

Assignment

Sujal - AI20BTECH11020

Download all latex codes from

<https://github.com/sujal100/EE3900/blob/main/Assignment3/Assignment3.tex>

Download all python codes from

<https://github.com/sujal100/EE3900/blob/main/Assignment3/codes/code.py>

The given information can be expressed as

$$\angle M = 75^\circ = \alpha \quad (2.0.4)$$

$$\angle I = 105^\circ = \beta \quad (2.0.5)$$

$$\angle S = 120^\circ = \gamma \quad (2.0.6)$$

$$\|\mathbf{M} - \mathbf{I}\| = 3.5 = a \quad (2.0.7)$$

$$\|\mathbf{I} - \mathbf{S}\| = 6.5 = b \quad (2.0.8)$$

So, Let

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} a \\ 0 \end{pmatrix}. \quad (2.0.9)$$

and we have to find \mathbf{S} and \mathbf{T} . Then, calculate θ

$$\theta = 360^\circ - (\beta + \gamma) \quad (2.0.10)$$

$$= 360^\circ - (105^\circ + 120^\circ) \quad (2.0.11)$$

$$= 135^\circ \quad (2.0.12)$$

So, For \mathbf{S} , we can use lemma 2.1. Thus, from (2.0.5) and (2.0.8) in (2.0.1),

$$\mathbf{S} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix} + 6.5 \begin{pmatrix} \cos 75^\circ \\ \sin 75^\circ \end{pmatrix} \quad (2.0.13)$$

$$= \begin{pmatrix} 5.18 \\ 6.28 \end{pmatrix} \quad (2.0.14)$$

For \mathbf{T} , we can use lemma 2.2. from (2.0.2), (2.0.3) and (2.0.13), we get

$$x\mathbf{Y} = y\mathbf{Z} + \mathbf{S} \quad (2.0.15)$$

$$\begin{pmatrix} \cos \alpha & -\cos \theta \\ \sin \alpha & -\sin \theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5.18 \\ 6.28 \end{pmatrix} \quad (2.0.16)$$

The corresponding augmented matrix is

$$\left(\begin{array}{cc|c} \cos \alpha & -\cos \theta & 5.18 \\ \sin \alpha & -\sin \theta & 6.28 \end{array} \right) \quad (2.0.17)$$

Using (2.0.4) and (2.0.12) we get

$$\left(\begin{array}{cc|c} 0.26 & 0.71 & 5.18 \\ 0.97 & -0.71 & 6.28 \end{array} \right) \quad (2.0.18)$$

We use the Gauss Jordan Elimination method as:

1 PROBLEM

(construction Q-2.7) Construct a quadrilateral $MIST$ where $MI = 3.5$, $IS = 6.5$, $\angle M = 75^\circ$, $\angle I = 105^\circ$ and $\angle S = 120^\circ$.

2 SOLUTION

Here, we have two sides and three angle (i.e. we know all angles). So, we can use following lemma,

Lemma 2.1.

$$\mathbf{S} = \mathbf{I} + b\mathbf{X} \text{ where } \mathbf{X} = \begin{pmatrix} \cos(180^\circ - \angle I) \\ \sin(180^\circ - \angle I) \end{pmatrix} \quad (2.0.1)$$

Proof. here, \mathbf{X} is unit vector in direction of line IS and then multiply it with b which is magnitude of line IS and last adding \mathbf{I} . \square

Lemma 2.2.

$$\mathbf{T} = x\mathbf{Y} \text{ where } \mathbf{Y} = \begin{pmatrix} \cos \alpha \\ \sin \alpha \end{pmatrix} \text{ and } x \in \mathbb{R}^+ \quad (2.0.2)$$

$$\text{Also, } \mathbf{T} = y\mathbf{Z} + \mathbf{S} \text{ where } \mathbf{Z} = \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \text{ and } y \in \mathbb{R}^+ \quad (2.0.3)$$

Proof. First, in equation (2.0.2) \mathbf{Y} is unit vector in direction of line MT , and we need to find x which is magnitude of line MT . Also, we use \mathbf{S} to find \mathbf{T} as in equation (2.0.3) \mathbf{Z} is unit vector in direction of line ST . so, θ is angle between ST and $+x$ -axis \square

$$\left(\begin{array}{cc|c} 0.26 & 0.71 & 5.18 \\ 0.97 & -0.71 & 6.28 \end{array} \right) \quad (2.0.19)$$

$$\xleftrightarrow{R_2 \rightarrow R_2 - \frac{0.97}{0.26} R_1} \left(\begin{array}{cc|c} 0.26 & 0.71 & 5.18 \\ 0 & -3.20 & -13.05 \end{array} \right) \quad (2.0.20)$$

$$\xleftrightarrow{R_2 \rightarrow -\frac{1}{3.20} R_2} \left(\begin{array}{cc|c} 0.26 & 0.71 & 5.18 \\ 0 & 1 & 4.08 \end{array} \right) \quad (2.0.21)$$

$$\xleftrightarrow{R_1 \rightarrow R_1 - 0.71 R_2} \left(\begin{array}{cc|c} 0.26 & 0 & 2.28 \\ 0 & 1 & 4.08 \end{array} \right) \quad (2.0.22)$$

$$\xleftrightarrow{R_1 \rightarrow \frac{1}{0.26} R_1} \left(\begin{array}{cc|c} 1 & 0 & 8.77 \\ 0 & 1 & 4.08 \end{array} \right) \quad (2.0.23)$$

Therefore, the values of x and y are:

$$x = 8.77 \quad (2.0.24)$$

$$y = 4.08 \quad (2.0.25)$$

And, from (2.0.2)

$$\mathbf{T} = 8.77 \begin{pmatrix} 0.26 \\ 0.97 \end{pmatrix} \quad (2.0.26)$$

$$= \begin{pmatrix} 2.28 \\ 8.51 \end{pmatrix} \quad (2.0.27)$$

Thus,

$$\mathbf{M} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} 3.5 \\ 0 \end{pmatrix}, \mathbf{S} = \begin{pmatrix} 5.18 \\ 6.28 \end{pmatrix}, \mathbf{T} = \begin{pmatrix} 2.28 \\ 8.51 \end{pmatrix} \quad (2.0.28)$$

and the quadrilateral *MIST* is plotted in Fig 0.

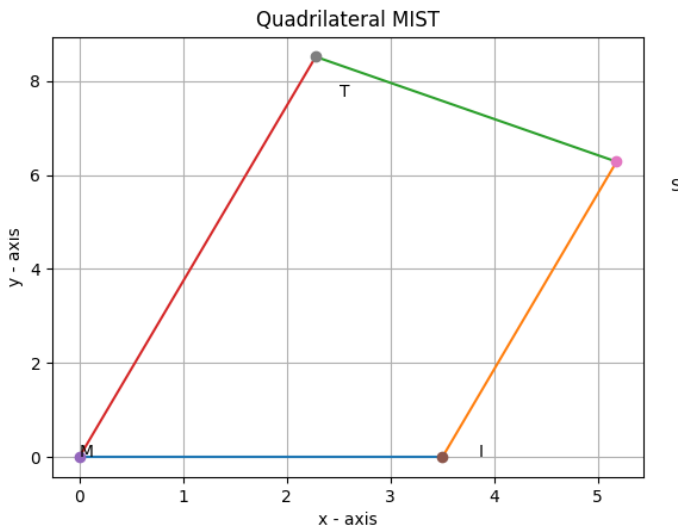


Fig. 0: Quadrilateral MIST