

4.4.31

AI25BTECH11035 - SUJAL RAJANI

QUESTION

Find the equation of the line joining **A**(1,3) and **B** (0,0).Also,find k if **D** (k,0) is a point such that the area of $\triangle ABD$ is 3 square units.

solution

as mentioned in the problem the position vector of the points is :

$$\mathbf{A} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} k \\ 0 \end{pmatrix}$$

the general equation of a line passing through two position vector **B** and **A** is :

$$\mathbf{L} = \mathbf{H} + z \begin{pmatrix} 1 \\ m \end{pmatrix}$$

H:is either the position vector of **A** OR **B**
m is the slope of the line .

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\mathbf{A} = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$$

$$m = 3$$

z is a constant .

the equation of line passing through **B** and **A** is :

$$\mathbf{L} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + z \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

area of $\triangle ABD$:

$$\frac{1}{2} \|(\mathbf{A} - \mathbf{B}) \times (\mathbf{D} - \mathbf{B})\| = 3$$

VECTOR PRODUCT

let **N** be a vector :

$$\mathbf{N} = \begin{pmatrix} n_1 \\ n_2 \\ 0 \end{pmatrix} \quad (1)$$

(2)

let **M** be a vector :

$$\mathbf{M} = \begin{pmatrix} m_1 \\ m_2 \\ 0 \end{pmatrix} \quad (3)$$

the vector product of two vectors \mathbf{N} and \mathbf{M} is

$$\mathbf{N} \times \mathbf{M} = \begin{pmatrix} 0 \\ 0 \\ n_1 m_2 - m_1 n_2 \end{pmatrix}$$

area of $\triangle ABD$ is :

$$\frac{1}{2} \|(\mathbf{A} - \mathbf{B}) \times (\mathbf{D} - \mathbf{B})\| = \frac{1}{2} \left\| \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} \times \begin{pmatrix} K \\ 0 \\ 0 \end{pmatrix} \right\| = \frac{1}{2} \sqrt{\begin{pmatrix} 0 \\ 0 \\ 3K \end{pmatrix}^\top \begin{pmatrix} 0 \\ 0 \\ 3K \end{pmatrix}} = 3$$

$$k = +2, -2$$

$$n_1=1, n_2=3, m_1=k, m_2=0$$

so the position vector of \mathbf{D} :

$$\mathbf{D} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$

