Problem 4.8.31

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Question

QUESTION

Given $\mathbf{a}=2\hat{i}-\hat{j}+\hat{k}$, $\mathbf{b}=3\hat{i}-\hat{k}$ and $\mathbf{c}=2\hat{i}+\hat{j}-2\hat{k}$ Find a vector $\mathbf{d}$ which is perpendicular to both $\mathbf{a}$ and $\mathbf{b}$and $\mathbf{c}.\mathbf{d}=3$.

Solution

SOLUTION as mention in the question :

$$\mathbf{a} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$

$$\mathbf{a}^\top \mathbf{d} = 0, \mathbf{b}^\top \mathbf{d} = 0, \mathbf{c}^\top \mathbf{d} = 3. \tag{1.1}$$

$$\begin{pmatrix} \mathbf{a} & \mathbf{b} \end{pmatrix}^\top \mathbf{d} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -1 & 1 \\ 3 & 0 & -1 \end{pmatrix} \mathbf{d} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

$$\mathbf{d} = k \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}$$

Solution

k is a constant
by equation (1.1):

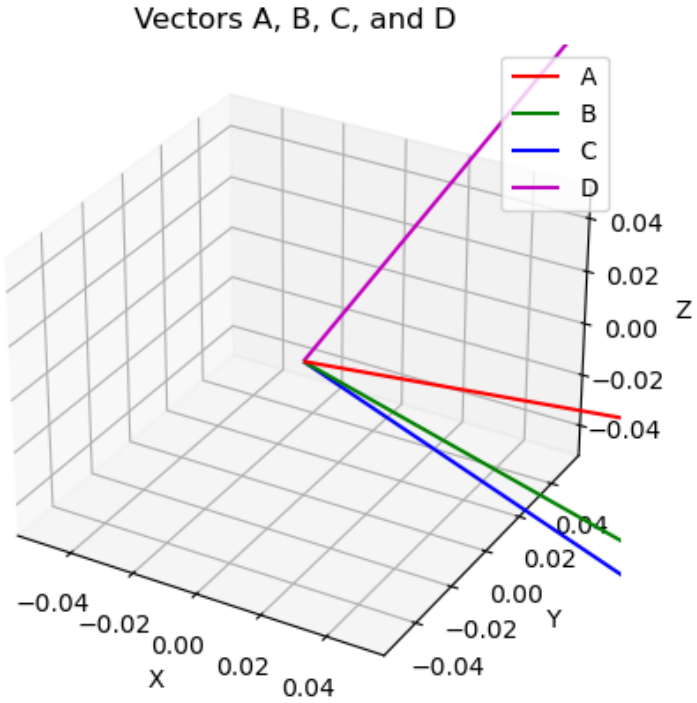
$$\mathbf{c}^\top \mathbf{d} = 3$$

$$\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}^\top \mathbf{d} = 3$$

$$k = \frac{3}{1}.$$

the position vector $\mathbf{d}$:

$$\mathbf{d} = \begin{pmatrix} 3 \\ 15 \\ 9 \end{pmatrix}$$


Vectors A, B, C, and D
A
B
C
D
0.04
0.02
0.00
−0.02
−0.04
Z
−0.04
−0.02
0.00
0.02
0.04
X
0.04
0.02
0.00
−0.02
−0.04
Y

C Code

```c
#include <stdio.h>

int main() {
    // Vectors
    int a[3] = {2, -1, 1};
    int b[3] = {3, 0, -1};
    int c[3] = {2, 1, -2};
    int cross[3];
    int d[3];
    int dot = 0;
    int lambda;
```

C Code

```
    // Cross product a x b
    cross[0] = a[1]*b[2] - a[2]*b[1];
    cross[1] = a[2]*b[0] - a[0]*b[2];
    cross[2] = a[0]*b[1] - a[1]*b[0];

    // Dot product c  (lambda * cross) = 3
    dot = c[0]*cross[0] + c[1]*cross[1] + c[2]*cross[2];

    lambda = 3 / dot; // Since dot = lambda * (c  cross)

    // Compute d = lambda * cross
    d[0] = lambda * cross[0];
    d[1] = lambda * cross[1];
    d[2] = lambda * cross[2];

    printf("Vector d = (%d, %d, %d)\n", d[0], d[1], d[2]);

    return 0;
}
```

Python Code for Plotting

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Define vectors
a = np.array([2, -1, 1])
b = np.array([3, 0, -1])
c = np.array([2, 1, -2])
d = np.array([1, 5, 3])
```

Python Code for Plotting

```
vectors = [a, b, c, d]
labels = ['a', 'b', 'c', 'd']

# Create 3D plot
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')

# Plot origin
origin = np.array([0, 0, 0])

# Plot each vector
for vec, label in zip(vectors, labels):
    ax.quiver(0, 0, 0, vec[0], vec[1], vec[2], arrow_length_ratio
        =0.1)
    ax.text(vec[0], vec[1], vec[2], label, fontsize=12)
```

Python Code for Plotting

```
# Set labels
ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')

ax.set_title("3D Plot of Vectors a, b, c, d")

# Set equal aspect ratio
max_range = np.array([a, b, c, d]).max()
min_range = np.array([a, b, c, d]).min()
ax.set_xlim([min_range-1, max_range+1])
ax.set_ylim([min_range-1, max_range+1])
ax.set_zlim([min_range-1, max_range+1])

plt.show()
```