# Assignment 7

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Download all latex codes from

https://github.com/https://github.com/sujal100/Probability_and_Random_variable/blob/main/exercise_6/exercise_6_main_tex.tex

## 1 Problem [CSIR NET(JUNE-2017) MATHS-STATISTICS (Q-104)]

Let $\{X_n, n \geq 1\}$ be i.i.d. uniform (-1,2) random variables. Which of the following statements are true?

a) $\frac{1}{n}\sum_{i=1}^{n} X_i \to 0$ almost surely
b) $\left\{\frac{1}{2n}\sum_{i=1}^{n} X_{2i} - \frac{1}{2n}\sum_{i=1}^{n} X_{2i-1}\right\} \to 0$ almost surely
c) $\sup\{X_1, X_2, \ldots\} = 2$ almost surely
d) $\inf\{X_1, X_2, \ldots\} = -1$ almost surely

## 2 Solution

We using convergence in almost surely and Strong law of large number (SLLN)

1) *Almost sure convergence :* Let $X_1, X_2, \ldots$ be an infinite sequence of random variables. We shall say that the sequence $\{X_i\}$ converges with probability 1 (or converges almost surely (a.s.)) to a random variable $Y$, if

$$\Pr\left(\lim_{n\to\infty} X_n = Y\right) = 1 \tag{2.0.1}$$

$$\text{and we write, } X_n \xrightarrow{a.s.} Y \tag{2.0.2}$$

2) *SLLN :* Let $X_n$ be i.i.d with $\mathbf{E}\left[|X_1|\right] < \infty$. Then, as $n \to \infty$ , we have

$$\frac{S_n}{n} \xrightarrow{\text{a.s.}} \mathbf{E}\left[X_1\right] \implies \frac{S_n}{n} \xrightarrow{\text{P}} \mathbf{E}\left[X_1\right] \tag{2.0.3}$$

$$\text{, where } S_n = X_1 + \cdots + X_n \tag{2.0.4}$$

also,

$$X_i \xrightarrow{a.s.} X \implies g(X_i) \xrightarrow{a.s.} g(X) \tag{2.0.5}$$

a)

$$\frac{1}{n}(X_1 + \cdots + X_n) \to E(X) \in (-1, 2) \tag{2.0.6}$$

$$\text{as } n \to \infty, \tag{2.0.7}$$

according to strong law of large numbers (SLLN).
So, option (A) is incorrect.

b) using this 2.0.5 , we solve as

$$\left\{\frac{1}{2n}\sum_{i=1}^{n} X_{2i} - \frac{1}{2n}\sum_{i=1}^{n} X_{2i-1}\right\} \xrightarrow{a.s.} \left\{\frac{nX}{2n} - \frac{nX}{2n}\right\} \tag{2.0.8}$$

$$= 0 \tag{2.0.9}$$

option (B) is correct.

c) Similarly, Let $M = \sup(S)$. Then,

$$x \leq M, \quad \forall x \in S \tag{2.0.10}$$

$$\forall \epsilon > 0, \quad (M - \epsilon, M] \cap S \neq \emptyset \tag{2.0.11}$$

Using $X_i \xrightarrow{a.s.} X$ this , we conclude that

$$\sup\{X_1, X_2, \ldots\} = 2 \textit{ almost surely} \tag{2.0.12}$$

d) Let $m = \inf(S)$. Then

$$x \geq m, \quad \forall x \in S \tag{2.0.13}$$

$$\forall \epsilon > 0, \quad [m, m + \epsilon] \cap S \neq \emptyset \tag{2.0.14}$$

Again using $X_i \xrightarrow{a.s.} X$ this , we conclude that

$$\inf\{X_1, X_2, \ldots\} = -1 \textit{ almost surely} \tag{2.0.15}$$

Hence (B), (C) and (D) are correct options.