Problem 8.3.15

Sujal Rajani

September 30, 2025

Question

Question: An equilateral triangle is inscribed in the parabola $y^2 = 4ax$, where one vertex is at the vertex of the parabola . Find the length of the side of the triangle .

Solution

SOLUTION let the three position vector of the equilateral triangle be **A**,**B** and **C** .
let

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} x \\ y \end{pmatrix}, \mathbf{C} = \begin{pmatrix} x \\ -y \end{pmatrix}$$

both parabola and equilateral triangle are symmetric about x axis that why the y coordinates of **B**and **C** are of different sign .

$$y^2 = 4ax$$

because the position vector of **B** and **C** lie of the parabola .

$$\begin{aligned} ||\mathbf{B} - \mathbf{A}||^2 &= ||\mathbf{B} - \mathbf{C}||^2 \\ (\mathbf{B} - \mathbf{A})^\top (\mathbf{B} - \mathbf{A}) &= (\mathbf{B} - \mathbf{C})^\top (\mathbf{B} - \mathbf{C}). \\ y^2 + x^2 &= 4y^2 \\ x &= 12a. \end{aligned}$$

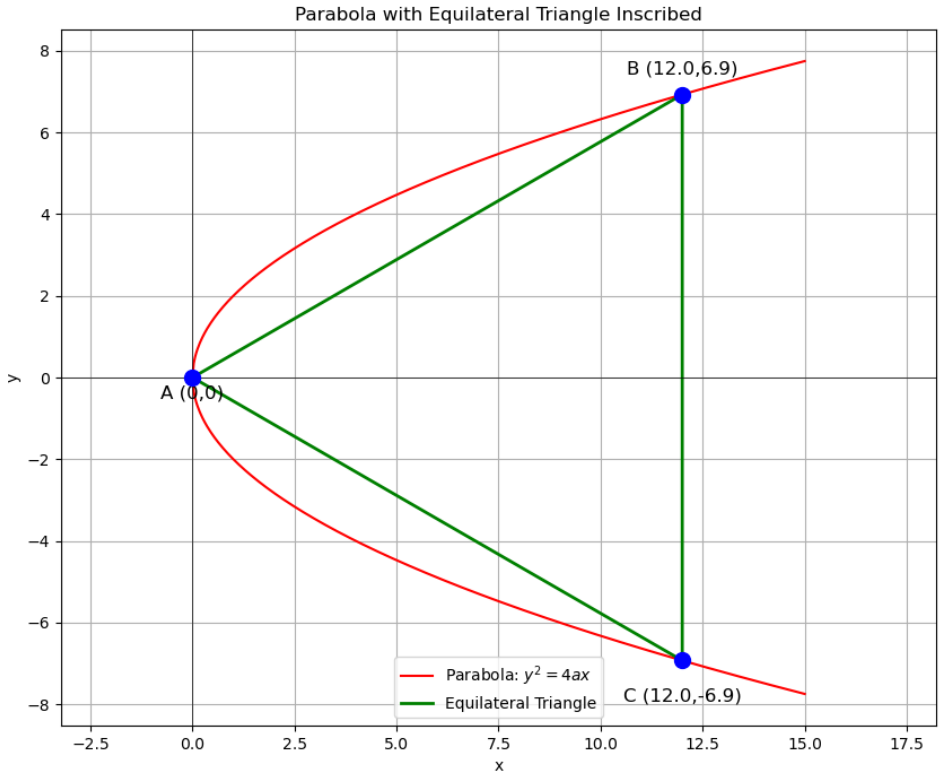
Frame Title

by replacing the value of x in (1) equation :

$$y^2 = 48a^2$$
$$y = \sqrt{48}a$$

the length of side of equilateral triangle is :

$$2y = 2\sqrt{48}a.$$


Parabola with Equilateral Triangle Inscribed
B (12.0,6.9)
A (0,0)
C (12.0,-6.9)
Parabola: $y^2 = 4ax$
Equilateral Triangle
x
y
8
6
4
2
0
−2
−4
−6
−8
−2.5
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5