4.4.31

AI25BTECH11035 - SUJAL RAJANI

QUESTION
Find the equation of the line joining **A**(1,3) and **B** (0,0).Also,find k if **D** (k,0) is a point such that the area of $\triangle ABD$ is 3 square units.
solution
as mentioned in the problem the position vector of the points is :

$$\mathbf{A} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} k \\ 0 \end{pmatrix}$$

the equation of the line passing through **A** and **B** is of the form

$$\mathbf{n}^\top x = 0.$$

because it is passing through origin .

$$\mathbf{n}^\top \mathbf{A} = 0.$$
$$\mathbf{n}^\top \begin{pmatrix} 1 \\ 3 \end{pmatrix} = 0.$$
$$\mathbf{n} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}.$$

so the equation of the line is :

$$\begin{pmatrix} -3 & 1 \end{pmatrix} \mathbf{x} = 0.$$

area of $\triangle ABD$:

$$\frac{1}{2} \| (\mathbf{A} - \mathbf{B}) X (\mathbf{D} - \mathbf{B}) \| = 3$$

VECTOR PRODUCT
let N be a vector :

$$\mathbf{N} = \begin{pmatrix} n_1 \\ n_2 \\ 0 \end{pmatrix} \tag{1}$$

$$\tag{2}$$

let M be a vector :

$$\mathbf{M} = \begin{pmatrix} m_1 \\ m_2 \\ 0 \end{pmatrix} \tag{3}$$

the vector product of two vectors $\mathbf{N}$ and $\mathbf{M}$ is

$$\mathbf{N}X\mathbf{M} = \begin{pmatrix} 0 \\ 0 \\ n_1m_2 - m_1n_2 \end{pmatrix}$$

area of $\triangle ABD$ is :

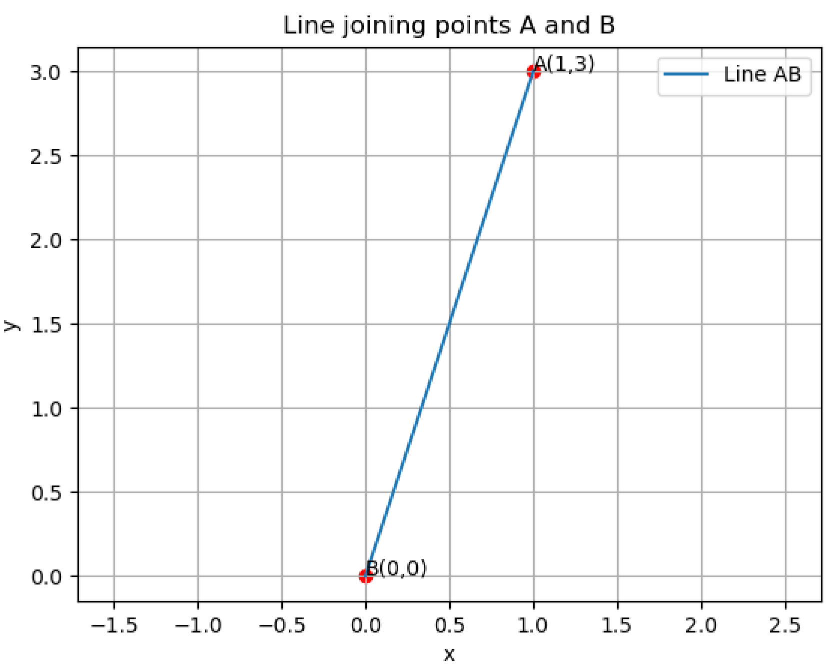
$$\frac{1}{2}\|(\mathbf{A}-\mathbf{B})X(\mathbf{D}-\mathbf{B})\| = \frac{1}{2}\|\begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} X \begin{pmatrix} K \\ 0 \\ 0 \end{pmatrix}\| = \frac{1}{2}\sqrt{\begin{pmatrix} 0 \\ 0 \\ 3K \end{pmatrix}^{\top}\begin{pmatrix} 0 \\ 0 \\ 3K \end{pmatrix}} = 3$$

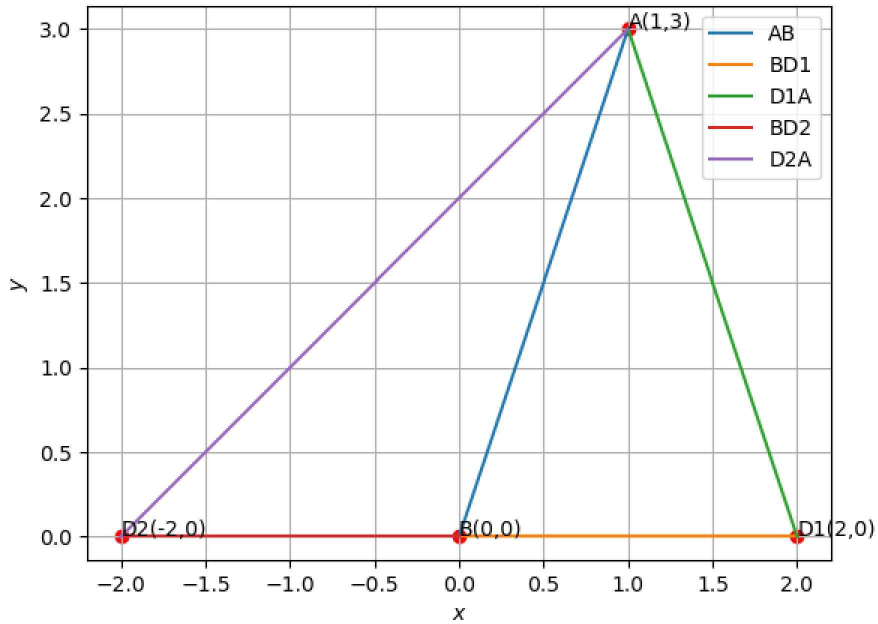
$$k = +2, -2$$

n1=1,n2=3,m1=k,m2=0

so the position vector of $\mathbf{D}$:

$$\mathbf{D} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$$




A(1,3)
B(0,0)
D1(2,0)
D2(-2,0)
AB
BD1
D1A
BD2
D2A
x
y
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0