Quiz 2

Sujal - AI20BTECH11020

1 Problem 3.18(b)

A causal LTI system has the system function

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{\left(1 + \frac{1}{2}z^{-1}\right)\left(1 - z^{-1}\right)}$$

Find the output of this system, $y[n]$, for the input

$$x[n] = e^{j(\pi/2)n}$$

2 Solution

$$H(z) = \frac{1 + 2^z - 1 + z^{-2}}{\left(1 + \frac{1}{2}z^{-1}\right)\left(1 - z^{-1}\right)} \quad (2.0.1)$$

$$= -2 + \frac{\frac{1}{3}}{1 + \frac{1}{2}z^{-1}} + \frac{\frac{8}{3}}{1 - z^{-1}} \quad (2.0.2)$$

Taking the inverse z-transform:

$$h[n] = -2\delta[n] + \frac{1}{3}\left(-\frac{1}{2}\right)^n u[n] + \frac{8}{3}u[n]$$

Here

$$x[n] = e^{j(\pi/2)n}$$

$z = 2$ is inside the ROC. Therefore,

$$y[n] = H(z)|_{z=2} \, 2^n \quad (2.0.3)$$
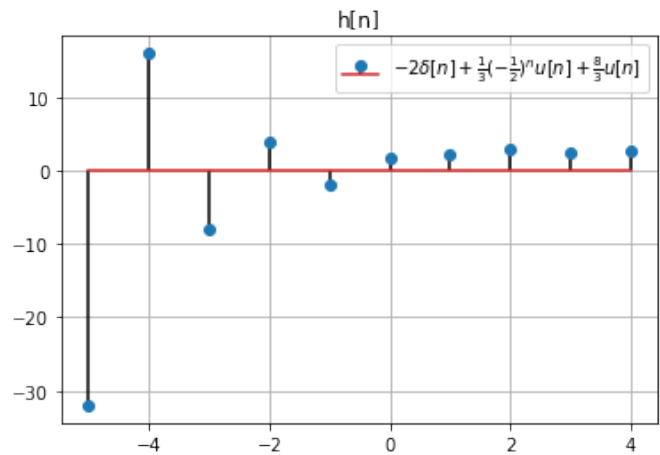
$$= \frac{18}{5}2^n \quad (2.0.4)$$

Fig. 0: plot of $h(n)$