Problem 9.4.7

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Question

Question: Find the roots of the following quadratic equation graphically:

$$16x^2 - 8x + 1 = 0$$

Solution

First, the equation is already in standard quadratic form:

$$16x^2 - 8x + 1 = 0$$

Input Variables:
The given quadratic can be written in the conic form:

$$\vec{x}^T \vec{V} \vec{x} + 2\vec{u}^T \vec{x} + f = 0$$

where

$$\vec{V} = \begin{pmatrix} 16 & 0 \\ 0 & 0 \end{pmatrix}, \quad \vec{u} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}, \quad f = 1$$

Solution

Since the roots correspond to intersections with the x-axis, we represent the line:

$$L : \vec{x} = \vec{h} + \kappa \vec{m}$$

with

$$\vec{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \vec{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Input Parameters

Symbol	Value
$\vec{V}$	$\begin{pmatrix} 16 & 0 \\ 0 & 0 \end{pmatrix}$
$\vec{u}$	$\begin{pmatrix} -4 \\ 0 \end{pmatrix}$
f	1
$\vec{h}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
$\vec{m}$	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Table: Input Parameters

Solution

The points of intersection of a line with a conic are:

$$\kappa = \frac{1}{\vec{m}^T \vec{V} \vec{m}} \left[-\vec{m}^T (\vec{V}\vec{h} + \vec{u}) \pm \sqrt{\left[\vec{m}^T (\vec{V}\vec{h} + \vec{u}) \right]^2 - g(\vec{h}) \vec{m}^T \vec{V} \vec{m}} \right]$$

where

$$g(\vec{h}) = \vec{h}^T \vec{V} \vec{h} + 2\vec{u}^T \vec{h} + f$$

Step-by-Step Evaluation

Step 1: Compute $\vec{m}^T\vec{V}\vec{m}$

$$\vec{m}^T\vec{V}\vec{m} = 16$$

Step 2: Compute $\vec{V}\vec{h} + \vec{u}$

$$\vec{V}\vec{h} + \vec{u} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$$

Step 3: Compute $\vec{m}^T(\vec{V}\vec{h} + \vec{u})$

$$\vec{m}^T(\vec{V}\vec{h} + \vec{u}) = -4$$

Step 4: Compute $g(\vec{h})$

$$g(\vec{h}) = 1$$

Final Roots

Step 5: Substitute into κ:

$$\kappa = \frac{4 \pm \sqrt{16 - 16}}{16} = \frac{4}{16} = 0.25$$

Step 6: Intersection point:

$$x = h + \kappa m = (0,0) + (0.25)(1,0) = (0.25, 0)$$

Thus, the quadratic $16x^2 - 8x + 1 = 0$ intersects the x-axis at

$$\boxed{x = 0.25}$$

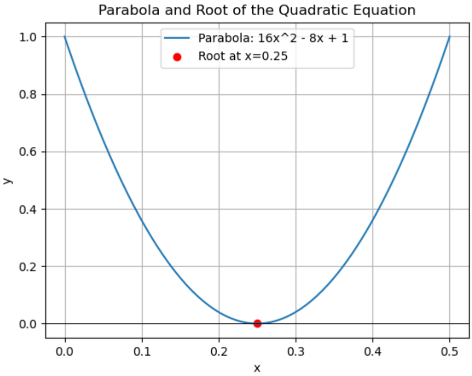
Parabola and Root of the Quadratic Equation
Parabola: 16x^2 - 8x + 1
Root at x=0.25
1.0
0.8
0.6
0.4
0.2
0.0
y
0.0
0.1
0.2
0.3
0.4
0.5
x

Figure: *