Numerical simulation of thermo-elasticity, inelasticity and rupture in membrane theory

by

Michael James Taylor

B.A.(The Johns Hopkins University) 2003
M.S. (University of California, Berkeley) 2005

A dissertation submitted in partial satisfaction of the
requirements for the degree of
Doctor of Philosophy

in

Engineering - Mechanical Engineering

in the

GRADUATE DIVISION
of the
UNIVERSITY OF CALIFORNIA, BERKELEY

Committee in charge:
Professor David J. Steigmann, Chair
Professor Tarek I. Zohdi
Professor Francisco Armero

Fall 2008

PREVIEW

UMI Number: 3386160

All rights reserved

INFORMATION TO ALL USERS
The quality of this reproduction is dependent upon the quality of the copy submitted.

In the unlikely event that the author did not send a complete manuscript
and there are missing pages, these will be noted. Also, if material had to be removed,
a note will indicate the deletion.

PREVIEW

UMI
Dissertation Publishing

UMI 3386160
Copyright 2009 by ProQuest LLC.
All rights reserved. This edition of the work is protected against
unauthorized copying under Title 17, United States Code.

ProQuest

ProQuest LLC
789 East Eisenhower Parkway
P.O. Box 1346
Ann Arbor, MI 48106-1346

The dissertation of Michael James Taylor is approved:

Chair Date

Date

Date

University of California, Berkeley

Fall 2008

PREVIEW

Numerical simulation of thermo-elasticity, inelasticity and rupture in membrane theory

Copyright 2008

by

Michael James Taylor

PREVIEW

Abstract

Numerical simulation of thermo-elasticity, inelasticity and rupture in membrane theory

by

Michael James Taylor

Doctor of Philosophy in Engineering - Mechanical Engineering

University of California, Berkeley

Professor David J. Steigmann, Chair

Two distinct two-dimensional theories for the modeling of thin elastic bodies are developed. These are demonstrated through numerical simulation of various types of membrane deformation.

The work includes a continuum thermomechanics-based theory for wrinkled thin films. The theory takes into account single-layer sheets as well as composite membranes made of multiple lamina. The resulting model is applied to the study of entropic elastic elastomers as well as Mylar/aluminum composite films. The latter has direct application in the area of solar sails. Several equilibrium deformations are illustrated numerically by applying the theory of dynamic relaxation to a finite difference discretization based on Green's theorem.

In addition, a shell theory based on the peridynamic theory of Silling is developed.

PREVIEW

Peridynamics is a reformulation of classical continuum theory particularly suited to the modeling of damage and fracture. This theory is extended to include viscoelasticity and viscoplasticity. Several dynamic simulations are presented using a mesh-free explicit code.

Professor David J. Steigmann
Dissertation Committee Chair

PREVIEW

To my parents, James and Linda.

Thank you.

PREVIEW

Contents

PREVIEW

PREVIEW

PREVIEW

List of Figures

PREVIEW

PREVIEW

PREVIEW

List of Tables

PREVIEW

Acknowledgments

I owe a debt of gratitude to my advisor and mentor, Professor David Steigmann. I consider working with someone with his kindness and mastery of mechanics to be a great privilege. What I have learned from him extends well beyond what is contained within the following pages.

Several other professors also deserve acknowledgment. Chief among those is Tarek Zohdi. He played a big role in many of the key moments of my graduate career ranging from my decision to come to Cal all the way to being a member of this dissertation committee. Thanks to him and Professors G.I. Barenblatt, George Johnson and David Bogey for their confidence in me to continue towards this goal. Many thanks to Professor Francisco Armero for being on this committee and for offering helpful suggestions.

As a graduate student, you often receive a boost (or several) from the people in the trenches with you. In that spirit, I'd like to thank David Powell. Over the years, we've had probably hundreds of conversations ranging from finite elements to soap bubbles. Each one helped me understand the subject, whatever it was, just a little bit more.

I had the pleasure of working with Scott McCormick, Pete Graham and Michael Neufer for a summer in the Hesse Hall shop and in two semesters teaching the shock absorber lab. These gentlemen do a terrific job supporting, maintaining, and improving student labs. From day one they made me feel a part of their team. I am grateful for them letting a theoretical fellow such as myself in on their fun.

On a more personal level, I'd like to offer big thanks to my parents. They always were behind me one hundred percent and encouraged me to work hard and aim high. That

PREVIEW

I am here writing this is due, in large part, to their hard work as well. For that, I am grateful.

Thanks muchly to Erin Haynes for her love, support, and encouragement. Graduate school was much better with her beside me.

Finally, I'd like to acknowledge the financial support of the Department of Mechanical Engineering through the Powley Fund for Ballistics Research, grants, and guaranteed teaching positions.

PREVIEW

Part I

Introduction

PREVIEW

This work is concerned with the modeling and numerical analysis of thin films and shells. Particularly, we are interested in deriving two-dimensional membrane and shell theories that include thermoelasticity, viscoelasticity, viscoplasticity and fracture. The models will be analyzed through numerical simulation of various equilibrium and dynamic deformations. The theories are developed and explored in the next two parts.

In Part II, we derive a theory of membrane thermoelasticity based on the familiar balance laws of continuum mechanics. The theory is developed for metallic and polymeric films and composite laminates made of combinations of both. Wrinkling and slackening are taken into account as well. A principal application of membrane thermoelasticity is to the analysis of solar sails, which consist of laminated films and reflective coatings that transfer the momentum of solar photons into thrust. Flexible polymer films with a number of industrial applications are also formed from laminae having various chemical compositions. The model is used to solve a number of equilibrium problems that demonstrate the effects of heating and mechanical loading on the stress field and consequent wrinkling pattern.

In Part III, we develop a shell model based on the peridynamic theory of S.A. Silling. The peridynamic theory was created as a reformulation of classical continuum mechanics and is especially useful in the model of damage and fracture. In peridynamics, the equation of motion is an integral equation as opposed to a differential equation. Thus, discontinuities at crack surfaces do not propose difficulties as they do in classical mechanics. This theory will be extended to encompass viscous effects (for certain rubber-like materials) and yielding (for viscoplastic materials that exhibit work-hardening). We apply a mesh-free numerical method to analyze the theory for several dynamic deformations of membranes.

PREVIEW

In Part IV, we summarize the work and discuss the numerical results of Parts II and III. This section concludes the dissertation.

PREVIEW

Part II

Membrane Thermoelasticity

PREVIEW

Chapter 1

Introduction

PREVIEW

This part is concerned with the development of a theory for thermoelastic membranes and numerical simulation of illustrative equilibrium deformations. In the second chapter, we summarize some basic results in the theory of continuum finite thermoelasticity. Weak forms based on the three-dimensional equations of thermoelastic equilibrium are derived in chapter three. Chapter four develops the leading-order (membrane) theory based on these weak forms.

In chapter five, we investigate the constitutive modelling of metallic materials, polymers which exhibit thermal expansion and a special class of elastomers called entropic elastic materials. Entropic elastic materials experience the Gough-Joule effect which causes them to behave counter-intuitively in the presence of heat. A typical example of this type of material is vulcanized natural rubber. In this chapter the theory is also modified to incorporate wrinkling and the presence of multiple laminates. The particular application of a laminated sheet to the design of solar sails is discussed.

The numerical method used, dynamic relaxation, is described in chapter six. In dynamic relaxation, equilibria are considered to be long-time limits of damped dynamic problem. We discretize the problem in space using finite difference approximations based on Green's theorem and in time using an explicit central differencing scheme. Chapter seven demonstrates our numerical results, with particular focus on the effect of heat on both stretch and wrinkling.

PREVIEW

Chapter 2

Finite Thermoelasticity

PREVIEW

We begin by laying down the basic results for a thermoelastic material in the manner of Casey and Krishnaswamy [7]. The local equation for the balance of energy in the referential form is:

$$\rho_\kappa \dot{\epsilon} \;=\; \tfrac{1}{2}\mathbf{S} \cdot \dot{\mathbf{C}} + \rho_\kappa r - Div\mathbf{Q}, \tag{2.1}$$

where $\mathbf{S}$ is the second Piola-Kirchoff Stress Tensor, $\mathbf{E}$ is the Lagrangian strain tensor, ϵ is the internal energy per unit mass, r is the internal heat supply and $\mathbf{Q}$ is the referential heat flux vector. Note, in addition, that the temperature in the deformed configuration is given by θ and the referential temperature gradient is given by $D\theta$.

Let the constitutive equations for a thermoelastic material be given as

$$\epsilon = \tilde{\epsilon}(\mathbf{F}, \theta) \qquad \mathbf{S} = \tilde{\mathbf{S}}(\mathbf{F}, \theta) \qquad \mathbf{Q} = \tilde{\mathbf{Q}}(\mathbf{F}, \theta, D\theta). \tag{2.2}$$

By enforcing invariance under superposed rigid body motion to the above constitutive laws, it can be shown that they must depend on the deformation gradient in the following way: