Construction of Quadrilateral with 2 sides and 3 angle

Sujal - AI20BTECH11020

August 24, 2021

Problem

Construct

A quadrilateral $ABCD$ with given information

$$\angle A = \alpha \tag{1}$$
$$\angle B = \beta \tag{2}$$
$$\angle C = \gamma \tag{3}$$
$$\|\mathbf{A} - \mathbf{B}\| = a \tag{4}$$
$$\|\mathbf{B} - \mathbf{C}\| = b \tag{5}$$

First, Consider

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} a \\ 0 \end{pmatrix} \tag{6}$$

and calculate angle between CD and $+$x-axis is θ

$$\theta = 360^\circ - (\beta + \gamma) \tag{7}$$

and we have to find **C** and **D**, which as shown in fig 1.

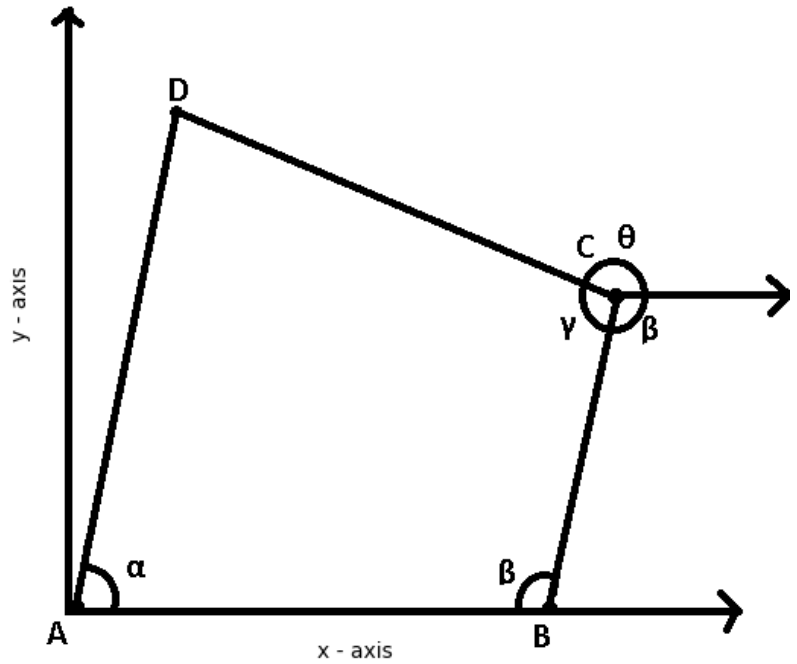

Figure: Quadrilateral MIST

Lemma

For $\mathbf{C}$,

$$\mathbf{C} = \mathbf{B} + b\mathbf{X} \text{ where } \mathbf{X} = \begin{pmatrix} \cos(180^\circ - \beta) \\ \sin(180^\circ - \beta) \end{pmatrix} \tag{8}$$

here, $\mathbf{X}$ *is unit vector in direction of line* BC *and then multiply it with* b *which is magnitude of line* BC *and last adding* $\mathbf{B}$. *For* $\mathbf{D}$,

$$\mathbf{D} = x\mathbf{Y} \text{ where } \mathbf{Y} = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} \text{ and } x \in R^+ \tag{9}$$

Here, $\mathbf{Y}$ *is unit vector in direction of line* AD, *and we need to find* x *which is magnitude of line* AD. *Also, we use* $\mathbf{C}$ *to find* $\mathbf{D}$ *as*

$$\mathbf{D} = y\mathbf{Z} + \mathbf{C} \text{ where } \mathbf{Z} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \text{ and } y \in R^+ \tag{10}$$

Here, $\mathbf{Z}$ *is unit vector in direction of line* CD *and we need to find* y *which is magnitude of line* CD.

Thus, from (2) and (5) in (8), we easily calculate $\mathbf{C}$ and from (9),(10) and $\mathbf{C}$, we get

$$x\mathbf{Y} = y\mathbf{Z} + \mathbf{C} \tag{11}$$

$$\begin{pmatrix} \cos\alpha & -\cos\theta \\ \sin\alpha & -\sin\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \mathbf{C} \tag{12}$$

and then find x and y
So, using x and (9) we get $\mathbf{D}$