

# Assignment 3

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Download all python codes from

[https://github.com/sujal100/  
Probability\\_and\\_Random\\_variable/tree/main/  
exercise\\_3/codes](https://github.com/sujal100/Probability_and_Random_variable/tree/main/exercise_3/codes)

and latex codes from

[https://github.com/sujal100/  
Probability\\_and\\_Random\\_variable/blob/main/  
/exercise\\_3/exercise\\_3\\_main\\_tex.tex](https://github.com/sujal100/Probability_and_Random_variable/blob/main/exercise_3/exercise_3_main_tex.tex)

## 1 PROBLEM [GATE(2015)MA-11]

In an experiment, a fair die is rolled until two sixes are obtained in succession. The probability that the experiment will end in the fifth trial is equal to

(A)  $\frac{125}{6^5}$  (B)  $\frac{150}{6^5}$  (C)  $\frac{175}{6^5}$  (D)  $\frac{200}{6^5}$

## 2 SOLUTION

Let Consider, Bernoulli random variables say  $X$ . Here,  $Pr(X = n)$  refer to the probability that experiment ends in exactly  $n^{th}$  rolls. Thus, problem is asking for  $Pr(X = 5)$ . We note that

$$Pr(X = 1) = 0, Pr(X = 2) = \frac{1}{6^2} \quad (2.0.1)$$

For  $n > 2$ , we remark that the first roll is either a 6 or it isn't. If it is, then the second roll can't be a 6. That leads to the recursion

$$Pr(X = n) = \frac{1}{6} \times \frac{5}{6} \times Pr(X = n - 2) + \frac{5}{6} \times Pr(X = n - 1) \quad (2.0.2)$$

So,

$$Pr(X = 3) = \frac{5}{6^2} Pr(X = 1) + \frac{5}{6} Pr(X = 2) = \frac{5}{6^3} \quad (2.0.3)$$

$$Pr(X = 4) = \frac{5}{6^2} Pr(X = 2) + \frac{5}{6} Pr(X = 3) = \frac{30}{6^4} = \frac{5}{6^3} \quad (2.0.4)$$

$$Pr(X = 5) = \frac{5}{6^2} Pr(X = 3) + \frac{5}{6} Pr(X = 4) = \frac{25 + 150}{6^5} = \frac{175}{6^5} \quad (2.0.5)$$

Hence (C) is correct option.