Assignment

Sujal - AI20BTECH11020

Download all latex codes from

https://github.com/https://github.com/sujal100/EE3900/blob/main/Assignment3/Assignment3.tex

Download all python codes from

https://github.com/https://github.com/sujal100/EE3900/blob/main/Assignment3/codes/code.py

1 Problem

(construction Q-2.7) Construct a quadrilateral $MIST$ where $MI = 3.5, IS = 6.5, \angle M = 75°, \angle I = 105°$ and $\angle S = 120°$.

2 Solution

The given information can be expressed as

$$\angle M = 75° = \alpha \tag{2.0.1}$$
$$\angle I = 105° = \beta \tag{2.0.2}$$
$$\angle S = 120° = \gamma \tag{2.0.3}$$
$$\|\mathbf{M} - \mathbf{I}\| = 3.5 = a \tag{2.0.4}$$
$$\|\mathbf{I} - \mathbf{S}\| = 6.5 = b \tag{2.0.5}$$

Let,

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} a \\ 0 \end{pmatrix} \tag{2.0.6}$$

and first calculate Angle between ST and +x-axis is

$$\theta = 360° - (\beta + \gamma) \tag{2.0.7}$$
$$= 360° - (105° + 120°) \tag{2.0.8}$$
$$= 135° \tag{2.0.9}$$

and we have to find $\mathbf{S}$ and $\mathbf{T}$. So, For $\mathbf{S}$,

Lemma 2.1.

$$\mathbf{S} = \mathbf{I} + b\mathbf{X} \textit{ where } \mathbf{X} = \begin{pmatrix} \cos(180° - \angle I) \\ \sin(180° - \angle I) \end{pmatrix} \tag{2.0.10}$$

Proof. here, $\mathbf{X}$ is unit vector in direction of line IS and then multiply it with b which is magnitude of line IS and last adding $\mathbf{I}$. □

Lemma 2.2.

$$\mathbf{T} = x\mathbf{Y} \textit{ where } \mathbf{Y} = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} \textit{ and } x \in R^+ \tag{2.0.11}$$

$$\textit{Also,}\mathbf{T} = y\mathbf{Z} + \mathbf{S} \textit{ where } \mathbf{Z} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \textit{ and } y \in R^+ \tag{2.0.12}$$

Proof. First, in equation (2.0.11) $\mathbf{Y}$ is unit vector in direction of line MT, and we need to find x which is magnitude of line MT. Also, we use $\mathbf{S}$ to find $\mathbf{T}$ as in equation (2.0.12) $\mathbf{Z}$ is unit vector in direction of line ST □

Thus, from (2.0.2) and (2.0.5) in (2.0.10),

$$\mathbf{S} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix} + 6.5 \begin{pmatrix} \cos 75° \\ \sin 75° \end{pmatrix} \tag{2.0.13}$$
$$= \begin{pmatrix} 5.18 \\ 6.28 \end{pmatrix} \tag{2.0.14}$$

Thus, from (2.0.11),(2.0.12) and (2.0.13), we get

$$x\mathbf{Y} = y\mathbf{Z} + \mathbf{S} \tag{2.0.15}$$
$$\begin{pmatrix} \cos\alpha & -\cos\theta \\ \sin\alpha & -\sin\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5.18 \\ 6.28 \end{pmatrix} \tag{2.0.16}$$

The corresponding augmented matrix is

$$\left(\begin{array}{cc|c} \cos\alpha & -\cos\theta & 5.18 \\ \sin\alpha & -\sin\theta & 6.28 \end{array}\right) \tag{2.0.17}$$

Using (2.0.1) and (2.0.9) we get

$$\left(\begin{array}{cc|c} 0.26 & 0.71 & 5.18 \\ 0.97 & -0.71 & 6.28 \end{array}\right) \tag{2.0.18}$$

We use the Guass Jordan Elimination method as:

$$\left(\begin{array}{cc|c} 0.26 & 0.71 & 5.18 \\ 0.97 & -0.71 & 6.28 \end{array}\right) \tag{2.0.19}$$

$$\xleftrightarrow{R_2 \to R_2 - \frac{0.97}{0.26} R_1} \left(\begin{array}{cc|c} 0.26 & 0.71 & 5.18 \\ 0 & -3.20 & -13.05 \end{array}\right) \tag{2.0.20}$$

$$\xleftrightarrow{R_2 \to -\frac{1}{3.20} R_2} \left(\begin{array}{cc|c} 0.26 & 0.71 & 5.18 \\ 0 & 1 & 4.08 \end{array}\right) \tag{2.0.21}$$

$$\xleftrightarrow{R_1 \to R_1 - 0.71 R_2} \left(\begin{array}{cc|c} 0.26 & 0 & 2.28 \\ 0 & 1 & 4.08 \end{array}\right) \tag{2.0.22}$$

$$\xleftrightarrow{R_1 \to \frac{1}{0.26} R_1} \left(\begin{array}{cc|c} 1 & 0 & 8.77 \\ 0 & 1 & 4.08 \end{array}\right) \tag{2.0.23}$$

Therefore, the values of x and y are:

$$x = 8.77 \tag{2.0.24}$$

$$y = 4.08 \tag{2.0.25}$$

And, from (2.0.11)

$$\mathbf{T} = 8.77 \begin{pmatrix} 0.26 \\ 0.97 \end{pmatrix} \tag{2.0.26}$$

$$= \begin{pmatrix} 2.28 \\ 8.51 \end{pmatrix} \tag{2.0.27}$$

Thus,

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} 5.18 \\ 6.28 \end{pmatrix}, \mathbf{T} = \begin{pmatrix} 2.28 \\ 8.51 \end{pmatrix} \tag{2.0.28}$$

and the quadrilateral $MIST$ is the plotted in Fig 0.

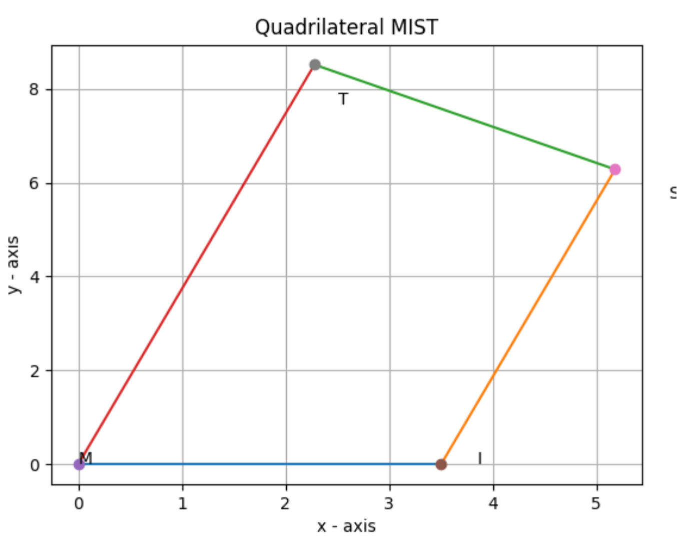

Fig. 0: Quadrilateral MIST