Assignment 3

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1 First Half

1.1 Do one of the followings:

1. Maximum Market Equilibrium Dynamics: Starting at truthful biddings, how would the player change their bids? Assuming best response, would the process converge?

2. Prove or disprove that the maximum revenue market equilibrium is a symmetric Nash Equilibrium in GSP.

3. What is the maximum revenue symmetric Nash equilibrium in GSP?

Question 1:
The players will always try to increase his utility. There are two possibilities :

- give higher bid and get more quality ;

- give lower bid and pay less.

When there are more than one items of different qualities, the process doesn't converge.
When there are only one item or all the items have the same quality, the process converge. For the market of N bidders and M items, the bids of the first M bidders converge to v_{M+1}

1.2 Do two of the followings:

1. How much one can gain from VCG to maximum VCG?

2. Give Examples where maximum VCG sells only one item. extend it to $k = 2, 3, ..., m$ items.

3. Call an case where only one item is sold VCG_1, VCG_k for k items where $k = 2, 3, ..., m$. How to compute them?

4. Compare maximum VCG revenue to $VCG_1, VCG_2, ..., VCG_k$. Derive the best bound you may obtain.

5. Design a truthful mechanism that derives a best bound in revenue against that of VCG_1. Compare your revenue against that of VCG_1. Extend to VCG_k, $k = 1, 2, ..., m$.

- Question 2:
  For the case k = 1, we have example :
  $$N = \left\{11, 7, 3, 2, 1\right\}, M = \left\{5, 1\right\}$$
  where we get the maximazation at $VCG(1,1) = 35$

  It can be extended by adding v_1, q_1 to to the vertors every time:
  For example, if k = 2 , $N = \left\{v_1, 11, 7, 3, 2, 1\right\}, M = \left\{q_1, 5, 1\right\}$ where $v_1 > 11$ and $q_1 > 5$.

- Question 3:
  To extend the conclusion, for maximum VCG, we have
  $$VCG(i, j_i) = VCG(i-1, j_{i-1}) + q_{j_i}[iv_{i+1} - (i-1)v_i]$$
  If it sells only k items, we have
  $$VCG(k+1, j_{k+1}) = VCG(k, j_k) + q_{j_{k+1}}[(k+1)v_{k+2} - kv_{k+1}] < VCG(k, j_k)$$
  Which gives the condition :
  - $iv_{i+1} - (i-1)v_i > 0$ for $i <= k$
  - $(k+1)v_{k+2} - kv_{k+1} < 0$ if there are more than k items