Assignment 7

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Download all latex codes from

https://github.com/https://github.com/sujal100/Probability_and_Random_variable/blob/main/exercise_6/exercise_6_main_tex.tex

1 Problem [CSIR NET(JUNE-2017) MATHS-STATISTICS (Q-104)]

Let $\{X_n, n \geq 1\}$ be i.i.d. uniform (-1,2) random variables. Which of the following statements are true?

a) $\frac{1}{n}\sum_{i=1}^{n} X_i \to 0$ almost surely
b) $\left\{\frac{1}{2n}\sum_{i=1}^{n} X_{2i} - \frac{1}{2n}\sum_{i=1}^{n} X_{2i-1}\right\} \to 0$ almost surely
c) $\sup\{X_1, X_2, \ldots\} = 2$ almost surely
d) $\inf\{X_1, X_2, \ldots\} = -1$ almost surely

2 Solution

We using convergence in almost surely and Strong law of large number (SLLN)

1) *Almost sure convergence :* Let $X_1, X_2, \ldots$ be an infinite sequence of random variables. We shall say that the sequence $\{X_i\}$ converges with probability 1 (or converges almost surely (a.s.)) to a random variable Y, if

$$\Pr\left(\lim_{n\to\infty} X_n = Y\right) = 1 \tag{2.0.1}$$

$$\text{and we write,} X_n \xrightarrow{a.s.} Y \tag{2.0.2}$$

2) *SLLN :* Let X_n be i.i.d with $\mathbf{E}[|X_1|] < \infty$. Then, as $n \to \infty$, we have

$$\frac{S_n}{n} \xrightarrow{\text{a.s.}} \mathbf{E}[X_1] \implies \frac{S_n}{n} \xrightarrow{\text{P}} \mathbf{E}[X_1] \tag{2.0.3}$$

$$\text{, where} S_n = X_1 + \cdots + X_n \tag{2.0.4}$$

also,

$$X_i \xrightarrow{a.s.} X \implies g(X_i) \xrightarrow{a.s.} g(X) \tag{2.0.5}$$

a)

$$\frac{1}{n}(X_1 + \cdots + X_n) \to E(X) \in (-1, 2) \tag{2.0.6}$$

$$\text{as} n \to \infty, \tag{2.0.7}$$

according to strong law of large numbers (SLLN).
So, option (A) is incorrect.

b) using this 2.0.5 , we solve as

$$\left\{\frac{1}{2n}\sum_{i=1}^{n} X_{2i} - \frac{1}{2n}\sum_{i=1}^{n} X_{2i-1}\right\} \xrightarrow{a.s.} \left\{\frac{nX}{2n} - \frac{nX}{2n}\right\} \tag{2.0.8}$$

$$= 0 \tag{2.0.9}$$

option (B) is correct.

c) A lower bound of a subset S of a partially ordered set $(P, \leq)$ is an element a of P such that

$$a \leq x \,\forall x \in S \tag{2.0.10}$$

A lower bound a of S is called an infimum (or greatest lower bound, or meet) of S if for all lower bounds y of S in $P, y \leq a$(a is larger than or equal to any other lower bound).Using $X_i \xrightarrow{a.s.} X$ this , we conclude that

$$\sup\{X_1, X_2, \ldots\} = 2 \textit{ almost surely} \tag{2.0.11}$$

d) Similarly, an upper bound of a subset S of a partially ordered set $(P, \leq)$ is an element b of P such that

$$b \geq x \,\forall x \in S \tag{2.0.12}$$

An upper bound b of S is called a supremum (or least upper bound, or join) of S if for all upper bounds z of S in $P, z \geq b$(b is less than any other upper bound). Again using $X_i \xrightarrow{a.s.} X$ this , we conclude that

$$\inf\{X_1, X_2, \ldots\} = -1 \textit{ almost surely} \tag{2.0.13}$$

Hence (B), (C) and (D) are correct option.