Construction of Quadrilateral

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Problem

Constuction Q-2.7

Construct a quadrilateral $MIST$ where $MI = 3.5, IS = 6.5, \angle M = 75^\circ, \angle I = 105^\circ$ and $\angle S = 120^\circ$.

The given information can be expressed as

$$\angle M = 75^\circ = \alpha \tag{1}$$

$$\angle I = 105^\circ = \beta \tag{2}$$

$$\angle S = 120^\circ = \gamma \tag{3}$$

$$\|\mathbf{M} - \mathbf{I}\| = 3.5 = a \tag{4}$$

$$\|\mathbf{I} - \mathbf{S}\| = 6.5 = b \tag{5}$$

Let,

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} a \\ 0 \end{pmatrix} \tag{6}$$

and Angle between ST and +x-axis is θ

$$\begin{aligned} \theta &= 360^\circ - (\beta + \gamma) && (7) \\ &= 360^\circ - (105^\circ + 120^\circ) && (8) \\ &= 135^\circ && (9) \end{aligned}$$

and we have to find **S** and **T**.

Lemma

$$\mathbf{S} = \mathbf{I} + b\mathbf{X} \textit{ where } \mathbf{X} = \begin{pmatrix} \cos(180^\circ - \angle I) \\ \sin(180^\circ - \angle I) \end{pmatrix} \tag{10}$$

$$\mathbf{T} = x\mathbf{Y} \textit{ where } \mathbf{Y} = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} \textit{ and } x \in R^+ \tag{11}$$

$$\textit{Also, } \mathbf{T} = y\mathbf{Z} + \mathbf{S} \textit{ where } \mathbf{Z} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \textit{ and } y \in R^+ \tag{12}$$

Thus, from (2) and (5) in (10),

$$\mathbf{S} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix} + 6.5 \begin{pmatrix} \cos 75^\circ \\ \sin 75^\circ \end{pmatrix} \tag{13}$$
$$= \begin{pmatrix} 5.18 \\ 6.28 \end{pmatrix} \tag{14}$$

Thus, from (11),(12) and (13), we get

$$x\mathbf{Y} = y\mathbf{Z} + \mathbf{S} \tag{15}$$
$$\begin{pmatrix} \cos \alpha & -\cos \theta \\ \sin \alpha & -\sin \theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5.18 \\ 6.28 \end{pmatrix} \tag{16}$$

The corresponding augmented matrix is

$$\left(\begin{array}{cc|c} \cos\alpha & -\cos\theta & 5.18 \\ \sin\alpha & -\sin\theta & 6.28 \end{array}\right) \tag{17}$$

Using (1) and (9) we get

$$\left(\begin{array}{cc|c} 0.26 & 0.71 & 5.18 \\ 0.97 & -0.71 & 6.28 \end{array}\right) \tag{18}$$

We use the Guass Jordan Elimination method as:

$$\left(\begin{array}{cc|c} 0.26 & 0.71 & 5.18 \\ 0.97 & -0.71 & 6.28 \end{array}\right) \tag{19}$$

$$\xleftrightarrow{R_2 \to R_2 - \frac{0.97}{0.26} R_1} \left(\begin{array}{cc|c} 0.26 & 0.71 & 5.18 \\ 0 & -3.20 & -13.05 \end{array}\right) \tag{20}$$

$$\xleftrightarrow{R_2 \to -\frac{1}{3.20} R_2} \left(\begin{array}{cc|c} 0.26 & 0.71 & 5.18 \\ 0 & 1 & 4.08 \end{array}\right) \tag{21}$$

$$\xleftrightarrow{R_1 \to R_1 - 0.71 R_2} \left(\begin{array}{cc|c} 0.26 & 0 & 2.28 \\ 0 & 1 & 4.08 \end{array}\right) \tag{22}$$

$$\xleftrightarrow{R_1 \to \frac{1}{0.26} R_1} \left(\begin{array}{cc|c} 1 & 0 & 8.77 \\ 0 & 1 & 4.08 \end{array}\right) \tag{23}$$

Therefore, the values of x and y are:

$$x = 8.77 \tag{24}$$

$$y = 4.08 \tag{25}$$

And, from (11)

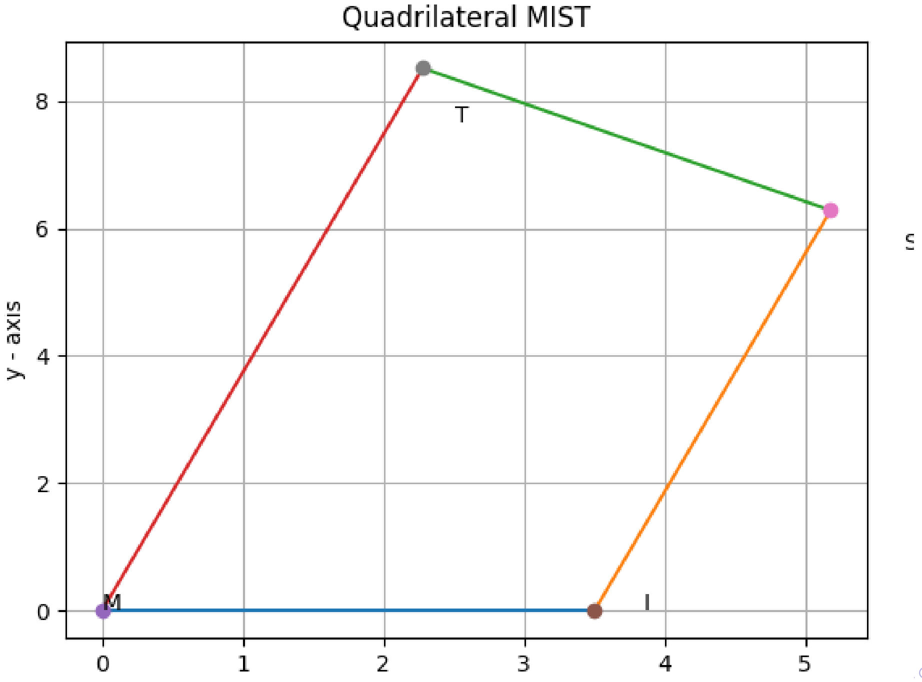
$$\mathbf{T} = 8.77 \begin{pmatrix} 0.26 \\ 0.97 \end{pmatrix} \tag{26}$$

$$= \begin{pmatrix} 2.28 \\ 8.51 \end{pmatrix} \tag{27}$$

Thus,

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} 5.18 \\ 6.28 \end{pmatrix}, \mathbf{T} = \begin{pmatrix} 2.28 \\ 8.51 \end{pmatrix} \tag{28}$$

and the quadrilateral *MIST* is the plotted in Fig 1.


Quadrilateral MIST
T
M
I
y - axis
8
6
4
2
0
0
1
2
3
4
5