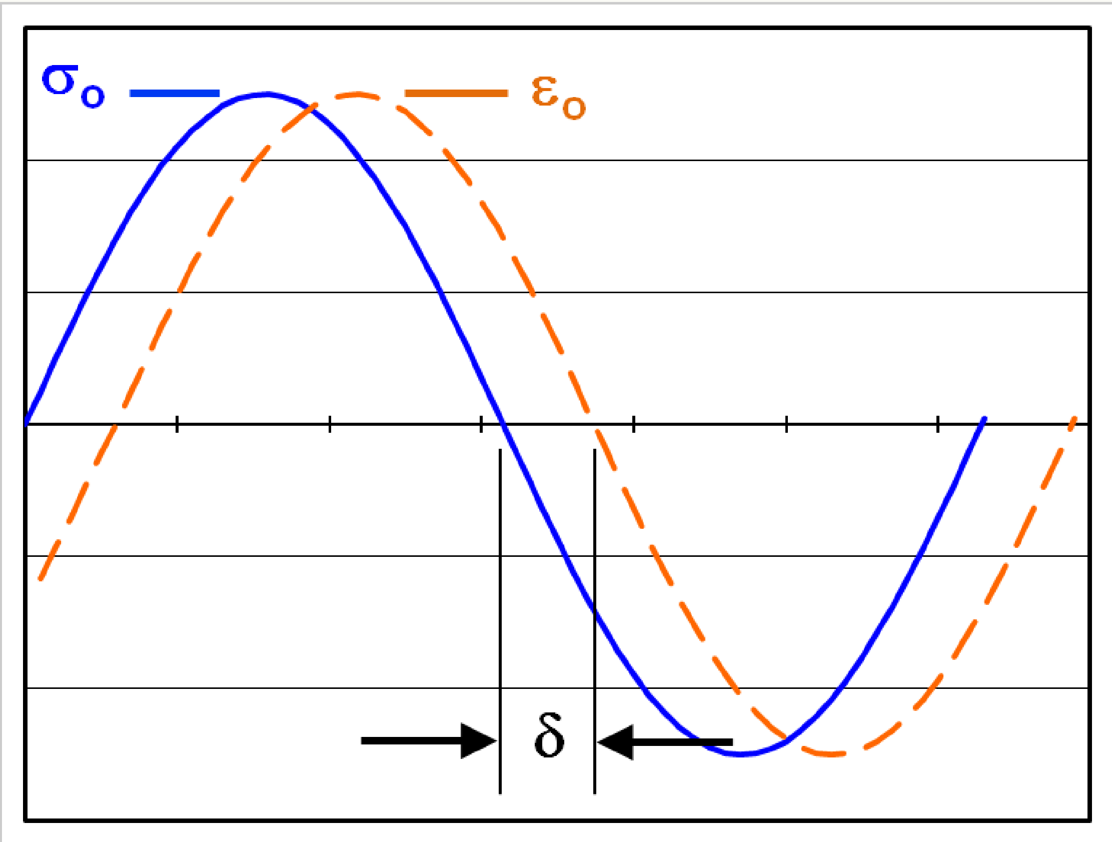
Dynamic Material Properties

Introduction

Classical dynamic material testing involves the application of a sinusoidal load to a sample and the recording of its displacement response. The load and displacement data are used to calculate stress and strain cycles. The ratio of the stress amplitude to the strain amplitude is the dynamic modulus. For shear loading, the usual symbol, G, is used. The phase lag, δ, between the stress input and strain response is also recorded and usually presented as $\tan(\delta)$. Various combinations of these parameters are plotted against strain amplitude, temperature, and/or frequency.



For shear loading, one could say that τ and γ should be used in the figure. Nevertheless, σ and ϵ are customarily used anyway.

Definitions

The stress amplitude, σ_o, strain amplitude, ϵ_o, and phase lag, δ are combined in many different ways to form different material property definitions. The most fundamental ones are simply ratios between stress and strain. The ratio of stress to strain gives the

ain to stress gives the dynamic compliance, J^*.

$$G^* = \frac{\sigma_o}{\epsilon_o} \qquad J^* = \frac{\epsilon_o}{\sigma_o}$$

Clearly $G^* = 1/J^*$ and vice-versa. The remaining fundamental quantity is the tangent of the phase lag, $\tan(\delta)$, often simply called "tan delta" and sometimes called the "loss tangent".

The in-phase and out-of-phase components of the dynamic modulus are known as the storage modulus and loss modulus, respectively.

Storage Modulus	$G' = G^* \cos(\delta)$
Loss Modulus	$G'' = G^* \sin(\delta)$

From this, it is clear that $\tan(\delta)$ is related to the ratio of G'' to G'.

$$\tan(\delta) = \frac{G''}{G'}$$

The in-phase and out-of-phase components of the dynamic compliance are known as the storage compliance and loss compliance, respectively.

Storage Compliance	$J' = J^* \cos(\delta)$
Loss Compliance	$J'' = J^* \sin(\delta)$

And it is clear that $\tan(\delta)$ is also related to the ratio of J'' to J'.

$$\tan(\delta) = \frac{J''}{J'}$$

Incorrect Relationships

Do not be tempted to say that $J' = 1/G'$ and $J'' = 1/G''$, because these are incorrect. The correct relationships are

$$J' = J^* \cos(\delta) = \frac{\cos(\delta)}{G^*} = \frac{\cos^2(\delta)}{G'}$$

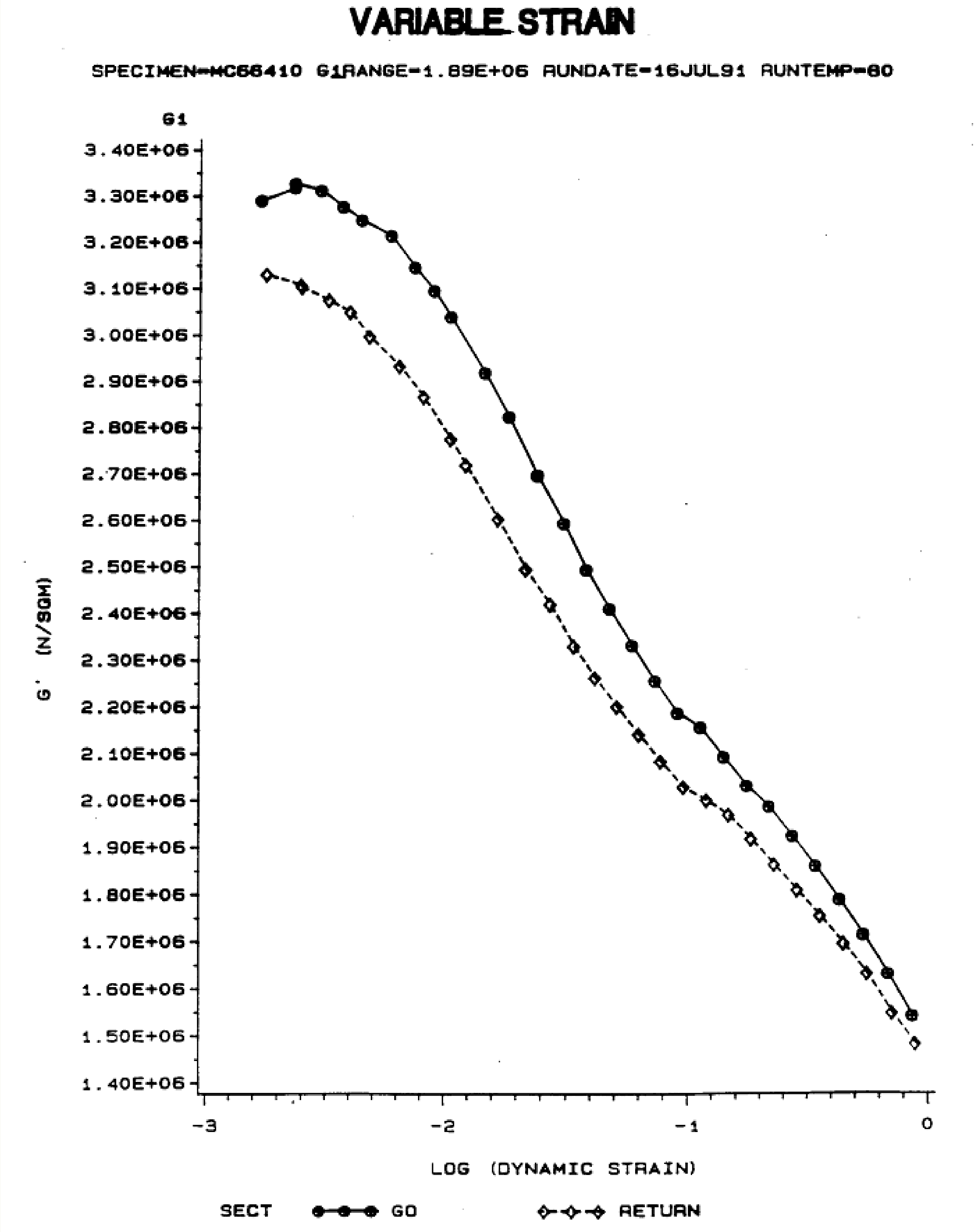
$$J'' = J^* \sin(\delta) = \frac{\sin(\delta)}{G^*} = \frac{\sin^2(\delta)}{G''}$$

It is perhaps easier to remember these as

$$G' * J' = \cos^2(\delta) \qquad \text{and} \qquad G'' * J'' = \sin^2(\delta)$$

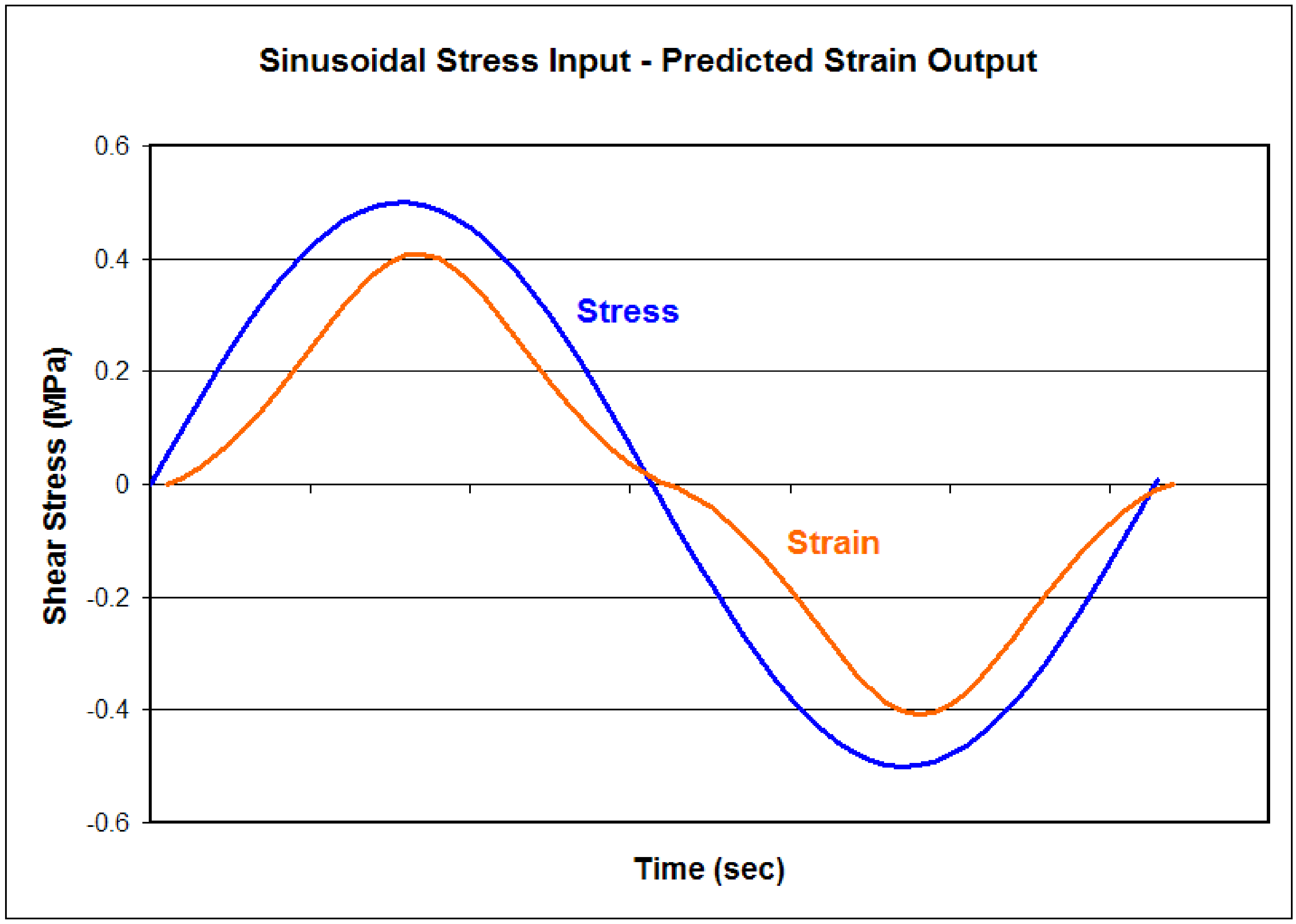
Strain Dependence

Here is some test data for a rubber sample. As with the uniaxial tension test data on the previous Mooney-Rivlin page, the stiffness of the rubber decreases as the strain amplitude increases. The curve labeled "GO" is for the portion of the test where the input load amplitude increases with time. The curve labeled "RETURN" is for the portion of the test where the input load amplitude decreases with time. The difference between the two is called the Mullins Effect.

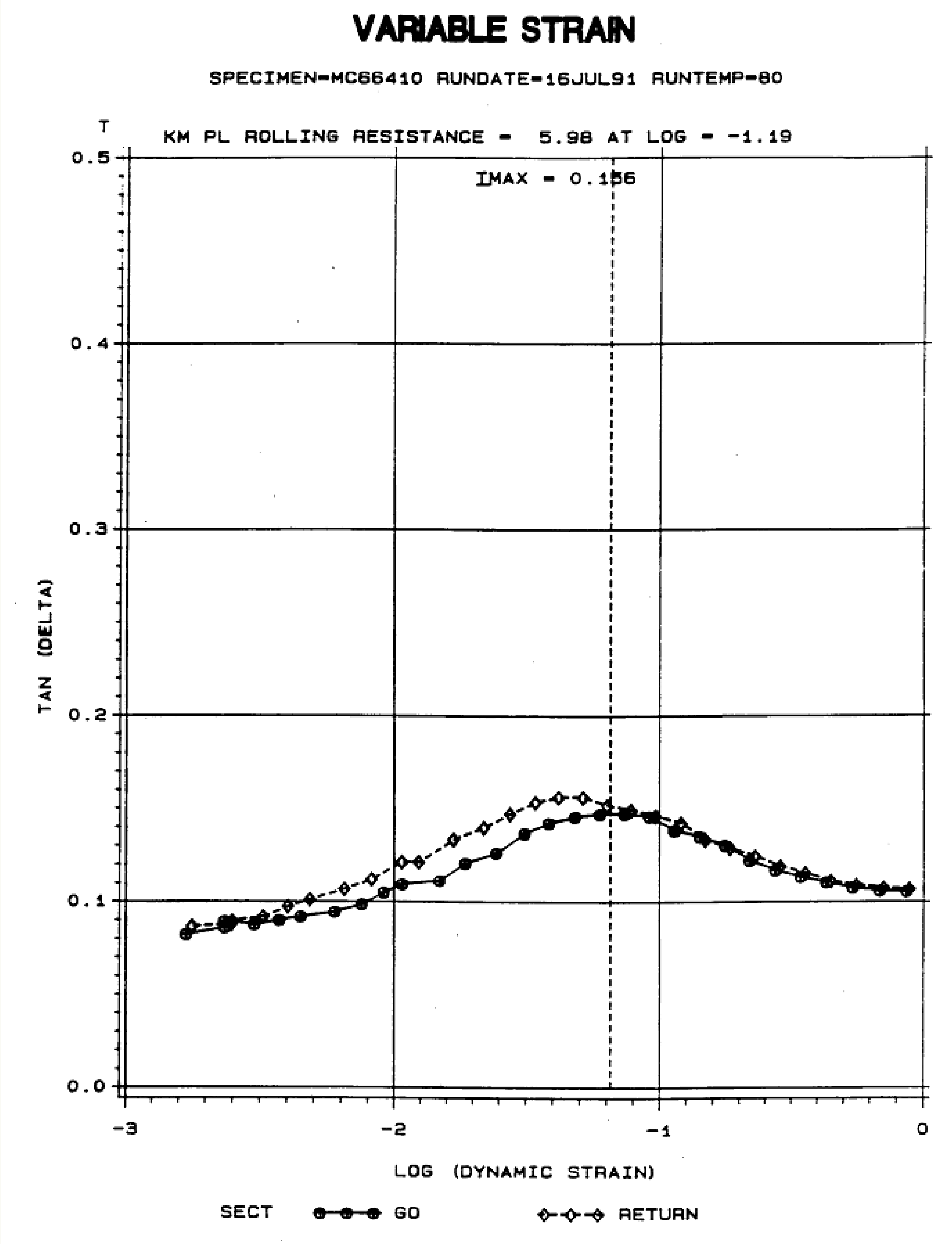


Below is a graph of the predicted shear strain for a sinusoidal shear stress input signal. The predictions are based on the material stiffness from the graph just above. Note how nonlinear the predicted shear strain is. In other words, the ratio of stress to strain as time passes is not constant (independent of the time delay due to the phase lag).

However, this is not the case for actual rubber behavior during dynamic tests. The actual strain signal is indistinguishable from the first figure at the top of this page.

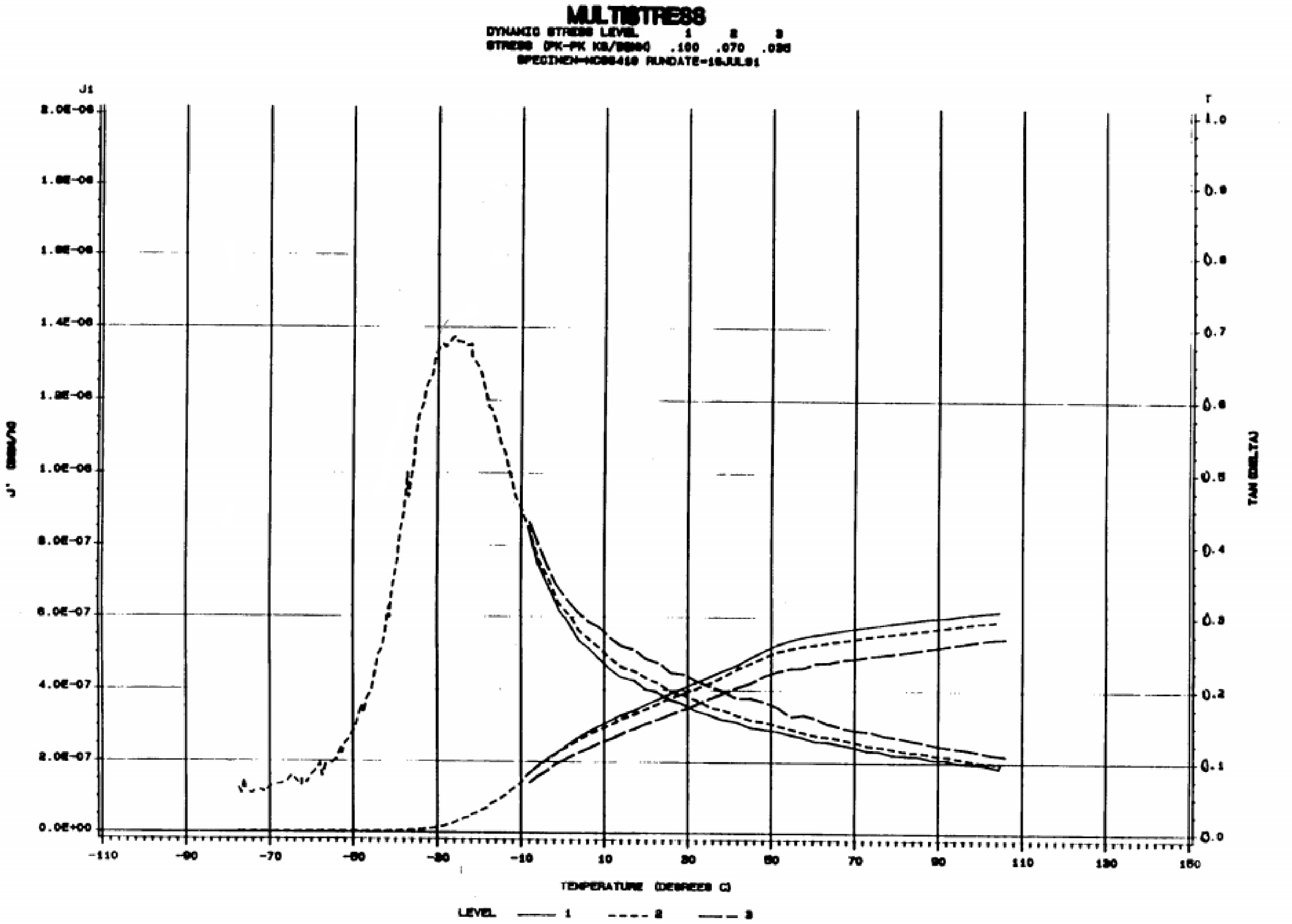


Here is test data for the phase lag of the same rubber sample.

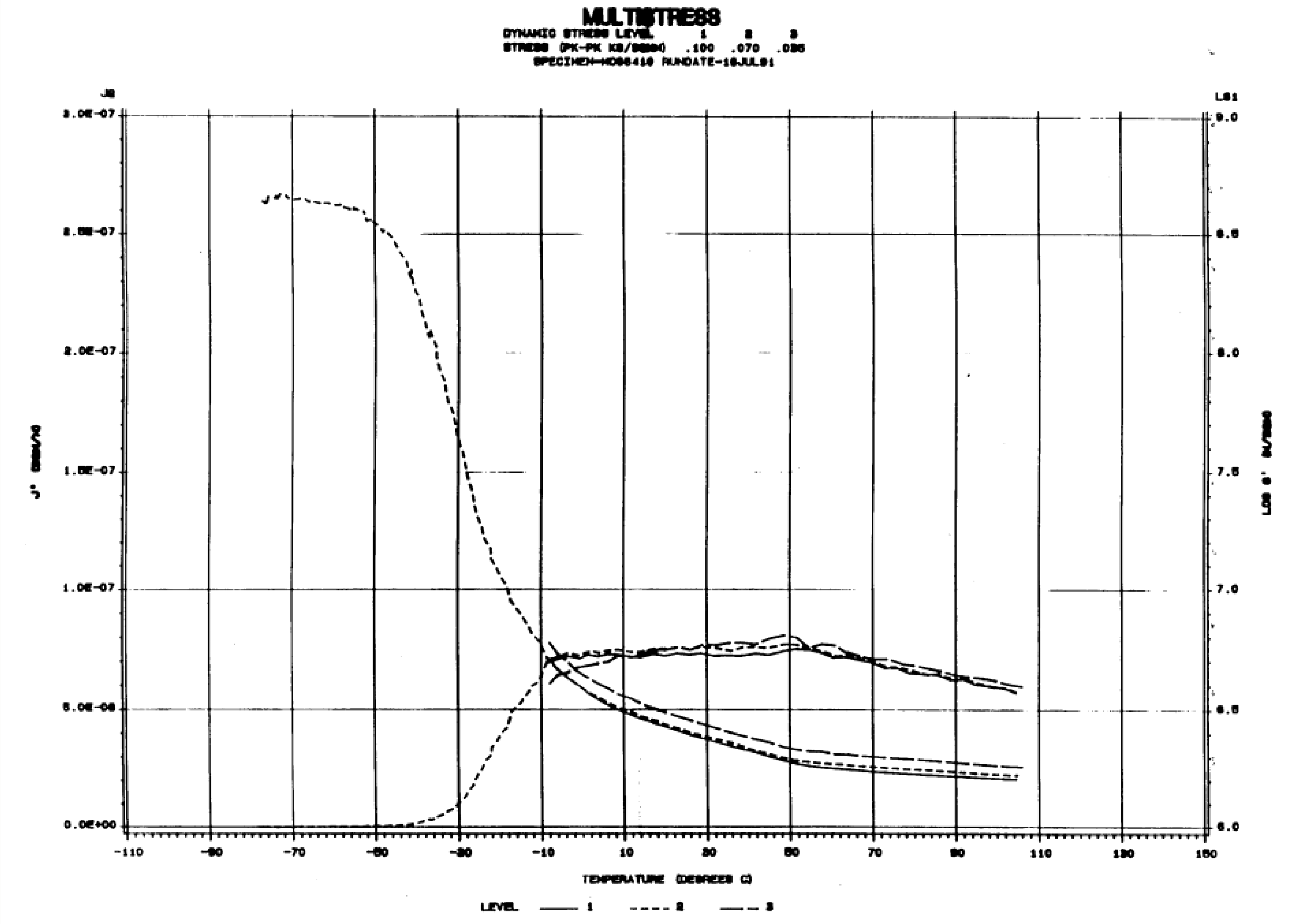


Temperature Dependence

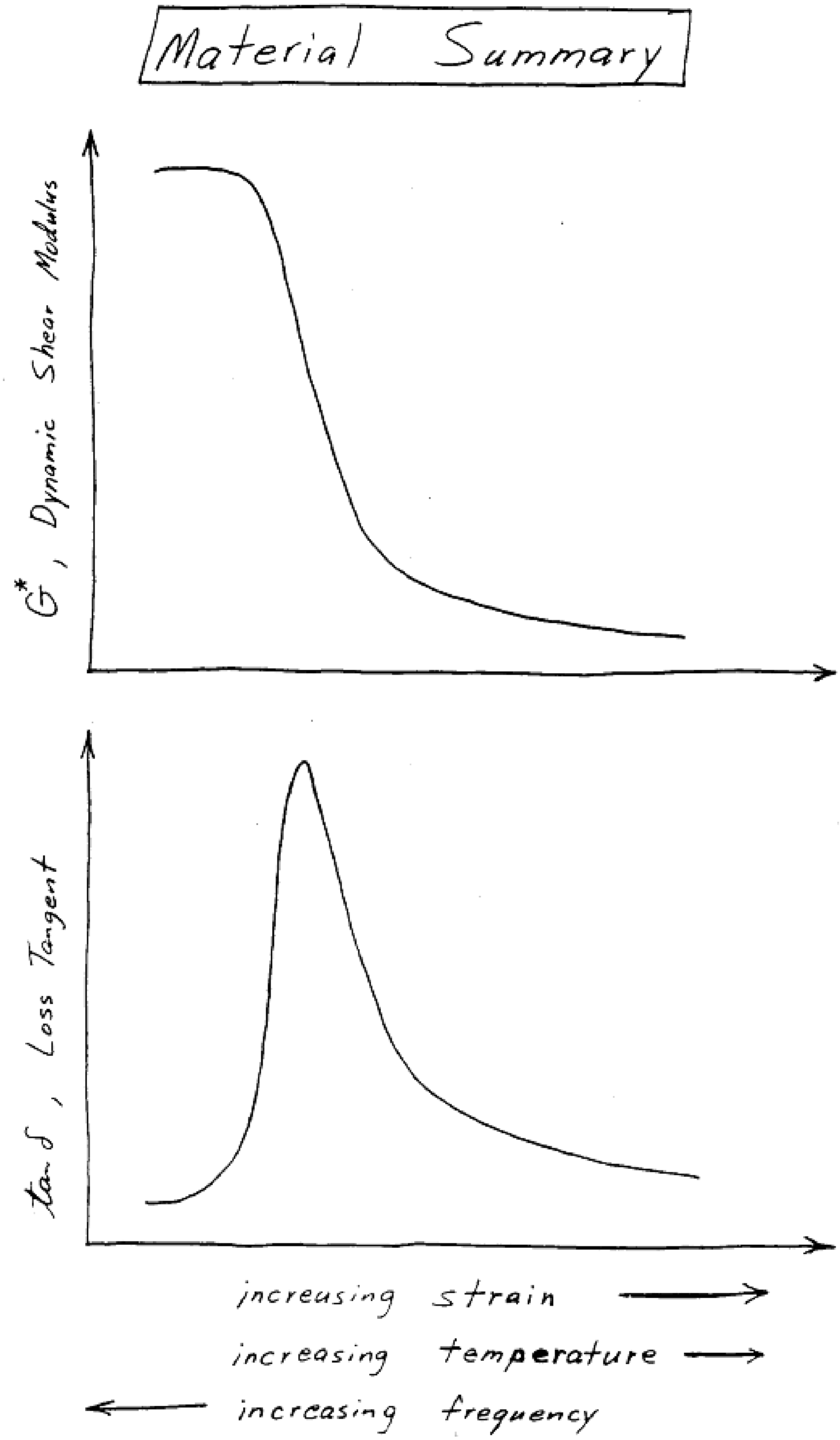
This is a plot of J' and $\tan(\delta)$ versus temperature.



This is a plot of J'' and $\log_{10}(G')$ versus temperature.



And here is a summary sketch. Note that temperature and frequency increase in opposite directions.

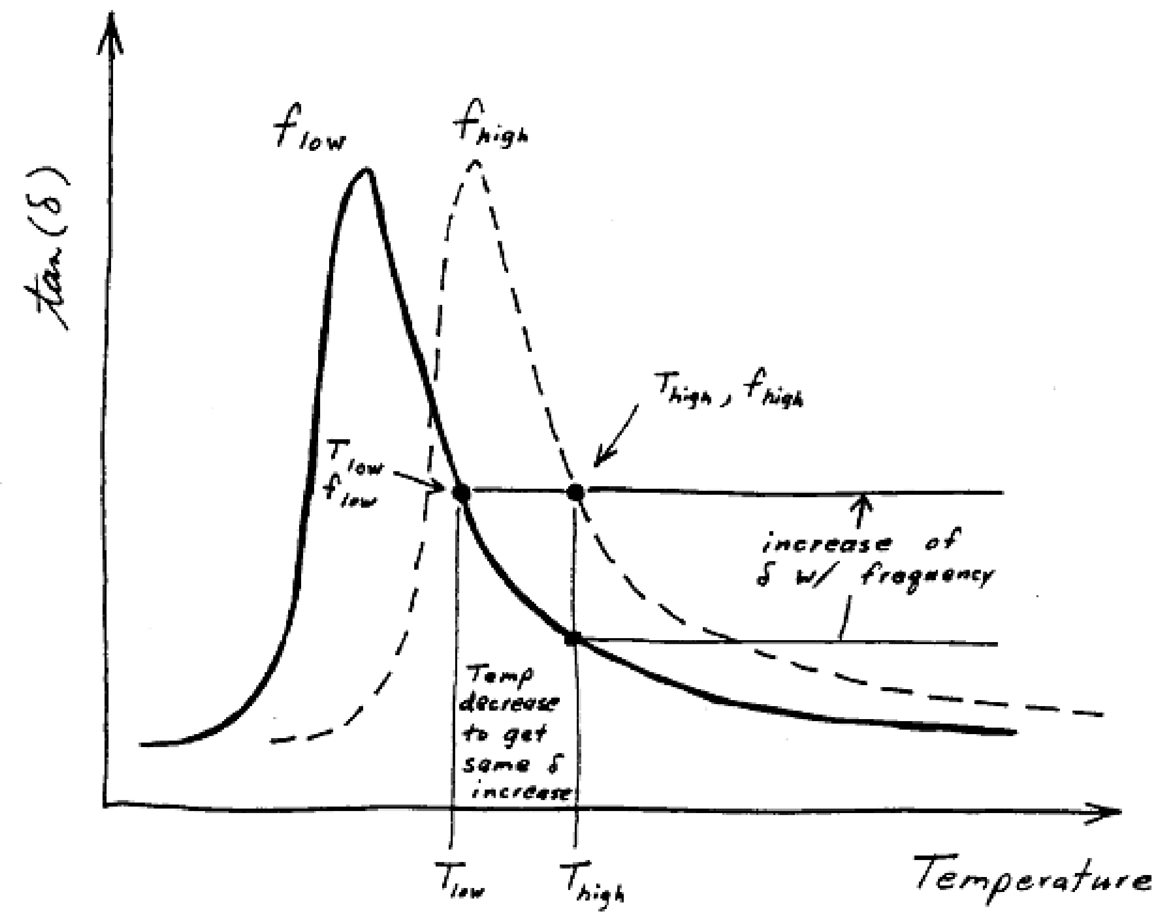


Time Temperature Equivalence

Around the time of World War II, Williams, Landel and Ferry reported a fascinating property of polymers: time-temperature equivalence. They observed that the dynamic material properties of a polymer at a reference temperature and frequency could be reproduced at a different combination of temperature and freeqency that are related to the first pair by one simple equation. That equation is now known as the WLF Equation or WLF Transform.

Time-temperature equivalence means that the stiffness and hysteresis of a polymer will be the same at the proper combination of low temperature and low frequency as at a given combination of high temperature and high frequency.

This property is very beneficial from an experimental viewpoint because it means that the viscoelastic properties of materials at thousands of Hertz can be estimated in the lab by testing at low frequencies and very low temperatures. (This is often done for traction studies.)



WLF transforms relate temp and frequency changes required to get the same properties.

WLF Transforms

The WLF equation is often published in many similar, though slightly different forms. These are just specific cases of the general form given here.

$$\log_{10}\left(\frac{f_1}{f_0}\right) = \frac{-C_1(\theta_1 - \theta_0)}{(C_2 + \theta_0)(C_2 + \theta_1)}$$

where: $\theta_0 = T_0 - T_g$
$\theta_1 = T_1 - T_g$

Usual values of the constants are C_1 = 900 and C_2 = 51.6 .

The coefficients, C_1 and C_2, are obtained by curve fitting the WLF equation to experiemental data. They change little regardless of the application. For example, estimates from test in MFG on PL KM's gave $C_1 = 900$ and $C_2 = 55$. But it must be noted that there was a great deal of uncertainty in these estimates.

Overall, the WLF transform is considered to perform quite well, though not perfectly, over many decades of frequency and at $T_g \leqslant T \leqslant T_g$+100° C. The WLF transform can be rearranged to solve for θ_1, which is more useful.

$$T_1 - T_g = \frac{C_1\theta_0 - C_2(C_2 + \theta_0)\log_{10}(f_1/f_0)}{C_1 + (C_2 + \theta_0)\log_{10}(f_1/f_0)}$$

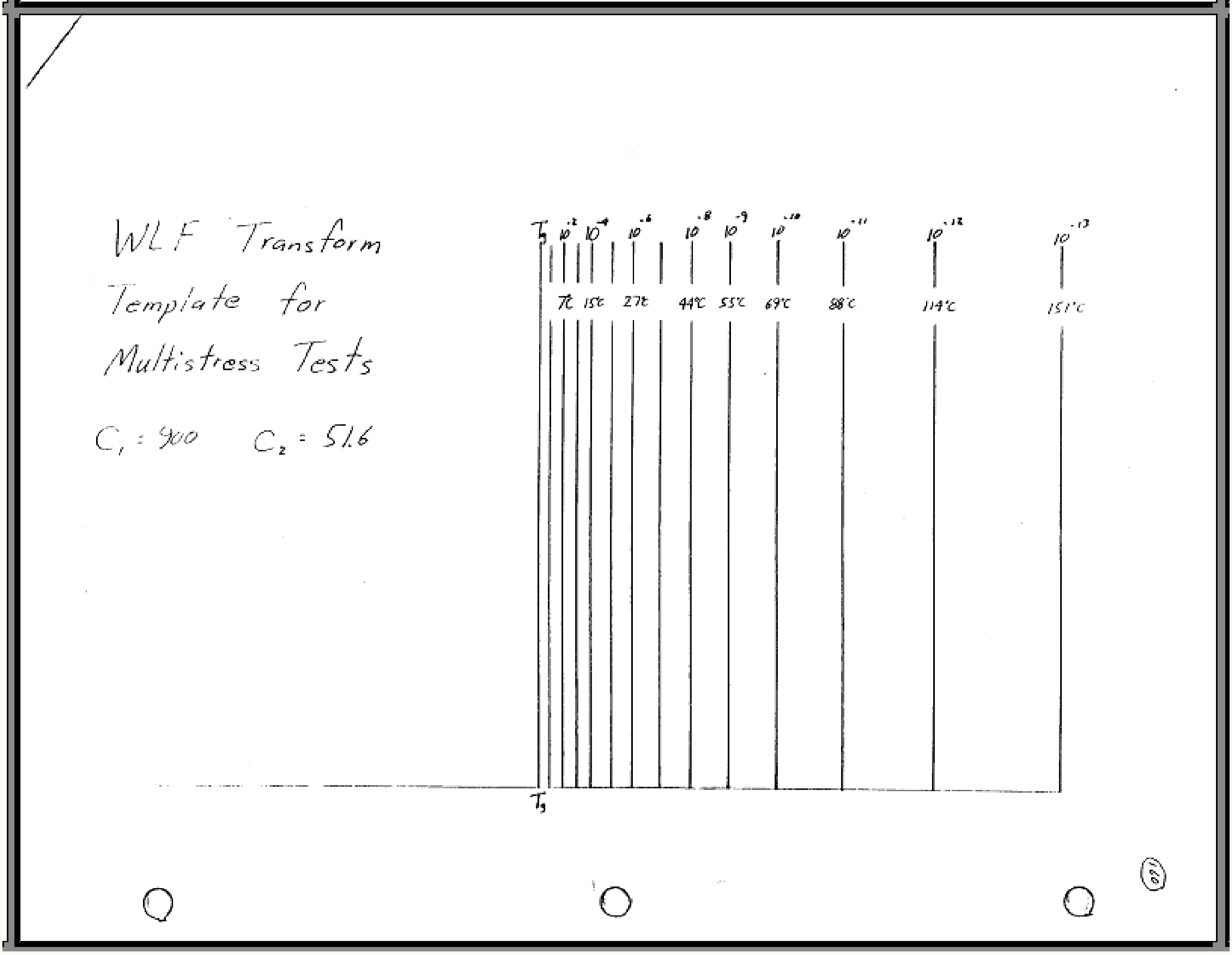
WLF Example

Consider the multistress test results above. The graph gives the material properties at 10 Hz over a wide range of temeratures. Let's (arbitrarily) start with $\tan(\delta)$ at 10 Hz, 70° C, and at the 0.035 kg/mm^2 stress level, (the long-dashed curve on the graph). At these conditions, $\tan\delta$ = 0.17 . To estimate $\tan\delta$ at 70° C and 100 Hz, use

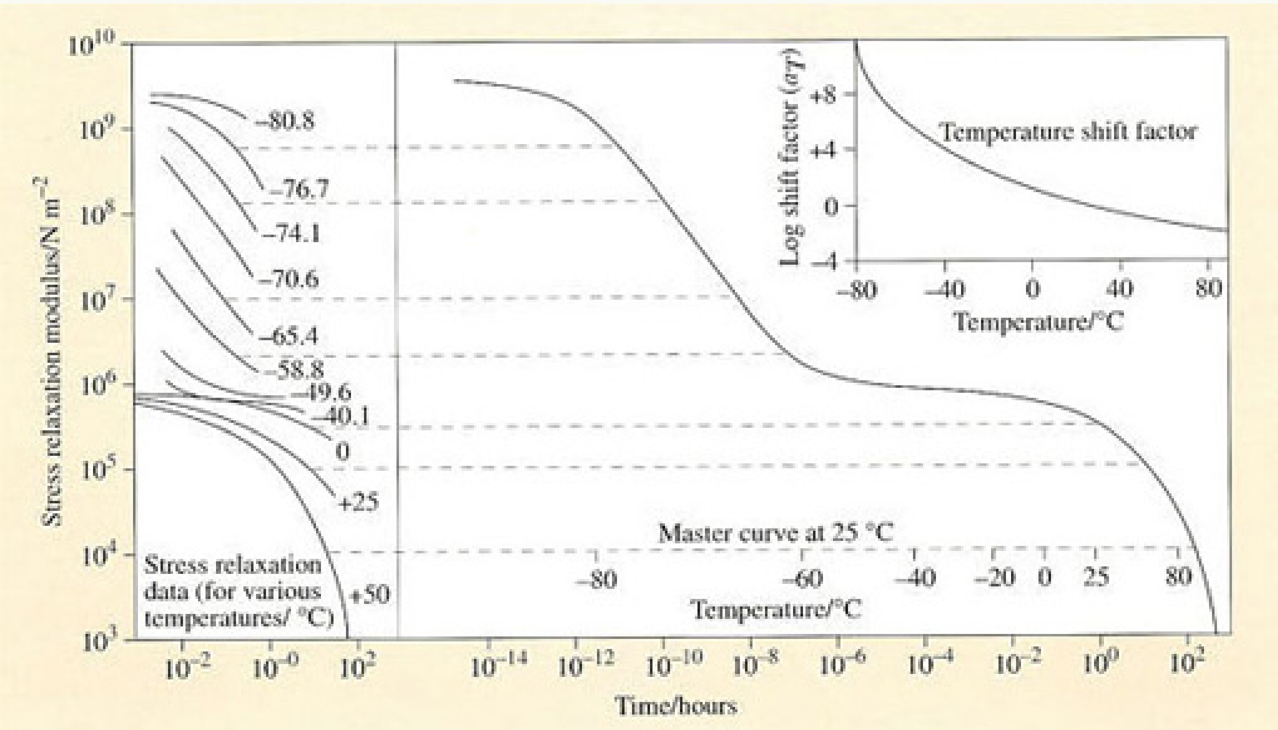
$$\begin{array}{lcl@{\qquad}lcl} C_1 & = & 900 & T_0 & = & 70^\circ\mathrm{C} \\ C_2 & = & 51.6 & f_0 & = & 10\ \mathrm{Hz} \\ T_g & = & \text{-}20^\circ\mathrm{C} & f_1 & = & 100\ \mathrm{Hz} \end{array}$$

to get T_1 = 51° C. At 51° C and 10 Hz, $\tan\delta$ = 0.2 . Therefore, at T_1 = 70° C and 100 Hz, $\tan\delta$ should also equal 0.2.

Print this file out as a transparency and it can be placed over a Multistress Test graph to perform approximate WLF transforms.



More extensive experimental data looks like this.



Ref: http://openlearn.open.ac.uk/mod/oucontent/view.php? id=397829§ion=5.3.1