Question of L4 for Midterm

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1 Linear Utility Market Equilibrium

Q1.1: Given a market where we have : 3 markets agents $M = \{1, 2, 3\}$, 5 goods at the market $N = \{1, 2, 3, 4, 5\}$. The initial endowment of the agents $w^1 = (0, 2, 0, 1, 0)$, $w^2 = (0, 2, 1, 0, 1)$, $w^3 = (1, 0, 0, 3, 0)$, $u_1(x^1) = 3x_1^1, u_2(x^2) = 4x_2^2 + 2x_3^2, u_3(x^3) = 2x_2^3 + x_4^3$. Given $p = (p_1, p_2, ..., p_5)$, give the expressions to get x^*, the optimal x for every agent. Explain the property: If $x_j^{i*} > 0$, then $\forall t \;\&\; p_t > 0 : \frac{u_j^i}{p_j} \geq \frac{u_t^i}{p_t}$

Q1.2: Do nomalization and atomization of the market. Why the demand graph is non-zero indegree ?

Q1.3: What are the differences betwee the initial setting of "Linear Utility Market" and "Fisher Market"?

Q1.1:

- Budget constraint :

  $px^i \leq pw^i$

- Individual optimality :

  $x^{i*} \in argmax\{u_i(x^i) : px^i \leq pw^{iT}, x^i \geq 0\}$, where $u^1 = [3, 0, 0, 0, 0]^T$ $u^2 = [0, 4, 2, 0, 0]^T$ $u^3 = [0, 2, 1, 0, 0]^T$

- Market clearance:

  $\sum_{i \in M} x^i \leq \sum_{i \in M} w^{iT}$

To get the maximum utility, the agent will put his money on the good where he get most utility with every unit of money since his budget constraints to the equation $px^i \leq pw^i$. So if $x_j^{i*} > 0$, that means $\frac{u_j^i}{p_j}$ is the maximum.

Q1.2:

- Normalization:
  - Everything is owned by someone: Condition fulfilled.
  - Everything is liked by someone: Elimite the 5_{th} good. Then we have 4 goods at the market $N = \{1,2,3,4\}$. The initial endowment of the agents $w^1 = \left(0,2,0,1\right), w^2 = \left(0,2,1,0\right), w^3 = \left(1,0,0,3\right)$,
  - Normalization: For the 2_{nd} and 4_{th} good, divide by total amount. The initial endowment of the agents $w^1 = \left(0,1/2,0,1/4\right), w^2 = \left(0,1/2,1,0\right), w^3 = \left(1,0,0,3/4\right)$,
- Atomization:
  - Every agent owns one item : replace agent 1, 2, 3 by $1_1, 1_2, 2_1, 2_2, 3_1, 3_2,$, then we have 6 markets agents $M = \{1_1, 1_2, 2_1, 2_2, 3_1, 3_2\}$, where $\begin{cases} u^{1_1} = \left[3,0,0,0\right]^T \\ u^{1_2} = \left[3,0,0,0\right]^T \\ u^{2_1} = \left[0,4,2,0\right]^T \\ u^{2_2} = \left[0,4,2,0\right]^T \\ u^{3_1} = \left[0,2,0,1\right]^T \\ u^{3_2} = \left[0,2,0,1\right]^T \end{cases}$

    The initial endowment of the agents $\begin{cases} w^{1_1} = \left(0,2,0,0\right) \\ w^{1_2} = \left(0,0,0,1\right) \\ w^{2_1} = \left(0,2,0,0\right) \\ w^{2_2} = \left(0,0,1,0\right) \\ w^{3_1} = \left(1,0,0,0\right) \\ w^{3_2} = \left(0,0,0,3\right) \end{cases}$
  - Every item is owned by one agent: Rename the same type of items own by different agents and equalize the utilities by an agent on them, then we have 6 goods at the market $N = \{1, 2_1, 2_2, 3_1, 3_2, 4\}$. $\begin{cases} w^{1_1} = \left(0,2,0,0,0,0\right) \\ w^{1_2} = \left(0,0,0,0,1,0\right) \\ w^{2_1} = \left(0,0,2,0,0,0\right) \\ w^{2_2} = \left(0,0,0,1,0,0\right) \\ w^{3_1} = \left(1,0,0,0,0,0\right) \\ w^{3_2} = \left(0,0,0,0,0,3\right) \end{cases}$ here $\begin{cases} u^{1_1} = \left[3,0,0,0,0,0\right]^T \\ u^{1_2} = \left[3,0,0,0,0,0\right]^T \\ u^{2_1} = \left[0,4,4,2,0,0\right]^T \\ u^{2_2} = \left[0,4,4,2,0,0\right]^T \\ u^{3_1} = \left[0,2,2,0,1,1\right]^T \\ u^{3_2} = \left[0,2,2,0,1,1\right]^T \end{cases}$

- Normalization revision: Divide by its size, we get $\begin{cases} w^{1_1} = \left(0,1,0,0,0,0\right) \\ w^{1_2} = \left(0,0,0,0,1,0\right) \\ w^{2_1} = \left(0,0,1,0,0,0\right) \\ w^{2_2} = \left(0,0,0,1,0,0\right) \\ w^{3_1} = \left(1,0,0,0,0,0\right) \\ w^{3_2} = \left(0,0,0,0,0,1\right) \end{cases}$

- Since everything is liked by someone (The goods that no one likes are eliminated), and an edge from i to j means agent i likes item j, there is non-zero indegree.

Q1.3:
In "Linear Utility Market", the goods are initialely distributed to the agents, but they don't have endowment of money.
In "Fisher Market", there is a seller who owns all the goods, but the agents have endowment of money to buy the goods.