

Assignment 6

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Download all latex codes from

https://github.com/sujal100/Probability_and_Random_variable/blob/main/exercise_6/exercise_6_main_text.tex

1 PROBLEM [CSIR NET(JUNE-2017) MATHS-STATISTICS (Q-50)]

X_1, X_2, \dots are independent identically distributed random variables having common density f . Assume $f(x) = f(-x)$ for all $x \in \mathbb{R}$. Which of the following statements is correct?

- a) $\frac{1}{n} (X_1 + \dots + X_n) \rightarrow 0$ in probability
- b) $\frac{1}{n} (X_1 + \dots + X_n) \rightarrow 0$ almost surely
- c) $P\left(\frac{1}{\sqrt{n}} (X_1 + \dots + X_n) < 0\right) \rightarrow \frac{1}{2}$
- d) $\sum_{i=1}^n X_i$ has the same distribution as $\sum_{i=1}^n (-1)^i X_i$

2 SOLUTION

In this question $f(x) = f(-x) \implies$ Symmetry about Zero, $x \in \mathbb{R}$.

We know that in almost surely

$$\frac{1}{n} (X_1 + \dots + X_n) \rightarrow E(X_i) < \infty, \quad (2.0.1)$$

as $n \rightarrow \infty$, according to strong law of large numbers (SLLN). So, also in probability also hold this equation.

Hence, (A) and (B) are incorrect option

X_1, X_2, \dots, X_n are independent identically distributed $\sim N[0, 2]$

Consider,

$$Y = \frac{X_1 + X_2 + \dots + X_n}{\sqrt{n}} \quad (2.0.2)$$

So,

$$E(Y) = E\left(\frac{X_1 + X_2 + \dots + X_n}{\sqrt{n}}\right) = 0 \quad (2.0.3)$$

$$V(Y) = V\left(\frac{X_1 + X_2 + \dots + X_n}{\sqrt{n}}\right) = \frac{1}{n} 2n = 2 \quad (2.0.4)$$

$$Y \sim N[0, 2] \quad (2.0.5)$$

we know that $f(x)$ is symmetric about zero. So,

$$P(Y < 0) = \frac{1}{2} \quad (2.0.6)$$

$$P\left(\frac{1}{\sqrt{n}} (X_1 + \dots + X_n) < 0\right) = \frac{1}{2} \quad (2.0.7)$$

Hence, (C) is incorrect option.

Now, for option (D) consider $\phi_x(t)$ is characteristic function of $X_i, i = 1, \dots, n$.

$$f(x) = f(-x) \implies \phi_x(t) = \phi_{-x}(t) \quad (2.0.8)$$

Therefore,

$$\phi_{\sum_{i=1}^n X_i}(t) = \phi_{X_1 + \dots + X_n}(t) = \phi_{X_1}(t) \cdot \phi_{X_2}(t) \cdots \phi_{X_n}(t) \quad (2.0.9)$$

$$= [\phi_x(t)]^n \quad (2.0.10)$$

similarly,

$$\phi_{\sum_{i=1}^n (-1)^i X_i}(t) = \phi_{-X_1} + \phi_{X_2} + \phi_{-X_3} + \dots + \phi_{(-1)^n X_n}(t) \quad (2.0.11)$$

$$= \phi_{-X_1}(t) \cdot \phi_{X_2}(t) \cdots \phi_{(-1)^n X_n}(t) \quad (2.0.12)$$

$$= [\phi_x(t)]^n \quad (2.0.13)$$

$$\phi_{\sum_{i=1}^n X_i}(t) = \phi_{\sum_{i=1}^n (-1)^i X_i}(t) \quad (2.0.14)$$

Therefore, $\sum_{i=1}^n X_i$ has same distribution as $\sum_{i=1}^n (-1)^i X_i$.

Hence, (D) is correct option.