2.10.24

AI25BTECH11035 - SUJAL RAJANI

Question:
Let **x**,**y** and **z** be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$. If **a** is a non-zero vector perpendicular to **x** and **y** x **z** and **b** is a non-zero vector perpendicular to **y** and **z** x **x**, then

1) $\mathbf{b} = (\mathbf{b} \cdot \mathbf{z})(\mathbf{z} - \mathbf{x})$
2) $\mathbf{a} = (\mathbf{a}.\mathbf{y})(\mathbf{y} - \mathbf{z})$
3) $\mathbf{a}.\mathbf{b} =$ -$(\mathbf{a}.\mathbf{y})(\mathbf{b}.\mathbf{z})$
4) $\mathbf{a} =$ -$(\mathbf{a}.\mathbf{y})(\mathbf{z} - \mathbf{y})$

solution :
defining : triple vector product

$$(\mathbf{x}X(\mathbf{y}X\mathbf{z})) = (\mathbf{x}^\top\mathbf{z})\mathbf{y} - (\mathbf{x}^\top\mathbf{y})\mathbf{z}$$

as mentioned in the question :

$$\|\mathbf{x}\| = \sqrt{2}, \|\mathbf{y}\| = \sqrt{2}, \|\mathbf{z}\| = \sqrt{2}$$
$$\mathbf{x}^\top\mathbf{y} = 1, \mathbf{x}^\top\mathbf{z} = 1, \mathbf{y}^\top\mathbf{z} = 1$$

If **a** is a non-zero vector perpendicular to **x** and **y** x **z** this implies :

$$\mathbf{a}^\top\mathbf{x} = 0, \mathbf{a}^\top(\mathbf{y}X\mathbf{z}) = 0$$

this implies that **x** and $\mathbf{y}X\mathbf{z}$ lie in the same plane and their vector product is parallel to **a**
so the vector product of

$$\mathbf{a}X(\mathbf{x}X(\mathbf{y}X\mathbf{z})) = 0$$

$$\mathbf{x}^\top\mathbf{z} = 1, \mathbf{x}^\top\mathbf{y} = 1$$
$$\mathbf{a}X(\mathbf{x}X(\mathbf{y}X\mathbf{z})) = (\mathbf{x}^\top\mathbf{z})(\mathbf{y}X\mathbf{a}) - (\mathbf{x}^\top\mathbf{y})(\mathbf{z}X\mathbf{a}) = \mathbf{a}X(\mathbf{y} - \mathbf{z})$$

as **a** is parallel to $\mathbf{y} - \mathbf{z}$ so we can say that :

$$\mathbf{a} = k(\mathbf{y} - \mathbf{z})$$

k is a constant
If **b** is a non-zero vector perpendicular to **y** and **z** x **x** this implies :

$$\mathbf{b}^\top\mathbf{y} = 0, \mathbf{b}^\top(\mathbf{z}X\mathbf{x}) = 0$$

this implies that **x** and $\mathbf{y}X\mathbf{z}$ lie in the same plane and their vector product is parallel to **a** so the vector product of

$$\mathbf{b}X(\mathbf{y}X(\mathbf{z}X\mathbf{x})) = 0$$

$$\mathbf{x}^\top\mathbf{y} = 1, \mathbf{z}^\top\mathbf{y} = 1$$
$$\mathbf{bX}(\mathbf{y}X(\mathbf{z}X\mathbf{x})) = (\mathbf{y}^\top\mathbf{x})(\mathbf{z}X\mathbf{b}) - (\mathbf{y}^\top\mathbf{z})(\mathbf{x}X\mathbf{b}) = \mathbf{b}X(\mathbf{z} - \mathbf{x}) = 0$$

as $\mathbf{b}$ is parallel to $\mathbf{z} - \mathbf{x}$ so we can say that :

$$\mathbf{b} = m(\mathbf{z} - \mathbf{x})$$

m is a constant
now we check options one by one :

option (a)
$\mathbf{b}$ = $(\mathbf{b}.\mathbf{z})(\mathbf{z} - \mathbf{x})$
we are putting b on both sides=

$$m(\mathbf{z} - \mathbf{x}) = (m(\mathbf{z} - \mathbf{x})^\top\mathbf{z})(\mathbf{z} - \mathbf{x})$$
$$m(\mathbf{z} - \mathbf{x}) = m(\mathbf{z}.\mathbf{z} - \mathbf{x}^\top\mathbf{z})(\mathbf{z} - \mathbf{x})$$

$$m(\mathbf{z} - \mathbf{x}) = m(\mathbf{z} - \mathbf{x})$$

option A is correct .
option (b)
$\mathbf{a}$ = $(\mathbf{a}^\top\mathbf{y})(\mathbf{y} - \mathbf{z})$
we are putting a on both sides=

$$k(\mathbf{y} - \mathbf{z}) = (k(\mathbf{y} - \mathbf{z})^\top\mathbf{y})(\mathbf{y} - \mathbf{z})$$
$$k(\mathbf{y} - \mathbf{z}) = (k(\mathbf{y}^\top\mathbf{y} - \mathbf{y}^\top\mathbf{z})(\mathbf{y} - \mathbf{z})$$

$$k(\mathbf{y} - \mathbf{z}) = k(\mathbf{y} - \mathbf{z})$$

option B is correct .
option (c)
$\mathbf{a}^\top\mathbf{b}$ = -$(\mathbf{a}^\top\mathbf{y})(\mathbf{b}^\top\mathbf{z})$
we are putting a and b on both sides=

$$k(\mathbf{y} - \mathbf{z})m(\mathbf{z} - \mathbf{x}) = -(m(\mathbf{z} - \mathbf{x}).\mathbf{z})(k(\mathbf{y} - \mathbf{z})^\top\mathbf{y})$$
$$-km = -km$$

option C is correct .
option D is correct because option B is correct.

for plotting we are assuming the position vector :

$$\mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \mathbf{y} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \mathbf{z} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \mathbf{a} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$