Assignment 3

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Download all python codes from

https://github.com/sujal100/Probability_and_Random_variable/tree/main/exercise_3/codes

and latex codes from

https://github.com/https://github.com/sujal100/Probability_and_Random_variable/blob/main/exercise_3/exercise_3_main_tex.tex

1 Problem [GATE(2015)MA-11]

In an experiment, a fair die is rolled until two sixes are obtained in succession. The probability that the experiment will end in the fifth trial is equal to
(A) $\frac{125}{6^5}$ (B) $\frac{150}{6^5}$ (C) $\frac{175}{6^5}$ (D) $\frac{200}{6^5}$

2 Solution

Let Consider, Bernoulli random variables say X. Here, $P_x(n)$ refer to the probability that experiment ends in exactly n^{th} rolls. Thus, problem is asking for $P_x(5)$. We note that

$$P_x(1) = 0, P_x(2) = \frac{1}{6^2} \tag{2.0.1}$$

For $n > 2$, we remark that the first roll is either a 6 or it isn't. If it is, then the second roll can't be a 6. That leads to the recursion

$$P_x(n) = \frac{1}{6} \times \frac{5}{6} \times P_x(n-2) + \frac{5}{6} \times P_x(n-1) \tag{2.0.2}$$

So,

$$P_x(3) = \frac{5}{6^2}P_x(1) + \frac{5}{6}P_x(2) = \frac{5}{6^3} \tag{2.0.3}$$

$$P_x(4) = \frac{5}{6^2}P_x(2) + \frac{5}{6}P_x(3) = \frac{30}{6^4} = \frac{5}{6^3} \tag{2.0.4}$$

therefore,

$$P_x(5) = \frac{5}{6^2}P_x(3) + \frac{5}{6}P_x(4) = \frac{25+150}{6^5} = \frac{175}{6^5} \tag{2.0.5}$$

Hence (C) is correct option.