Problem 2.10.24

Sujal Rajani

September 29, 2025

Question

QUESTION

Find the equation of the line joining **A**(1,3) and **B** (0,0).Also,find k if **D** (k,0) is a point such that the area of $\triangle ABD$ is 3 square units.

Solution

SOLUTION as mentioned in the problem the position vector of the points is :

$$\mathbf{A} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} k \\ 0 \end{pmatrix}$$

the equation of the line passing through **A** and **B** is of the form

$$\mathbf{n}^\top x = 0.$$

because it is passing through origin .

$$\mathbf{n}^\top \mathbf{A} = 0.$$
$$\mathbf{n}^\top \begin{pmatrix} 1 \\ 3 \end{pmatrix} = 0.$$
$$\mathbf{n} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}.$$

so the equation of the line is :

Solution

so the equation of the line is :

$$\begin{pmatrix} -3 & 1 \end{pmatrix} \mathbf{x} = 0.$$

area of $\triangle ABD$:

$$\frac{1}{2}||(\mathbf{A} - \mathbf{B})X(\mathbf{D} - \mathbf{B})|| = 3$$

VECTOR PRODUCT

VECTOR PRODUCT

let N be a vector :

$$\mathbf{N} = \begin{pmatrix} n_1 \\ n_2 \\ 0 \end{pmatrix} \tag{1.1}$$

$$\tag{1.2}$$

let M be a vector :

$$\mathbf{M} = \begin{pmatrix} m_1 \\ m_2 \\ 0 \end{pmatrix} \tag{1.3}$$

the vector product of two vectors **N** and **M** is

$$\mathrm{NXM} = \begin{pmatrix} n_{23} & m_{23} \\ n_{31} & m_{31} \\ n_{12} & m_{12} \end{pmatrix} = \begin{pmatrix} n_2 m_3 - n_3 m_2 \\ n_3 m_1 - n_1 m_3 \\ n_1 m_2 - n_2 m_1 \end{pmatrix} = \begin{pmatrix} 3 \times 0 - 0 \times 0 \\ 0 \times k - 1 \times 0 \\ 1 \times 0 - 3 \times k \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 3k \end{pmatrix}$$

solution

area of $\triangle ABD$ is :

$$\frac{1}{2}||(\mathbf{A}-\mathbf{B})X(\mathbf{D}-\mathbf{B})|| = \frac{1}{2}||\begin{pmatrix}1\\3\\0\end{pmatrix} X \begin{pmatrix}K\\0\\0\end{pmatrix}|| = \frac{1}{2}\sqrt{\begin{pmatrix}0\\0\\3K\end{pmatrix}^\top \begin{pmatrix}0\\0\\3K\end{pmatrix}} = 3$$

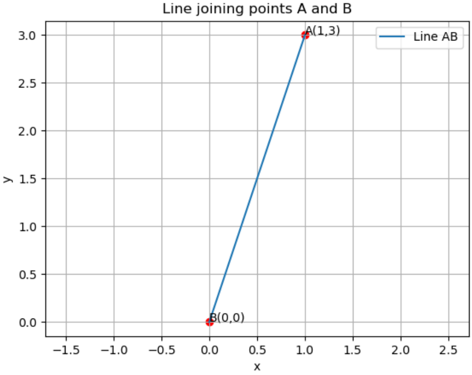
$$k = +2, -2$$

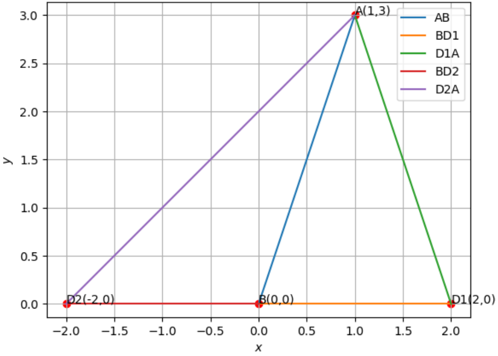
n1=1,n2=3,m1=k,m2=0

for the conveniences we are taking $\mathbf{D_1}$and $\mathbf{D_2}$

so the position vector of $\mathbf{D}$:

$$\mathbf{D_1} = \begin{pmatrix}2\\0\end{pmatrix}, \mathbf{D_2} = \begin{pmatrix}-2\\0\end{pmatrix}$$


Line joining points A and B
A(1,3)
B(0,0)
Line AB
3.0
2.5
2.0
1.5
1.0
0.5
0.0
y
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
2.5
x


A(1,3)
B(0,0)
D1(2,0)
D2(-2,0)
AB
BD1
D1A
BD2
D2A
x
y
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0

C Code

```c
#include <stdio.h>
#include <stdlib.h>
#include <math.h>

int main() {
    // Coordinates for A, B
    int x1 = 1, y1 = 3;
    int x2 = 0, y2 = 0;

    // Calculate slope and intercept
    float m = (float)(y2 - y1) / (x2 - x1); // Slope
    float c = y1 - m * x1; // Intercept
    printf("Equation of line AB: y = %.2fx + %.2f\n", m, c);
```

C Code

```c
        // Area of triangle formula in coordinates:
    // Area = (1/2) * |x1(y2-y3) + x2(y3-y1) + x3(y1-y2)|
    // Let D(k,0) = (x3, y3)
    int y3 = 0;
    float area_target = 3.0;
    // Substitute:
    // area = 0.5 * |1*(0-0) + 0*(0-3) + k*(3-0)|
    // = 0.5 * |3*k|
    // Set 0.5 * |3*k| = 3 => |k| = 2
    float k1 = 2.0, k2 = -2.0;

    printf("Possible values of k for D(k,0): %.2f and %.2f\n", k1
        , k2);

    return 0;
}
```

Python Code for Plotting

```python
import numpy as np
import matplotlib.pyplot as plt
from line.funcs import *
# from triangle.funcs import *
# from conics.funcs import circ_gen
# if using termux
import subprocess
import shlex
# end if
```

Python Code for Plotting

```
# Triangle vertices
A = np.array([1,3]).reshape(-1,1)
B = np.array([0,0]).reshape(-1,1)
D = np.array([2,0]).reshape(-1,1)
D' = np.array([-2,0]).reshape(-1,1
coords = np.block([[A,B,D,D']])
# Generate triangle sides
AB = line_gen(A,B)
BD = line_gen(B,D)
DA = line_gen(D,A)
BD' = line_gen(B,D')
D'A = line_gen(D',A)
```

Python Code for Plotting

```


# Plot sides
plt.plot(AB[0,:],AB[1,:], label='AB')
plt.plot(BD[0,:],BD[1,:], label='BD')
plt.plot(DA[0,:],DA[1,:], label='DA')
plt.plot(BD'[0,:],BD'[1,:], label='BD'')
plt.plot(D'A[0,:],D'A[1,:], label='D'A)
```

Python Code for Plotting

```
# Scatter vertices
plt.scatter(coords[0,:],coords[1,:])
plt.text(A[0],A[1],"A(1,3)")
plt.text(B[0],B[1],"B(0,0)")
plt.text(D[0],D[1],"D(2,0)")
plt.text(D'[0],D'[1],"D'(-2,0)")
# Styling
plt.xlabel('$x$')
plt.ylabel('$y$')
plt.legend(loc='best')
plt.grid(True)
plt.axis('equal')

plt.savefig('../figs/triangle.png')
plt.show()
```