

Assignment 5

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Download all python codes from

https://github.com/sujal100/Probability_and_Random_variable/tree/main/exercise_5/codes

and latex codes from

https://github.com/sujal100/Probability_and_Random_variable/blob/main/exercise_5/exercise_5_main_tex.tex

1 PROBLEM [GATE(2003)EC-61]

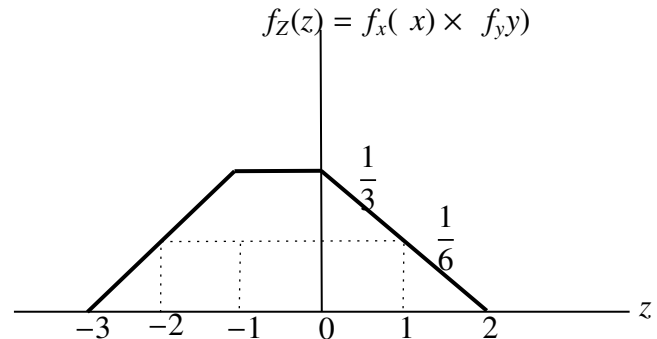
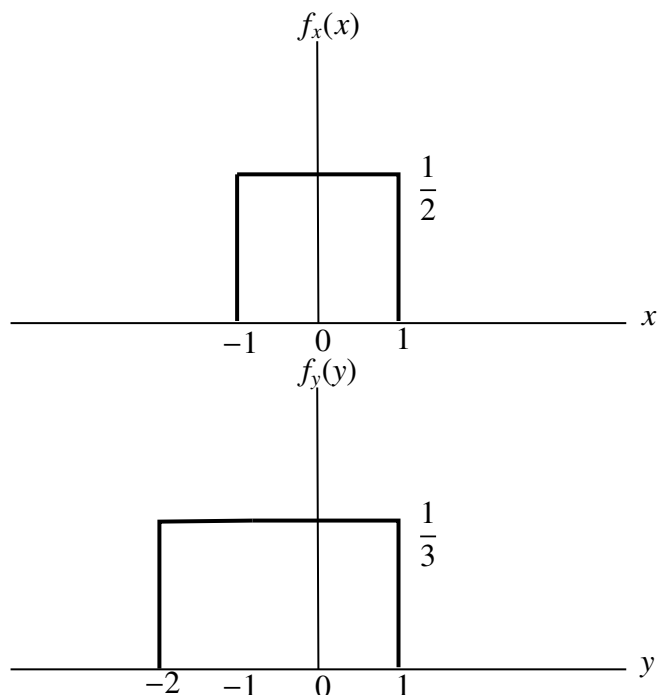
Let X and Y be two statistically independent random variables uniformly distributed in the ranges $(-1, 1)$ and $(-2, 1)$ respectively. Let $Z = X + Y$. then the probability that $[Z \leq -2]$ is

- (a) zero (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{1}{12}$

2 SOLUTION

The pdf of $Z(= X + Y)$ will be convolution of pdf of X and pdf of Y as shown below.

$$f_x(x) \times f_y(y) = f_z(z) \quad (2.0.1)$$



Now

$$\begin{aligned} \Pr(Z \leq z) &= \int_{-\infty}^z f_z(z) dz \\ \Pr(Z \leq -2) &= \int_{-\infty}^{-2} f_z(z) dz \\ &= \text{Area } [z \leq -2] \\ &= \frac{1}{2} \times \frac{1}{6} \times 1 = \frac{1}{12} \end{aligned}$$

Hence (D) is correct option.