

Assignment 3

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Download all python codes from

[https://github.com/sujal100/
Probability_and_Random_variable/tree/main/
exercise_3/codes](https://github.com/sujal100/Probability_and_Random_variable/tree/main/exercise_3/codes)

and latex codes from

[https://github.com/sujal100/
Probability_and_Random_variable/blob/main/
/exercise_3/exercise_3_main_tex.tex](https://github.com/sujal100/Probability_and_Random_variable/blob/main/exercise_3/exercise_3_main_tex.tex)

1 PROBLEM [GATE(2015)MA-11]

In an experiment, a fair die is rolled until two sixes are obtained in succession. The probability that the experiment will end in the fifth trial is equal to

(A) $\frac{125}{6^5}$ (B) $\frac{150}{6^5}$ (C) $\frac{175}{6^5}$ (D) $\frac{200}{6^5}$

2 SOLUTION

Let $P(n)$ be the probability that experiment ends in exactly n rolls. Thus your problem is asking for $P(5)$. We note that

$$P(1) = 0, P(2) = \frac{1}{6^2} \quad (2.0.1)$$

For $n > 2$, we remark that the first roll is either a 6 or it isn't. If it is, then the second roll can't be a 6. That leads to the recursion

$$P(n) = \frac{1}{6} \times \frac{5}{6} \times P(n-2) + \frac{5}{6} \times P(n-1) \quad (2.0.2)$$

So,

$$P(3) = \frac{5}{6^2}P(1) + \frac{5}{6}P(2) = \frac{5}{6^3} \quad (2.0.3)$$

$$P(4) = \frac{5}{6^2}P(2) + \frac{5}{6}P(3) = \frac{30}{6^4} = \frac{5}{6^3} \quad (2.0.4)$$

$$P(5) = \frac{5}{6^2}P(3) + \frac{5}{6}P(4) = \frac{25 + 150}{6^5} = \frac{175}{6^5} \quad (2.0.5)$$

Hence (C) is correct option.