

Assignment 4

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Download all latex codes from

<https://github.com/sujal100/EE3900/blob/main/Assignment4/Assignment4.tex>

Download all python codes from

<https://github.com/sujal100/EE3900/blob/main/Assignment4/codes/code.py>

Row reducing the augmented matrix,

$$\begin{pmatrix} 7 & 1 & 4 \\ -6 & -2 & 6 \\ 1 & 1 & 8 \end{pmatrix} \xrightarrow{R_3 \leftrightarrow R_1} \begin{pmatrix} 1 & 1 & 8 \\ -6 & -2 & 6 \\ 7 & 1 & 4 \end{pmatrix} \quad (2.0.5)$$

$$\xrightarrow{\begin{matrix} R_2=6R_1+R_2 \\ R_3=-7R_1+R_3 \end{matrix}} \begin{pmatrix} 1 & 1 & 8 \\ 0 & 4 & 54 \\ 0 & -6 & -52 \end{pmatrix} \xrightarrow{R_2=\frac{R_2}{4}} \begin{pmatrix} 1 & 1 & 8 \\ 0 & 1 & \frac{27}{2} \\ 0 & -6 & -52 \end{pmatrix} \quad (2.0.6)$$

$$\xrightarrow{R_3=6R_2+R_3} \begin{pmatrix} 1 & 1 & 8 \\ 0 & 1 & \frac{27}{2} \\ 0 & 0 & 29 \end{pmatrix} \quad (2.0.7)$$

1 PROBLEM

(Linear forms Q-2.22) Find the shortest distance between the lines

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}, \quad (1.0.1)$$

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \quad (1.0.2)$$

2 SOLUTION

In the given problem

$$\mathbf{A}_1 = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}, \mathbf{m}_1 = \begin{pmatrix} 7 \\ -6 \\ 1 \end{pmatrix}, \mathbf{A}_2 = \begin{pmatrix} 3 \\ 5 \\ 7 \end{pmatrix}, \mathbf{m}_2 = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}. \quad (2.0.1)$$

The lines will intersect if

$$\begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix} + \lambda_1 \begin{pmatrix} 7 \\ -6 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 7 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \quad (2.0.2)$$

$$\Rightarrow \lambda_1 \begin{pmatrix} 7 \\ -6 \\ 1 \end{pmatrix} - \lambda_2 \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \\ 7 \end{pmatrix} - \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix} \quad (2.0.3)$$

$$\Rightarrow \begin{pmatrix} 7 & 1 \\ -6 & -2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ 8 \end{pmatrix} \quad (2.0.4)$$

The above matrix has $rank = 3$. Hence, the lines do not intersect. Note that the lines are not parallel but they lie on parallel planes. Such lines are known as *skew* lines as can be seen in Fig 0.

\therefore The distance between given two lines is

$$\frac{|(\mathbf{A}_2 - \mathbf{A}_1)^T (\mathbf{m}_1 \times \mathbf{m}_2)|}{\|\mathbf{m}_1 \times \mathbf{m}_2\|} = 10.77 \quad (2.0.8)$$

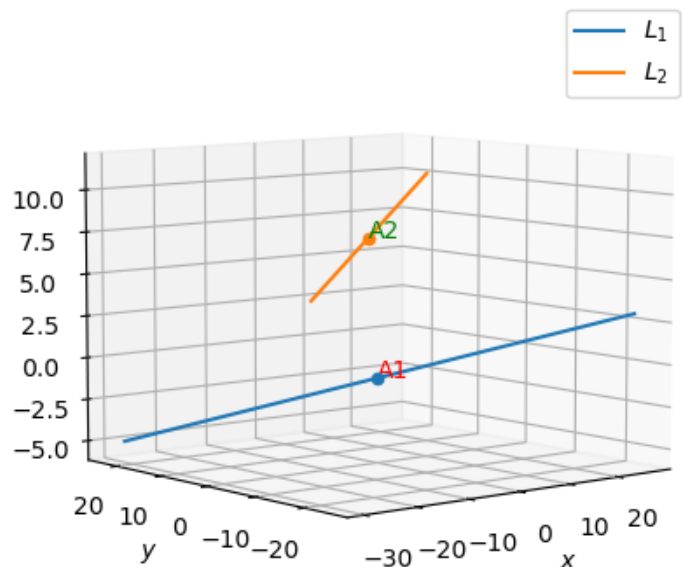


Fig. 0: plot of lines