Problem 2.7.7

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September 16, 2025

Question

Question:
Find the area of the quadrilateral ABCD whose vertices are **A**(-4,-3), **B**$(3, -1)$,**C**(0,5), and **D**(-4,2).

Solution

Solution:
as given in the question :

$$\mathbf{A} = \begin{pmatrix} -4 \\ -3 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ 5 \\ 0 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -4 \\ -2 \\ 0 \end{pmatrix}$$

the position vector joining **B** and **D** = **B** - **D** = $\begin{pmatrix} 7 \\ 1 \\ 0 \end{pmatrix}$

the position vector joining **C** and **A** = **C** - **A** = $\begin{pmatrix} 4 \\ 8 \\ 0 \end{pmatrix}$

the area of quadrilateral ABCD is the vector product of $\frac{1}{2}$(**B** - **D**)X(**C** - **A**)

Solution

VECTOR PRODUCT

let N be a vector :

$$\mathbf{N} = \begin{pmatrix} n_1 \\ n_2 \\ 0 \end{pmatrix} \tag{2.1}$$

$$\tag{2.2}$$

let M be a vector :

$$\mathbf{M} = \begin{pmatrix} m_1 \\ m_2 \\ 0 \end{pmatrix} \tag{2.3}$$

the vector product of two vectors **N** and **M** is

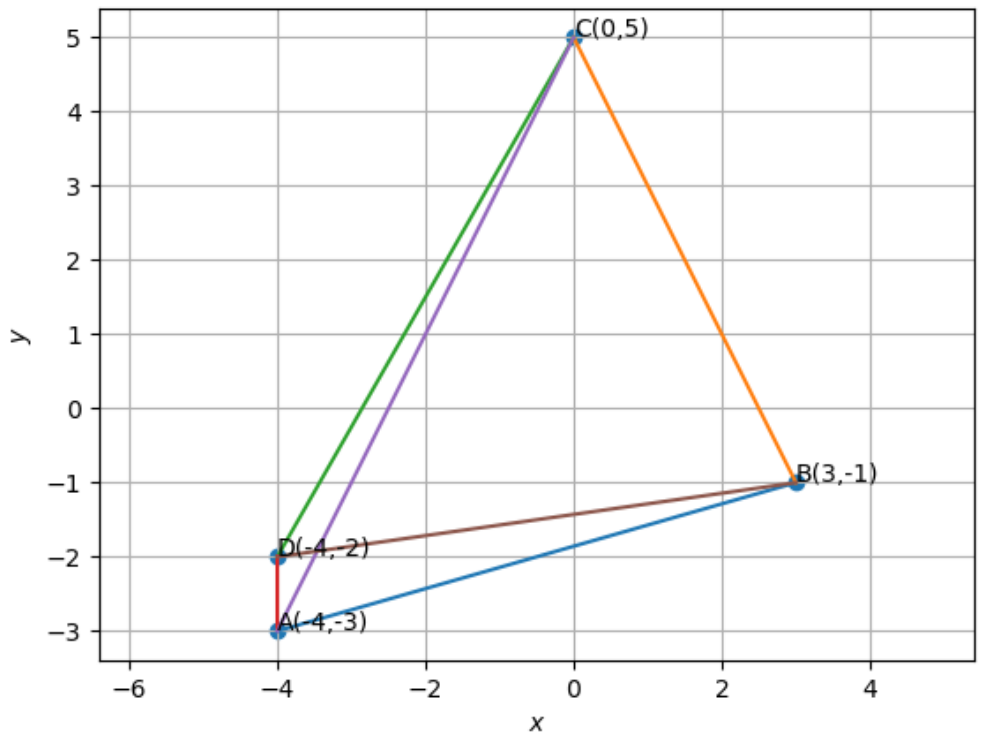
$$\text{NXM} = \begin{pmatrix} n_{23} & m_{23} \\ n_{31} & m_{31} \\ n_{12} & m_{12} \end{pmatrix} = \begin{pmatrix} n_2 m_3 - n_3 m_2 \\ n_3 m_1 - n_1 m_3 \\ n_1 m_2 - n_2 m_1 \end{pmatrix} = \begin{pmatrix} 1 \times 0 - 0 \times 8 \\ 0 \times 7 - 7 \times 0 \\ 7 \times 8 - 1 \times 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 52 \end{pmatrix}$$

Solution

area of ABCD is :

$$\frac{1}{2}||(\mathbf{B}-\mathbf{A})X(\mathbf{C}-\mathbf{A})|| = \frac{1}{2}|| \begin{pmatrix} 7 \\ 1 \\ 0 \end{pmatrix} X \begin{pmatrix} 4 \\ 8 \\ 0 \end{pmatrix} || = \frac{1}{2}\sqrt{\begin{pmatrix} 0 \\ 0 \\ 52 \end{pmatrix}^T \begin{pmatrix} 0 \\ 0 \\ 52 \end{pmatrix}} = 26$$

n1=7,n2=1,m1=4,m2=8


C(0,5)
B(3,-1)
D(-4,-2)
A(-4,-3)
y
x
5
4
3
2
1
0
−1
−2
−3
−6
−4
−2
0
2
4

C Code

```c
#include <stdio.h>
#include <stdlib.h>

int main() {
    // Vertices
    int Ax = -4, Ay = -3;
    int Bx = 3, By = -1;
    int Cx = 0, Cy = 5;
    int Dx = -4, Dy = 2;

    // Diagonals as vectors: AC and BD
    int ACx = Cx - Ax; // 0 - (-4) = 4
    int ACy = Cy - Ay; // 5 - (-3) = 8
    int BDx = Dx - Bx; // -4 - 3 = -7
    int BDy = Dy - By; // 2 - (-1) = 3
```

C Code

```c
// Cross product of AC and BD (scalar value)
    int cross = ACx * BDy - ACy * BDx;

    // Area is half the magnitude of the cross product
    double area = 0.5 * abs(cross);

    printf("Area of the quadrilateral by scalar product = %.2lf
        square units\n", area);

    return 0;
}
```

Python Code for Plotting

```
import math
import sys

import numpy as np
import numpy.linalg as LA
import matplotlib.pyplot as plt
import matplotlib.image as mpimg
from line.funcs import *
#from triangle.funcs import *
#from conics.funcs import circ_gen
#if using termux
import subprocess
import shlex
#end if

A = np.array([-4,-3]).reshape(-1,1)
B = np.array([3,-1]).reshape(-1,1)
```

Python Code for Plotting

```
C = np.array([0,5]).reshape(-1,1)
D = np.array([-4,-2]).reshape(-1,1)
coords = np.block([[A,B,C,D]])


AB = line_gen(A,B)
BC = line_gen(B,C)
CD = line_gen(C,D)
DA = line_gen(D,A)
AC = line_gen(A,C)
BD = line_gen(B,D)

plt.plot(AB[0,:],AB[1,:])
plt.plot(BC[0,:],BC[1,:])
plt.plot(CD[0,:],CD[1,:])
```

Python Code for Plotting

```
plt.plot(DA[0,:],DA[1,:])
plt.plot(AC[0,:],AC[1,:])
plt.plot(BD[0,:],BD[1,:])

plt.scatter(coords[0,:],coords[1,:])



plt.text(A[0],A[1],"A(-4,-3)")
plt.text(B[0],B[1],"B(3,-1)")
plt.text(C[0],C[1],"C(0,5)")
plt.text(D[0],D[1],"D(-4,-2)")

plt.xlabel('$x$')
plt.ylabel('$y$')
plt.legend(loc='best')
plt.grid() # minor
plt.axis('equal')

plt.savefig('../figs/img.png')
```

Python Code for Plotting

```
plt.legend(loc='best')
plt.grid() # minor
plt.axis('equal')

plt.savefig('../figs/img.png')
```