

CS28010 Question for Final

Guoxin SUI

December 20, 2017

1 Multivariate least squares

Given a training set with multiple outputs for each example:

$$\{(x^{(i)}, y^{(i)}), i = 1, \dots, m\}; x^{(i)} \in \mathbb{R}^n; y^{(i)} \in \mathbb{R}^p$$

Similar to linear regression, to use a linear model to predict the outputs: $y = \Theta^T x$, where $\Theta \in \mathbb{R}^{n \times p}$. We define the cost function as

$$J(\Theta) = \frac{1}{2} \sum_{i=1}^m \sum_{j=1}^p \left((\Theta^T x^{(i)})_j - y_j^{(i)} \right)^2$$

Write $J(\Theta)$ in matrix-vector notation and find the closed form solution for Θ which minimizes $J(\Theta)$

Answer:

The objective function can be expressed as

$$J(\Theta) = \frac{1}{2} \text{tr} \left((X\Theta - Y)^T (X\Theta - Y) \right)$$

The closed form is

$$\Theta = (X^T X)^{-1} X^T Y$$

where $Y \in \mathbb{R}^{m \times p}$. This is quite similar to the case in univariate case.