# Engineering Assignment Coversheet

**Student Number(s)**

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Group Code (if applicable): 12

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|---|---|
| **Assignment Title:** | Probability and Random Models Workshop 4 |
| **Subject Number:** | ELEN90054 |
| **Subject Name:** | Probability and Random Models |
| **Student Name:** | Devika Vasanth, Yue Chang |
| **Lecturer/Tutor:** | Pasha Tolmachev |
| **Due Date:** | Fridav 11 Mav 2018 |

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Has an extension been granted? Yes / No (circle)

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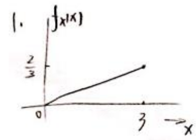
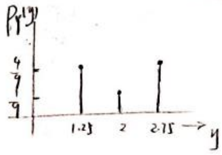
Student signature ........Devika Vasanth, Yue Chang........ Date ........10 May 2018........

PRM workshop 4

1.




$\mu_X = \int_0^3 x \cdot \frac{2}{9}x \, dx = \frac{2}{9} \frac{1}{3}x^3 \Big|_0^3 = 2$

$E[X^2] = \int_0^3 x^2 \frac{2}{9}x \, dx = \frac{2}{9} \frac{1}{4}x^4 \Big|_0^3 = \frac{9}{2}$

$Var(X) = E[X^2] - \mu_X^2 = \frac{9}{2} - 4 = \frac{1}{2}$

$\mu_Y = 1.25 \times \frac{4}{9} + 2 \times \frac{1}{9} + 2.75 \times \frac{4}{9} = 2$

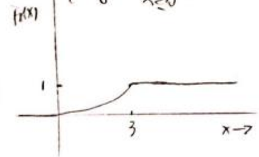
$E[Y^2] = (1.25)^2 \times \frac{4}{9} + (2)^2 \times \frac{1}{9} + (2.75)^2 \times \frac{4}{9} = \frac{9}{2}$

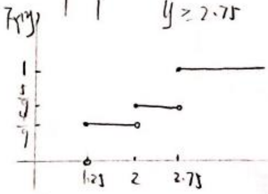
$Var[Y] = E[Y^2] - \mu_Y^2 = \frac{9}{2} - 4 = \frac{1}{2}$

2. (a) $0 < x < 3$

$F_X(x) = \begin{cases} \int_0^x \frac{2}{9}x \, dx = \frac{1}{9}x^2 & (0 < x < 3) \\ 1 & x \geq 3 \\ 0 & x \leq 0 \end{cases}$

$F_Y(y) = \begin{cases} \frac{4}{9} & 1.25 \leq y < 2 \\ \frac{5}{9} & 2 \leq y < 2.75 \\ 0 & \text{otherwise} \\ 1 & y \geq 2.75 \end{cases}$




(b) $F_X(x) = F_U(u)$

$U = \frac{1}{9}x^2$

$X = 3\sqrt{U}$

$\therefore h(U) = 3\sqrt{U}$

from CDF plot we get

$k(U) = \begin{cases} 1.25 & 0 < U \leq \frac{4}{9} \\ 2 & \frac{4}{9} < U \leq \frac{5}{9} \\ 2.75 & \frac{5}{9} < U \leq 1 \end{cases}$

Q3)

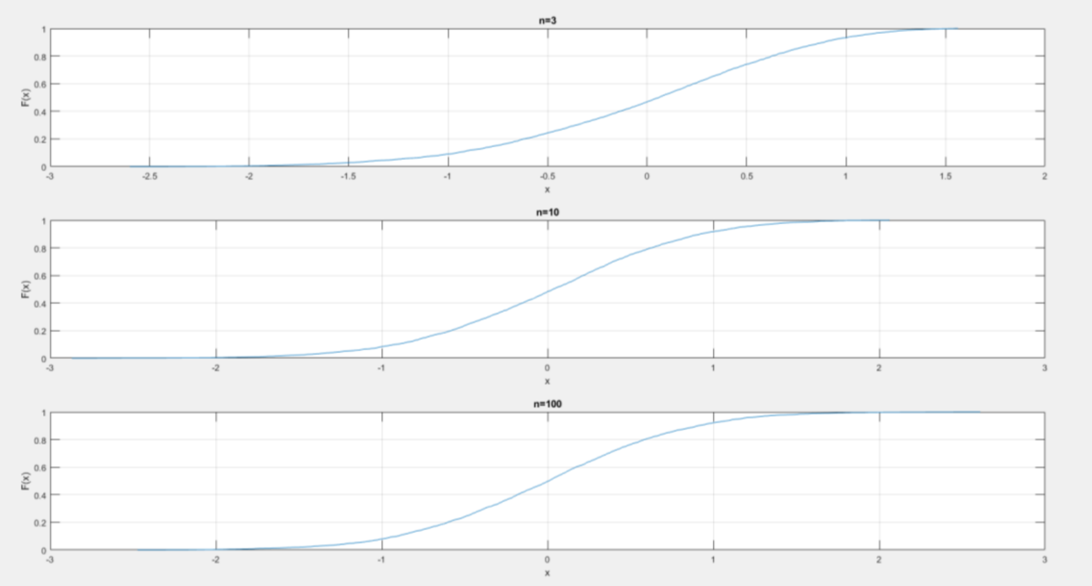
a) The results are very consistent around the mean of 2

```
>> question3
average distance per step = 2
average distance per step = 2.0005
average distance per step = 2.0019
average distance per step = 2.0002
average distance per step = 2.0023
```

b) The law of large numbers state that an empirical value of sample mean is an approximation of E[x] , when n is large it's becomes very unlikely to get an outliner. This is consistent with our experiment with n=100,000 sample mean stays around 2.

Q4

The CDF plots are shown for a sample size of 5000 and steps of 3,10 and 100



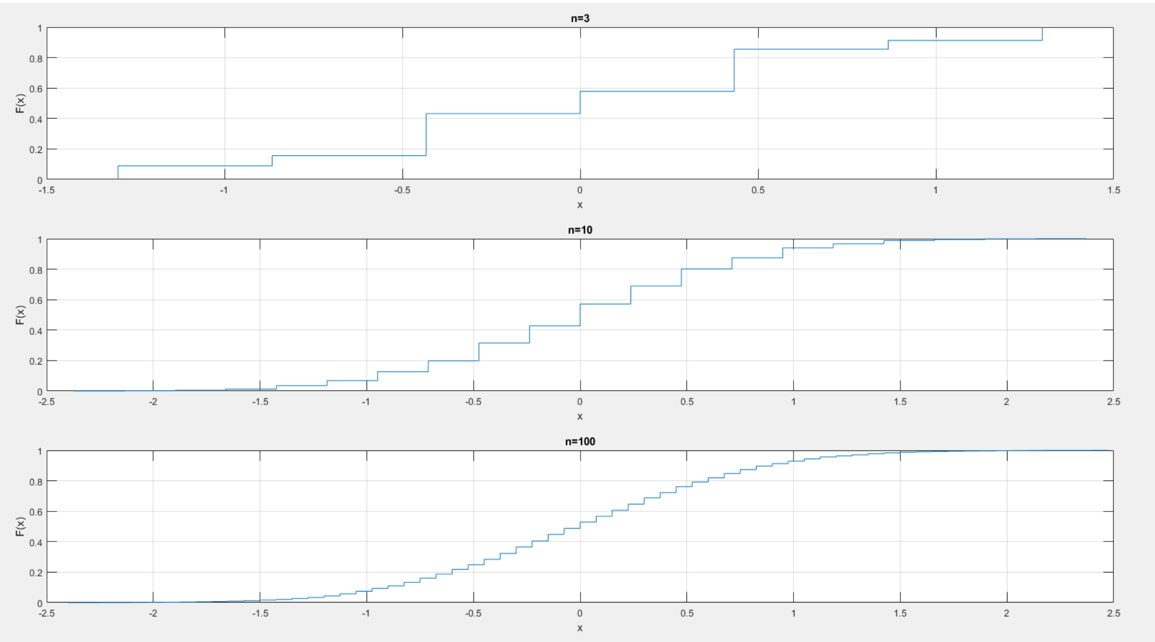

Q5

```
>> question5
average distance per step = 1.9993
average distance per step = 1.9988
average distance per step = 2.0031
average distance per step = 2.0022
average distance per step = 1.9989
```

The above results show the output for $\frac{1}{n} Y_{sum}^{(n)}$ when n=100,000. The results are again very close to the theoretical mean of 2.

Figures below show the CDFs for n=3,10,100.

We observe that as n increases the graph becomes smoother, in other words closer to Gaussian's distribution's cdf.



Q6

Q6.

According to CLT:

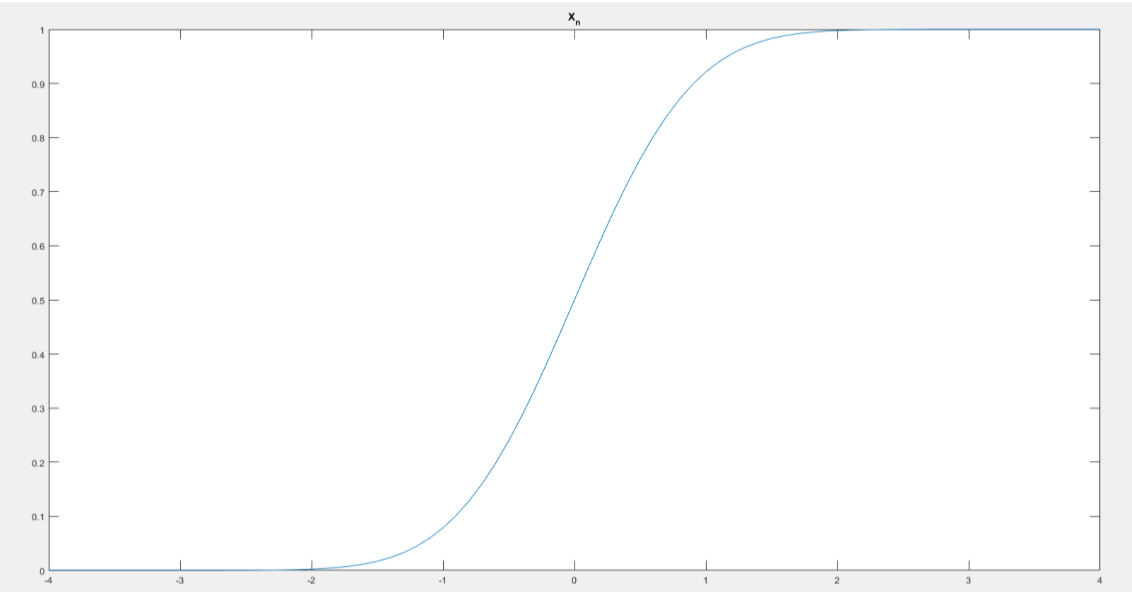
$$Z_n = \frac{(X_1+X_2+\cdots+X_n) - nE[X]}{\sqrt{n}} \qquad E[Z_n]=0 \qquad Var[Z_n] = \sigma_X^2$$

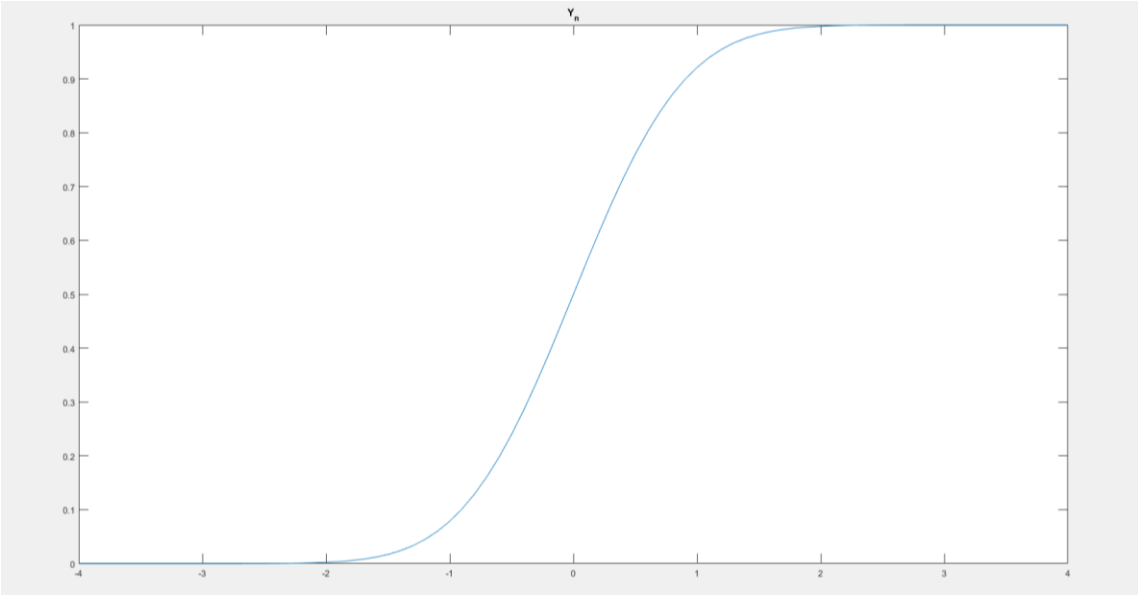
$$\lim_{n\to\infty} F_{Z_n}(z) = F_G(z) \qquad G \sim N(0, \sigma_X^2)$$

So as n increasing, the cdf will coverge to the normal cdf form which has mean of 0 and variance of $\sigma_X^2$.

As $\sigma_X^2 = \sigma_Y^2 = \frac{1}{2}$, so both $X^{(n)}$ and $Y^{(n)}$'s cdf will coverge as the form of Gaussian $(0, \frac{1}{2})$'s cdf form.

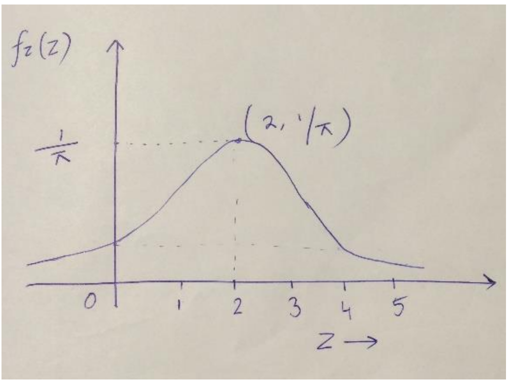
From the graphs below we observe that as n increases the graphs in Q4) and Q5) become very close to the normal cdf graph obtained below, which is consistent with CLT.






Q7

a) The mean and variance of Z is undefined. As the definition of mean won't apply for this case, because it's not integrable.
The pdf of Z is :



(b)

$$F_Z(z) = \int_{-\infty}^{z} \frac{1}{\pi} \cdot \frac{1}{1+(z-2)^2}\, dz$$

$$= \frac{1}{\pi} \cdot \arctan(z-2) \Big|_{-\infty}^{z}$$

$$= \frac{1}{\pi} \left( \frac{\pi}{2} + \arctan(z-2) \right)$$

$$= \frac{1}{2} + \frac{1}{\pi} \arctan(z-2)$$

$$F_Z(z) = U$$

$$\frac{1}{2} + \frac{1}{\pi} \arctan(z-2) = U$$

$$\arctan(z-2) = \pi\left(U - \frac{1}{2}\right)$$

$$z = \tan\left(\pi\left(U-\frac{1}{2}\right)\right) + 2$$

$\because$ $U$ and $U-\frac{1}{2}$ are identically distributed

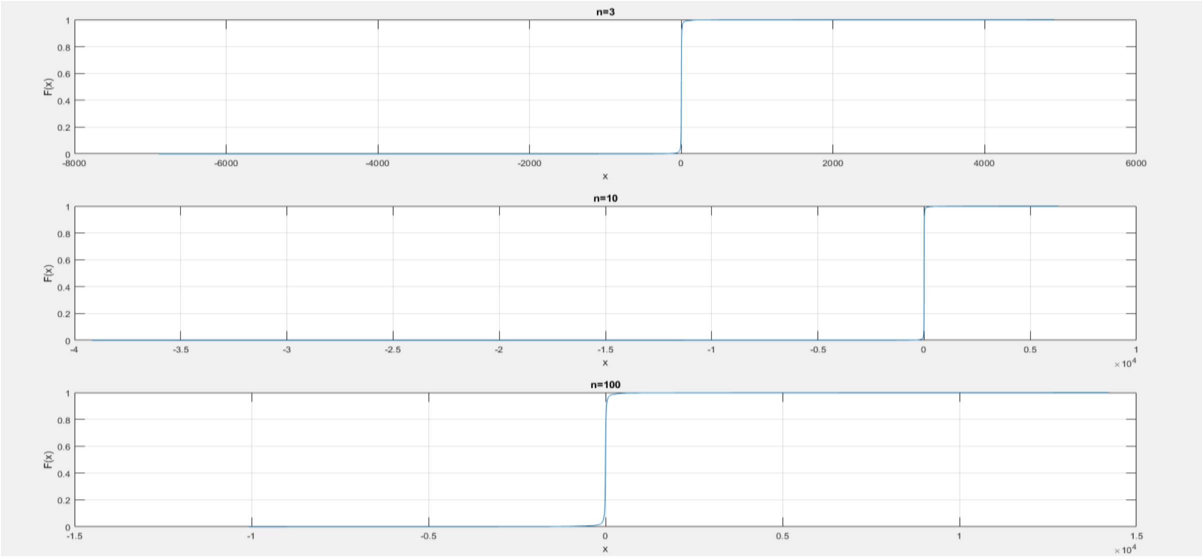
$$g(U) = \tan(\pi U) + 2$$

C) outputs of Z when n=100,000 steps

```
>> question7
average distance per step = 16.3056
average distance per step = 1.9619
average distance per step = 1.4896
average distance per step = 1.7597
average distance per step = 2.8556
>> question7
average distance per step = -19.7532
average distance per step = 3.2732
average distance per step = 1.3418
average distance per step = 2.4229
average distance per step = 2.0186
```

d) The results obtained are very random. We expect the results to be around the mean value of distribution Z. But as the mean and variance of Z is undefined as a result inconsistent results are obtained.

e) Because the CLT only applies when the mean of Z exist and variance of Z is nonzero and infinite. Since Cauchy's rv has no mean, the CLT doesn't hold, so we wouldn't expect the cdf plot look like Q4 and Q6's normcdf.


n=3
F(x)
x
n=10
F(x)
x
$\times 10^4$
n=100
F(x)
x
$\times 10^4$