Assignment 2

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1 First Half

1. Find a mixed strategy Nash equilibrium using Lemke-Howson algorithm for the game below:

events	opera	football
opera	4,2	3,1
football	1,3	2,4

Initiate LH Algorithm:

$$A = \begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}, B = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}, L(x) = \{1, 2\}, L(y) = \{3, 4\}$$

P	x1	x2	t3	t4	=
3	2	3	1	0	1
4	1	4	0	1	1

Q	r1	r2	y3	y4	=
1	1	0	4	3	1
2	0	1	1	2	1

Second step:

$$L(x) = \{2, 3\}, L(y) = \{3, 4\}$$

P	x1	x2	t3	t4	=
3	2	3	1	0	1
4	0	5	-1	2	1

Q	r1	r2	y3	y4	=
1	1	0	4	3	1
2	0	1	1	2	1

Third step:

$$L(x) = \{2, 3\}, L(y) = \{1, 4\}$$

P	x1	x2	t3	t4	=
3	2	3	1	0	1
4	0	5	-1	2	1

Q	r1	r2	y3	y4	=
1	1	0	4	3	1
2	-1	4	0	5	3

So we have $x_1^* = 1, x_2^* = 0, y_1^* = 1, y_2^* = 0$

2. Find a mixed strategy Nash equilibrium using Linear Programming for the game below. Also compare your solution with one obtained using Lemke-Howson algorithm.

events	opera	football
opera	4,-4	-3,3
football	-1,1	2,-2

We have:

$$A = \begin{pmatrix} 4 & -3 \\ -1 & 2 \end{pmatrix}$$

Set the max value as v :

$$\begin{cases} 4x_1 - x_2 \geq v \\ -3x_1 + 2x_2 \geq v \end{cases} \rightarrow \begin{cases} 4x_1/v - x_2/v \geq 1 \\ -3x_1/v + 2x_2/v \geq 1 \end{cases}$$

Suppose $p_1 = x_1/v, p_2 = x_2/v \rightarrow p_1 + p_2 = 1/v$

Minimize $1/v \rightarrow \begin{cases} x_1 = 3/10 \\ x_2 = 7/10 \\ v = 1/2 \end{cases}$

Set the min value as w :

$$\begin{cases} 4y_1 - 3y_2 \leq w \\ -y_1 + 2x_2 \leq w \end{cases} \rightarrow \begin{cases} 4y_1/w - 3y_2/w \leq 1 \\ -y_1/w + 2x_2/w \leq 1 \end{cases}$$

Suppose $q_1 = y_1/w, q_2 = y_2/w \rightarrow q_1 + q_2 = 1/w$

Minimize $1/w \rightarrow \begin{cases} y_1 = 1/2 \\ y_2 = 1/2 \\ w = 1/2 \end{cases}$

3. Implement Lemke-Howson Algorithm (and to solve the above problem, using Pythen or Java). Test it using the above problems.

The python file is in attachment. We get the same result. The algorithm is also suitable for lager matrix.

2 Second Half

Let f $(\cdot)$ be continuous. Is there always a fixed point in the following? Prove it or give a counter example.

1. Continuous Fixed Point $f(x) : [0, 1] \rightarrow [0, 4], f(0) = 1, f(1) = 3$.

   No.

   Couter example: $f(x) = 2x + 1$

2. Continuous Fixed Point $f(x) : [0, 1] \rightarrow [0, 1], f(0) > 0, f(1) < 1$.

   Yes.

   Construct a function $g(x) = f(x) - x$, then we have $g(0) \leq 0, g(1) \leq 0$.

   So there is a point $x^* \subset (0, 1)$ that $g(x^*) = 0$ since g is continuous.

   $x^*$is the fixed point.

3. Continuous Fixed Point $f(x) : [0, 1] \rightarrow [1/3, 1/2]$

   Yes.

   Construct a function $g(x) = f(x) - x$, then we have $g(x) \subset (-1/2, 1/2)$.

   So there is a point $x^* \subset (0, 1)$ that $g(x^*) = 0$ since g is continuous.

   $x^*$is the fixed point.

4. Continuous Fixed Point $f(x) : [0, 1) \rightarrow [0, 1)$.

   Yes.

   Construct a function $g(x) = f(x) - x$, then we have $g(x) \subset (-1, 1)$.

   So there is a point $x^* \subset (0, 1)$ that $g(x^*) = 0$ since g is continuous.

   $x^*$is the fixed point.

5. Continuous Fixed Point $f(x) : [0, \infty) \rightarrow [0, \infty)$

   Yes.

   Construct a function $g(x) = f(x) - x$, then we have $g(x) \subset (-\infty, \infty)$.

   So there is a point $x^* \subset (0, \infty)$ that $g(x^*) = 0$ since g is continuous.

   $x^*$is the fixed point.