Assignment 5

Sujal - AI20BTECH11020

Download all python codes from

https://github.com/sujal100/Probability_and_Random_variable/tree/main/exercise_5/codes

and latex codes from

https://github.com/https://github.com/sujal100/Probability_and_Random_variable/blob/main/exercise_5/exercise_5_main_tex.tex

1 Problem [GATE(2003)EC-61]

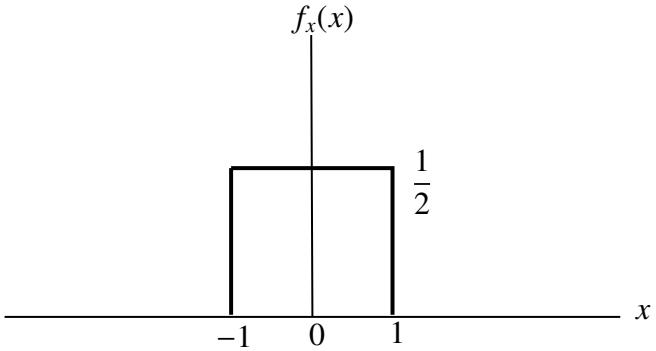
Let X and Y be two statistically independent random variables uniformly distributed in the ranges $(-1, 1)$ and $(-2, 1)$ respectively. Let $Z = X + Y$. then the probability that $[Z \leq -2]$ is

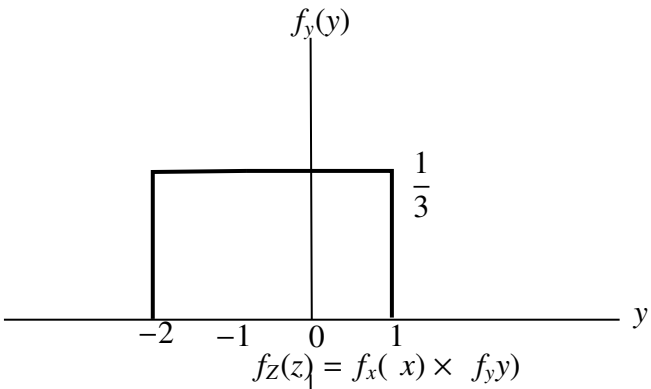
a) zero
b) $\frac{1}{6}$
c) $\frac{1}{3}$
d) $\frac{1}{12}$

2 Solution

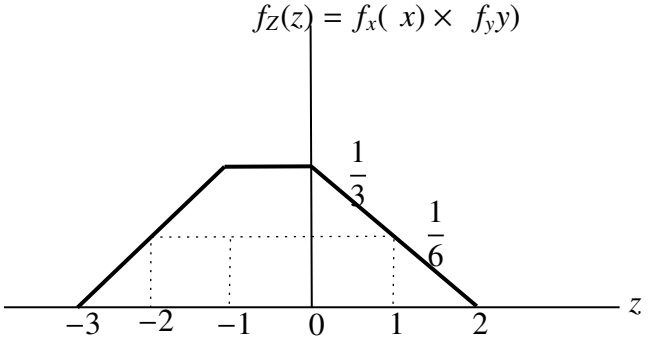
The pdf of $Z(= X + Y)$ will be convolution of pdf of X and pdf of Y as shown below.

$$f_x(x) \times f_y(y) = f_z(z) \tag{2.0.1}$$





$f_Z(z) = f_x(\ x) \times \ f_y y)$



Now

$$\Pr(Z \leq z) = \int_{-\infty}^{z} f_Z(z)dz$$

$$\Pr(Z \leq -2) = \int_{-\infty}^{-2} f_Z(z)dz$$

$$= \text{ Area } [z \leq -2]$$

$$= \frac{1}{2} \times \frac{1}{6} \times 1 = \frac{1}{12}$$

Hence (D) is correct option.