The document serves as a comprehensive resource on the mathematical principles underlying life insurance, primarily derived from Hans U. Gerber's "Life Insurance Mathematics." It emphasizes the transition from deterministic to probabilistic models in actuarial mathematics, beginning with an overview of compound interest and extending to the future lifetime of individuals, various forms of life insurance, and life annuities. Key topics include net premiums, net premium reserves, and the implications of multiple decrements in life insurance policies. The text discusses the calculation of total claim amounts in a portfolio, expense loadings, and methods for estimating probabilities of death, including classical and Bayesian approaches.

The document elaborates on financial concepts related to compound interest, including effective and nominal interest rates, and the modeling of funds with continuous contributions. It introduces differential equations to describe fund growth and discusses interest in advance. The exploration of perpetuities and annuities is detailed, with formulas for calculating present values and the relationship between annuities and perpetuities. The repayment of debts is also addressed, illustrating how present values relate to outstanding debts.

In terms of life insurance, the document categorizes various types, such as whole life, term, and endowment insurance, detailing the calculation of net single premiums based on expected present values of benefits. It emphasizes the importance of understanding the timing of benefit payments and the impact of interest rates on these calculations. The text also covers life annuities, defining them as sequences of payments for the duration of a beneficiary's life, and provides formulas for calculating their present value and net single premium.

The document further discusses net premiums, defined as those that satisfy the equivalence principle, ensuring that the expected loss to the insurer is zero. It includes examples of calculating net annual premiums based on mortality assumptions and the present value of benefits. The treatment of interest rates in insurance is noted, with a fixed interest rate assumption typically used

for calculations.

Additionally, the document outlines the net premium reserve, defined as the conditional expectation of the liability given that the time exceeds a certain point. It emphasizes the need for insurers to maintain adequate reserves to cover future liabilities, detailing the recursive relationships for calculating reserves in various types of insurance. The analysis extends to multiple life insurance, introducing joint-life and last-survivor statuses, and provides formulas for calculating net single premiums for various benefits.

The document also addresses statistical methods for estimating probabilities of death, discussing classical and Bayesian approaches, and the challenges of incomplete observations. It highlights the importance of life tables and commutation functions in actuarial calculations, providing a framework for understanding the financial dynamics of insurance policies.

Overall, the document serves as a detailed guide to the mathematical foundations of life insurance and annuities, emphasizing the interplay between mortality rates, present value calculations, and the application of actuarial principles in determining premiums and benefits. It includes exercises and practical applications to reinforce the concepts discussed, making it a valuable resource for students and practitioners in the field of actuarial science.