Cloth and Fur Energy Functions

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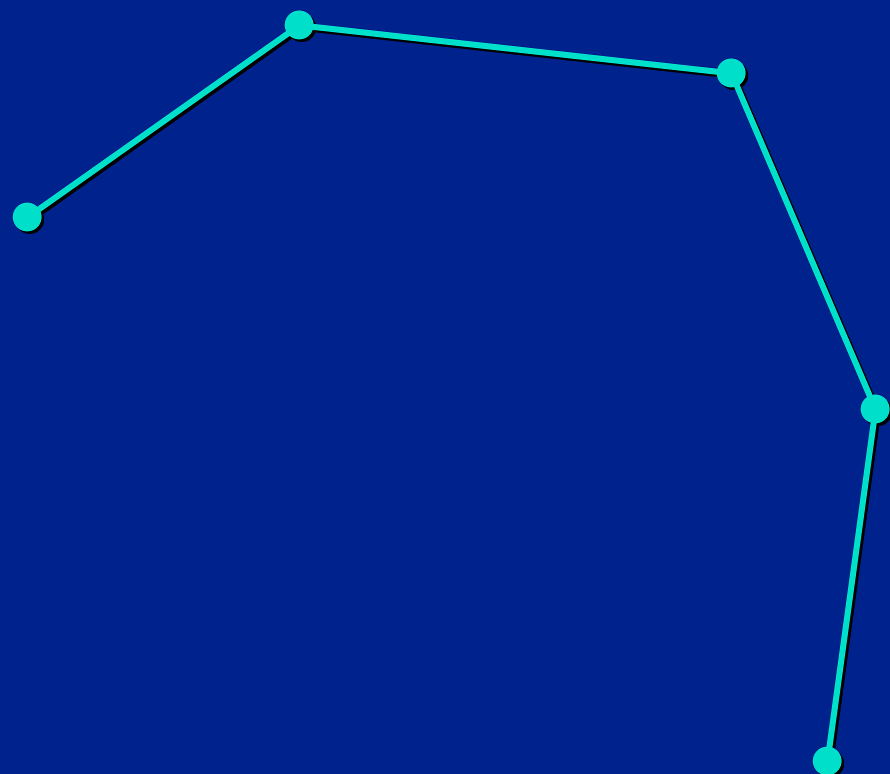
PIXAR
ANIMATION STUDIOS

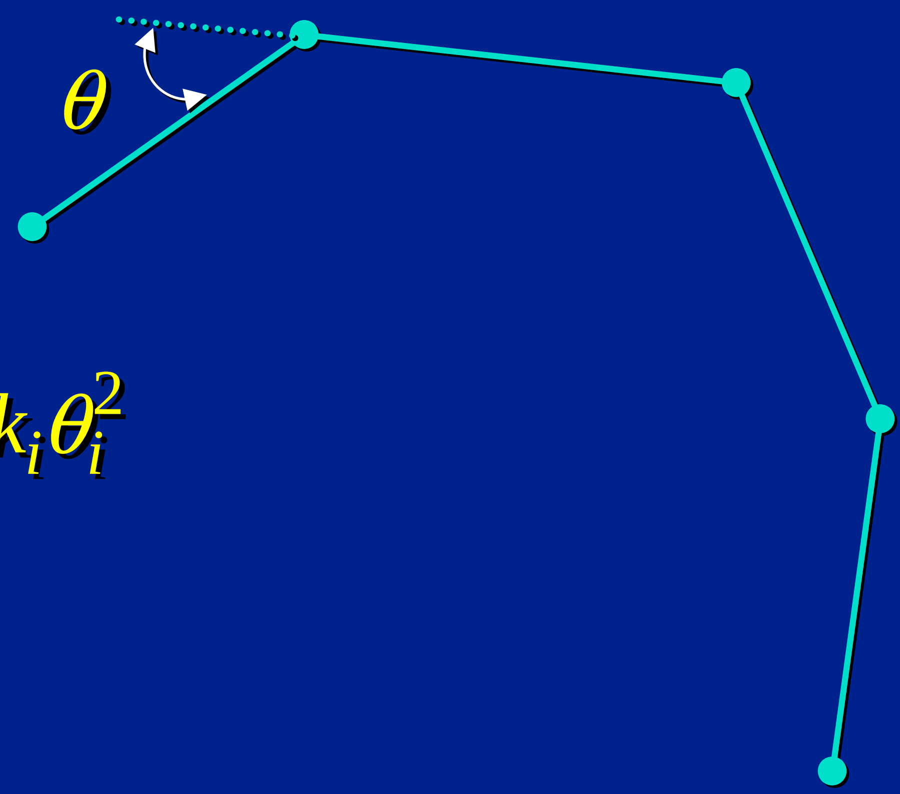
Hair Model

Limp hair: Just a set of springs.



Hair Model

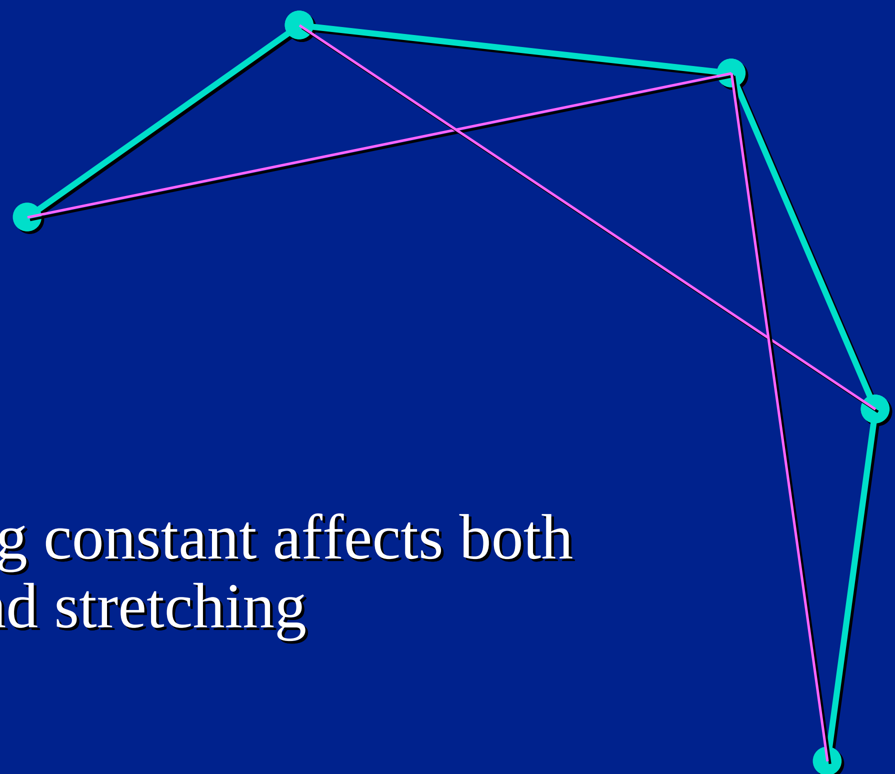
Add body: Angular Springs



$$E = \frac{1}{2}\sum_i k_i \theta_i^2$$

Hair Model

Alternative: More Linear Springs



Difficulty:

Each spring constant affects both bending and stretching

Discretization

Make sure energy independent of sampling.

Total energy: $E = \frac{1}{2} k \sum (l - l_{\text{rest}})^2$

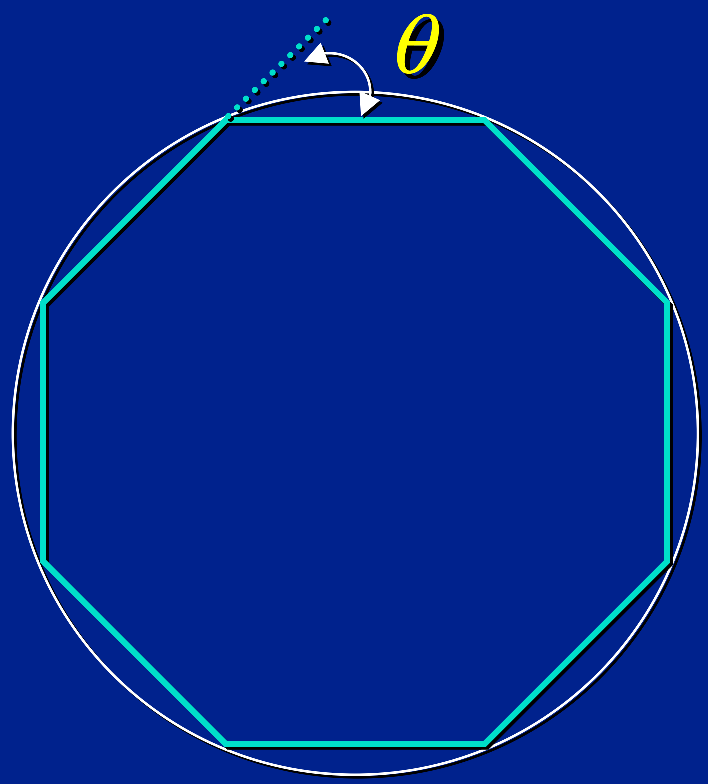
Stretch 100%: $E = \frac{1}{2} nk \left(\frac{L}{n} \right)^2$

Constant energy implies:

$k \propto n$ or $k_i \propto \frac{1}{l_i}$

Note: High sampling --> stiffness

Discretization



Consider a discretized circle.

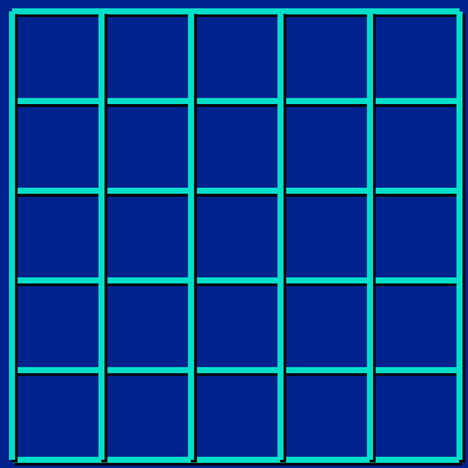
$$E = \frac{1}{2} k \sum \theta^2$$

Again, constant energy implies:

$$k \propto n \qquad \text{or} \qquad k_i \propto \frac{1}{l_i}$$

Clothing

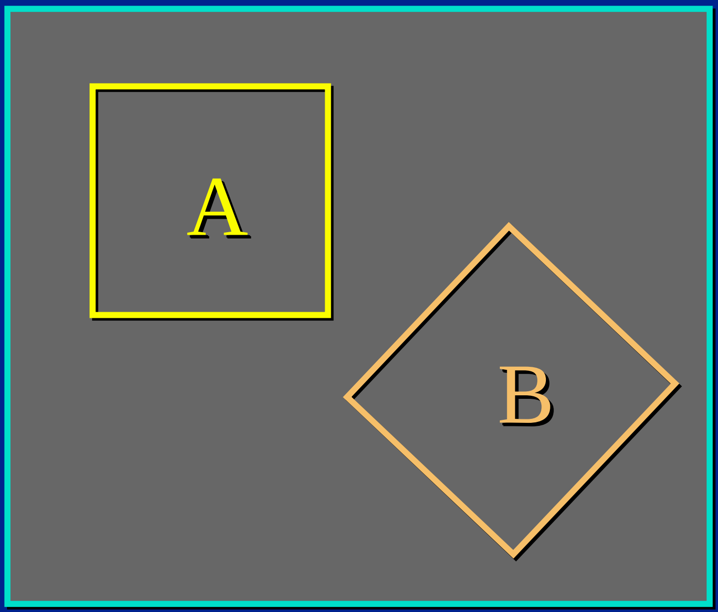
- Start with warp and weft threads.
- Weave them together.
- Add angular springs so threads want to stay perpendicular.



Cloth Properties

Cloth Resists
- Stretching
- Shearing
- Bending

Warp and Weft directions are special.



A and B will move differently

Rest Mesh Options

Model in 3D

- Clothing already on characters.
- Can directly craft desired 3D shape.
- Annotate warp/weft directions.
- Clothing probably will not locally flatten.

Model in 2D

- Must put clothing on characters
- Hire a tailor to get the pattern right.
- Sew parts together.
- Clothing guaranteed to flatten locally.
- Greater realism.

Non-flat Cloth

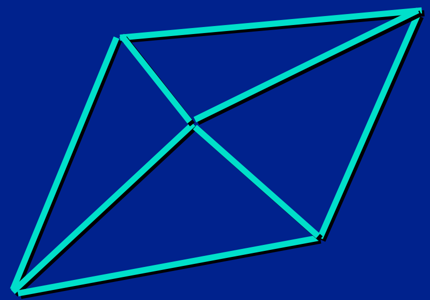
Non-flat cloth is strange stuff:

A baseball with no seams?

Wrinkles give strength?

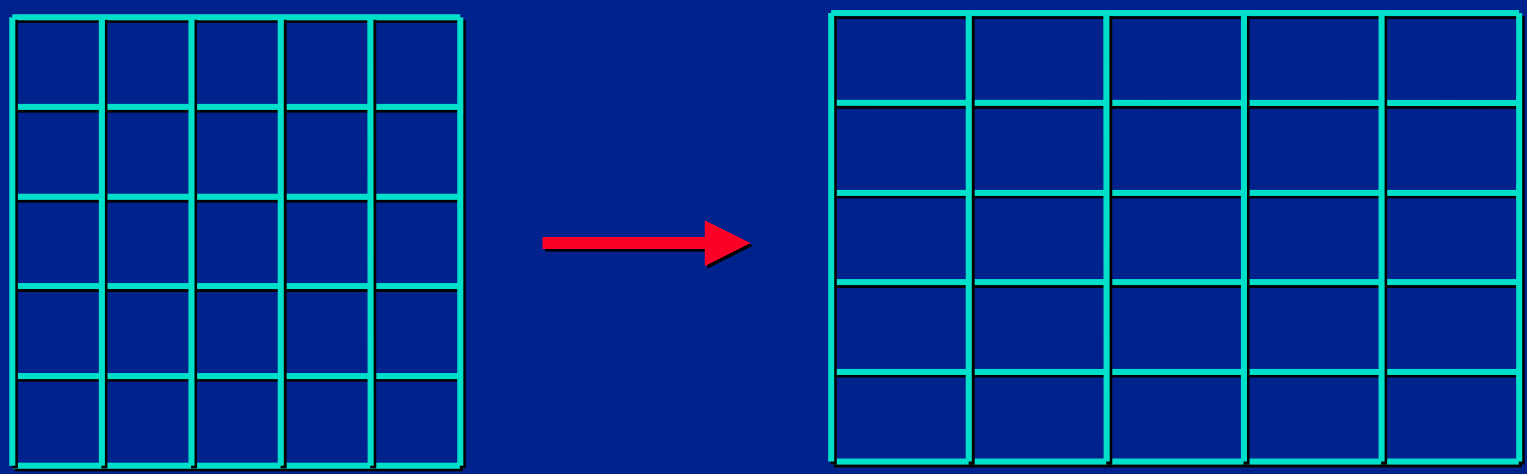
Clothing cut out of a volume?

Convexities that pop?



Even 4 Triangles are over-constrained:
16 rest angles, 8 rest lengths.
24 constraints on 15 dofs.
Must be consistent!

Stretch (Continuum Version)

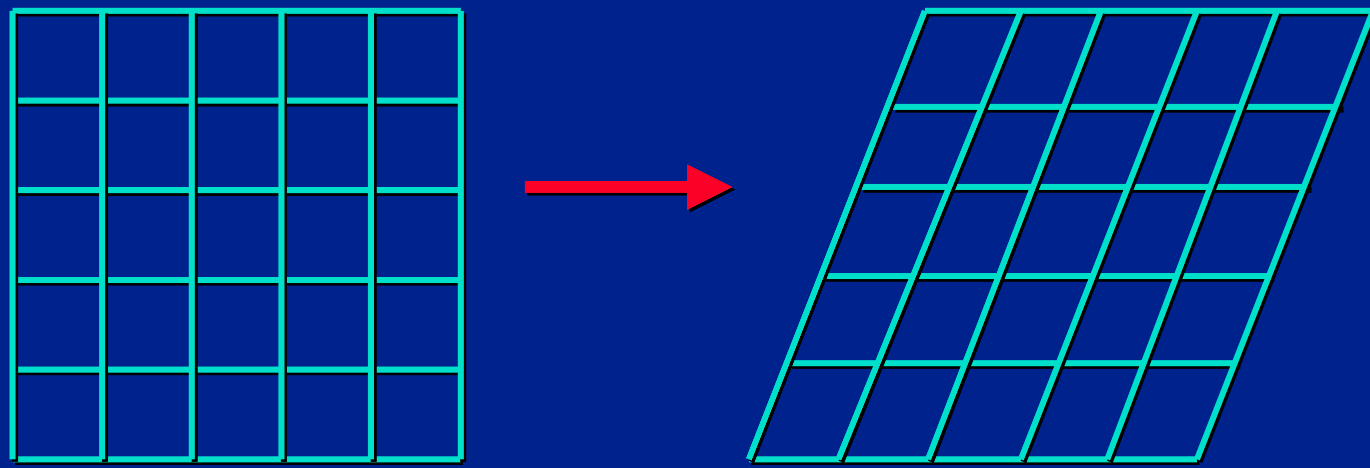


(u, v) $\qquad\qquad$ $\vec{x}$

$$S_u = \left\| \frac{\partial \vec{x}}{\partial u} \right\| - 1$$

$$E = \frac{1}{2} k \int (S_u^2 + S_v^2)\, du\, dv$$

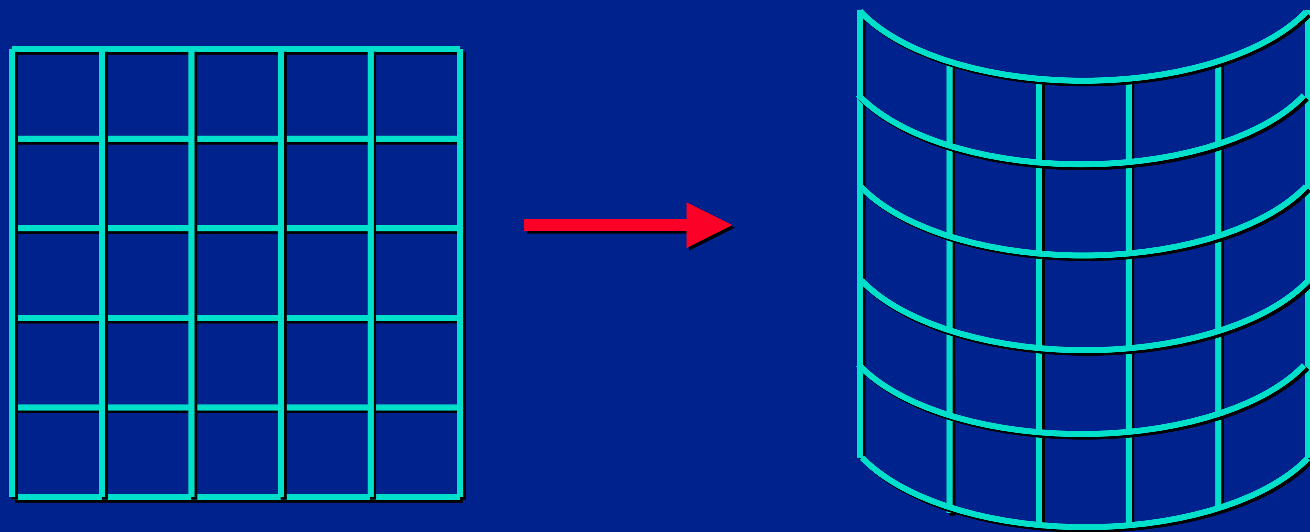
Shear (Continuum Version)



$$(u,v) \qquad \vec{x}$$

$$\theta = \cos^{-1}\left(\frac{\widehat{\partial \vec{x}}}{\partial u} \cdot \frac{\widehat{\partial \vec{x}}}{\partial v} \right) \qquad E = \frac{1}{2} k \int \theta^2 \, du \, dv$$

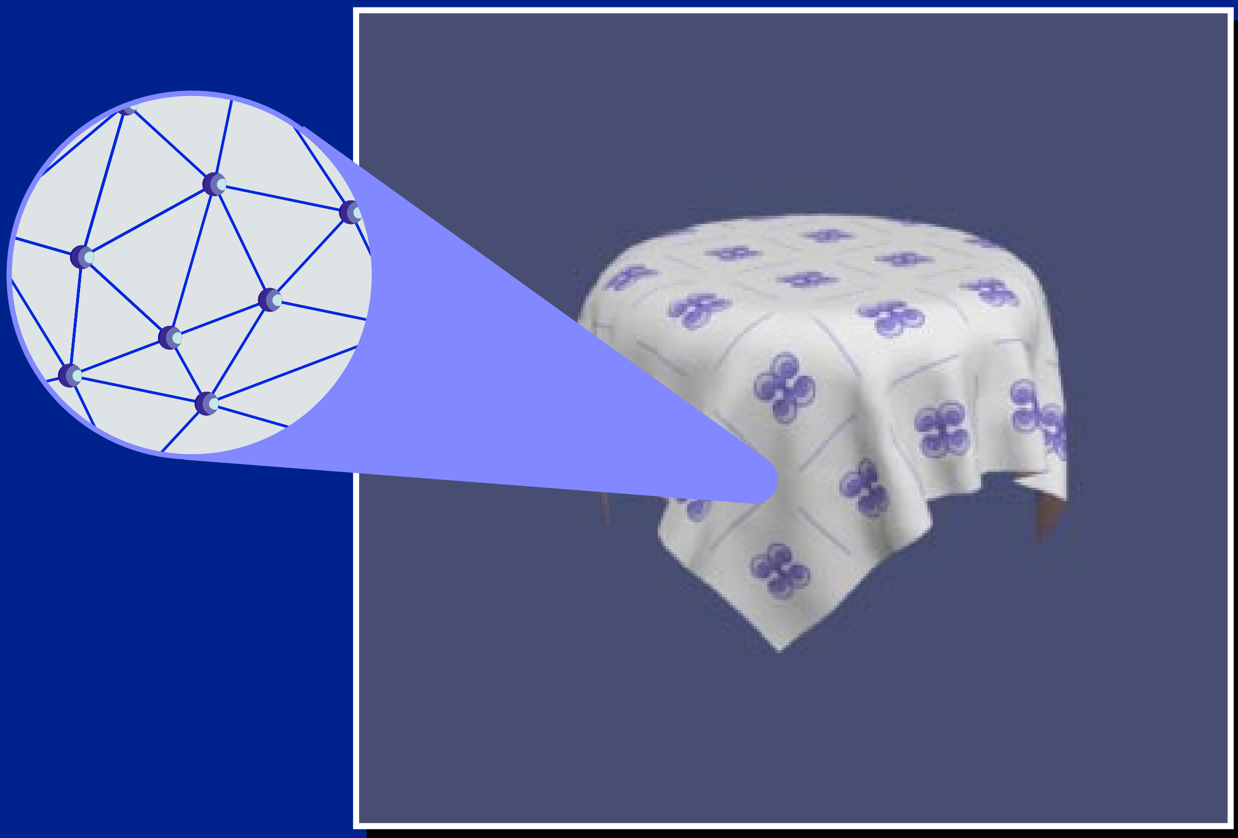
Bend (Continuum Version)



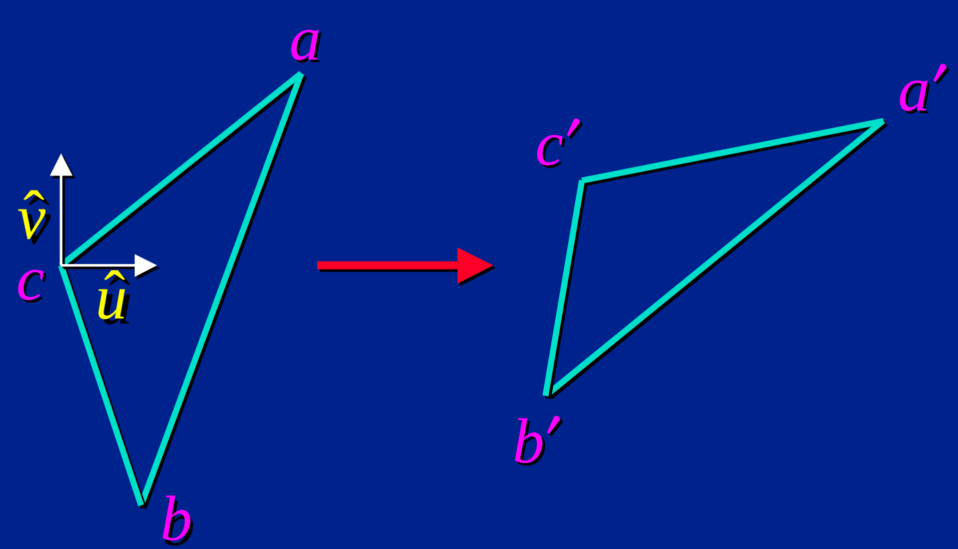
(u,v) $\qquad$ $\vec{x}$

$$E = \frac{1}{2}k\int(\kappa_u^2 + \kappa_v^2)\,du\,dv$$

Discretization



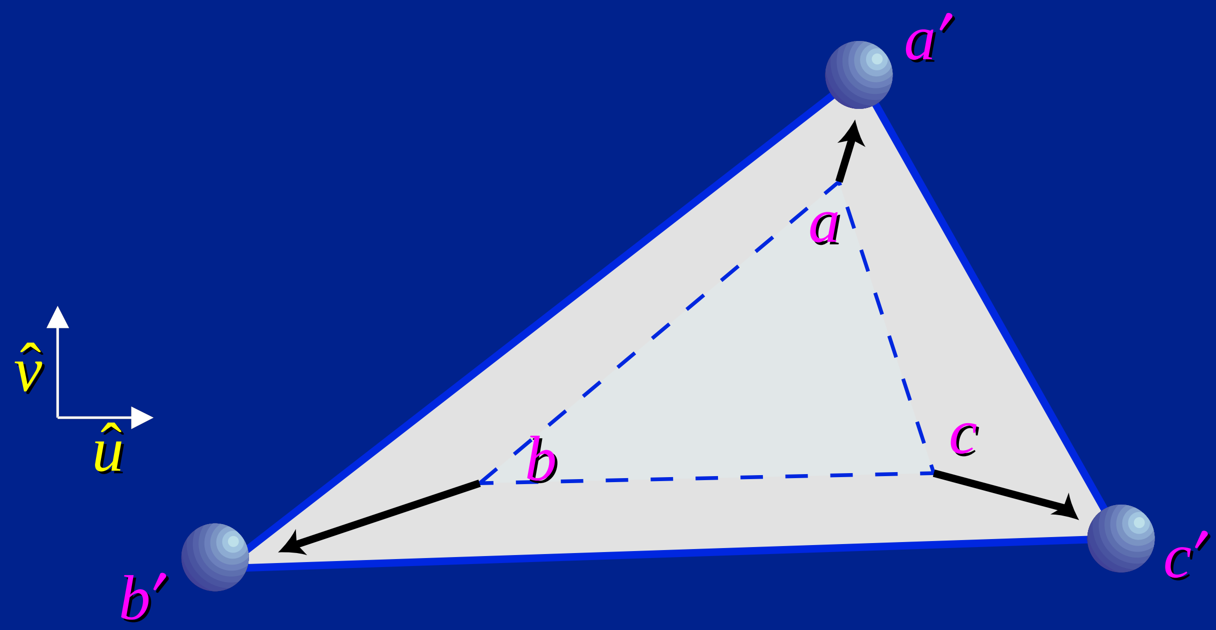
Triangle Energy



First, compute the affine transformation T that maps:

$$T: a \to c'$$
$$b \to b'$$
$$c \to c'$$

Triangle Stretch Energy

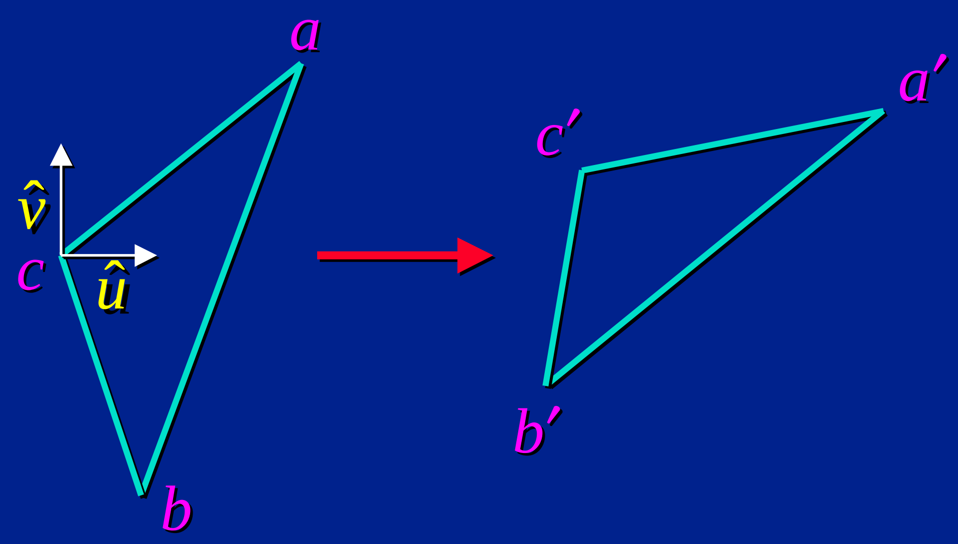


Now compute the stretch energy.

$$S_u = \left\| T(\hat{u}) \right\| - 1$$

$$E_{\text{stretch}} = \frac{1}{2} k (S_u^2 + S_v^2) A$$

Triangle Shear Energy

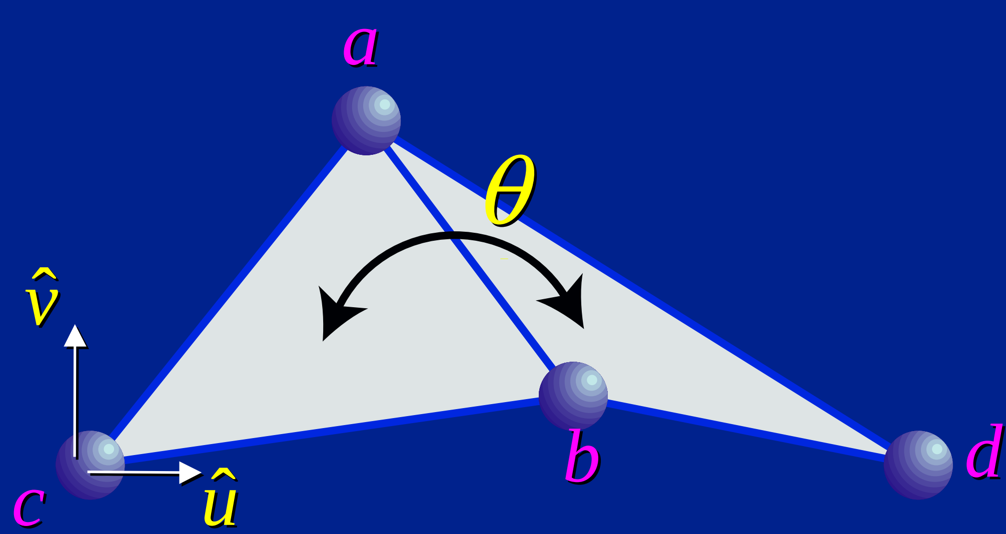


Next compute the shear energy.

$$\theta = \cos^{-1}(T(\hat{u}) \bullet T(\hat{v}))$$

$$E_{\text{shear}} = \frac{1}{2} k \theta^2 A$$

Triangle Bend Energy



$$\kappa = \frac{\theta}{l_{perp}}$$

Finally compute the bend energy.

$$E_{\text{bend}} = \frac{k}{2}(\kappa^2)A$$