Formale Methoden in Software Entwurf

Verification ⇔ Validation
- Impl = spec
- requirements as intended

Defects

dangerous
expensive
hard to patch

: Digitalisation
↗ security critical systems

⇒ prevent

+ confidence in system

civil engineering

calculations/estimations
Hardware redundancy
robust design (no single point of failure)
Seperation of subsystems
Design patterns

but

bit-flip changes behavior

complexity underestimated, non-continuous func.
redundancy as replication same bug
signals propagate to whole system
Properties not compositional
untrained engineers, immature designs
time-to-market > reliability
cost

Formal

mathematics, symbolic logic
formality basis for tools

check if impl. satisfies spec — no validation

+ dev time
- gen test cases
- support debugging

Testing

against internal SW errors (bugs)
external faults by simulating them and tracing propagation

inputs
- fuzzing - no guarantees
- chosen - expensive by hand
- automatic - need spec/model

test coverage
test oracle = spec
↳ desirable

- can only show presence of bugs
- labor intensive

formal methods can partially replace testing
+ potentially validating

Specification

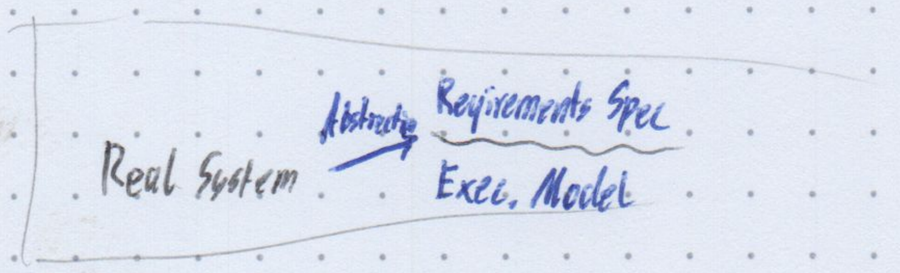
Simple properties
Safety " : something bad will never happen
"Liveness" : " good " happen eventually

Schematic "
Deadlock-freedom, no starvation, fairness
Reachability, no uncought except./errors

Non-functional "
Runtime, Mem, Useability

Full behavioural spec
impl. satisfies contract
data consistency/system invariants
Modularity, Encapsulation
Program equivalenz e.g. compiler correctness

Relational properties
refinement, information flow, fault propagation



Proving system properties

Abstract ↔ Concrete level (model)

e.g. finite automaton
+ automatic proofs
- unfaithful modeling inevitable
may capture essence (e.g. of protocols)

e.g. JAVA/C program
- unbounded data types
complex
in general not automatic proofs
precise characterization of semantics

Simple ↔ Complex properties (Spec)

simple, schematic properties
finite case distinctions
approximation propositional logic
automatted proofs

full behavioural spec
quantification over infinite domains
first order logic

e.g. SPIN — KeY

Automatic ↔ Semi-Automatic Proof

batch/auto-active mod
may fail or be inconclusive
tune param
patch program, spec

interactive
intermediate inspection
proof checked by tool
— needs knowledge of tool

Hardware ↔ Software Verification

good match
limitations of tech and application

specialized software
check of abstraction not code directly

Pitfalls formal models

Creation

(Abstraction) e.g.
over-simplified — no latency
missing requirement — max length
incorrect model — int - float

machine checkable
declare signatures (symbols) to spot incompleteness

Errors give deep insights

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Deductive Verification
proofing spec Conformity

e.g. for
Hardware Verification
(FPGA)
Software
safety critical
parts of OS
...
Libaries
JDK LinkedList
...
tools
Frama-C
...

SPIN verify Promela model against temporal logic specs
- meaningfed properties

KEY Java against JML contracts
- proof size
finding loop invariant

=> Tool supper
for repetitive tasks
+ avoid clerical errors
+ cope large programms
certifiable

free, well documented

PROMELA – Process meta-Language

based on ω-automata, linear temporal logic

Exec
spin ...pml
ispin ...pml

modeling
- finite state space
- pure expressions
- few control structs
- Multi-threaded
- shared Mem
- Sync & Message passing

not programming language
- no floats
- encapsulation
- methods
- ref. types
- API, Libary
- input, GUI

but Non-deterministic
scheduling policy configurable

∀ init mit 0

arithmetic Data types

- int
- short, byte
- bit = {0,1} = bool

unsigned

Ops +, −, *, /, %

computed as int then converted

Arrays

static constant

type name[N]
- only 1-dim
- no a==b – always diff types

" name[5]

Enumerations

mtype[:tag] = { name, ... }; // multiple declarations ⇒ union of them
max. 255 literals

mtype[:tag] var = name;

Records

typedef NAME {
  type attr, ...;
  ...
};

NAME a;
a.attr...

only previous declared records ⇒ no self-references

multi-dim arrays
typedef VECTOR { int vector[10] };
VECTOR matrix[5];
...

Control Statements

Sequencing

; seperator

conditional expr. atomic
(boolean-guard → then : else)

Guarded Commands

not atomic

if
:: <guard-statements>+ -:: bool exp | else → ; command
fi | od

- ≥1 true : exec one arbitrary
- =0 true : block until one true

can lead to timeout

only way to exit
break / goto

Syntactic sugar
for (i : 1 .. N) { ... }
for (i in array) { ... }
i durch indizes

Jumps

goto Label – unique per process
=
Label: // not in front of guards

Inlining

inline name(a, b, ...) { // dekl. before use
  ... // new scope
}
=
name(...)

anonymous var –
+ no state
+ ~~vers~~übersicht

Non-Deterministic Assignment

if
:: range = 1
:: range = 2
...
fi

do
:: range < HIGH → range++
:: break
od

not equal chance
but doesn't matter
for verification

select(i : LOW .. HIGH)

printf("... %d", decimalvar)

Concurrent Processes

```
active prototype Name(){
  //code
}
active prototype Name(){
  //code
}
```

up to 255 uniquely named Processes

executed interleaved on single processor

```
prototype Name (type name, ...){...}
```

type name: global / local-in P

Sets of Processes

```
active [N] prototype P(){
  // ... _pid //own local vars
}
```

State of Process

program counter
var values

Computation with $P_1, \dots P_n$

$c^i = (s_0^i, \dots)$ $i \in (1, \dots n)$

time →

s_0^i : start of P_i
s_{j+1}^i : successor of s_j^i

Terminating
$\forall c^i$ finite $\wedge$ final state reached (end of code)

Blocking
$\forall c^i$ finite $\wedge \exists i$: final state not end of code

Infinite
$\exists c^i$ infinite

End locations of process
textual end
label end×××:

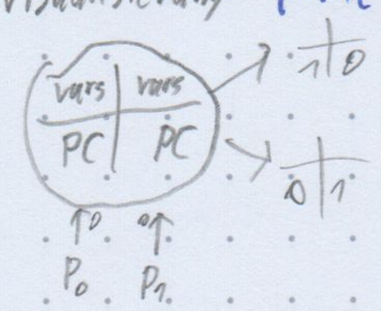
finite domains
fixed #processes
finite #statements

Interleaving

$(s_0, s_1, \dots)$ interleaving of $(c^1, \dots c^n)$

$\Rightarrow$ $s_j = s_{j'}^1, s_k = s_{k'}^1, j < k \Rightarrow j' < k'$
$\wedge$ $0 \le r \le k \Rightarrow s_j^i$ in interleaving

Visualisierung: finite directed graph



Weakly / Strongly atomic seq.

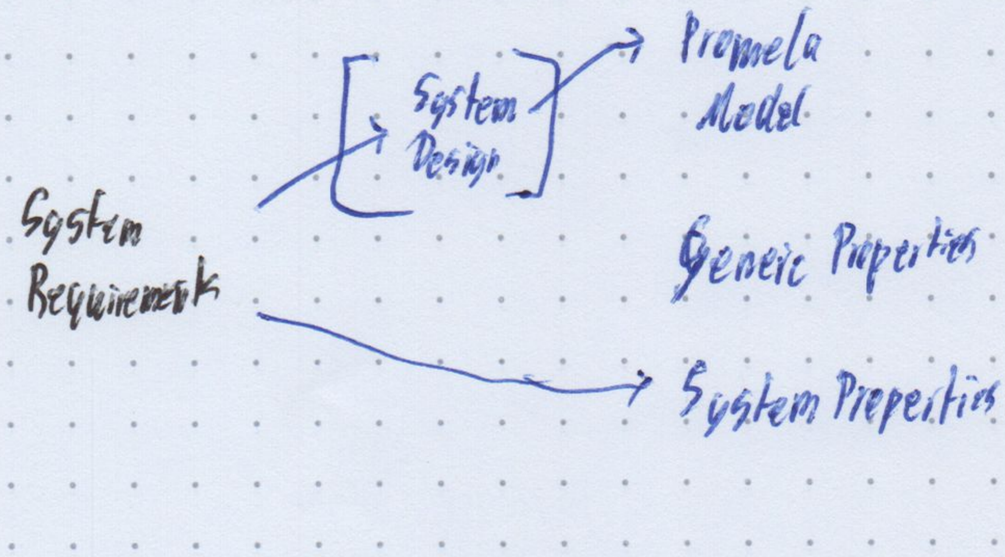
atomic {...} ⚠ not first (guard) blocking stmt ⇒ interleaving

d_step {...} never interrupted
- non-determinism always takes first option
- only first stmt may block, else error (guard)

Usage

Model essential

abstract complex computations (non-deterministic choices)
finite approx for unbound data structures
assumes fair scheduler



Select properties

generic: mutex, deadlock, starvation
system-specific: event sequences ...

Verify

$\forall$ possible runs sat properties
$\Rightarrow$ counter example

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Verification

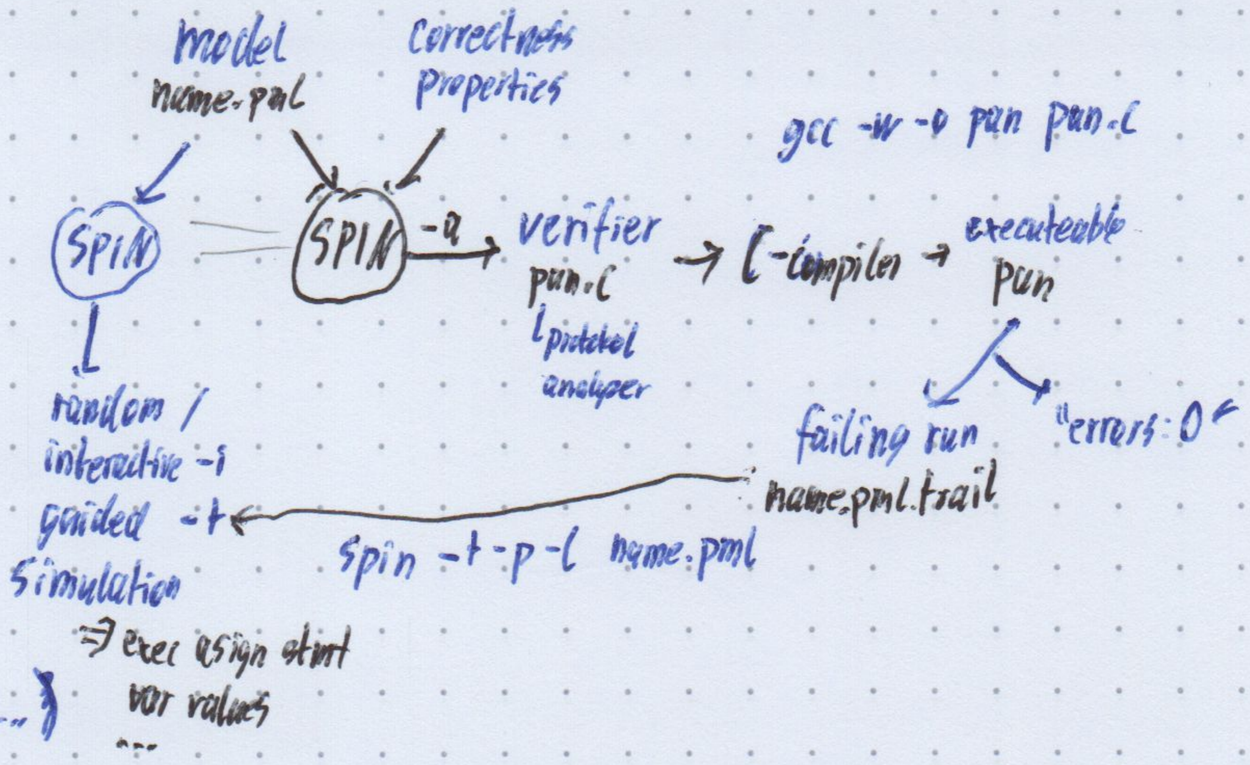
Simulation : one run

Verification
MC - Model Checker
∀ runs ∀ properties
exhaustive counter example search

R_M = Menge ∀ possible runs Model M

$R_{M,C_i} \subseteq R_M$ satisfying C_i

M correct Relative $C_1, \ldots \Leftrightarrow R_M = \{R_{M,C_1} \cap \ldots\}$



Stating Correctness Properties
outside model
never claims
temperal logic formulas
within model
Mete labels
end, accept, progress
Assertion statements — no quantification
⇒ z.B. GCD not fully verifiable
but sanity check
assert(expr)
└ if ≠0 ⇒ error

Modellierung
Seperation of concers
system ↔ property
verify should have not allready has

Richness ↔ simplicity

Concurrent / distributed systems

- combinatorial explosion
- data races, counts measured: deadlocks
- limited control (scheduling, speed
  performance, reliability communication)
- Testing - controllability ⇒ miss failures
  - reproduceability
  - resources for exhaustive

Initial Process pid=0

declared starts all p.'s
implicitly using »active«
explicitly

```
init {
  ... //z.B. starting p's
}      init global vars
```

run P() - starts copy of P drüber deklariert
(in atomic: only start exec next stmt after atomic)

(_nr_pr == 1) → ... - waits until its last p. running
└ # running processes

shared mem

⇒ global var
res ⇒ global var of bool / Enum
communication ⇒ global var of chan (channel)

Synchronization

no semaphores, locks, monitor primitives

Executability ⟺ blocking

assignments, assertions, print stmts	always
expr. stmt	iff ≠0
send / receive	
compound stmt atomic, d_step	iff guard
if, do	iff any alternative iff guard
for	always

process iff location counter

scheduler non-deterministic
non-blocking process

Critical Section CS

block of code accessing shared state

Problem Race Condition
⟺ result depends on exec order

Solution Mutual Exclusion ≤1 p exec it's CS at a time
! deadlock ∃p must eventually exec CS
! starvation ∃p''

Patterns

Busy waiting - wasteful

```
enterCritical P = true
do
:: ! enterCritical Q → break
:: else → skip
od
```

Block

```
enterCritical P = true;
! enterCritical Q
```

! deadlock
⇒ test and set

```
atomic {
  ! enterCritical Q;
  enterCritical P = true
}
```

! realistic in real system?
(inefficient)

Variations
at most n P's in CS
⇒ counter, semaphores, ...

refined mutex conditions
diff CS's
readers don't exclude readers
FIFO queue for entering (semaphores)

Verifying Mutex

system global property but assert code local

⇒ Invariants in Temporal Logic

Ghost Variables
vars only for spec / verifying
z.B. ein critical counter counting p's in CS
}

Verteilte Systeme

- Nodes ~> process
- communication channels ~> type chan
- protocols ~> algorithm

Global ↔ Lokal channel je nach us. Referenz
- ∀p: can send/receive
- even if passed to one other

```
chan name = [capacity] of {type_1, ... type_n}
            ↑ referenz teil          ↑ message Tupel
```

capacity = 0
=> Rendezvous channel

cap := capacity > 0
=> Buffered (FIFO)
part of state => keep small

checking status

full(ch)	cannot negate these => avoid not/else
nfull	
empty	
nempty	

send:

```
name ! expr_1, ... expr_n
↑ chan   ↑ types match
```

receive

```
name ? expr_1, ... expr_n
↑ chan   | expr_i is value => only receives msg_i == expr_i
         | expr_i is var => var := msg_i
           eval(var) to interpriete as value
```

Random receive

```
name ?? expr_1, ... n
```
first matching msg in queue

copy without removal

```
name ? <expr_1, ... n>
```

Prefix Syntax msg

```
expr_1 (expr_2, ... n)   für expr_1, ... n
```

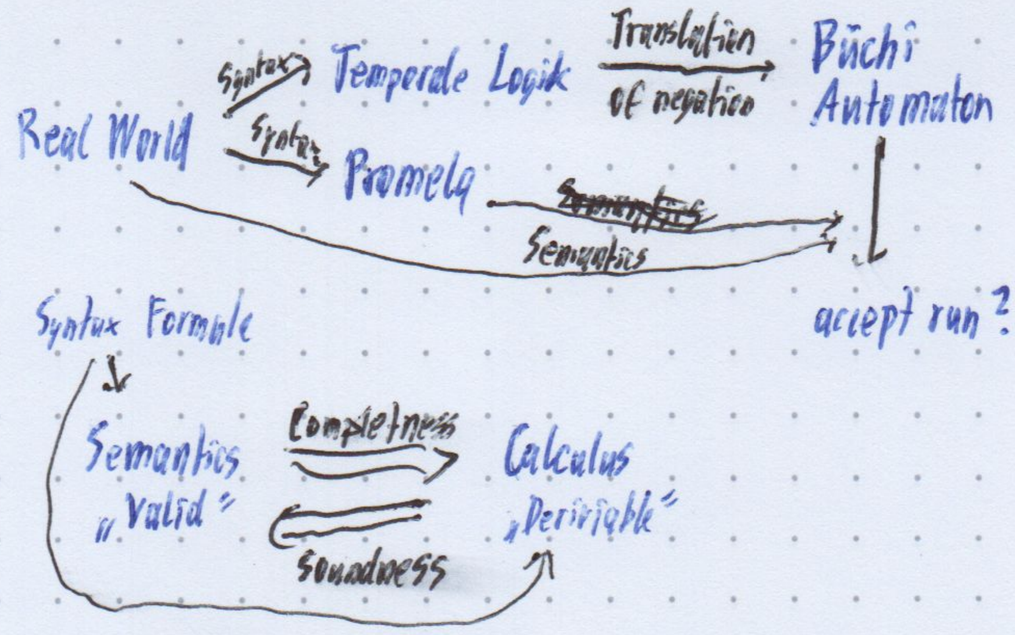
Executability
- Rendezvous
- Buffered async.
  - msg queue not full
- send/receive
- Synchron
  - « empty

Client-Server global req, reply channel
multiple servers => ! ensure responding to sender

=> sending _pid to server,
reply with it, pattern matching

=> sending local resp chan to server

Lineare Temporale Logik



Propositional Logic

Syntax

P - set of variables

Formulas For_0

true, false - Truth constants

Φ - vars connectives: Klammern verzichtbar

$\neg\phi, (\phi \wedge \psi), (\phi \vee \psi), (\phi \rightarrow \psi), (\phi \leftrightarrow \psi)$

in SPIN: ! && || -> <->

Semantics

Interpretation $I: P \rightarrow \{T, F\}$ ~ extension $\{q \in P \mid I(q) = T\}$

$val_I: For_0 \rightarrow \{T, F\}$ $val_I(p_i) = I(p_i)$ $val_I(true) = T$ ... $val_I(\phi \wedge \psi) = \begin{cases} T & \text{if } val_I(\phi) = T \text{ and } \dots \\ F & \text{sonst} \end{cases}$

$val_I(\phi) = T \Rightarrow I$ satisfies ϕ $I \models \phi$

$\forall I: (\forall \psi \in \Gamma: I \models \psi) \Rightarrow I \models \phi \Rightarrow \phi$ follows from Γ $\Gamma \models \phi$

Formalise Promela in Prob. Logic?

vars: e.g. byte n $\Rightarrow N_0, \dots N_7$ prob Vars

PC: PC_{ij} $\forall$ process i, line j

$\Rightarrow \Phi_{Program} := ((PC_{0,1} \wedge \neg PC_{0,2} \wedge \dots) \vee \dots) \wedge$ Startzustand, Übergänge

└ all reachable states $\mathbb{D} \models \psi \Rightarrow \psi$ true in $\forall$ reachable states

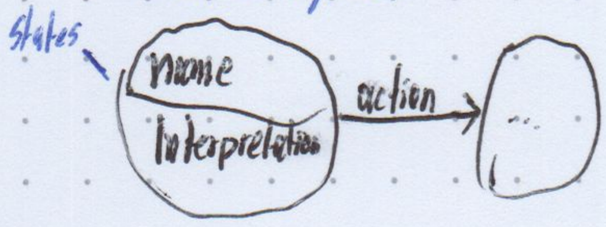
– cant express evolving state

e.g. n will become 0 eventually

changes value infinetly often ...

⇓ more expressiv Logic

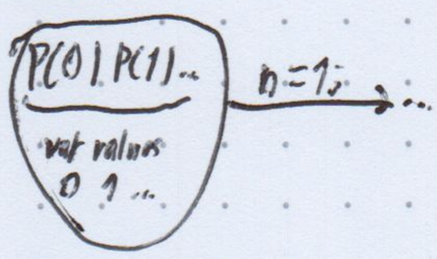
Transition Systems



Computations/runs - infinit paths

(finite stuck at final state)

Promela



$\mathcal{T} = (S, Init, \delta, I)$

- States
- initial states $\emptyset \neq Init \subseteq S$
- TransitionRelation $\delta \subseteq S \times S$
- Labeling $\forall s \in S$ I_s

run $\sigma = s_0 s_1 \dots$ mit $s_0 \in Init$ $\forall i: (s_i, s_{i+1}) \in \delta$

ϕ valid in $\mathcal{T} \Leftrightarrow \forall$ runs σ of $\mathcal{T}: \sigma \models \phi$

hard to automate

Temporal Logic extension of prob. logic

spec. sets of runs

ISPIN	Syntax		Semantic: sequence of interpretations
[]	$\Box\phi$	Always	Run $\sigma = s_0 s_1 \dots$ $\sigma_i = s_i s_{i+1} \dots$ (suffix)
<>	$\Diamond\phi$	Eventually	$\sigma \models true$ $\sigma \not\models false$
U	$\phi U \psi$	Until	$\sigma \models p$ iff $I_0(p) = T$ p∈P - in initial state
			$\sigma \models \neg\phi$ iff $\sigma \not\models \phi$
			...
			$\sigma \models \Box\phi$ iff $\sigma_k \models \phi$ $\forall k \geq 0$
			$\sigma \models \Diamond\phi$ iff $\exists k \geq 0$ $\sigma_k \models \phi$
			$\sigma \models \phi U \psi$ iff $\exists k \geq 0$ $\sigma_k \models \psi$ $\wedge$ $\forall 0 \leq j < k$ $\sigma_j \models \phi$

Validity $\models \phi$, ϕ valid $\Leftrightarrow \forall \sigma \in \{(2^P)^\omega$ - infinit seq. of interpretations$\}: \sigma \models \phi$

Represent set of runs e.g. $(\neg\phi\phi\phi\dots) \models \Diamond\Box\phi$

$\phi_0 \phi_1 \dots$ represents $\{s_0 s_1 \dots \mid I_j \models \phi_j \ \forall j \geq 0\}$

Properties z.B roulette mit ghost var

Safety - bad never happens $\Box$bad

Liveness - good eventually $\Diamond$good

infinetly often $\Box\Diamond\phi$

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Regeln $\Box\Box\phi \leftrightarrow \Box\phi$ reflexiv

$(\neg\Box\phi) \Leftrightarrow (\Diamond\neg\phi)$ dual

$\Diamond\Box\phi \leftrightarrow (true\ U\ \phi)$

Formal Languages

alphabet/vocabulary Σ

word $w \in \Sigma^* = a_0 \dots a_n$

$L \subseteq \Sigma^*$ language over Σ^*

Formal ω-Language

ω-word $w \in \Sigma^\omega$

infinite seq. $a_0 \dots$

$L^\omega \subseteq \Sigma^\omega$ ω-language over Σ

Büchi Automaton over Σ

$B = (Q, I, F, S)$

Q locations/states $\neq \emptyset$ finite

$I \subseteq Q$ initial start locations $\neq \emptyset$

$F = \{F_1, \dots F_n\} \subseteq Q$ accepting locations

$S \subseteq Q \times \Sigma \times Q$ transition relation

in contrast to runs transitions not states labeld

Acceptance of $w = a_0 \dots$

$\Leftrightarrow \exists\, q_i \in S(q_i, a_i)$ $\forall i \geq 0$

$q_0 \in I$

$\exists f \in F$: infinetly often visited

$L^\omega(B) = \{w \in \Sigma^\omega \mid w$ accepted run of $B\}$

$L^\omega(B)$ accepted by B

ω-regular Language $\Leftrightarrow \exists B$ accepting language

ω-Regular expression

ab - a then b

$a+b$ - a or b

a^* - finitly often a

a^ω - infinitly often a

Theorem

Decidabifity whether $L^\omega(B)$ empty

Closure properties

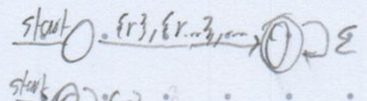
ω-languages closed under intersection, union, complement

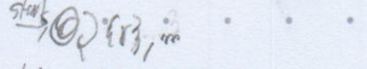
! deterministic Büchi less expressive

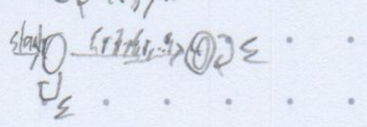
cannot accept all ω-expressions

LTL formula → Büchi Automaton

$\Sigma = \mathcal{P}(P)$ $I_{a \in \Sigma}(p) = p \in a \begin{cases} T & \text{if } p \in a \\ F & \text{const} \end{cases}$

$r \rightarrow$ 

$\Box r \rightarrow$ 

$\Diamond r \rightarrow$ 

Transitions System = Büchi Automaton

$\top \rightarrow B_\top$

$p := 1$ $p := 0$	p $\neg p$
start state	initial state
locations with end-label	accepting states
runs	words

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Model Checking - gilt $\tau \models \varphi$?

check $h^u(B_\tau) \cap h^u(B_{\neg\varphi}) = \emptyset$

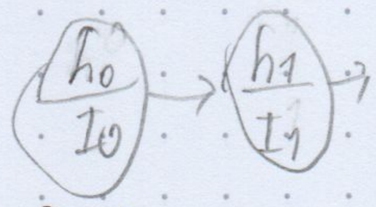
if not: send as counter example

Hies Temporal Logic = Linear temporal logic

Promela stating correctness properties

within model
- end labels endXXX:
- accept labels acceptXXX:
  - allows Acceptance Cycle
- assertions
  - + convenient always mutex, Array index bounds
  - + expressiveness
  - Absence Deadlock, Starvation
  - - no seperation of concerns
  - - easily out of sync (errors pone) forget
  - - be only state at their location

outside
- temporal logic
  - Boolean temporal logic
    - global vars, constants ~~of bool/bit $\in For_{BTL}$~~
    - of bool/bit $\in For_{BTL}$
    - numerical expressions
      - $e1 == e2, !=, <, <=, >, >= \in For_{BTL}$
    - Process@label $\in For_{BTL}$



h_j : Process → location
I_j : vars → value

$h_j I_j \models x < y \Leftrightarrow I_j(x) < I_j(y)$
$h_j I_j \models P@l \Leftrightarrow h_j(P) = l$ - e.g. for mutex
...

in file
- LTL <claim name> $\in$ {<formula>} ! ensure it/the right one is used
  - check use claim acceptance cycle
  - übersetzt in never claim $\neg\varphi \Rightarrow$ process with accept labels ! only one Process advances each step
    - accept ⇒ fail

acceptance cycle
- spin -a file.pml
- gcc -DSAFETY -o pan pan.c
- ./pan -N name

never { Process with accept labels }

$h^w(M) \cap h^w(NC_{\neg\varphi}) > \emptyset \Rightarrow \varphi$ holds in M

Weak Fairness check option spin -f

∀ continuously executeable stmt executed eventually

Strong Fairness

∀ oft executeable stmt exe eventually
harder to check not impl by most model checkers

First Order Logic

Signatur Σ for $TSym \neq \emptyset$ Set of types

- FSym set function symbols $f: A_1 \times .. A_n \to A$, $A_i, A \in TSym$ — Spezialfall ($n=0$) constant
- PSym predicate $p(A_1, ... A_n)$ — ϕ propositional variable
- VSym variable $v: A$

Syntax — type e.g. int
- type symbol (type, ..); ...
- symbol((type, ..); ...
- type symbol; ...

KeY \sorts{ a; ..; }

\predicates {
p(.., ..); ..; }

\problem{
//formula }

First order term of $A \in TSym$ over Σ Trm_A

$v \in VSym$ of A
$f(t_1, ... t_n)$ of A
└ typkorrekt

Semantic

Evaluation $val_{s,\beta}: Trm_A \to D^A$
$val_{s,\beta}(v) = \beta(v)$
$val_{s,\beta}(f(t_1, ..)) = I(f)(val_{s,\beta}(t_1), ..)$

universe / domain $D \neq \emptyset$ — $D^A = \{d \in D \mid \delta(d) = A\} \neq \emptyset$
type $\delta: D \to TSym$ — $D = \bigcup D^A$; $A \neq B \Rightarrow D^A \cap D^B = \emptyset$

Interpretation I of $f: D^{A_1} \times .. \to D^A$
of $p \subseteq D^{A_1} \times ... D^{A_n}$
of $\doteq = \{(d, d) \mid d \in D\}$ fixed

$S = (D, \delta, I)$
fo state/model

First order formulas Fml

Atomic formula
- true, false — $val_{s,\beta}(true) = T$
- $t_1 \doteq t_2$ equality — $val_{s,\beta}(p(t_1, ..)) = T \Leftrightarrow (val_{s,\beta}(t_1), ..) \in I(p)$
- $p(t_1, ... t_n)$ predicate

Var Assignment $\beta: VSym \to D$ respecting type

modified $\beta_y^d(x) = \begin{cases} \beta(x) & x \neq y \\ d & x = y \end{cases}$

$[x/t]\phi$ replace occurence var x with t

KeY-FOL

$\neg\phi$, $\phi \wedge \psi$, $\phi \vee \psi$ — !, &, | — $val_{s,\beta}(\phi \wedge \psi) = T \Leftrightarrow val_{s,\beta}(\phi) = T$ and ...
$\phi \to \psi$, $\phi \leftrightarrow \psi$ — ->, ...
$\forall x; \phi$, $\exists x; \phi$ — \forall A x; ϕ, .. exists — $val_{s,\beta}(\forall x; \phi) = T \Leftrightarrow val_{s,\beta_x^d}(\phi) = T$ for all $d \in D^A$, $x: A$ ($\exists$: for at least one)
└ x bound ↔ free
no free vars ⇒ formula closed

Semantic Concepts
- Satisfiability $val_{s,\beta}(\phi) = T$
- Truth $s \models \phi$ — ϕ true in s
- Validity $\models \phi$
- closed formula sat ⇒ true

close $\dfrac{}{\Gamma, \phi \Rightarrow \phi, \Delta}$ true $\dfrac{}{\Gamma \Rightarrow true, \Delta}$ false ... closeEq $\dfrac{}{\Gamma \Rightarrow t \doteq t, \Delta}$

notLeft $\dfrac{\Gamma \Rightarrow \phi, \Delta}{\Gamma, \neg\phi \Rightarrow \Delta}$ orLeft $\dfrac{\Gamma, \phi \Rightarrow \Delta \quad \Gamma, \psi \Rightarrow \Delta}{\Gamma, \phi \vee \psi \Rightarrow \Delta}$

andLeft $\dfrac{\Gamma, \phi, \psi \Rightarrow \Delta}{\Gamma, \phi \wedge \psi \Rightarrow \Delta}$ impLeft $\dfrac{\Gamma \Rightarrow \phi, \Delta \quad \Gamma, \psi \Rightarrow \Delta}{\Gamma, \phi \to \psi \Rightarrow \Delta}$

...Right

Semantic Argements
- uncountably states
- may infinite Domain
- at least exponential

allLeft / instAll $\dfrac{\Gamma, \forall_\tau x; \phi, [x/t']\phi \Rightarrow \Delta}{\Gamma, \forall_\tau x; \phi \Rightarrow \Delta}$ — t' varfree term

applyEqLeft $\dfrac{\Gamma, t \doteq t', [t/t']\phi \Rightarrow \Delta}{\Gamma, t \doteq t' \Rightarrow \Delta}$ applyEqRight $\dfrac{\Gamma, t \doteq t' \Rightarrow [t/t']\phi, \Delta}{\Gamma, t \doteq t' \Rightarrow \Delta, \phi}$

exLeft $\dfrac{\Gamma, [x/c]\phi \Rightarrow \Delta}{\Gamma, \exists_\tau x; \phi \Rightarrow \Delta}$ — c fresh constant

eqSymmLeft $\dfrac{\Gamma, s \doteq t \Rightarrow \Delta}{\Gamma, t \doteq s \Rightarrow \Delta}$

...Right

allRight.. , exRight/instEx

⇒ Syntactic Transformation
Sequent Calculus

Γ, Δ - set of formulas

Sequent: $\psi_1, ..., \psi_m \Rightarrow \phi_1, .. \phi_n$
Antecedent — Succedent
$\Leftrightarrow (\psi_1 \wedge .. \psi_m) \to (\phi_1 \vee .. \phi_n)$

Schema
$\Gamma \Rightarrow \phi \wedge \psi, \Delta$
main formula — side formulas
match top level occurance in Succedent

Soundness
closed proof ⇒ validity
Completness
valid formula ⇒ closed proof

Theorem sequent calculus is both

Rule

Premisse
Rule Name $\dfrac{\Gamma_1 \Rightarrow \Delta_1 \quad .. \quad \Gamma_r \Rightarrow \Delta_r}{\Gamma \Rightarrow \Delta}$
Conclusion

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Proof tree structure, goal as root
closed ⇔ ∀ branches closed ⇐ rule without Premisse
durch ∀ ?

~~JML - Java Modeling Language~~

~~Fokus~~ Fokus: Unit Spezification
- interfaces
- classes
- methods
  - res. val.
  - initial val of formal param
  - accessible part of pre-/poststate

Design by
Spezification as Contract between caller & callee — Application / Library

Set of pre/post condition pairs
satisfied ⇔ (called in state sat. prec. ⇒ terminating ⇒ postcondition
- normal
- abrupt (exception)

JML - Java Modeling Language

Java Extension in comments

```
/*@ ... ; comment
  [@] ... ;
  [@] */
```

Spezification cases

~~/*@ ... behavior~~

```
/*@ public ... behavior
    [requires expr.;]*          mehrere ∧
    ([ensures expr.;]* - no Except thrown
    |[signals_only Exc1, ..., Excn;] - thrown Exceptiontype
     [signals (Exctype) expr.;]*) - post state depending on
    [diverges true;] - may not-termination   [assignable location, ....;] - frame condition
                                              \nothing
                                              \everything
                                              array[0..h]
    [also next case]*/
```

~~Data~~ Modifiers

```
/*@ spec_public @*/ - in ~~data~~ Definition public for spec   (auch mehrere hintereinander ↓)
    pure           - method: no sideeffects, always terminates
    nullable
    non_null       - default   bei array direct auch Elemente
```

Invariant - global state constraints

```
/*@ public invariant expr. */
    [static] ↔ Instance Invariant
```

- anywhere in class
  convention: before fields it talks about

added to pre-/post condition methods
post ∧ constructor

Inheritance
Spec from superclass

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JML expr ↔ Java expr

pure Java expr.

\old (expr.) - from prestate

boolean JML expr. a b

!a , &&, ||

a ==> b

a <==> b

Quantified

(\forall type varName; expr.) [varName2]

\exists

Range Predicates

(\forall type varName; $expr_1$; $expr_2$)

\exists   sugar for   ==> - bei ∀

&& - bei ∃

\created (object) - true on created obj

\result

Dynamic Logic – Java in FOL

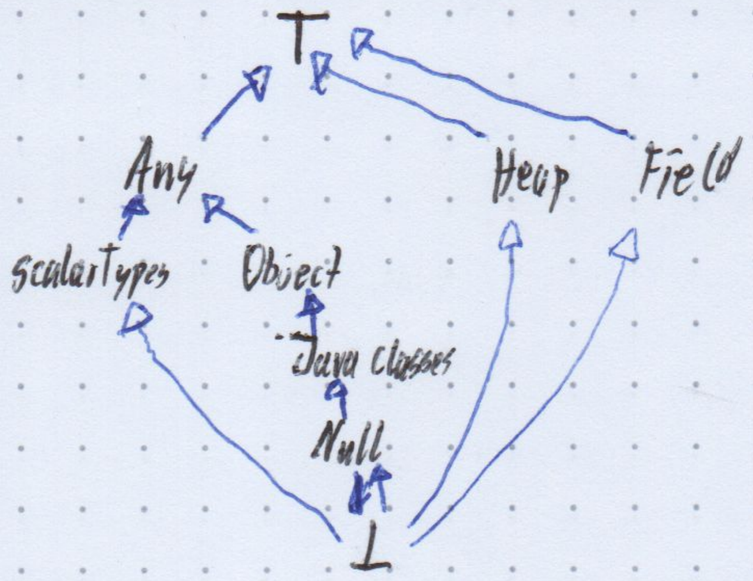
Type Hierarchy $\mathcal{T} = (T_{Sym}, \sqsubseteq)$

T_{Sym} – set of type symbols

$\sqsubseteq \subseteq T_{Sym} \times T_{Sym}$ subtype relation

$\bot$ empty type – no therms of $\bot$

$\top$ universal – untyped FOL



FO/Logical Vars ↪ Program Vars ProgVSym

rigid — non-rigid/flexible

in quantifiers declared — state dependent

not in programms — declared in code

not quantifiable

DL Signature $\Sigma = (P_{Sym}, F_{Sym}, V_{Sym}, ProgV_{Sym})$ $F_{Sym} \cap ProgV_{Sym} = \emptyset$

Predicate, Function, flexible

KeY Input file

```
\sorts {
\functions {
\predicates {
\programVariables { // non-rigid, global
\problems { // formula to verify
```

JavaSource "path";

Modalities

exceptional/abrupt ↕ normally

diamond box

$\langle p \rangle \phi$ – total correctness — p terminates, ϕ holds in final state

$[p]$ partial

Semantik

state $s \in (D, \delta, I)$

for D, δ : S = states with

D, δ – constant Domain assumption

I differs only in flexible vars

Kripke structure / Labeled Transition System

$K = (S, \rho)$

ρ: Program $\to (S \to S)$ – defined by syntactic induction

$\rho(p)(s_1) = s_2 \Leftrightarrow$ p exec in s_1 terminates normally in s_2

Formula validity (for K initial programm var values)

$(K, s, \beta) \models \langle p \rangle \phi \Leftrightarrow \rho(p)(s)$ defined $\wedge (K, \rho(p)(s), \beta) \models \phi$

$[\,] \Rightarrow$

$\models \phi \Leftrightarrow val_{s,\beta}(\phi) = T$

valid $\Leftrightarrow \forall K \forall s \in S \forall \beta : (K, s, \beta) \models \phi$

Form

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$\langle p \rangle \phi \Leftrightarrow \neg [p] \neg \phi$

$\langle p \rangle \phi \Rightarrow [p] \phi$

DL sequenz

$\{\phi_1, \dots, \phi_n\} \Rightarrow \{\psi_1, \dots \psi_m\} \Leftrightarrow (\phi_1 \land \dots \land \phi_n) \rightarrow (\psi_1 \lor \dots \lor \psi_m)$

Symbolic Execution

follow control flow

Rule Schema

$\Gamma \Rightarrow \langle \pi \ stmt; \ \omega \rangle \phi, \Delta$ (also with [ ])

- ω – postfix
- stmt – active stmt – first executeable
- π – inactive prefix

if $\dfrac{\Gamma, b \doteq true \Rightarrow \langle \pi \ p \ \omega \rangle \phi, \Delta \qquad \Gamma, b \doteq false \Rightarrow \langle \pi \ q \ \omega \rangle \phi \Delta}{\Gamma \Rightarrow \langle \pi \ if(b) \{p\} \ else \{q\} \ \omega \rangle \phi, \Delta}$

Symbolic updates +simplify, combine effects

$\{v := t\}(t' | \phi)$

- v – $\in$ ProgVSym
- t – $\in$ $Term_T$

$p(\{v := t\})(s) = s'$ mit $I_{s'}(v) = val_{s,\beta}(t)$, Rest gleich

Program var $\{x := t\}y \rightsquigarrow y \quad x \neq y$

$x \rightsquigarrow t$

$f(t_1, \dots t_n) \rightsquigarrow f(\{x := t\}t_1, \dots)$

...

no rewrite rule for $\{x := t\}(\langle p \rangle \phi)$

$\forall i_0; \{i := i_0\} \langle \_ i \_ \rangle \phi$

quantify over ProgramVars

Parallel Update

$\{v_1 := r_1 \ || \dots || \ v_n := r_n\}$ @conflict last wins

$\{v_1 := r_1\}\{v_2 := r_2\} = \{v_1 := r_1 \ || \ v_2 := r_2\}$

Heap in FOL

Heap store(Heap, Object, Field, Any)
(Any – value)

Any select(")
(Any – often needs to be cast z.B. int :: select(...))

u.f@heap

Pretty Printing in KeY

- u.f@heap = select
- heap[o.f := t] = store

$\Rightarrow$ ClassName $\in$ TSym

$D^{ClassName}$

unique constant field $\in$ FSym of type Field

ClassName :: $field

assign $\dfrac{o \doteq null, \Gamma \Rightarrow \langle throw \ new \ NullPointerException(); \ \omega \rangle \phi, \Delta \qquad o \,!\!= null, \Gamma \Rightarrow \{heap := store(heap, o, f, t)\} \langle \pi \ \omega \rangle \phi, \Delta}{\Gamma \Rightarrow \langle \pi \ o.f = t; \ \omega \rangle \phi, \Delta}$

null.f defined but unknown

$null.f \doteq null.f$

Aliasing

Static fields

- final static – Compiletime constant $\Rightarrow$ FOL constant $C.f : A$
- else $C.f$: Field store(heap, null, C.f, v)

this : Object, != null
unique program var
often called self

Arrays
flexible function arr : int $\rightarrow$ Field
store(heap, a, arr(i), v)

a.length

Object Creation $\phi \in$ rigid FOL formula

$\vdash \forall x; \phi \rightarrow [p](\forall x; \phi)$

implizit created : Boolean Field

Decomposition of complex terms e.g. expressions with side effects

Principles

Local code Transformation

e.g. $$\frac{\ldots\ x = t;\ x = y; \ldots}{\ldots\ x = y = t; \ldots}$$

fresh var for intermediate res

e.g. $$\frac{\ldots \text{boolean } v0;\ v0 = b;\ \text{if}(v0)\ p; \ldots}{\ldots\ \text{if}(b)\ p; \ldots}$$

Abrupt termination

try{ - inactive Prefix

$$\frac{\Rightarrow \langle \pi\ \text{if}(e \text{ instanceof } T)\{ \text{try}\{x=e;q\} \text{finally}\{r\}\} \text{else}\{r; \text{throw } e;\}\ \omega\rangle \phi}{\Rightarrow \langle \pi\ \text{try}\{\text{throw } e; p\} \text{catch}(T\ x)\{q\}\ \text{finally}\{r\}\ \omega\rangle \phi}$$

Method call

- source needed
  ! commercial API's
  system-level "
- invoked multiple times
  gets big fast
- no unbound recursion

fresh vars for param p#i

Lookup possible impl. class, split proof on ~~dynamic dispatch~~

Polymorphism
dynamic dispatch
Scoping
visibility

Inlining

(rename param)

methodBodyExpand $$\frac{\Gamma \Rightarrow \langle \pi\ \text{method-frame}(\text{source}=C, \text{this}=c):\{\text{body}\}\ \omega\rangle \phi, \Delta}{\Gamma \Rightarrow \langle \pi\ c.m(p\#1,\ldots)@C;\ \omega\rangle \phi, \Delta}$$

Object initialization

chain to new Object

⇒ methods
<createObject>()
<init>()

⇒ (use) Method contracts

$\Gamma \Rightarrow \mathcal{U}(F(preNormal) \vee F(preExc)), \Delta$ (precondition)

" $\mathcal{V}_{mod_{normal}}(\{exc \doteq null \wedge \phi_{post}) \rightarrow \langle \pi\ p\ \omega\rangle \phi), \Delta$ (normal)

$\quad_{exc}$ $\neq$ $\langle \pi\ \text{throw } exc; p\ \omega\rangle$ (exceptional)

$$\overline{\Gamma \Rightarrow \mathcal{U}\langle \pi\ result = m(args, \ldots);p\ \omega\rangle \phi, \Delta}$$

Anonymising Heap of @assignable modified

$\mathcal{V}_{modified} = \{ l_1 = c_1 \| \ldots \}$
- $l \in$ mod., c fresh constant
- $ls \subseteq$ Object × Fields

anon(heap, LocationSet, freshHeap)
= heap like heap if of $\in ls$
like freshHeap sonst

~~requires~~ Proving Method Contracts

$F(preNormal) \rightarrow \langle$ exc = null; try{o.m();} catch (Exception e) {exc = e;} $\rangle$ (exc $\doteq$ null $\wedge$ F(postNormal) $\wedge$ assignable (correct))

exceptional ..

unclosed proofs

Reasons
- buggy/incomplet spec
  program
- max #proof steps
- to weak automatic proof search

Understand/Fix
1. follow flow to open goal
2. Whats unproveable?

Loop invariants for loops with many iterations (hard with unwind)

at start
preserved by loop guard+body ⇒ holds afterwards
terminates

↳ by induction
Hypothesis, Base, Stepcase

How to find Inv.
desired postcondition?
Strengthening
Generalization partial results?

preserve context unmodified by loop

! default assignable \everything

$$\frac{\Gamma \Rightarrow \mathcal{U}(Inv \wedge b \doteq TRUE \to [p]Inv),\Delta \quad \Gamma \Rightarrow FALSE \to [\pi\, w]\emptyset,\Delta}{\Gamma \Rightarrow \mathcal{U}[\pi\, \text{while}(b)\, p\, w]\emptyset,\Delta}$$

initially valid
preserved
use case

```
/*@ loop_invariant
  Ø;
  assignable ...;
  diverges true;/
  decreasing v;
```

{ v ≥ 0 ; v strictly dec. } by loop body

e.g. a.length - i

Termination?

Well-Founded Order over decreasing term/variant