Assignment 7

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Download all latex codes from

https://github.com/https://github.com/sujal100/Probability_and_Random_variable/blob/main/exercise_6/exercise_6_main_tex.tex

1 Problem [CSIR NET(JUNE-2017) MATHS-STATISTICS (Q-104)]

Let $\{X_n, n \geq 1\}$ be i.i.d. uniform (-1,2) random variables. Which of the following statements are true?

a) $\frac{1}{n}\sum_{i=1}^{n} X_i \to 0$ almost surely
b) $\left\{\frac{1}{2n}\sum_{i=1}^{n} X_{2i} - \frac{1}{2n}\sum_{i=1}^{n} X_{2i-1}\right\} \to 0$ almost surely
c) $\sup\{X_1, X_2, \dots\} = 2$ almost surely
d) $\inf\{X_1, X_2, \dots\} = -1$ almost surely

2 Solution

We using convergence in almost surely and Strong law of large number (SLLN)

1) Let $X_1, X_2, \dots$ be an infinite sequence of random variables. We shall say that the sequence $\{X_i\}$ converges with probability 1 (or converges almost surely (a.s.)) to a random variable Y, if $P\left(\lim_{n\to\infty} X_n = Y\right) = 1$ and we write $X_n \xrightarrow{a.s.} Y$
2) Let X_n be i.i.d with $\mathbf{E}\left[|X_1|\right] < \infty$. Then, as $n \to \infty$, we have $\frac{S_n}{n} \xrightarrow{\text{a.s.}} \mathbf{E}\left[X_1\right] \implies \frac{S_n}{n} \xrightarrow{\text{P}} \mathbf{E}\left[X_1\right]$, where $S_n = X_1 + \cdots + X_n$.
3) $X_i \xrightarrow{a.s.} X \implies g(X_i) \xrightarrow{a.s.} g(X)$

a)

$$\frac{1}{n}\left(X_1 + \cdots + X_n\right) \to E(X) \in (-1, 2) \quad (2.0.1)$$

as $n \to \infty$, according to strong law of large numbers (SLLN).
So, option (A) is incorrect.

b)

$$\left\{\frac{1}{2n}\sum_{i=1}^{n} X_{2i} - \frac{1}{2n}\sum_{i=1}^{n} X_{2i-1}\right\} \xrightarrow{a.s.} \left\{\frac{nX}{2n} - \frac{nX}{2n}\right\} \quad (2.0.2)$$

$$= 0 \quad (2.0.3)$$

option (B) is correct.

c) using $X_i \xrightarrow{a.s.} X$ this , we also conclude that

$$\sup\{X_1, X_2, \dots\} = 2 \textit{ almost surely} \quad (2.0.4)$$

$$\inf\{X_1, X_2, \dots\} = -1 \textit{ almost surely} \quad (2.0.5)$$

Hence (B), (C) and (D) are correct option.