Engineering Assignment Coversheet

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Assignment Title:	workshop 5
Subject Number:	ELEN90054
Subject Name:	Probability and Random Models
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Lecturer/Tutor:	Pasha Tolmachev
Due Date:	18/05/18

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Student signature Yue Chang Hongyu Du Date 12/05/18

ELEN90054 Probability and Random Models
MATLAB Workshop 5 Hypothesis Testing

Question 1

Hypothesis: H_0 = Reader is not faulty; H_1 = Reader is faulty.

$P[H_0] = 0.78$, $P[H_1] = 0.22$.

The first main rv is Y, and Y = quality of readers. $S_Y = \{Ok, Faulty\} = \{H_0, H_1\}$.

The second main rv is X, X = the first failure occurs on Nth trial (X = N).

$X|H_0 \sim$ Geometric (0.02), $f_{x|H0}(X) = (1\text{-}0.02)^{N\text{-}1}*(0.02) = 0.98^{N\text{-}1}*0.02$;

$X|H_1 \sim$ Geometric (0.1), $f_{x|H0}(X) = (1\text{-}0.1)^{N\text{-}1}*(0.1) = 0.9^{N\text{-}1}*0.02$;

Question 2

Under this case we can say that $P[\text{accept } H_1|H_0 \text{ is true}]_{min}=0$.

Minimizing the conditional probability means that H_1 should never get accepted, so the decision rule is: **decide not faulty if and only if N>=1.**

The probability called $\mathbf{P_{FA}}$

Question 3

Q3.

ML: "Accept H_0 if and only if the likelihood ratio $\Lambda(x) \geq 1$"

$$\Lambda(x) = \frac{f_{x|H_0}(x)}{f_{x|H_1}(x)} = \frac{(0.98)^{N-1}(0.02)}{(0.9)^{N-1}(0.1)} = \left(\frac{49}{45}\right)^{N-1}\left(\frac{1}{5}\right) \geq 1$$

$$(N-1)\ln\left(\frac{49}{45}\right) \geq \ln 5$$

$$N \geq 19.7$$

decision rule: decide not faulty if $N \geq 20$, otherwise decide faulty.

The decision rule is: **decide not faulty if and only if N >= 20, otherwise decide faulty.**

Question 4

Q4 MAP: "Accept H_0 if and only if the likelihood ratio $\Lambda(x) \geq \gamma$"

$$\Lambda(x) = \left(\frac{49}{45}\right)^{N-1}\left(\frac{1}{5}\right) \qquad \gamma = \frac{P[H_1]}{P[H_0]} = \frac{0.22}{0.78} = \frac{11}{39}$$

$$\left(\frac{49}{45}\right)^{N-1}\left(\frac{1}{5}\right) \geq \frac{11}{39}$$

$$(N-1)\ln\left(\frac{49}{45}\right) \geq \ln\left(\frac{55}{39}\right)$$

$$N \geq 5.04$$

decision rule:
decide not faulty if $N \geq 6$, otherwise decide faulty.

(a) The decision rule is: **decide not faulty if and only if N >= 6, otherwise decide faulty.**

(b)

Yes it's different. Because in ML rule, we assume prior $P[H_0]=P[H_1]=\frac{1}{2}$, $\Lambda(x)\gtrless\frac{P[H_1]}{P[H_0]}=1$

but in MAP, we take the real prior $\Lambda(x)\gtrless\frac{P[H_1]}{P[H_0]}=\frac{0.22}{0.78}$

Question 5

Q5.

accept H_1 when H_0 is true \$10 $C_{FA}=10$

accept H_0 when H_1 is true \$50 $C_M=50$

to minimize the expected overall cost, we adjust the decision rule " Accept H_0 if and only if the likelihood ratio $\Lambda(x)\gtrless\frac{C_M}{C_{FA}}\cdot\frac{P[H_1]}{P[H_0]}$ "

$\left(\frac{49}{45}\right)^{N-1}\left(\frac{1}{5}\right)\gtrless\frac{50}{10}\cdot\frac{0.22}{0.78}$ decision rule: decide not faulty if $N\geq 24$, otherwise decide faulty.

$(N-1)\ln\left(\frac{49}{45}\right)\gtrless\ln\left(\frac{275}{39}\right)$

$N\geq 23.9$

$P_{miss}=P(\text{accept } H_0 \mid H_1 \text{ is true})=P(N\geq 24 \mid H_1 \text{ is true})=1-(1-(1-0.1)^{23})=0.0886$

$P_{FA}=P(\text{accept } H_1 \mid H_0 \text{ is true})=P(N<24 \mid H_0 \text{ is true})=1-(1-0.02)^{23}=0.3716$

expected cost: $C_M\cdot P_{miss}\cdot P[H_1]+C_{FA}\cdot P_{FA}\cdot P[H_0]=50\times 0.0886\times 0.22+10\times 0.3716\times 0.78=3.873$

The decision rule is: **decide not faulty if N >= 24, otherwise decide faulty.**

P_{miss} = 0.0886, P_{FA} = 0.3717, expected cost = 3.87.

Yes, it's different from MAP rule, because minimum cost decision rule takes the cost for 'FA' and 'miss' into the consideration, to minimize the weighted expected cost, the priorities between 'FA' and 'miss' need to be tuned. $C_{FA}=10<C_M=50$, so in this case, 'miss' is a more significant error, we have to change the decision rule.

Question 6

Q6. $P_{miss}\leq 0.02$ Assume decision rule: decide not faulty if $N\geq k$, otherwise faulty.

$P_{miss}=P(\text{accept } H_0 \mid H_1 \text{ is true})=P(N\geq k \mid H_1 \text{ is true})=1-(1-(1-0.1)^{k-1})\leq 0.02$

$(0.9)^{k-1}\leq 0.02 \Rightarrow k\geq 39$

decision rule: decide not faulty if $N\geq 39$, otherwise faulty.

$P_{miss}=1-(1-(1-0.1)^{38})=0.0182$

$P_{FA}=1-(1-0.02)^{38}=0.536$

expected cost $=C_M\cdot P_{miss}\cdot P[H_1]+C_{FA}\cdot P_{FA}\cdot P[H_0]=50\times 0.018\times 0.22+10\times 0.536\times 0.78=4.3802$

For Q5, we need to get the min cost, but for this question we need to minimize P_{FA} under the constraint $P_{miss}\leq 0.02$, so the decision rule is different.

The decision rule is: **decide nor faulty if N >= 39, otherwise decide faulty.**

P_{miss}= 0.0182, P_{FA} = 0.536, expected cost = 4.3802

For Q5, we need to get the minimum cost, but for this question we need to minimize the P_{FA} under the constraint P_{miss}<=0.02, so the decision rule is different.

Question 7

a)

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>> Q7_a
the reader was not faulty
according to the decision rule in Q5, the reader was not faulty(correct)
according to the decision rule in Q6, the reader was not faulty(correct)
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b)

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>> Q7_b
according to the decision rule in Q5,
the expected cost = 3.858990, the miss rate = 0.087408, the false alarm rate = 0.371467
according to the decision rule in Q6
the expected cost = 4.377300, the miss rate = 0.018027, the false alarm rate = 0.535820
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With the results we can say that the empirical values of the expected cost, the miss rate and the false alarm rate are very close to the theoretical values in Q5 and Q6, so basically the results are consistent with Q5 and Q6.