Assignment 5

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Download all python codes from

https://github.com/sujal100/Probability_and_Random_variable/tree/main/exercise_5/codes

and latex codes from

https://github.com/https://github.com/sujal100/Probability_and_Random_variable/blob/main/exercise_5/exercise_5_main_tex.tex

1 Problem [GATE(2003)EC-61]

Let X and Y be two statistically independent random variables uniformly distributed in the ranges $(-1, 1)$ and $(-2, 1)$ respectively. Let $Z = X + Y$. then the probability that $[Z \leq -2]$ is

(a) zero (b)$\frac{1}{6}$ (c)$\frac{1}{3}$ (d)$\frac{1}{12}$

2 Solution

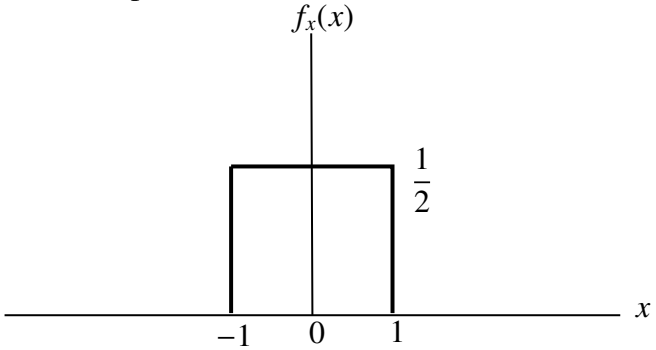
Let be the probability densities of random variables $X, Y$and Z.

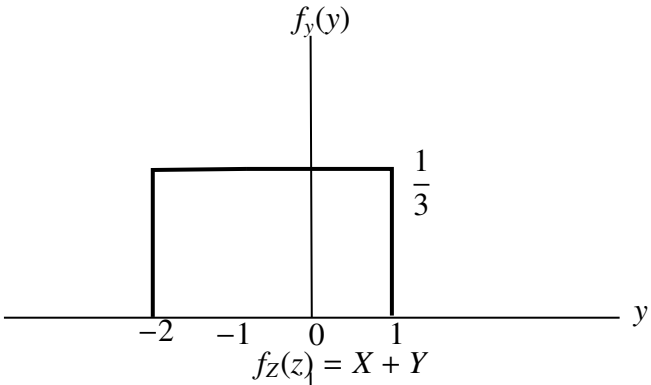
$$p_X(x) = \Pr(X = x) \tag{2.0.1}$$

$$p_Y(y) = \Pr(Y = y) \tag{2.0.2}$$

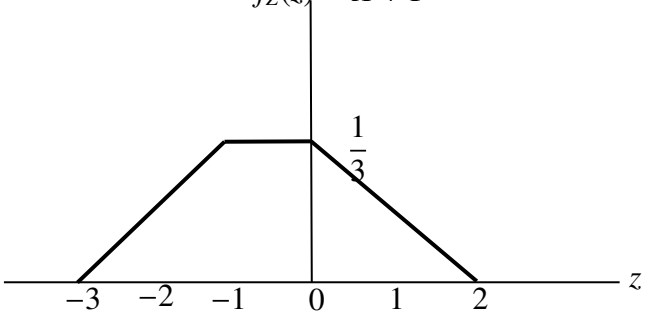
$$p_Z(z) = \Pr(Z = z) \tag{2.0.3}$$

The pdf of $Z(= X + Y)$ will be convolution of pdf of X and pdf of Y as shown below.





$f_Z(z) = X + Y$



Now

$$p[Z \leq z] = \int_{-\infty}^{z} f_Z(z)dz$$

$$p[Z \leq -2] = \int_{-\infty}^{-2} f_Z(z)dz$$

$$= \text{Area } [z \leq -2]$$

$$= \frac{1}{2} \times \frac{1}{6} \times 1 = \frac{1}{12}$$

Hence (D) is correct option.