Assignment 7

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Download all latex codes from

https://github.com/https://github.com/sujal100/Probability_and_Random_variable/blob/main/exercise_6/exercise_6_main_tex.tex

1 Problem [CSIR NET(JUNE-2017) MATHS-STATISTICS (Q-104)]

Let $\{X_n, n \geq 1\}$ be i.i.d. uniform (-1,2) random variables. Which of the following statements are true?

a) $\frac{1}{n}\sum_{i=1}^{n} X_i \to 0$ almost surely
b) $\left\{\frac{1}{2n}\sum_{i=1}^{n} X_{2i} - \frac{1}{2n}\sum_{i=1}^{n} X_{2i-1}\right\} \to 0$ almost surely
c) $\sup\{X_1, X_2, \ldots\} = 2$ almost surely
d) $\inf\{X_1, X_2, \ldots\} = -1$ almost surely

2 Solution

We know that in almost surely

$$\frac{1}{n}(X_1 + \cdots + X_n) \to E(X) \in (-1, 2) \tag{2.0.1}$$

as $n \to \infty$, according to strong law of large numbers (SLLN).
So, option (A) is incorrect.
In almost surely

$$X_i \xrightarrow{a.s.} X \implies g(X_i) \xrightarrow{a.s.} g(X) \tag{2.0.2}$$

So, in almost surely,

$$\left\{\frac{1}{2n}\sum_{i=1}^{n} X_{2i} - \frac{1}{2n}\sum_{i=1}^{n} X_{2i-1}\right\} \to \left\{\frac{1}{2n}nX - \frac{1}{2n}nX\right\} \tag{2.0.3}$$

$$= 0 \tag{2.0.4}$$

So, using $X_i \xrightarrow{a.s.} X$ this , we also conclude that

$$\sup\{X_1, X_2, \ldots\} = 2\, a.s. \tag{2.0.5}$$

$$\inf\{X_1, X_2, \ldots\} = -1\, a.s. \tag{2.0.6}$$

Hence (B), (C) and (D) are correct option.