Assignment 2

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1 First Half

1. Find a mixed strategy Nash equilibrium using Lemke-Howson algorithm for the game below:

events	opera	football
opera	4,2	3,1
football	1,3	2,4

Initiate LH Algorithm:

$$A = \begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}, B = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}, L(x) = \{1, 2\}, L(y) = \{3, 4\}$$

P	x1	x2	t3	t4	=
3	2	3	1	0	1
4	1	4	0	1	1

Q	r1	r2	y3	y4	=
1	1	0	4	3	1
2	0	1	1	2	1

Second step:

$$L(x) = \{2, 3\}, L(y) = \{3, 4\}$$

P	x1	x2	t3	t4	=
3	2	3	1	0	1
4	0	5	-1	2	1

Q	r1	r2	y3	y4	=
1	1	0	4	3	1
2	0	1	1	2	1

Third step:

$$L(x) = \{2, 3\}, L(y) = \{1, 4\}$$

P	x1	x2	t3	t4	=
3	2	3	1	0	1
4	0	5	-1	2	1

Q	r1	r2	y3	y4	=
1	1	0	4	3	1
2	-1	4	0	5	3

So we have $x_1^* = 1, x_2^* = 0, y_1^* = 1, y_2^* = 0$

2. Find a mixed strategy Nash equilibrium using Linear Programming for the game below. Also compare your solution with one obtained using Lemke-Howson algorithm.

events	opera	football
opera	4,-4	-3,3
football	-1,1	2,-2

We have:

$$A = \begin{pmatrix} 4 & -3 \\ -1 & 2 \end{pmatrix}$$

Set the max value as v :

$$\begin{cases} 4x_1 - x_2 \geq v \\ -3x_1 + 2x_2 \geq v \end{cases} \rightarrow \begin{cases} 4x_1/v - x_2/v \geq 1 \\ -3x_1/v + 2x_2/v \geq 1 \end{cases}$$

Suppose $p_1 = x_1/v, p_2 = x_2/v \rightarrow p_1 + p_2 = 1/v$

Minimize $1/v \rightarrow \begin{cases} x_1 = 3/10 \\ x_2 = 7/10 \\ v = 1/2 \end{cases}$

Set the min value as w :

$$\begin{cases} 4y_1 - 3y_2 \leq w \\ -y_1 + 2x_2 \leq w \end{cases} \rightarrow \begin{cases} 4y_1/w - 3y_2/w \leq 1 \\ -y_1/w + 2x_2/w \leq 1 \end{cases}$$

Suppose $q_1 = y_1/w, q_2 = y_2/w \rightarrow q_1 + q_2 = 1/w$

Minimize $1/w \rightarrow \begin{cases} y_1 = 1/2 \\ y_2 = 1/2 \\ w = 1/2 \end{cases}$

3. Implement Lemke-Howson Algorithm (and to solve the above problem, using Pythen or Java). Test it using the above problems.

The result holds since our classes were arbitrary and distinct.

2 Second Half

Let f $(\cdot)$ be continuous. Is there always a fixed point in the following? Prove it or give a counter example.

1. Continuous Fixed Point $f(x) : [0, 1] \to [0, 4], f(0) = 1, f(1) = 3$.

   No.

   Couter example: $f(x) = 2x + 1$

2. Continuous Fixed Point $f(x) : [0, 1] \to [0, 1], f(0) > 0, f(1) < 1$.

   Yes.

   Construct a function $g(x) = f(x) - x$, then we have $g(0) \leq 0, g(1) \leq 0$.

   So there is a point $x^* \subset (0, 1)$ that $g(x^*) = 0$ since g is continuous.

   $x^*$is the fixed point.

3. Continuous Fixed Point $f(x) : [0, 1] \to [1/3, 1/2]$

   Yes.

   Construct a function $g(x) = f(x) - x$, then we have $g(x) \subset (-1/2, 1/2)$.

   So there is a point $x^* \subset (0, 1)$ that $g(x^*) = 0$ since g is continuous.

   $x^*$is the fixed point.

4. Continuous Fixed Point $f(x) : [0, 1) \to [0, 1)$.

   Yes.

   Construct a function $g(x) = f(x) - x$, then we have $g(x) \subset (-1, 1)$.

   So there is a point $x^* \subset (0, 1)$ that $g(x^*) = 0$ since g is continuous.

   $x^*$is the fixed point.

5. Continuous Fixed Point $f(x) : [0, \infty) \to [0, \infty)$

   Yes.

   Construct a function $g(x) = f(x) - x$, then we have $g(x) \subset (-\infty, \infty)$.

   So there is a point $x^* \subset (0, \infty)$ that $g(x^*) = 0$ since g is continuous.

   $x^*$is the fixed point.