Impact of Synaptic Strength on Propagation of Asynchronous Spikes in Biologically Realistic Feed-Forward Neural Network

Sayan Faraz, Idir Mellal, and Milad Lankarany

***Abstract*—We consider the problem of reliable information propagation in the brain using biologically realistic models of spiking neurons. Biological neurons use action potentials, or spikes, to encode information. Information can be encoded by the rate of asynchronous spikes or by the (precise) timing of synchronous spikes. Reliable propagation of synchronous spikes is well understood in neuroscience and is relatively easy to implement by biologically-realistic models of neurons. However, reliable propagation of rate-modulated asynchronous spikes is poorly understood and remains difficult to implement by those models. In this paper, we formulate how a multi-layered feedforward neural network (mlFNN) comprising biologically-plausible model neurons enables propagation of time-varying asynchronous spikes. Gradient descent algorithm is developed to estimate the connectivity between neurons (i.e., synaptic weights) in mlFNN. Furthermore, we propose an abstract network model to replicate information propagation in mlFNN with substantially less complexity in estimating synaptic weights. The abstract model has a great implication for neuromorphic computing, as it can be implemented in neuromorphic circuits with less complexity, less energy, and more speed. Simulation results demonstrate that (i) the mlFNN with optimal synapses transmits asynchronous spikes reliably, and (ii) the abstract network model reproduces information propagation performed by mlFNN with high accuracy (coding fraction $= 0.97 \pm 0.02$). We anticipate that this study will facilitate the design and implementation of biologically realistic mlFNN in neuromorphic circuits as well as cross-fertilizations between the fields of neuromorphic engineering, computational neuroscience and artificial intelligence.**

***Index Terms*—Information propagation, asynchronous spikes, optimal synaptic weights, abstract representation, biological neural network.**

I. Introduction

THE brain processes (i.e., represents, propagates, and computes) phenomenal amounts of information. It does so efficiently despite several biological constraints on the total number of neurons and the maximal firing rate (i.e., the average number of spikes in certain time windows) as well as other factors like the component size that limit the signal-to-noise ratio [1]. Neuron, as a building block of the brain, uses action potentials, or spikes, to encode information. Computation based on spikes is the most inspirational factor in the design and the development of biologically plausible artificial intelligence (AI) and deep learning algorithms. Machine learning (ML) interpretations of the learning rules that exists in biological neurons – e.g., considering spike-timing-depended plasticity (STDP) (the main form of synaptic change in neurons [2], [3]) as stochastic gradient descent algorithm – is garnering substantial interest in AI-related research and industry. This interest is extending to hardware implementations in the form of neuromorphic circuits. The structure of these circuits mimics neuro-physiological architectures of the nervous system [4]. Neuromorphic circuits are required to (i) better decipher the computational properties of neural systems using models implemented in integrated circuits and (ii) design and implement efficient devices for engineering applications by incorporating biological properties of neural systems [5], [6].

Despite the recent efforts in linking ML-based leaning rules to those exist in the brain [4], [7], [8], there are still fundamental gaps in the engineering realization of information processing performed by biological neurons and networks. As the brain is highly modular [9], effective communication in the brain relies on reliable transmission of information between the modules. Multi-layered feedforward neural networks (mlFNNs), with all-to-all connectivity, are thought as a biologically-plausible architecture that transmits information from the upstream neurons to the downstream neurons (see [10], [11] for other alrenative network models), either across different layers within the same cortical region or between different cortical regions [9]. Such information is represented by different types of spikes, namely synchronous and asynchronous.

The implementation of reliable transmission of spikes through mlFNNs is one of the most challenging issues in the field of *Computational Neuroscience* because biological constraints applied to the model neurons preclude accurate propagation of spikes from one layer to the other [9].

In this paper, we investigate how information can be reliably propagated in mlFNN comprising biologically-realistic model neurons connected via synapses in all-to-all fashion. Unlike

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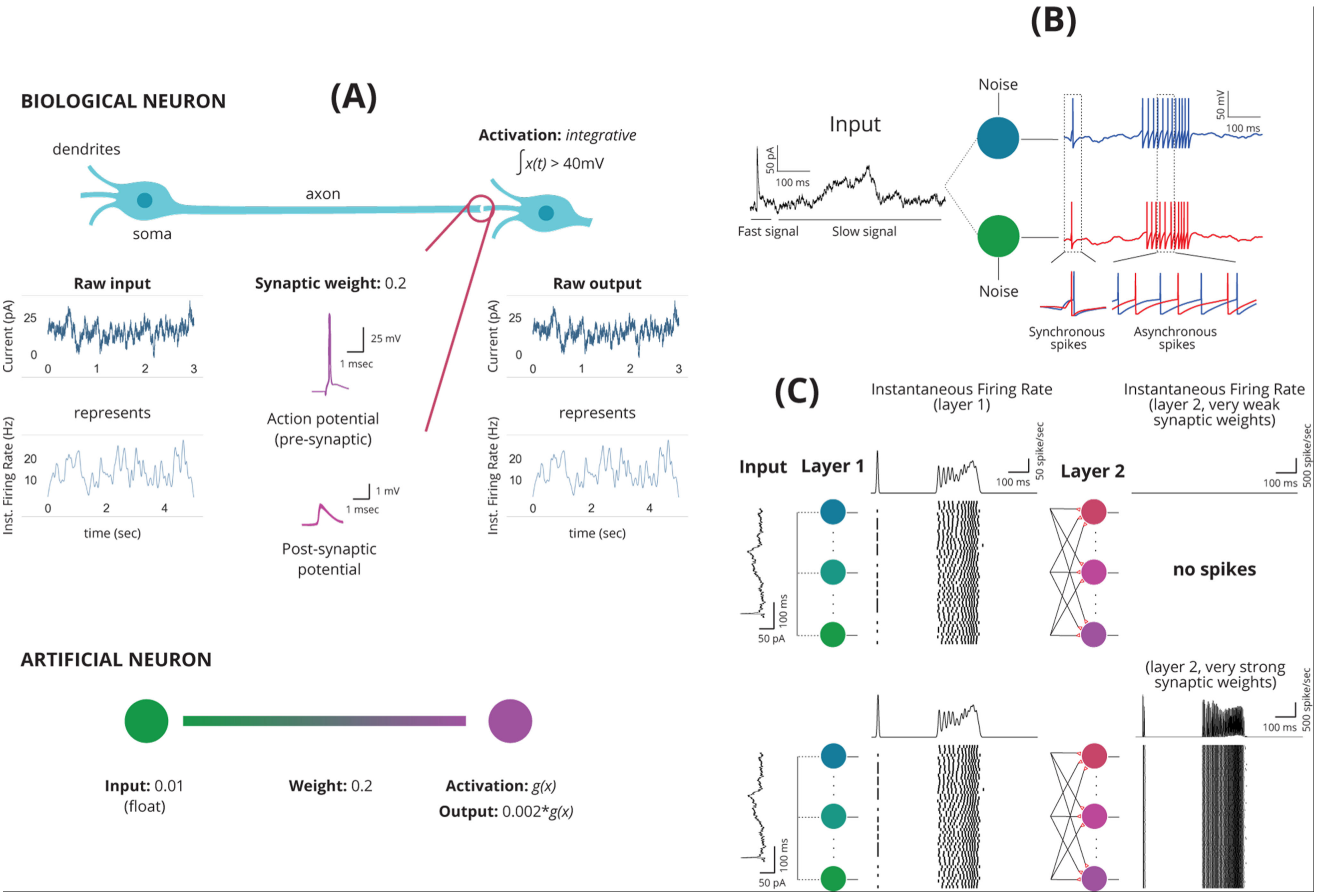

Fig. 1. (A) Biological versus artificial neurons. Note that biological neurons represent information (current (pA), in this example) in terms of *instantaneous firing rate* (which is calculated by the trial average of spikes over several neurons receiving the same input), and that the activation function (in LIF model) is a threshold on the *integral* of the previous input. (B) Synchronous vs asynchronous spikes. Each neuron receives an identical input plus independent background synaptic noise. The output of the LIF model neurons show membrane potential (mV). (C) Effect of synaptic weight values on the propagation of spikes. Spikes vanish in the presence of weak synaptic weights and turn synchronous in the presence of strong synaptic weights. Note that the output of neurons, at each layer, is shown by spikes, and the instantaneous firing rate is calculated by convolving those spikes with a Gaussian kernel ($\sigma_{\text{kernel}} = 25$ msec).

other related works in the literature where the connectivity between neurons, i.e., synaptic weights, is constant and within a physiologically realistic range, we derive an objective function, used with gradient descent optimization, to estimate the optimal synaptic weights in mlFNN. Furthermore, we propose an abstract network model that replicates information propagation in the mlFNN. The abstract model not only fulfills neurophysiological constraints but also converts the *ad-hoc* problem of estimating N^2 synaptic weights in the mlFNN (N is the number of neurons at each layer) to a tractable (closed-form) solution. Our extensive simulation studies demonstrate that reliable transmission of information can be implemented by mlFNN with optimal synaptic weights. As well, our results capitalize on the capability of the abstract network model in reproducing such reliable transmission.

The organization of this paper is as follows. In Section II, a thorough background on biological neurons as well as the state of the art of information propagation in biological neurons is provided. In Section III, preliminaries and problem formulation of the mlFNN are described. The abstract network model is developed in Section IV, and the methods for calculating synaptic weights of the mlFNN and the abstract model are presented in Section V. Simulation results are provided in Section VI, and finally, concluding remarks and future directions are provided in Section VII.

II. Background and Prior Work

A biological neuron model – spiking neuron model – represents functional properties of single cells in the brain using mathematical equations. These equations, due to the level of description, consist several stochastic differential equations (see [12] for more details). A typical biological neuron, as shown in Fig. 1.A, comprises dendrites (input to the neuron), cell body (integrate the received inputs), and axon which carries information (if action potential is generated). This is different from a typical artificial neuron whose activation function acts on individual float values.

Two neurons can be connected through a synapse which enables signal transfer (either chemical or electrical) from one neuron to another. Every action potential generates a post-synaptic potential in the target neuron (it is either excitatory that depolarizes, or inhibitory that hyperpolarizes the target neuron). These basic processes of biological neurons pose certain functional constraints that should be incorporated into the model neurons. The major neurophysiological constraints for information propagation through mlFNN are pointed as follows.

1) *Firing rate* (i.e., an average of spike counts expressed in Hz or spike/sec), the firing rate of biological neurons does not generally exceed 20 Hz (equivalent to 20 spike/sec) in response to a consistent stimulus (very few neurons can consistently fire fast (see [13]).
2) *Network size* (i.e., number of neurons), almost all of the studies in computational neuroscience consider the network size of (100 – 500) for an mlFNN (see [14]–[22]).
3) *Synaptic weight*, it indicates the strength of a synaptic connection. Indeed, one spike in a single presynaptic cell (soma) cannot generate more than 2 mV post-synaptic potential in the target neuron. Therefore, multiple presynaptic spikes are required to drive a postsynaptic spike.
4) *Network architecture*, an mlFNN with all-to-all connectivity with excitatory synapses represents the biological network model of information propagation in the brain (see [14]–[22]).

Neurons encode information by the rate of *asynchronous* spikes – rate code – or by precise timing of *synchronous* spikes – temporal code [14]. Fig. 1.B shows the generated spikes of two neurons in response to a common input. Spikes are characterized as *synchronous* and *asynchronous*. Synchronous spikes occur almost simultaneously in both neurons. In the temporal coding, information is carried by groups of neurons that fire more or less synchronously, as in synfire chains [22], [23].

Although *asynchronous* spikes do not preserve information of the spike timing, they encode the intensity of the stimulus.

In the rate coding, neuronal firing ideally remains asynchronous across neurons [24]–[26]. Reliable propagation of synchronous spikes is well understood and relatively easy to implement by biologically-realistic models of neurons [22], [26]–[28]. In contrast, reliable propagation of rate-modulated asynchronous spikes (rate code) is poorly understood and remains difficult to implement by those models [10], [11], [24], [29]. Several studies have addressed the conditions required for spike-rate propagation [9]–[11], [24]–[27], [29]–[31]. Feasibility of the rate transmission using leaky integrate and fire (LIF) models receiving balanced excitatory and inhibitory inputs is shown in [26]. The authors of [25] studied conditions for propagating synchronous spiking and asynchronous firing rate using more complicated and biologically realistic network models. They showed that the coexistence of firing rate and synchrony propagation can be achieved under precise combinations of synaptic strength and connection probability [9]. It is to be noted that in all the previous studies, the synaptic strengths between neurons were considered constant and within a physiologically realistic range.

Fig. 1.C emphasizes on the importance of synaptic weights in the propagation of spikes. The input to the first layer of the mlFNN comprises slow and fast signals. While the latter is represented by the precise timing of the synchronous spikes, asynchronous spikes encode the intensity of the slow signal (see the instantaneous firing rate of the first layer of an mlFNN). Weak synapses eliminate spikes to propagate to the next layer (Fig. 1.C (top)) whereas strong synapses result in excessive synchrony (Fig. 1.C (bottom)). If asynchronous spikes were to become synchronous in subsequent layers, the instantaneous firing rate would increase and therefore, the spikes would represent a higher value than the original spikes being transmitted.

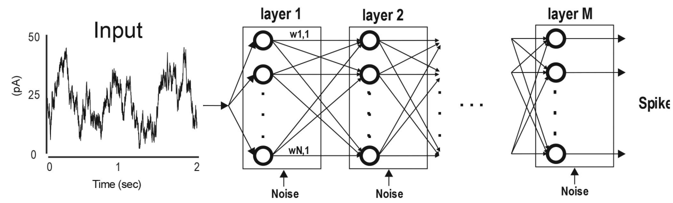

Fig. 2. **mlFFN model of information propagation**. Schematic representation of information propagation in a biologically-realistic feed-forward neural network with all-to-all connectivity (via synapses) comprising N neurons and M layers.

Therefore, more considerations should be given to the propagation of time-varying slow signals whose information is reflected by the rate of asynchronous spikes. Nevertheless, reliable propagation of the time-varying firing rate under biologically realistic assumptions, and where synaptic weights are not constant for all neurons over different layers, is rarely considered in the literature.

The focus of this paper is to address how asynchronous spikes can be reliably propagated through the mlFNN comprising biologically plausible model neurons.

III. Problem Formulation of Multi-Layered FNN Comprising Biological Neurons

In this section, we incorporate the abovementioned neurophysiological constraints in the mlFNN and create a feedforward network composed of biological neurons, modeled by leaky integrate and fire model (LIF, see Appendix). Fig. 2 shows a schematic representation of an all-to-all connected mlFNN with N neurons and M layers. Each model neuron, in the first layer, receives a common input that projects the neural activity of converging upstream neurons (see [32]) plus background synaptic noise (see [33] for more details).

The background synaptic noise is modeled by the Ornstein-Uhlenbeck (OU) process (see Appendix). For each neuron in layer k ($k > 1$), the post-synaptic potential (that is equivalent to the input to the neuron, I) is produced by passing (convolving) the spikes of the preceding layer, through an identical synaptic waveform. Note that the original input is added as a current to all neurons in layer 1 (similar to that in Fig. 1.C).

Spikes, for each (post-synaptic) neuron, are generated by feeding the weighted sum of those post-synaptic potentials into the LIF model. The spike train of neuron i in layer k, $s_i^{layer\,k}$, can be written as follows.

$$s_i^{layerk} = F_i\left[I_i^{layerk}\right] \tag{1}$$

where I_i and F_i indicate the (synaptic) input and the function of the model neuron i. As neurons, in this paper, are homogeneous,

$F_i = F$ for all $i = 1, \ldots,$ N. And, $I_i^{layer\, k}$ can be expressed by:

$$\begin{aligned} I_i^{layerk} &= w_{1,i}\phi_1^{layerk-1} + \cdots + w_{N,i}\phi_N^{layerk-1} + I_i^{Noise} \\ &= \sum_{j=1}^{N} w_{j,i}\phi_j^{layerk-1} + I_i^{Noise} \\ &= W_{(:,i)}^H \Phi^{layerk-1} + I_i^{Noise} \end{aligned} \tag{2}$$

where I_i^{Noise} stands for background synaptic noise received by neuron *i*, and modeled by Gaussian noise with zero mean and standard deviation σ (similar to [33], $\sigma = 25$ pA used in this paper generated $\sim$ 5mV sub-threshold membrane potential).

$$I_i^{Noise} \overset{D}{\equiv} Normal\left(0, \sigma\right), i \in [1, \ldots, N] \tag{3}$$

In (2), *W* represents the synaptic weight matrix.

$$W = \begin{bmatrix} w_{1,1}, \ldots, w_{1,N} \\ \vdots \\ w_{N,1}, \ldots, w_{N,N} \end{bmatrix}^H \tag{4}$$

And, $\Phi^{layerk-1}$ (non-weighted post-synaptic potentials) indicates a vector comprising spikes (layer *k-1*) convolved with the synaptic waveform, and it can be expressed as follow.

$$\Phi^{layerk-1} = [\phi_1^{layerk-1}, \ldots, \phi_N^{layerk-1}]^H \tag{5}$$

where,

$$\phi_i^{layerk-1} = s_i^{layerk-1} * \alpha\left(.\right), i \in [1, \ldots, N] \tag{6}$$

$\alpha(.)$ is an identical synaptic waveform modeled by a double exponential function of $\tau_{rise} = 0.5\, msec$ & $\tau_{fall} = 3\, msec$, and '$*$' stands for the convolution function.

Here, we define the objective function underlying maximum information representation in the k^{th} layer of an mlFNN, that is equivalent to minimization of the L_2 norm between the instantaneous firing rate in the k^{th} layer and that in the first layer (see Fig. 2).

$$\min_{\text{s.t. } N,w,\sigma} \left\{ \left| K_{FR}\left(S^{layer\ k}\right) - K_{FR}\left(S^{layer\ 1}\right)\right|_2^2 \right\} \tag{7}$$

where, $\mathrm{K_{FR}(S)} = \mathrm{S} * Kn$, represents the instantaneous firing rate, and *Kn* is a Gaussian kernel with a standard deviation of 25 *msec*. $S^{layerk} = [s_1^{layerk}, \cdots, s_N^{layerk}]$ shows the vector of all neuron's spikes, and *k* indicates the layer index. Since the exact spike timing cannot be reproduced in even purely reliable information propagation, we consider the instantaneous firing rate with a moderate time window of 25 *msec* to compare the spikes in the initial and first layers.

IV. Abstract Network Model

The abstract network model, as shown in Fig. 3, is derived to reproduce information propagation performed by the mlFNN. Here, we show that this abstract network model is equivalent to the mlFNN under the following assumptions. First, all the neurons in the network (including all the layers) are homogeneous. Second, the synaptic function (alpha-function $\alpha(.)$) is identical for all synapses. Both assumptions have been considered in other related studies [14]–[22].

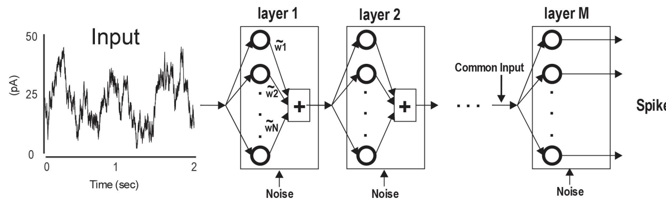

Fig. 3. Schematic representation of the abstract network model of mlFNN.

Given the abovementioned assumptions, one can express the common input to the neurons of an arbitrary layer as:

$$I_{Common} \equiv Normal\left(E\left\{I_{Common}\right\}, \sigma\right) \tag{8}$$

where E{.} is the expectation function, and σ is the level of background synaptic noise (as before). One can see that the common input has a Gaussian distribution whose variability is produced by the synaptic noise that each (homogeneous) neuron receives, i.e., $I_{Common} = E\{I_{Common}\} + Normal(0, \sigma)$ (see (2)). Considering the homogeneity of each neuron, the mean of the common input is equal to that of each neuron, that is

$$E\left\{I_{Common}\right\} = E\left\{I_i\right\}, \text{for } i \in [1, \ldots, N] \tag{9}$$

From (2), we have,

$$E\left\{I_i\right\} = \sum_{j=1}^{N} w_{j,i}\phi_j, \text{for } i \in [1, \ldots, N] \tag{10}$$

In fact, (10) implies that the mean of the input signal to all neurons in a specific layer is identical. We can write:

$$w_{j,i} = w_{j,k}\,, \text{for } i\,,\ k\ \in\ [1\,,\ \ldots,\ N] \tag{11}$$

Thus, (11) offers a network model reduction with respect to the abovementioned assumptions. Given (9), (10) and (11), one can replace the synaptic matrix *W* with the synaptic vector $\tilde{W}$, where,

$$\tilde{W} = \begin{bmatrix} \tilde{w}_1 \\ \vdots \\ \tilde{w}_N \end{bmatrix} \tag{12}$$

The average of the input to a neuron, i.e., identical for all neurons in a specific layer, can be written as:

$$E\left\{I_{Common}\right\} = \sum_{k=1}^{N} \tilde{w}_k\phi_k = \Phi\tilde{W} \tag{13}$$

Therefore, unlike *W* that reflects the synaptic weights, $\tilde{W}$ expresses the contribution of neurons in constructing (decoding) the input signal, i.e., each neuron has a certain contributing weight uniformly distributed to all following neurons.

V. Estimating Synaptic Weights in Multi-Layered FNN and Abstract Model

To calculate synaptic weights using the objective function (7), the function of the biological neurons (see (1)) should be known a priori. However, the nonlinearities underlying this function complicates the derivation of tractable optimization algorithms. To overcome this, we incorporate the previously explained assumptions and derive a new objective function that is equivalent to (7) and enables the development of the tractable optimization algorithms.

The common input to each layer of an mlFNN can be described by a matrix of post-synaptic potentials,Φ, in the preceding layer, multiplied by synaptic weights (see (2) for mlFNN and (13) for the abstract model). Thus, the generated spikes by a neuron in the k^{th} layer, as expressed in (1), can be rewritten as follows.

$$s_i^{layer\ k} = F_i\left[E\left\{I_{common}^{layer\ k}\right\} + Normal\left(0,\sigma\right)\right] \tag{14}$$

where $i \in [1,\ldots,N]$. Equivalently, the spike matrix of all neuron in this layer will be expressed by:

$$S^{layer\ k} \approx F\left[Normal\left(E\left\{I_{Common}^{layer\ k}\right\},\sigma\right)\right] tag15$$

Therefore, the objective function (7) is equal to:

$$\begin{aligned}&\min_{\text{s.t. } N,w,\sigma}\left\{\left|K_{FR}\left(S^{layer\ k}\right) - K_{FR}\left(S^{layer\ 1}\right)\right|_2^2\right\} = \\ &= \min_{\text{s.t. } N,w,\sigma}\left\{\left|K_{FR}\left(F\left[Normal\left(E\left\{I_{Common}^{layer\ k}\right\},\sigma\right)\right]\right)\right.\right. \\ &\qquad\left.\left. -\ K_{FR}\left(F\left[Normal\left(input,\ \sigma\right)\right]\right)\right|_2^2\right\}\end{aligned} \tag{16}$$

Here, we simplify (16), and solve it in an indirect but innovative way. Our rationale is based upon the fact that minimization of (16) implies that the *decoded* input by spikes of neurons in the k^{th} layer of the mlFNN should be equal (in an ideal case) to the original input. In fact, reliable transmission of firing rate across several layers of an mlFNN is achievable if the rate of asynchronous spikes can be maintained almost unchanged in the preceding layers. This interpretation is equivalent to information maximization from a decoding perspective [14]. Using this interpretation, we write a new objective function as below.

$$\begin{aligned}&\min_{\text{s.t. } N,w,\sigma}\left\{\left|K_{FR}\left(S^{layer\ k}\right) - K_{FR}\left(S^{layer\ 1}\right)\right|_2^2\right\} \\ &\approx \min_{\text{s.t. } N,\ w,\ \sigma}\left\{\left|1/N\sum_{i=1}^{N} W_{(:,i)}^{H}\Phi^{layerk} - input\right|_2^2\right\}\end{aligned} \tag{17}$$

To calculate the weights in an mlFNN, we use gradient descent method where the derivatives of the loss function (17) – with respect to the weights – are used to update the weights iteratively (see Simulation Results for more details).

To calculate the synaptic weights in the abstract network model, we write (17) as follows.

$$\min_{\text{s.t. } N,\ w,\ \sigma}\left\{\left|\Phi^{layer\ k}\tilde{W}^{layer\ k} - input\right|_2^2\right\} \tag{18}$$

Since the weights are non-negative, one can derive constrained and unconstrained optimization solutions for (18). In this paper, we use unconstrained optimization (Least-square method) where the negative weights (after calculation) are set to zero. In this case, the weights can be calculated in a closed-form.

$$\tilde{W}^{layerk} = \left(\Phi^{layer\ k^H}\Phi^{layerk}\right)^{-1}\Phi^{layer\ k^H} input \tag{19}$$

where H indicates a matrix Hessian.

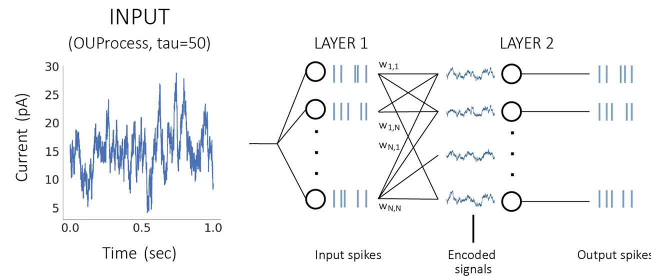

Fig. 4. Simulating mlFNN. Input derived from an OU-process generates asynchronous spikes in layer 1, which in turn generate asynchronous spikes after being weighted with optimal synapses and passed through layer 2.

VI. Simulation Results

In this section, we use numerical simulations to verify if (**i**) an mlFNN with biological neurons and optimal synaptic weights enables reliable propagation of asynchronous spikes, and (**ii**) the abstract network model can capture such reliable information propagation.

In this simulation study, mlFNN and abstract model are composed of excitatory neurons, modeled by leaky integrate and fire (LIF) model, receiving shared input from the previous layer plus background synaptic noise. All the models and corresponding analysis are performed in Python. Fig. 4 shows the main steps underlying the numerical simulations of the mlFNN comprising 2 layers of 200 LIF neurons.

For both mlFNN and abstract model, a 10-second signal (common input is produced by an OU process of $\mu = 16$ pA, $\sigma = 15$ pA, and $\tau = 50$ msec) is fed into the first layer of LIF neurons, generating spike trains. As well, each neuron receives an independent background synaptic noise of 25 pA, modeled by an OU-process with $\tau = 5$ msec. The spike trains, generated in layer 1, are convolved with an identical synaptic waveform and multiplied by each network's respective weights (matrix-weight in mlFNN, and vector-weight in the abstract model). For mlFNN, the multiplication yields a voltage output per neuron, which is fed into the second layer of LIF neurons. For the abstract model, the resulting common output is fed (as an injected current) into each neuron of the second layer of LIF neurons.

Fig. 5 shows the results of the mlFNN.

First, we show that the mlFNN with optimal synaptic weights propagates asynchronous spikes reliably. We utilized Adam's stochastic gradient descent optimization approach, implemented in PyTorch [34], [35], to obtain the synaptic weight-matrix. Note that the dimension of synaptic weights is expressed by pA/mV based on (17). The matrix-weight is trained on 3 seconds of data $(dt = 1 \times 10^{-4}s)$ which was obtained from the common input in layer 1.

As shown in Fig. 5 (top & middle), we observe a reliable propagation of asynchronous spikes in the second layer of

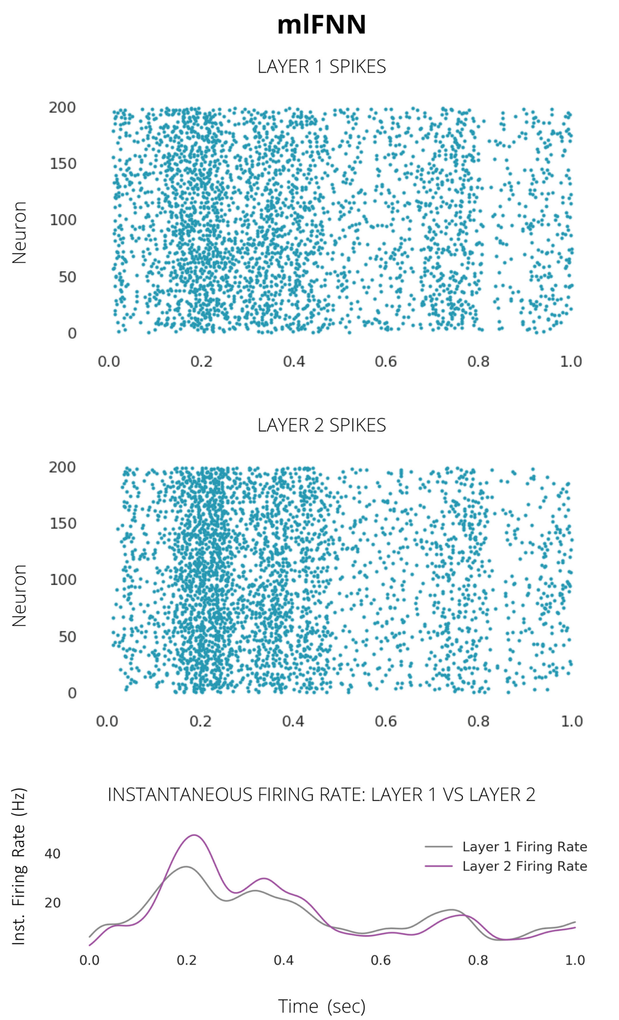

Fig. 5. Information propagation in mlFNN (up to 2 layers). Spikes of 200 neurons, corresponding to 1 sec simulation, in the 1st layer (top) and the 2nd layer (middle) are plotted. The instantaneous firing rate of spikes in both layers (bottom) indicates that the rate of asycnronous spikes is reliably propagated.

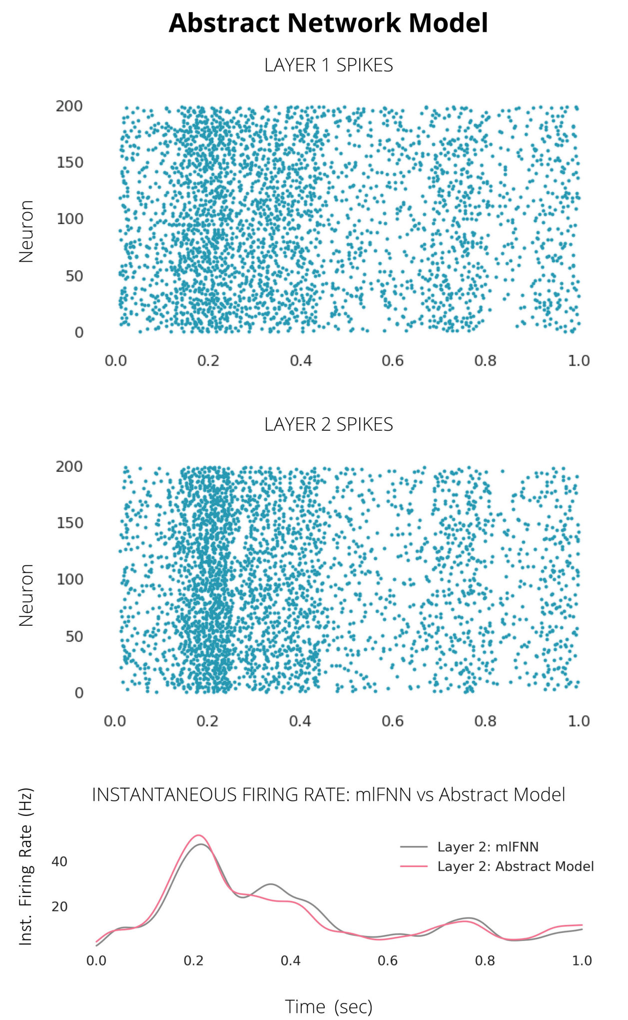

Fig. 7. Information propagation in the abstract model (up to 2 layers). Spikes of 200 neurons, in the 1st layer (top) and the 2nd layer (middle), are shown. The instantaneous firing rate of spikes in the 2nd layer of the abstract model is plotted versus that of the mlFNN (bottom).

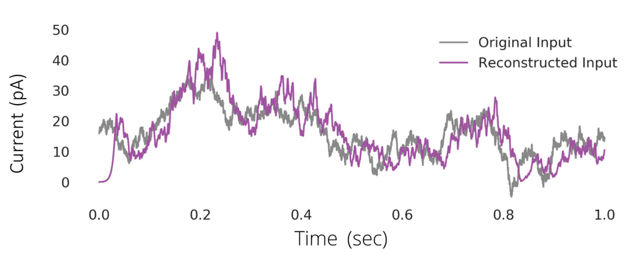

Fig. 6. Original (gray) vs. reconstructed input (purple) by spikes of layer 2 of the mlFNN.

mlFNN with optimal synaptic weights. The instantaneous firing rate of the spike trains of layers 1 and 2, which is calculated by convolving the spike trains with a Gaussian kernel ($\sigma = 25ms$), is shown in Fig. 5 (bottom). The similarity of the instantaneous firing rates confirms that the rate of asynchronous spikes remains almost unchanged across the layers of mlFNN. Fig. 6 shows the input generated by the spikes of the 2nd layer of the mlFNN versus the original common input. The robust reconstruction of the input of layer 2 (compared to the original input) confirms that the mlFNN with optimal synapses preserves (input) information.

Second, we show that the abstract network model reproduces information propagation observed in mlFNN. To this end, we consider the mlFNN with the estimated synaptic matrix-weight as the ground truth and investigate how well the abstract model with synaptic vector-weight can approximate it. Fig. 7 shows that asynchronous spikes are reliably propagated through the layers of the abstract model. As shown in Fig. 7 (bottom), the similarity between the instantaneous firing rates of the 2nd layers of the abstract model and the mlFNN indicates that the abstract model preserves the rate of asynchronous spikes.

Moreover, we use coding fraction (CF) to quantify how well the abstract network model can reproduce information propagation demonstrated by the mlFNN. Given the rate of asynchronous spikes in the 1st layer as the information to be transmitted, CF is calculated as follows [36].

$$CF = 1 - \frac{Firing\,(Layer\ 2) - Firing(Layer\ 1)_2}{Firing(Layer\ 1)_2} \qquad (20)$$

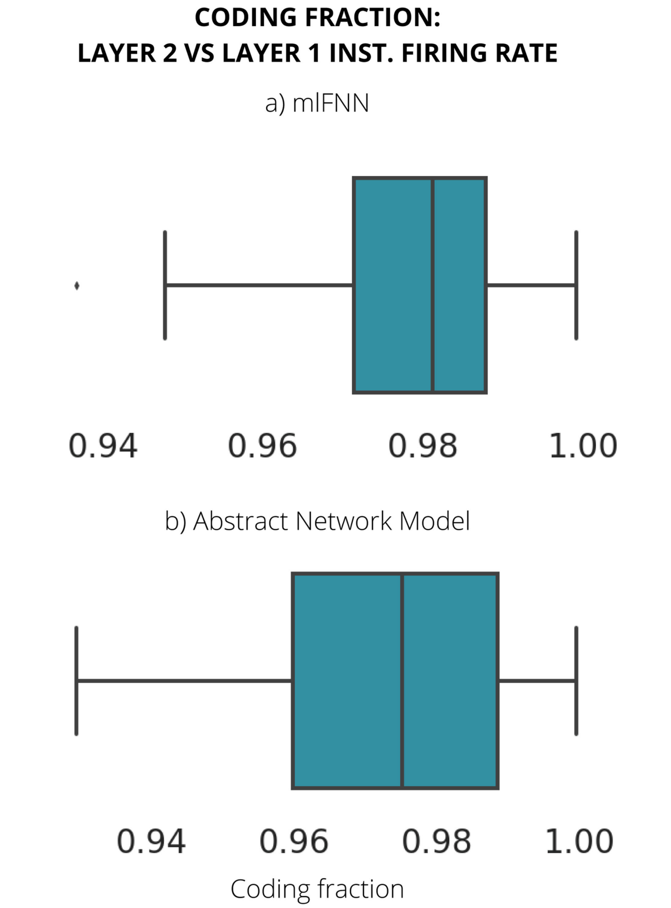

Fig. 8. (a) Coding fraction between the spikes in the 1st and 2nd layers of the mlFNN (c.f. mean: 0.978, std-dev 0.014, 50 trials). (b) Coding fraction between the spikes in the 1st and 2nd layers of the abstract model (c.f. mean: 0.973, std-dev 0.019; 50 trials). The abstract network model preserves the instantaneous firing rate of spikes propagated through the network.

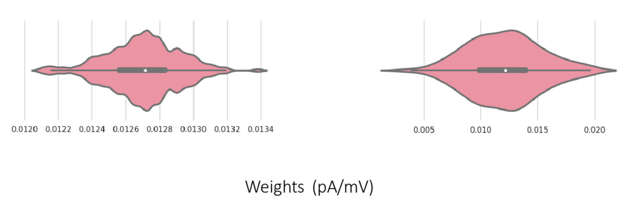

Fig. 9. Synaptic weights estimated by gradient descent (left) and closed-form solution (right). The synaptic weights in the latter closely resemble those found for the former. Weights are greater than 0, and center around a similar mean, and have a small standard deviation in both representations.

where *Firing (Layer 1)* and *Firing (Layer 2)* denote the instantaneous firing rate of the 1st and the 2nd layers, respectively. And, $\|.\|_2$ indicates the norm 2. CF lies within $[-1, 1]$, where 1 represents perfect reconstruction.

Fig. 8(a) shows that CF of the mlFNN is near to 1, meaning that information of asynchronous spikes in the 1st layer of the mlFNN is almost completely preserved by the spikes of the 2nd layer. As well, CF of the abstract model (Fig. 8(b)) is as high as that of the mlFNN. This confirms that the abstract network model can robustly reproduce the propagation of asynchronous spikes performed by the mlFNN. Note that the spikes of the 1st layers of both mlFNN and abstract model are the same.

Finally, we test if the estimated synaptic weights are biologically feasible. The distributions of synaptic matrix-weight (mlFNN) and synaptic vector-weight (abstract model) are plotted in Fig. 9.

Both distributions have a small spread around the mean, which generates post-synaptic potentials smaller than 2mV, meaning that the weights are biologically feasible. In addition, the estimated weights in both cases are centered around a mean of 0.0125 pA/mV. Therefore, one can conclude that the synaptic vector-weight closely approximates the synaptic matrix-weight.

Note: all the codes in our simulation study are provided in Pyton and can be found here: https://github.com/nsbspl/async-spike-propagation.

VII. Concluding Remarks

In this paper, we addressed the problem of reliable propagation of information encoded by the rate of asynchronous spikes using biologically realistic models of spiking neurons. A biologically realistic mlFNN with all-to-all connectivity is formulated for this problem by incorporating neurophysiological constraints on the total number of neurons, the maximal firing rate, the synaptic strength, and the network architecture.

We further developed an abstract network model fulfilling the mentioned constraints to replicate information propagation in mlFNN with substantially less complexity in estimating synaptic weights, i.e., reducing the parameter space of synaptic weights from N^2 to N (N is the number of neurons at each layer). Our simulation studies demonstrated that information of asynchronous spikes can be reliably propagated in the mlFNN (two layers have been used in this paper). And more importantly, the proposed abstract network model reproduced information propagation performed by the mlFNN. To the best of our knowledge, the abstract network model (including neurophysiological constraints) is the first of this kind, and can be used for further explorations on information propagation in neuroscience. Compared to the existing biological models for studying information propagation in the brain, the abstract network model offers two major contributions. First, it can be utilized in a comprehensive understanding of information propagation in mlFNN by systematically studying necessary conditions (e.g., network size, level of back synaptic noise, etc) for reliable propagation of time-varying firing rates. Second, the abstract network model can be efficiently implemented in neuromorphic circuits. Reducing the size of the neural network will have a significant impact on the accuracy, the speed, and the flexibility of the implemented models in neuromorphic circuits. Moreover, the power consumption for implementing the abstract model compared to the mlFNN with all-to-all connectivity will be significantly decreased.

The proposed abstract network model can replicate information propagation in the mlFNN with substantially less complexity, thus suggesting a reliable biological network model – biologically realistic feedforward neural network – for information propagation. Further considerations on the use of more efficient objective functions (information-theoretic) that ensure the maximization of information in the consecutive layers of an mlFNN shape our future line of research.

APPENDIX

LIF model. Neurons were modeled as follows.

$$\frac{dV}{dt} = \frac{-(V - E_L) + RI_{inj}}{\tau_V}$$

where $E_{\mathrm{L}} = -70$ mV, $R = 1$ MΩ, and $\tau_V = 10$ msec. I_{inj} indicates the injected current (slow signal in this paper). Spike occurs when V≥Vth, where $V_{th} = -40$ mV and the reset voltage is -90 mV.

OU-process. This process can be written as:

$$\frac{dx}{dt} = -\frac{x(t) - \mu}{\tau} + \sigma\sqrt{\frac{2}{\tau}}\,\xi(t)$$

where ξ is a random number drawn from a Gaussian distribution with 0 average and unit variance. τ is the time constant, μ and σ indicate the mean and standard deviation of variable *x,* respectively. The slow signal (common signal) and background synaptic noise are generated using OU process with time constant of 50 *msec* and 5 *msec*, respectively. The mean and variance of the slow signal (in layer 1) is 16 pA and 15 pA, respectively.

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