GATE Assignment 1

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Download all latex codes from

https://github.com/https://github.com/sujal100/EE3900/

1 Problem

(GATE EC 2015 - Q51) In the system shown in Figure(a), $m(t)$ is a low-pass signal with bandwidth W Hz. The frequency response of the band-pass filter $H(f)$ is shown in Figure(b). If it is described that the output signal $z(t) = 10x(t)$, the maximum value of W (in Hz) should be strictly less than

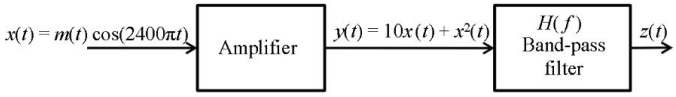

Fig. 0: (a)

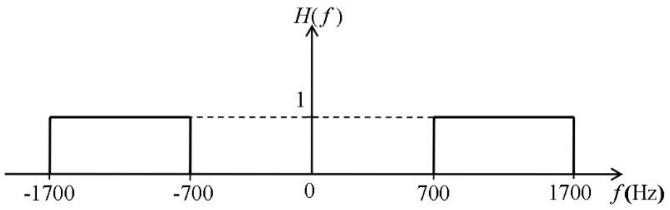

Fig. 0: (b)

2 Solution

We have the input signal

$$x(t) = m(t)\cos(2400\pi t) = m(t)\cos(\omega t)$$

And, $\omega = 2400\pi$ rad then, $f = 1200$ Hz

$$y(t) = 10x(t) + x^2(t) \tag{2.0.1}$$

$$= 10m(t)\cos(2400\pi t) + m^2(t)\cos^2(2400\pi t) \tag{2.0.2}$$

$$= 10m(t)\cos(\omega t) + m^2(t)\left[\frac{\cos(2\omega t)+1}{2}\right] \tag{2.0.3}$$

$$= \underbrace{\frac{m^2(t)}{2}}_{\text{+we frequency}} + \underbrace{10m(t)\cos(\omega t)}_{[\omega-W,\omega+W]} + \underbrace{\frac{m^2(t)\cos(2\omega t)}{2}}_{[2\omega-2W,2\omega+2W]} \tag{2.0.4}$$

If a signal x(t) is multiplied by a sinusoidal signal then the Fourier transform of x(t) gets shifted by the frequency of the sinusoid.

So, From the frequency plot, we conclude the

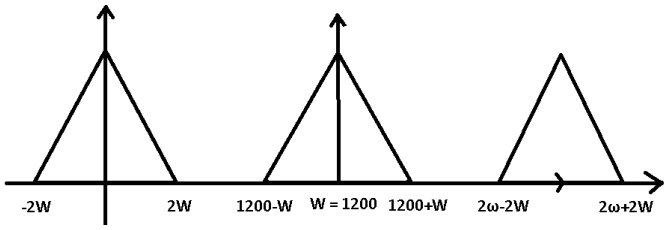

Fig. 0: Frequency plot

following results.

$$\begin{aligned}
\text{Results } 1: \quad & \omega - W > 700 && (2.0.5)\\
& 1200 - W > 700 && (2.0.6)\\
& W < 500 && (2.0.7)\\
\text{Results } 2: \quad & \omega + W < 1700 && (2.0.8)\\
& 1200 + W < 1700 && (2.0.9)\\
& W < 500 && (2.0.10)\\
\text{Results } 3: \quad & \omega - W > 2W && (2.0.11)\\
& 1200 > 3W && (2.0.12)\\
& W < 400 && (2.0.13)\\
\text{Results } 4: \quad & 2\omega - 2W > 1700 && (2.0.14)\\
& 2400 - 1700 > 2W && (2.0.15)\\
& 2W < 700 && (2.0.16)\\
& W < 350 && (2.0.17)
\end{aligned}$$

Thus, the above conclusions result in $W < 350$.
For $z(t) = 10x(t)$, maximum value of W must be less than 350 Hz.