Problem 12.9

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Question

Question : given the wavelet , a=$\{3,-2\}$ and b=$\{1,-2\}$,the cross-correlation , ϕ_{ab},is given by .

Solution

SOLUTION Given the sequences

$$\mathbf{a} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

The cross-correlation between $a[n]$ and $b[n]$ is defined as

$$\phi_{ab}[k] = \sum_n a[n]\, b[n+k].$$

For sequences of length 2, the cross-correlation length is

$$L = 2 + 2 - 1 = 3, \quad k = -1, 0, 1.$$

Frame Title

The cross-correlation can be represented as a matrix multiplication:

$$\boldsymbol{\phi_{ab}} = \begin{pmatrix} \phi_{ab}[-1] \\ \phi_{ab}[0] \\ \phi_{ab}[1] \end{pmatrix}$$

This matrix corresponds to the shifted overlaps between $a[n]$ and $b[n]$:

First row $\rightarrow$ shift -1 (overlap with $b[1]$)

Second row $\rightarrow$ shift 0 (full overlap)

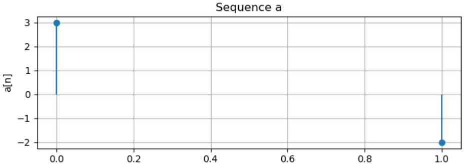
Third row $\rightarrow$ shift $+1$ (overlap with $b[0]$)

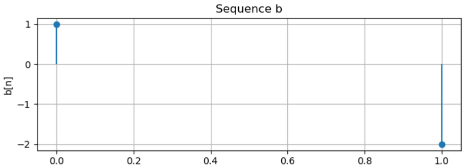
Step-by-Step Calculation

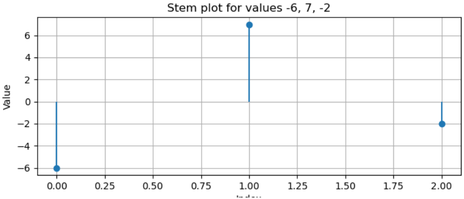
$$\begin{aligned}
\phi_{ab}[-1] &= a[0]\, b[1] = 3 \times (-2) = -6, \\
\phi_{ab}[0] &= a[0]\, b[0] + a[1]\, b[1] = 3 \times 1 + (-2) \times (-2) = 3 + 4 = 7, \\
\phi_{ab}[1] &= a[1]\, b[0] = (-2) \times 1 = -2.
\end{aligned}$$

$$\boldsymbol{\phi_{ab}} = \begin{pmatrix} \phi_{ab}[-1] \\ \phi_{ab}[0] \\ \phi_{ab}[1] \end{pmatrix} = \begin{pmatrix} -6 \\ 7 \\ -2 \end{pmatrix}.$$

$$\boxed{\phi_{ab}[k] = \{-6,\ 7,\ -2\}}$$


Sequence a
a[n]
3
2
1
0
−1
−2
0.0
0.2
0.4
0.6
0.8
1.0


Sequence b
b[n]
1
0
−1
−2
0.0
0.2
0.4
0.6
0.8
1.0


Stem plot for values -6, 7, -2
Value
6
4
2
0
−2
−4
−6
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00