

12.9

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PROBLEM: CROSS-CORRELATION USING MATRIX METHOD

Question : given the wavelet , $a=\{3,-2\}$ and $b=\{1,-2\}$,the cross-correlation , ϕ_{ab} ,is given by .

solution:

Given the sequences

$$\mathbf{a} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}.$$

The cross-correlation between $a[n]$ and $b[n]$ is defined as

$$\phi_{ab}[k] = \sum_n a[n] b[n+k].$$

For sequences of length 2, the cross-correlation length is

$$L = 2 + 2 - 1 = 3, \quad k = -1, 0, 1.$$

Matrix Representation

The cross-correlation can be represented as a matrix multiplication:

$$\boldsymbol{\phi}_{ab} = \begin{pmatrix} \phi_{ab}[-1] \\ \phi_{ab}[0] \\ \phi_{ab}[1] \end{pmatrix}$$

This matrix corresponds to the shifted overlaps between $a[n]$ and $b[n]$:

- First row \rightarrow shift -1 (overlap with $b[1]$)
- Second row \rightarrow shift 0 (full overlap)
- Third row \rightarrow shift $+1$ (overlap with $b[0]$)

Step-by-Step Calculation

$$\phi_{ab}[-1] = a[0] b[1] = 3 \times (-2) = -6,$$

$$\phi_{ab}[0] = a[0] b[0] + a[1] b[1] = 3 \times 1 + (-2) \times (-2) = 3 + 4 = 7,$$

$$\phi_{ab}[1] = a[1] b[0] = (-2) \times 1 = -2.$$

Final Answer

$$\boldsymbol{\phi}_{ab} = \begin{pmatrix} \phi_{ab}[-1] \\ \phi_{ab}[0] \\ \phi_{ab}[1] \end{pmatrix} = \begin{pmatrix} -6 \\ 7 \\ -2 \end{pmatrix}.$$

$$\boxed{\phi_{ab}[k] = \{-6, 7, -2\}}$$