5.13.104

AI25BTECH11035-SUJAL RAJANI

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QUESTION

Let $\mathbf{R}^2$ denote the Euclidean space . Let
S=(a, b, c):$a, b, c \epsilon \mathbf{R}$,$ax^2 + 2bxy + cy^2 > 0 \forall (x, y) \epsilon \mathbf{R}^2 - (0, 0)$.Then which of the following statement is (are) TRUE ?

(a) $(2, \frac{7}{2}, 6) \epsilon S$

(b) IF $(3, b, \frac{1}{12}) \epsilon S$,then $|2b| < 1$.

(c)For any given $(a, b, c) \epsilon S$,the system of linear equations

$$ax + by = 1$$
$$bx + cy = -1$$

has a unique solution .

(d) For any given $(a, b, c) \epsilon S$, the system of linear equations

$$(a + 1)x + by = 0$$
$$bx + (c + 1)y = 0$$

has a unique solution .

Theoretical Solution

solution

as mentioned in the question :

$$ax^2 + 2bxy + cy^2 > 0 \forall (x, y) \epsilon \mathbf{R}^2 - (0, 0)$$

let Q be a matric :

$$\mathbf{Q} = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$$

then:

$$ax^2 + 2bxy + cy^2 = \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} a & b \\ b & c \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

for this to happen :

$$a > 0, c > 0, ||\mathbf{Q}|| = ac - b^2 > 0$$

So,S:((a,b,c):$a > 0, c > 0$,$ac - b^2 > 0$)

Theoretical solution

option (a):

$a = 2 > 0$,c=6,b=$\frac{7}{2}$

$$ac - b^2 = 12 - (\frac{7}{2})^2 = -\frac{1}{4} < 0$$

option a is not correct .

Theoretical Solution

option b is correct :

$$a = 3 > 0, ac - b^2 = \frac{1}{4} - b^2 > 0, |2b| < 1$$

option b is correct .

SOLUTION

option c :

$$ax + by = 1, bx + cy = -1.$$
$$\begin{pmatrix} a & b \\ b & c \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

for the solution to be unique:

$$||\mathbf{Q}|| = ac - b^2 \neq 0.$$

from the definition of S,$ac - b^2 > 0$.
option c is correct .

Theoretical Solution

option d is not correct

$$(a+1)x + by = 0, bx + (c+1)y = 0.$$

this type of homogeneous equation either have one solution or infinite solution .
in case of one solution the solution is (0,0) which we do not get for any

$$(a, b, c) \epsilon S$$

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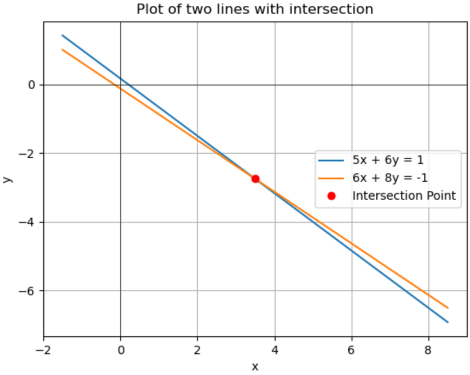
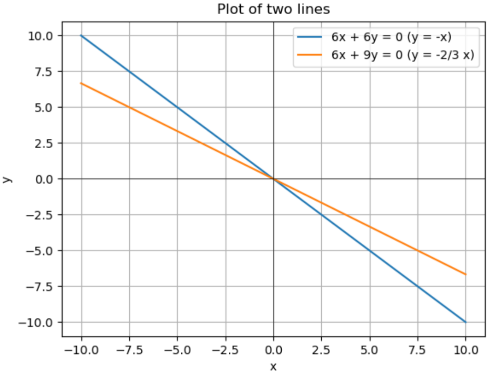
Plot



Plot of two lines with intersection
5x + 6y = 1
6x + 8y = -1
Intersection Point
x
y
0
−2
−4
−6
−2
0
2
4
6
8

Plot



Plot of two lines
6x + 6y = 0 (y = -x)
6x + 9y = 0 (y = -2/3 x)
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
x
y