GATE Assignment 1

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Download all latex codes from

https://github.com/https://github.com/sujal100/EE3900/

1 Problem

(GATE EC 2015 - Q51) In the system shown in Figure(a), $m(t)$ is a low-pass signal with bandwidth W Hz. The frequency response of the band-pass filter $H(f)$ is shown in Figure(b). If it is described that the output signal $z(t) = 10x(t)$, the maximum value of W (in Hz) should be strictly less than

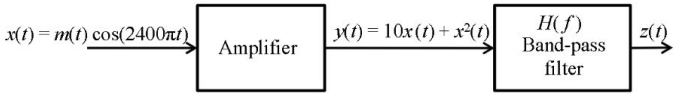

Fig. 0: (a)

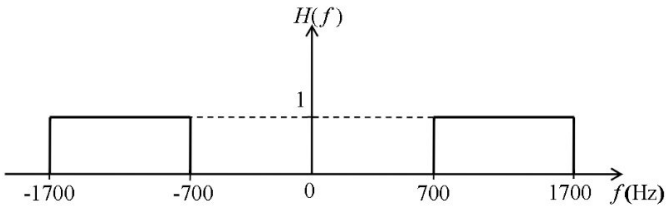

Fig. 0: (b)

2 Solution

We have the input signal

$$x(t) = m(t)\cos(2400\pi t) = m(t)\cos(\omega t)$$

Take m(t) to be a sinusoid with bandwidth W. But $m(t)$ is low-pass signal. So, $\omega = 2400\pi$ rad then, $f = 1200$ Hz.

$$y(t) = 10x(t) + x^2(t) \tag{2.0.1}$$

$$= 10m(t)\cos(2400\pi t) + m^2(t)\cos^2(2400\pi t) \tag{2.0.2}$$

$$= 10m(t)\cos(\omega t) + m^2(t)\left[\frac{\cos(2\omega t)+1}{2}\right] \tag{2.0.3}$$

$$= \underbrace{\frac{m^2(t)}{2}}_{\text{+we frequency}} + \underbrace{10m(t)\cos(\omega t)}_{[\omega-W,\omega+W]} + \underbrace{\frac{m^2(t)\cos(2\omega t)}{2}}_{[2\omega-2W,2\omega+2W]} \tag{2.0.4}$$

If a signal x(t) is multiplied by a sinusoidal signal then the Fourier transform of x(t) gets shifted by the frequency of the sinusoid. So, From the frequency plot in fig. 0, we conclude the following results.

$$\text{Results 1}: \omega - W > 700 \tag{2.0.5}$$
$$1200 - W > 700 \tag{2.0.6}$$
$$W < 500 \tag{2.0.7}$$
$$\text{Results 2}: \omega + W < 1700 \tag{2.0.8}$$
$$1200 + W < 1700 \tag{2.0.9}$$
$$W < 500 \tag{2.0.10}$$
$$\text{Results 3}: \omega - W > 2W \tag{2.0.11}$$
$$1200 > 3W \tag{2.0.12}$$
$$W < 400 \tag{2.0.13}$$
$$\text{Results 4}: 2\omega - 2W > 1700 \tag{2.0.14}$$
$$2400 - 1700 > 2W \tag{2.0.15}$$
$$2W < 700 \tag{2.0.16}$$
$$W < 350 \tag{2.0.17}$$

Thus, the above conclusions result in $W < 350$.
For $z(t) = 10x(t)$, maximum value of W must be less than 350 Hz.

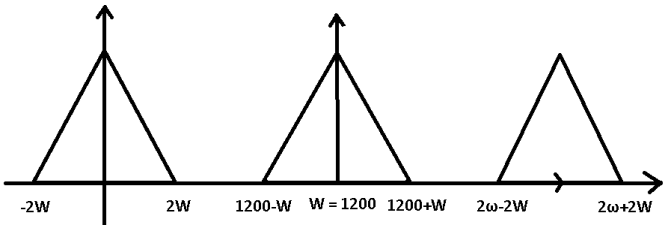

Fig. 0: Frequency plot