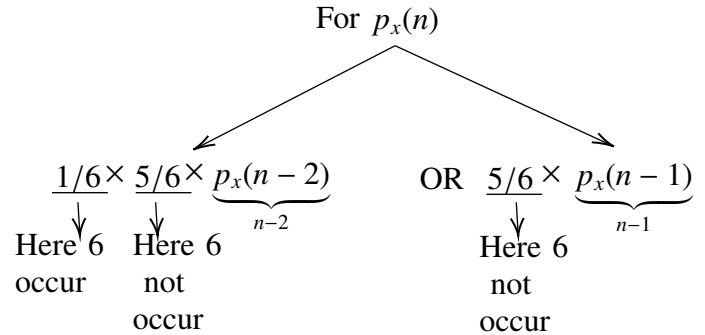# Assignment 3

Sujal - AI20BTECH11020

Download all python codes from

https://github.com/sujal100/
  Probability_and_Random_variable/tree/main/
  exercise_3/codes

and latex codes from

https://github.com/https://github.com/sujal100/
  Probability_and_Random_variable/blob/main
  /exercise_3/exercise_3_main_tex.tex

For $p_x(n)$

$\underbrace{1/6\times\ 5/6\times\ p_x(n-2)}_{n-2}$ OR $5/6\times\ \underbrace{p_x(n-1)}_{n-1}$

Here 6   Here 6   Here 6
occur   not   not
         occur   occur

1 PROBLEM [GATE(2015)MA-11]

In an experiment, a fair die is rolled until two sixes are obtained in succession. The probability that the experiment will end in the fifth trial is equal to

a) $\dfrac{125}{6^5}$

b) $\dfrac{150}{6^5}$

c) $\dfrac{175}{6^5}$

d) $\dfrac{200}{6^5}$

2 SOLUTION

Let Consider, Bernoulli random variables say X. Here, $P(X = n)$ refer to the probability that experiment ends in exactly n^{th} rolls. Thus, problem is asking for p_5. We note that

$$p_1 = 0, p_2 = \frac{1}{6^2} \qquad (2.0.1)$$

For $n > 2$, we remark that the first roll is either a 6 or it isn't. If it is, then the second roll can't be a 6. That leads to the recursion

$$p_n = \frac{1}{6} \times \frac{5}{6} \times p_{n-2} + \frac{5}{6} \times p_{n-1} \qquad (2.0.2)$$

Put $n = 0$, we get

$$p_2 = \frac{5}{6^2}p_0 + \frac{5}{6}p_1 \qquad (2.0.3)$$

$$\frac{1}{6^2} = \frac{5}{6^2}p_0 \qquad (2.0.4)$$

$$p_0 = \frac{1}{5} \qquad (2.0.5)$$

Now, put n=n+2 We have,

$$p_{n+2} = \frac{5}{6^2}p_n + \frac{5}{6}p_{n+1}, p_0 = \frac{1}{5}, p_1 = 0 \qquad (2.0.6)$$

Taking, Z-transform both side,

$$Z[p_{n+2}] = \frac{5}{6^2}Z[p_n] + \frac{5}{6}Z[p_{n+1}] \qquad (2.0.7)$$

$$z^2 \Pr(z) - z^2 p_0 - z p_1 = \frac{5}{6^2}\Pr(z) + \frac{5}{6}[z\Pr(z) - z p_0] \qquad (2.0.8)$$

Inserting initial conditions,

$$z^2 \Pr(z) - \frac{z^2}{5} = \frac{5}{6^2} \Pr(z) + \frac{5z}{6} \Pr(z) - \frac{z}{6} \quad (2.0.9)$$

$$\Pr(z) = \frac{\left(\dfrac{z^2}{5} - \dfrac{z}{6^2}\right)}{\left(z^2 - \dfrac{5z}{6} - \dfrac{5}{6^2}\right)} \quad (2.0.10)$$

$$\frac{5 \Pr(z)}{z} = \frac{36z - 5}{36z^2 - 5z - 5} \quad (2.0.11)$$

$$\frac{5\sqrt{745}\,\Pr(z)}{36} = \frac{(5 + \sqrt{745})z}{(72z + \sqrt{745} - 5)} \quad (2.0.12)$$

$$- \frac{(\sqrt{745} - 5)z}{(-72z + \sqrt{745} + 5)} \quad (2.0.13)$$

We know that $Z^{-1}\left(\dfrac{z}{z-a}\right) = a^n$

So, taking Z-inverse transform both side,we get

$$\frac{5\sqrt{745}}{36} Z^{-1} \Pr(z) = \frac{(5 + \sqrt{745})}{72} Z^{-1}\left(\frac{z}{z - \frac{(5 - \sqrt{745})}{72}}\right) \quad (2.0.14)$$

$$- \frac{(5 - \sqrt{745})}{72} Z^{-1}\left(\frac{z}{z - \frac{(\sqrt{745}+5)}{72}}\right) \quad (2.0.15)$$

$$\frac{5\sqrt{745}}{36} p_n = \frac{(5 + \sqrt{745})}{72}\left(\frac{(5 - \sqrt{745})}{72}\right)^n \quad (2.0.16)$$

$$- \frac{(5 - \sqrt{745})}{72}\left(\frac{(\sqrt{745} + 5)}{72}\right)^n \quad (2.0.17)$$

$$p_n = \left(\frac{1}{2\sqrt{745}} + \frac{1}{10}\right)\left(\frac{(5 - \sqrt{745})}{72}\right)^n \quad (2.0.18)$$

$$- \left(\frac{1}{2\sqrt{745}} - \frac{1}{10}\right)\left(\frac{(\sqrt{745} + 5)}{72}\right)^n \quad (2.0.19)$$

so,

$$p_5 = \left(\frac{1}{2\sqrt{745}} + \frac{1}{10}\right)\left(\frac{(5 - \sqrt{745})}{72}\right)^5 \quad (2.0.20)$$

$$- \left(\frac{1}{2\sqrt{745}} - \frac{1}{10}\right)\left(\frac{(\sqrt{745} + 5)}{72}\right)^5 \quad (2.0.21)$$

$$p_5 = 0.001146095297972871 \quad (2.0.22)$$

Hence (C) is correct option.