Problem 2.2.28

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Question

Question: Find the angle between the two planes 2x+y-2z=5 and 3x-6y-2z=7 using vector method.

Solution

Solution:

the normal vector of plane 2 x+y-2z=5 is : $\mathbf{n_1}$

$$\mathbf{n_1} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$$

the normal vector of plane 3 x-6y-2z=7 is : $\mathbf{n_2}$

$$\mathbf{n_2} = \begin{pmatrix} 3 \\ -6 \\ -2 \end{pmatrix}$$

The value of $||\mathbf{n_1}||$:

$$(\mathbf{n_1})^T(\mathbf{n_1}) = ||\mathbf{n_1}||^2 = 9$$

SOLUTION

The value of $||\mathbf{n_2}||$:

$$(\mathbf{n_2})^T(\mathbf{n_2}) = ||\mathbf{n_2}||^2 = 49$$

The angle between two plane is same as the angle between their normal vectors , which is θ .

the angle between $\mathbf{n_1}$ and $\mathbf{n_2}$ is :

$$\cos\theta = \frac{(\mathbf{n_1})^T(\mathbf{n_2})}{||\mathbf{n_1}|| \quad ||\mathbf{n_2}||} = \frac{4}{21}$$
$$\theta = cos^{-1}\frac{4}{21}$$

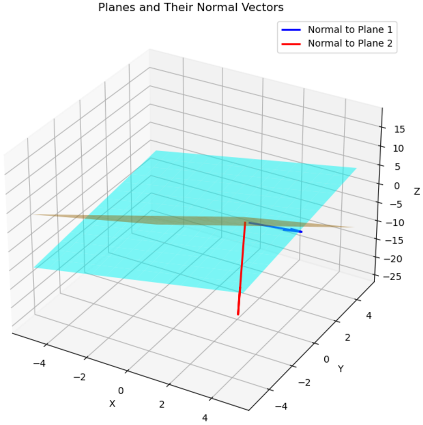
Planes and Their Normal Vectors
Normal to Plane 1
Normal to Plane 2
15
10
5
0
−5
−10
−15
−20
−25
Z
−4
−2
0
2
4
X
4
2
0
−2
−4
Y

Figure: *

C Code

```c
#include <stdio.h>
#include <math.h>

// Function to compute dot product
double dot(double a[3], double b[3]) {
    return a[0]*b[0] + a[1]*b[1] + a[2]*b[2];
}

// Function to compute magnitude of a vector
double magnitude(double v[3]) {
    return sqrt(v[0]*v[0] + v[1]*v[1] + v[2]*v[2]);
}

int main() {
    // Normal vectors of the two planes
    double n1[3] = {2, 1, -2}; // for plane 2x + y - 2z = 5
    double n2[3] = {3, -6, -2}; // for plane 3x - 6y - 2z = 7
```

C Code

```c
// Dot product
   double dot_val = dot(n1, n2);
   // Magnitudes
   double mag1 = magnitude(n1);
   double mag2 = magnitude(n2);

   // Cos(theta)
   double cos_theta = dot_val / (mag1 * mag2);

   // Angle in radians and degrees
   double theta_rad = acos(cos_theta);
   double theta_deg = theta_rad * 180.0 / M_PI;
```

C Code

```c
 // Output
    printf("Dot product = %.2f\n", dot_val);
    printf("|n1| = %.2f, |n2| = %.2f\n", mag1, mag2);
    printf("cos(theta) = %.4f\n", cos_theta);
    printf("Angle between planes = %.4f radians = %.2f degrees\n"
        , theta_rad, theta_deg);

    return 0;
}
```

Python Code for Plotting

```
import sympy as sp
import numpy as np
import matplotlib.pyplot as plt
import matplotlib as mp
mp.use("TkAgg")

# -------------------------
# Step 1: Define planes
# -------------------------
n1, d1 = sp.Matrix([2, 1, -2]), 5 # 2x + y - 2z = 5
n2, d2 = sp.Matrix([3, -6, -2]), 7 # 3x - 6y - 2z = 7

# -------------------------
# Step 2: Function to plot a plane
# -------------------------
def plot_plane(ax, n, d, color, alpha=0.4):
    xx, yy = np.meshgrid(np.linspace(-5,5,15), np.linspace
        (-5,5,15))
    zz = (d - n[0]*xx - n[1]*yy) / n[2]
```

Python Code for Plotting

```
# ------------------------
# Step 3: Get points on planes for plotting normals
# ------------------------
x, y, z = sp.symbols('x y z')
plane1_eq = n1[0]*x + n1[1]*y + n1[2]*z - d1
plane2_eq = n2[0]*x + n2[1]*y + n2[2]*z - d2

p1 = sp.solve(plane1_eq.subs({y:0, z:0}), x)
p2 = sp.solve(plane2_eq.subs({y:0, z:0}), x)

a1 = np.array([float(p1[0]), 0, 0]) if p1 else np.zeros(3)
a2 = np.array([float(p2[0]), 0, 0]) if p2 else np.zeros(3)

# ------------------------
# Step 4: Plot planes + normals
# ------------------------
fig = plt.figure(figsize=(8,8))
ax = fig.add_subplot(111, projection='3d')
```

Python Code for Plotting

```python

plot_plane(ax, n1f, d1f, "cyan", 0.5)
plot_plane(ax, n2f, d2f, "orange", 0.5)

# Plot normal vectors
ax.quiver(a1[0], a1[1], a1[2], n1f[0], n1f[1], n1f[2],
         length=3, color="blue", linewidth=2, label="Normal to
             Plane 1")
ax.quiver(a2[0], a2[1], a2[2], n2f[0], n2f[1], n2f[2],
         length=3, color="red", linewidth=2, label="Normal to
             Plane 2")

# Labels
ax.set_xlabel("X")
ax.set_ylabel("Y")
ax.set_zlabel("Z")
ax.set_title("Planes and Their Normal Vectors")
ax.legend()
```