanced code interpreters and mathematical solvers, will push the boundaries of what AI models can achieve in terms of problem-solving.
- **Synthetic Data Validation:** In our synthetic data generation pipeline, we use a single-stage approach with GPT-4O at high temperature (T=0.8). While this generates diverse problems and solutions, it is challenging to validate their correctness. Future work will involve developing methods to further annotate or modify these synthetic problems to improve fine-tuning performance.
- **Improving CoT Solutions for Complex Problems:** For some very challenging problems, the current CoT solutions may lack sufficient detail. Further cleaning up and validating these annotations, possibly with the assistance of domain experts, will improve model performance.
- **Enhancing Instruction Following:** Models fine-tuned on the Numina dataset may lose some capacity for general instruction following. Addressing this issue will involve balancing the dataset to maintain the ability for few-shot learning and general instruction compliance.

- **Minimizing Dependency on Proprietary Models:** The dataset's creation heavily relies on GPT-4O, which introduces some restrictions. Future efforts will focus on utilizing more open models to perform necessary tasks, thereby reducing dependency on proprietary solutions.
- **Leveraging Proof Problems:** Approximately 40% of the dataset consists of proof-based problems, particularly from the IMO. Future work will explore translating these problems into formal mathematical proofs, fully leveraging their potential to enhance the reasoning capabilities of AI models.
- **Preventing Data Contamination:** To prevent training data contamination, we included canary strings in the dataset. This practice will be continued and refined in future iterations to ensure the integrity of the dataset.

We invite contributions from the community to further enhance the dataset and its applications. Collaboration will be key to overcoming the current limitations and advancing the state of AI in mathematical reasoning.

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