

# Assignment 7

Sujal - AI20BTECH11020

Download all latex codes from

[https://github.com/sujal100/Probability\\_and\\_Random\\_variable/blob/main/exercise\\_6/exercise\\_6\\_main\\_tex.tex](https://github.com/sujal100/Probability_and_Random_variable/blob/main/exercise_6/exercise_6_main_tex.tex)

## 1 PROBLEM [CSIR NET(JUNE-2017) MATHS-STATISTICS (Q-104)]

Let  $\{X_n, n \geq 1\}$  be i.i.d. uniform  $(-1, 2)$  random variables. Which of the following statements are true?

- a)  $\frac{1}{n} \sum_{i=1}^n X_i \rightarrow 0$  almost surely
- b)  $\left\{ \frac{1}{2n} \sum_{i=1}^n X_{2i} - \frac{1}{2n} \sum_{i=1}^n X_{2i-1} \right\} \rightarrow 0$  almost surely
- c)  $\sup \{X_1, X_2, \dots\} = 2$  almost surely
- d)  $\inf \{X_1, X_2, \dots\} = -1$  almost surely

## 2 SOLUTION

We use convergence in almost surely and Strong law of large number (SLLN)

- 1) *Almost sure convergence* : Let  $X_1, X_2, \dots$  be an infinite sequence of random variables. We shall say that the sequence  $\{X_i\}$  converges with probability 1 (or converges almost surely (a.s.)) to a random variable  $Y$ , if

$$\Pr\left(\lim_{n \rightarrow \infty} X_n = Y\right) = 1 \quad (2.0.1)$$

$$\text{and we write, } X_n \xrightarrow{a.s.} Y \quad (2.0.2)$$

- 2) *SLLN* : Let  $X_n$  be i.i.d with  $\mathbf{E}[|X_1|] < \infty$ . Then, as  $n \rightarrow \infty$ , we have

$$\frac{S_n}{n} \xrightarrow{a.s.} \mathbf{E}[X_1] \implies \frac{S_n}{n} \xrightarrow{P} \mathbf{E}[X_1] \quad (2.0.3)$$

$$\text{, where } S_n = X_1 + \dots + X_n \quad (2.0.4)$$

also,

$$X_i \xrightarrow{a.s.} X \implies g(X_i) \xrightarrow{a.s.} g(X) \quad (2.0.5)$$

a)

$$\frac{1}{n} (X_1 + \dots + X_n) \rightarrow E(X) \in (-1, 2) \quad (2.0.6)$$

$$\text{as } n \rightarrow \infty, \quad (2.0.7)$$

according to strong law of large numbers (SLLN).

So, option (A) is incorrect.

- b) using this 2.0.5, we solve as

$$\left\{ \frac{1}{2n} \sum_{i=1}^n X_{2i} - \frac{1}{2n} \sum_{i=1}^n X_{2i-1} \right\} \xrightarrow{a.s.} \left\{ \frac{nX}{2n} - \frac{nX}{2n} \right\} \quad (2.0.8)$$

$$= 0 \quad (2.0.9)$$

option (B) is correct.

- c) Similarly, Let  $M = \sup(S)$ . Then,

$$x \leq M, \quad \forall x \in S \quad (2.0.10)$$

$$\forall \epsilon > 0, \quad (M - \epsilon, M] \cap S \neq \emptyset \quad (2.0.11)$$

where,  $S$  be a nonempty subset of  $\mathbb{R}$  with an upper bound. Using  $X_i \xrightarrow{a.s.} X$  this, we conclude that

$$\sup \{X_1, X_2, \dots\} = 2 \text{ almost surely} \quad (2.0.12)$$

- d) Let  $m = \inf(S)$ . Then

$$x \geq m, \quad \forall x \in S \quad (2.0.13)$$

$$\forall \epsilon > 0, \quad [m, m + \epsilon] \cap S \neq \emptyset \quad (2.0.14)$$

where,  $S$  be a nonempty subset of  $\mathbb{R}$  with an lower bound. Again using  $X_i \xrightarrow{a.s.} X$  this, we conclude that

$$\inf \{X_1, X_2, \dots\} = -1 \text{ almost surely} \quad (2.0.15)$$

Hence (B), (C) and (D) are correct options.