International Journal of Computational Methods
Vol. 2, No. 4 (2005) 543–553
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NUMERICAL ANALYSIS ON PROGRESSIVE FRACTURE BEHAVIOR BY USING ELEMENT-FREE METHOD BASED ON FINITE COVERS

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Received 20 December 2004
Revised 21 June 2005
Accepted 2 July 2005

The finite-cover element-free method FCEFM is applied to simulate the fracture and damage evolution process of geo-materials. This method is mathematically based on the finite-cover technique of manifold method and the multiple weighted moving least-square method to solve the continuous and discontinuous problems without meshing or re-meshing. The damage heterogeneity and evolutionary processes of rock mass with initial cracks are analyzed and numerically simulated by FCEFM. Using the method of probability to generate the parameters of materials randomly, the physical and mechanical properties of materials are randomly distributed in nodes or Gaussian points. And an alternating damage model together with numerical implementation which is adapted to microscopic elasto-brittle fracture analysis is proposed. Through analysis of several numerical examples, the validity and efficiency of progressive fracture analysis with use of the proposed FCEFM is demonstrated.

Keywords: Meshless; manifold method; discontinuous; crack growth; heterogeneity; damage analysis.

1. Introduction

In the studies of rock mass and brittle materials, the macroscopic fracture mechanics is based on classical fracture mechanics to research the macroscopic cracks. Many numerical methods were developed for crack growth and fracture of materials, such as FEM, BEM, MM [Shi (1996)], EFGM [Belytschko (1994)], FCEFM [Luan (2001)], PIM [Liu (2001)], and so on. Although these numerical methods have been applied in practical engineering, they have some limitations when dealing with problems which have serious initial heterogeneity and discontinuity, and the initiation, propagation and interaction of complex cracks. Other methods are

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based on the microscopic damage mechanics [Namet-Nasser (1993)], which mainly focus on the fracture of microscopic constitution of material with the continuum mechanic analysis method and the theory of fracture statistics. It aims to access to the macroscopic fracture process through the study of initiation, evolution and merging of the microscopic constitution and cracks of solid materials. The preexisted damage is the key factor that influences mechanical property of materials for the heterogeneity and discontinuity. To various present numerical methods based on the traditional fracture mechanics in solving problems of multi-cracks, the disadvantage is that the heterogeneity of materials is not considered. The macroscopic nonlinearity exhibited in deformation is mainly depended on the microscopic heterogeneity of materials.

The meshless or mesh-free method uses nodes to disperse the domain of problem, the construction of approximation and the computation and the energy functional are all based on a set of limited nodes. Because the connection between nodes is only built up during the construction of the approximation function, the nodes can be added in or subtracted at need. Therefore the meshless method is very suitable to analyze the propagation of crack. But there are some problems in dealing with the interaction of multi-crack and the propagation of secondary interstices or cracks. In this paper, the viewpoints of damage mechanics and the idea of progressive fracture [Bui (1984); Chang (1987)] are introduced, and the element-free method based on finite-cover of progressive fracture analysis is presented and applied to study the fracture process and the damage evolution of brittle materials such as rock, concrete and so on.

2. Finite-Cover Element-Free Galerkin Method

The Element-free Galerkin method is a mesh-free method which is based on the moving least squares method and Galerkin weak form. Moving least squares method (MLS) was proposed by Lancaster and Salkauskas, 1981, for data fitting and surface constructing. MLS should be categorized as a linear programming method. MLS approximation was initially used to construct shape functions for diffuse element method by Nayroles *et al.* [1992]. The MLS approximation method has two main characteristics: (1) the approximation field function is continuous and smooth in the domain; (2) it is capable of constructing shape function with arbitrary order of consistency. The constructing procedure of shape function by using MLS approximation is not given in this paper in detail due to the limited space.

Manifold method (MM) was developed by Shi Genhua to solve the problems of continuous and discontinuous medium. The finite-cover method applies the finite circle cover technique of the manifold method, to ensure that the shapes of approximation fields constant on the continuous problems or the discontinuous problems. For this, the constructions of approximation function for continuous and discontinuous problems will be unified in the same approximation field.

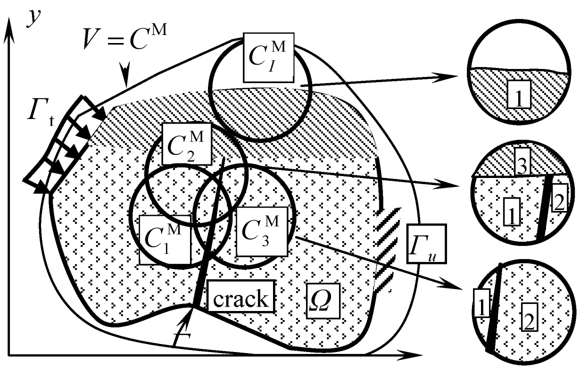

Fig. 1. The circular cover system and the mathematical cover (C_i^{M}) and the physical cover (C_i^{P} marked with $1, 2, 3$ in the right part of the figure).

Consider that the domain Ω is limited by its boundary Γ. A flakelet or sphere respectively in 2D or 3D problem that overlaps the domain Ω is defined as a mathematical cover denoted as C^{M}. As cut by one or more physical boundaries, such as cracks or interfaces of different media, a mathematical cover may be divided into two or more parts which are defined as physical cover denoted as C^{P}. For continuous deformation, a C^{M} can only reproduce a physical cover C^{P}, and such a mathematical cover C^{M} is called as a continuous cover. For discontinuous deformation problems, however, a C^{M} may be cut into two or more C^{P}s and such a C^{M} is called discontinuous cover. All C^{P}s form a finite physical cover system of the domain that is a discretization pattern of the domain of problem. The technique of decomposing a domain by the finite physical cover system is called the finite cover technique. In the meantime, a C^{P} can be called a node, denoted as $\{\mathbf{x}_I\}$, $(I = 1, 2, \ldots, P)$. The freedom of a node is defined on its center according to C^{M}. The forming process of the cover system is illustrated in Fig. 1.

2.1. *Multiple weighted moving least-squares method*

In the light of moving least-squares method, the fundamental of the multiple weighted moving least-squares method (MWMLS) is to be illuminated as follows. At first, define a cover weighted function for every mathematical cover C_i^{M}. It is denoted as $\bar{w}_i(x)$, $x \in C_i^{\mathrm{M}}$ and satisfied the following conditions for continuity.

$$\begin{cases} \bar{w}_i(x_i) = 1, \\ \bar{w}_i(x) = 0, \quad x \notin \mathrm{int}\ \partial C_i^{\mathrm{M}}, \\ \bar{w}_i(x) > 0, \quad x \in \mathrm{int}\ \partial C_i^{\mathrm{M}}, \\ \text{at least } C^1 \text{ continuously differentiable.} \end{cases} \tag{1}$$

As the cover weighted functions were defined on the mathematical cover, the shapes of cover weighted functions will not be changed no matter what the specified

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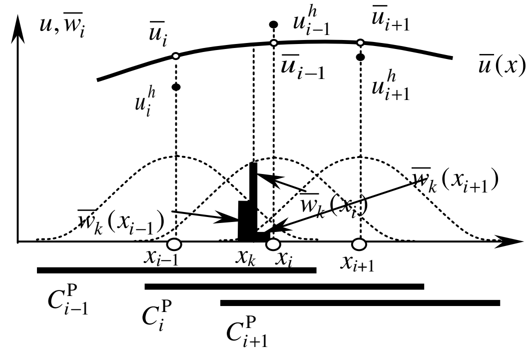

Fig. 2. The multiple weighted MLS method.

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materials are, continuous or discontinuous. It should be noted that a mathematical cover may has many different physical covers. Therefore this cover weighted function has one or more different influence ranges, and it has many different definition domains as shown in Fig. 2.

The approximation function $\bar{\mathbf{u}}(\mathbf{x})$ of the real field function $\mathbf{u}(\mathbf{x})$ at any point can be defined as:

$$\mathbf{u}(\mathbf{x}) \approx \bar{\mathbf{u}}(\mathbf{x}) = \sum_{j=1}^{m} P_j(x) a_j(x) = \mathbf{P}^{\mathrm{T}}(\mathbf{x})\, \mathbf{a}(\mathbf{x}), \tag{2}$$

where the $\mathbf{P}(\mathbf{x})$ is a linearly independent complete function sequence, and $\mathbf{a} = [a_1\ a_2\ a_3 \cdots a_m]^{\mathrm{T}}$ is undetermined coefficients, where m is the number of undetermined coefficients dependent on $\mathbf{P}(\mathbf{x})$, $m = 3$ as $\mathbf{P}(\mathbf{x})$ is first-order polynomial and $m = 6$ as $\mathbf{P}(\mathbf{x})$ is second-order polynomial. The functional of weighted residual of MWMLS is:

$$\begin{aligned} \Pi(\mathbf{x}) &= \sum_{i}^{n_k} \bar{w}_k(\mathbf{x} - \mathbf{x}_i)\big[u^h(\mathbf{x}_i) - \bar{u}(\mathbf{x}_i)\big]^2 \\ &= \sum_{i}^{n_k} \bar{w}_k(\mathbf{x} - \mathbf{x}_i)\big[u^h(\mathbf{x}_i) - \mathbf{P}^{\mathrm{T}}(\mathbf{x}_i)\, \mathbf{a}(\mathbf{x})\big]^2. \end{aligned} \tag{3}$$

The difference of the traditional MLS with the MWMLS is that the MLS defines the weighted function only at the sample point, but the MWMLS applies all Cover functions $\bar{w}_k(\mathbf{x})$ of these n_k physical covers which all cover the sample point, to construct the weighted function. The cover weight functions $\bar{w}(\mathbf{x})$ are employed to weight the square distances at an arbitrary $\mathbf{x}$. It should be noticed that the weighting scheme in Eq. (3) is different from the way it is used in MLS in that many different weight factors $\bar{w}_k(\mathbf{x})$ are taken as the value of the Ith cover weight function at x participated in weighting the square distance in $\prod(\mathbf{x})$. Taking advantage of this characteristic, the MWMLS has its superiority in solving discontinuous problems and it is also termed as multiple fixed least-square (MFLS) approximation by Onate (1996).

2.2. *Construction of the shape function*

At an arbitrary point x, which is covered by n_k physical covers, according to the Eq. (2), the approximation function $\mathbf{u}(\mathbf{x})$ of the real field function $\bar{\mathbf{u}}(\mathbf{x})$ is defined as:

$$\mathbf{u}(\mathbf{x}) \approx \bar{\mathbf{u}}(\mathbf{x}) = \sum_{i}^{n_k} P_i(\mathbf{x}) a_i(\mathbf{x}) = \mathbf{P}^{\mathrm{T}}(\mathbf{x})\,\mathbf{a}(\mathbf{x}), \quad \mathbf{x} \in C_i^{\mathrm{P}}, \tag{4}$$

Form the functional of weighted residual of MWMLS as given in Eq. (3), the approximation function of $\mathbf{u}(\mathbf{x})$ can be derived according to its extreme value criterion:

$$\mathbf{u}(\mathbf{x}) \approx \bar{\mathbf{u}}(\mathbf{x}) = \sum_{i}^{n_k} \mathbf{N}_i(\mathbf{x})\,\mathbf{u}_i, \tag{5}$$

$$\mathbf{u}(\mathbf{x}) = [\bar{u}_1(x), \bar{u}_2(x), \ldots, \bar{u}_{n_k}(x)], \quad \mathbf{u}_i = \left[u_{i1}^h,\, u_{i2}^h, \ldots, u_{in_k}^h\right] \tag{6}$$

Where $\mathbf{N}(\mathbf{x})$ is the shape function of approximation of field function at a given point x:

$$\mathbf{N}_i(\mathbf{x}) = \mathbf{P}^{\mathrm{T}}(\mathbf{x})\mathbf{A}^{-1}(\mathbf{x})\,\mathbf{Q}(\mathbf{x}_i) \tag{7a}$$

$$\mathbf{A}(\mathbf{x}) = \sum_{i=1}^{n_k} \bar{w}_k(\mathbf{x}_i)\,\mathbf{P}(\mathbf{x}_i)\,\mathbf{P}^{\mathrm{T}}(\mathbf{x}_i), \quad \mathbf{Q}(\mathbf{x}-\mathbf{x}_i) = \bar{w}_k(\mathbf{x}_i)\,\mathbf{P}(\mathbf{x}_i) \tag{7b}$$

And the approximation function $\mathbf{N}(\mathbf{x})$ has the property of partition of unity.

2.3. *The cover system and computational model at the crack-tip*

Generally, for the accuracy of calculation, the mathematical covers should be distributed intensively in the field of crack-tip. The scheme of distribution is that, a close-packing circle with a center at the crack tip is placed in the local field of crack tip, so a number of mathematical cover s are distributed homogeneously along the circumference of circle with different polar angles, $\theta \in [0°,\ 360°)$.

3. Heterogeneity and Randomness of Material Represented by Meshless Method

Though many present numerical methods are used to solve the macroscopic heterogeneity and discontinuity, such as MM, discontinuous deformation analysis, discrete element method and so on, they are not able to take the microscopic characters of materials into consideration. In the meshless methods, the heterogeneity and randomness of materials can be described by various parameters of nodes and Gaussian points. The physical and mechanic parameters at Gaussian points will be

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randomly processed to indicate the microscopic heterogeneity of materials [Tang *et al.* (1997)]. These parameters are randomly distributed to each Gaussian point. According to the Weibull's distribution, the distribution function of a random variable A is expressed as:

$$F(\alpha) = P_{\mathrm{f}}(A < \alpha) = \begin{cases} 1 - \exp\left[-\left(\dfrac{\alpha - \alpha_0}{\alpha_{\mathrm{u}}}\right)^m\right] & \alpha \geq \alpha_0, \\ 0 & \alpha < \alpha_0. \end{cases} \tag{8}$$

The distribution density function $f(\alpha)$ can be obtained by a differential calculus as follows:

$$f(\alpha) = \begin{cases} \dfrac{m}{\alpha_{\mathrm{u}}}\left(\dfrac{\alpha - \alpha_0}{\alpha_{\mathrm{u}}}\right)^{m-1} \exp\left[-\left(\dfrac{\alpha - \alpha_0}{\alpha_{\mathrm{u}}}\right)^m\right] & \alpha \geq \alpha_0, \\ 0 & \alpha < \alpha_0, \end{cases} \tag{9}$$

where α is the value of the random variable A (to brittle materials, the random variable may have elastic ratio, intensity and so on); α_0 is the minimum value of random variable A; α_{u} is the average value of α; m is named Weibull's model, which can reflect the heterogeneity of materials. The bigger the value of m, which is called the Homogeneity Index of materials is, the more homogeneous the materials are. As it drives to infinite, the materials will be completely homogeneous. Furthermore, it can be derived from Eq. (9):

$$\alpha = \alpha_0 + \alpha_u[-\ln(1 - F(\alpha))]^{\frac{1}{m}}, \quad F(\alpha) \in [0, 1). \tag{10}$$

The random variable α can be generated by some probability calculation methods, such as the Monte Carlo method. To generate a group of random number sequence $\{F_i(\alpha)\}$, $(i = 0, 1, 2, \ldots, n_{\mathrm{g}})$, a group of parameters of material $\{\alpha_i\}$ is calculated by Eq. (8). Using this method, the parameters of materials are obtained and endowed to each Gaussian point. This method can satisfied the heterogeneity and randomness of the physical parameters of brittle materials. The different heterogeneity of rocks with various values of heterogeneity index m are shown in Fig. 3.

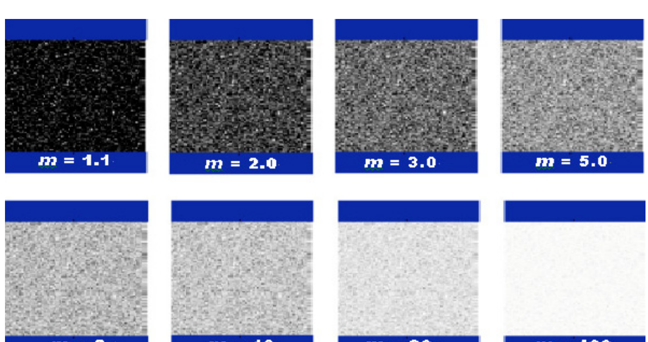

Fig. 3. Heterogeneity of rock with different values of heterogeneity index m represented by gray-scale diagrams.

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4. Constitutive Relation and Fracture Criterions

It is supposed, in this paper, that the macroscopic discontinuity and heterogeneity of material is caused by a serial of simple elastic damages and accumulation of these damages. To the infinitesimal volume elements in the brittle material, some assumptions are made: (1) the constitutive relation of infinitesimal volume element is linear elastics before fracture and damage; (2) the constitutive relation is still linear and the infinitesimal volume elements have some residual strength after fracture. It should be noticed that the macroscopic constitutive relation is dependent on the heterogeneity of materials. This damage model is illustrated in Fig. 4.

β_{c} and β_{t} are called the compressive and the tensile infinitesimal avianized parameter respectively. They are ranged from 0 to 1. σ_{c} and σ_{t} are the uniaxial compression strength and the uniaxial tension strength of infinitesimal of material. They submit to the random distribution for the heterogeneity of materials. It has been indicated that the constitutive relation is linear elastics before and after the damage of infinitesimal. And the infinitesimal still have some residual strength after the sudden elastic damage.

Though only the Gaussian points are endowed with these attributes of material, the neighborhood of the Gaussian points is supposed to possess the same attributes. When Gaussian points are close enough, the microscopic damage attributes of materials can be represented by that of Gaussian point. In the model, the fracture criterion of Gaussian points adopts the Mohr-Coulomb criterion with three parameters [Anadei (1996); Koo (1997)], bond strength c, friction angle ϕ, and tensile strength σ_{t}. Supposing the compression is positive, the principal stress space of σ_1 and σ_3 is shown in the Fig. 5.

In Fig. 5, it is indicated that the criterion is able to take into account both the tensile failure and the shear failure, σ_1 and σ_3 represent the maximum and the minimum principle stress respectively:

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\tau_{xy}^2 + \frac{1}{4}(\sigma_x - \sigma_y)^2}, \tag{11}$$

$$\sigma_3 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\tau_{xy}^2 + \frac{1}{4}(\sigma_x - \sigma_y)^2}. \tag{12}$$

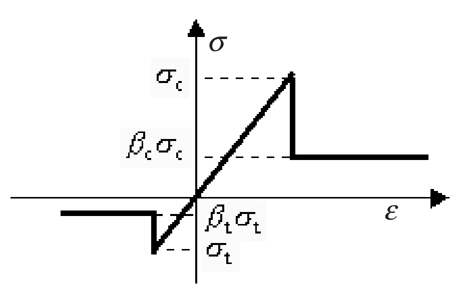

Fig. 4. Infinitesimal damage model — Elastic discontinuous/mutational model.

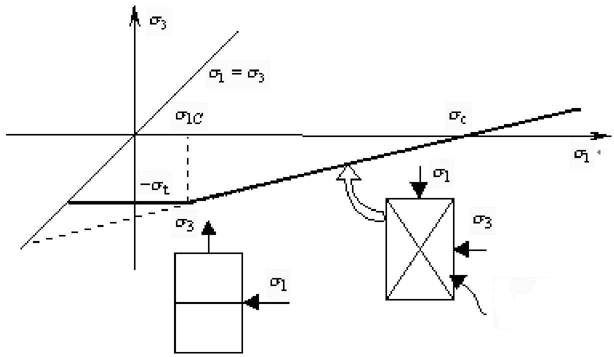

Fig. 5. Three-parameter Mohr-Coulomb criterion in $\sigma_1 - \sigma_3$ space.

Denoting σ_{ic} as the critical value of the minimum principal stress when the tensile failure and shear failure take place, given as:

$$\sigma_{\mathrm{ic}} = \sigma_{\mathrm{c}} - \sigma_{\mathrm{t}} \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) \quad \text{or} \quad \sigma_{\mathrm{ic}} = \sigma_{\mathrm{c}} \left(1 - \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) \frac{1}{\lambda}\right), \tag{13}$$

where σ_{c} is the uniaxial compression strength of material, which can be obtained through experiments or from $\sigma_c = 2c \tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right)$. The critical condition for shear failure is:

$$\sigma_1 \geq \sigma_{\mathrm{ic}} \quad \text{and} \quad \sigma_1 - \sigma_3 \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) - \sigma_{\mathrm{c}} \geq 0, \tag{14}$$

and the critical condition for tensile failure is:

$$\sigma_1 \leq \sigma_{\mathrm{ic}} \quad \text{and} \quad \sigma_1 \leq \sigma_{\mathrm{t}} + \sigma_3. \tag{15}$$

For displaying the degree of fracture of material at the neighborhood of Gaussian points, the fracture degree factor β is introduced, where β is defined as:c

$$\beta = 1 - \langle 1 - \beta' \rangle \quad \text{and} \quad \beta' = \begin{cases} \dfrac{\sigma_1}{\sigma_{\mathrm{c}}} - \dfrac{\sigma_3}{\sigma_{\mathrm{c}}} \tan^2\left(\dfrac{\pi}{4} + \dfrac{\phi}{2}\right), & \sigma_1 \geq \sigma_{\mathrm{ic}}, \\ -\dfrac{\sigma_3}{\sigma_{\mathrm{t}}}, & \sigma_1 \leq \sigma_{\mathrm{ic}}, \end{cases} \tag{16}$$

and $\langle\ \rangle$ is Macauley's bracket sign which is defined as $\langle x \rangle = 0.5(|x| + x)$.

In general, the fractured material will gradually constitute a shear zone with intensively high shear stresses. In order to prevent the incorrect mutual intrusion of the infinitesimals at neighborhood of shear zone, the rigidity of fractured Gaussian points should be rebuilt up and thereafter the stiffness matrix is re-established.

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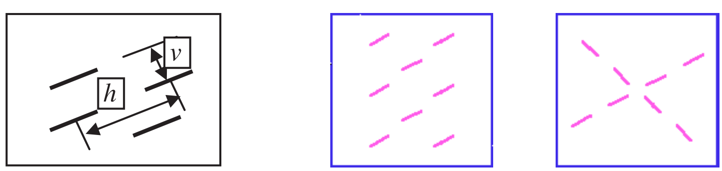
5. Numerical Simulations

5.1. *Numerical analysis of rock samples with regularly-distributed cracks*

Two-dimensional rock samples with the size of 300 mm × 300 mm are considered. The length of crack is 40 mm; the distance of cracks h is 120 mm; the array pitch v between cracks is 100 mm; elastic ratio is 152.3 MPa. Poisson's ratio is 0.2. Uniaxial compressive strength σ_c is 3.85 MPa, and uniaxial tensile strength σ_t is 0.33 MPa. The ratio of compression to tension strengths is 11.67. The internal friction angle and cohesion are respectively $\phi = 57.34°$ and $c = 0.564$ MPa. The weight of the sample is ignored. The distribution of cracks is regularly arranged as shown in Fig. 6.

In the process of simulation, the loading mode is uniaxial compression and displacement control. Each displacement increment is 1.0×10^{-4} mm. The Heterogeneity Index m is set as $m = 10$. The computed results are shown in Fig. 7.



(a) The distance h and array pitch v (b) Parallel cracks (c) Intersected cracks.

Fig. 6. Cracks in the rock samples with various distribution modes.

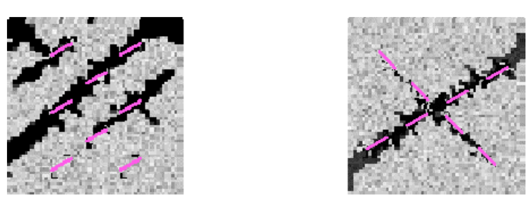


Fig. 7. Numerical results of fracture behavior for two distribution modes of cracks.

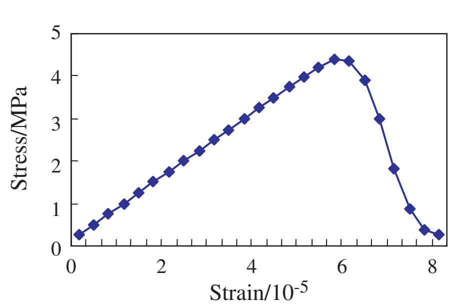


Fig. 8. Stress-strain relationship for the rock with crack II-mode computed by the proposed method.

Fig. 9. The sample with random distributed cracks and the fracture simulation under uniaxial compressive load.

From the results, it can be recognized that final fracture directions are along the surfaces of cracks, the main failure surface is formed by a reciprocal cut through among cracks. Therefore the fracture of rock body is directly related to the cracks and the location and direction of failure surfaces is, in a great degree, dependent on the distribution of cracks. A measure point is set at the loading end of the sample with parallel cracks, and the stress-strain relation measured and computed during the uniaxial loading process is illustrated in Fig. 8.

5.2. *Numerical analysis of rock samples with cracks random distribution*

In this section, the sample used for numerical analysis is same to the above-mentioned samples. There are 20 cracks randomly distributed in the sample. The length, location and orientation of cracks are all generated stochastically, shown in Fig. 9.

According to the results, it is indicated that the fracture formation is affected by the distribution of cracks integrally. More local fractures generate and develop more quickly in the neighborhood of mainly distributed cracks than the other place on the sample. Through the relationship between the orientation of main failure surface and the distribution of cracks, it is obvious that the fractures mainly occur in the regions with dense cracks distribution. In general, the fractures of samples with parallel cracks and with intersected cracks will slip along the main failure surface but the fracture of sample with random distributed cracks will not.

6. Concluding Remarks

FCEFM coupled with microscopic material fracture analysis can effectively analyze and simulate the fracture and damage of brittle materials with joints and cracks. It has good adaptability and versatility in deal with multiple discontinuous interfaces and various distribution types. With the high-order approximation and the close-packing technique at crack-tips, the accuracy of computation is raised. In the process of analyzing and simulating the fracture and damage evolution of brittle materials, intensive attention to the initial heterogeneity and randomness of materials should be paid.

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Acknowledgments

Financial supports from National Natural Science Foundation of China through the grant No. 10172022 are gratefully acknowledged.

References

Anadei, B., Lin, C. and Dwyer, J. (1996). Recent extension to the DDA method. Proceeding of the first *International Conference on Discontinuous Deformation Analysis (DDA) and Simulation of Discontinuous Media*, 1–30, June, 1996, Berkeley, California, USA.

Belytschko, T., Liu, Y. and Gu, L. (1994). Element free Galerkin method. *International Journal of Numerical Methods in Engineering*, **37**: 229–256.

Bui, H. D., Ehrlacher, A. and Renard, C. (1984). The steady state propagation of a damaged zone of an elastic brittle solid. *Proceeding of the 6th Conference of Fracture*, Valluri S. R. *et al.* (eds.), Pergamon, Oxford, pp. 1061–1067.

Koo, C. Y. and Chern, J. C. (1997). Modeling of progressive fracture in jointed rock by DDA method. Proceeding of the second *International Conference on Analysis of Discontinuous Deformation*, 186–200, July, 1997, Kyoto, Japan.

Liu, G. R. and Gu, Y. T. (2001). A point interpolation method for two-dimensional solid. *Int. J. Numer. Meth. Eng.* **50**: 937–951.

Luan, M. T., Tian, R. and Yang, Q. (2001). A new mesh-free method — Finite-cover element-free method. Proceeding of the *10th International Computer Methods and Advances in Geomechanics*, January 2001, University of Arizona, Tucson, Arizona, USA.

Nemat-Nasser, S. and Hori, M. (1993). *Micromechanics: Overall Properties of Heterogeneous Materials*, Elsevier, The Netherlands.

Shi, G. H. (1996). Manifold method. Proceedings of the first *International Forum on Discontinuous Deformation Analysis (DDA) and Simulation of Discontinuous Media* Salami, M. R. and Banks, D. (eds.), Berkeley, California, USA, 52–204.

Tang, C. A. and Zhao, W. (1997). RFPA^{2D} System for Rock Failure Process Analysis. *Chinese Journal of Rock Mechanics and Engineering*, **16**(5): 507–508.

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