8.3.15

AI25BTECH11035 - SUJAL RAJANI

Question: Find the roots of the following quadratic equation graphically:

$$16x^2 - 8x + 1 = 0$$

Solution:
First, we expand and observe that the equation is already in standard quadratic form:

$$16x^2 - 8x + 1 = 0$$

Input Variables:
The given quadratic can be written in the conic form:

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0$$

where

$$\mathbf{V} = \begin{pmatrix} 16 & 0 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}, \quad f = 1$$

Since the roots correspond to intersections with the x-axis, we represent the line:

$$L : \mathbf{x} = \mathbf{h} + \kappa\mathbf{m}$$

with

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Symbol	Value
$\mathbf{V}$	$\begin{pmatrix} 16 & 0 \\ 0 & 0 \end{pmatrix}$
$\mathbf{u}$	$\begin{pmatrix} -4 \\ 0 \end{pmatrix}$
f	1
$\mathbf{h}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
$\mathbf{m}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

The points of intersection of a line with a conic are given by:

$$\kappa = \frac{1}{\mathbf{m}^T\mathbf{V}\mathbf{m}}\left[-\mathbf{m}^T(\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^T(\mathbf{V}\mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})\mathbf{m}^T\mathbf{V}\mathbf{m}}\right]$$

where

$$g(\mathbf{h}) = \mathbf{h}^T\mathbf{V}\mathbf{h} + 2\mathbf{u}^T\mathbf{h} + f$$

Step 1: Compute $\mathbf{m}^T\mathbf{V}\mathbf{m}$

$$\mathbf{m}^T\mathbf{V}\mathbf{m} = 16$$

Step 2: Compute Vh + u

$$\mathbf{Vh} + \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -4 \\ 0 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$$

Step 3: Compute $\mathbf{m}^T(\mathbf{Vh} + \mathbf{u})$

$$\mathbf{m}^T(\mathbf{Vh} + \mathbf{u}) = (1, 0) \cdot (-4, 0) = -4$$

Step 4: Compute $g(\mathbf{h})$

$$g(\mathbf{h}) = 0 + 0 + 1 = 1$$

Step 5: Substitute into formula for κ

$$\kappa = \frac{-(-4) \pm \sqrt{(-4)^2 - 16 \cdot 1}}{16} = \frac{4 \pm \sqrt{16 - 16}}{16} = \frac{4}{16} = 0.25$$

Step 6: Find intersection point

The intersection is

$$x = h + \kappa m = (0, 0) + (0.25)(1, 0) = (0.25, 0)$$

Thus, the quadratic $16x^2 - 8x + 1 = 0$ intersects the x-axis at

$$\boxed{x = 0.25}$$

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Parabola and Roots of the Quadratic Equation