

2.7.7

AI25BTECH11035 - SUJAL RAJANI

Question:

Find the area of the quadrilateral ABCD whose vertices are A(-4,-3), B(3,-1), C(0,5), and D(-4,2).

Solution

as given in the question :

$$\mathbf{A} = \begin{pmatrix} -4 \\ -3 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ 5 \\ 0 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} -4 \\ 2 \\ 0 \end{pmatrix}$$

the position vector joining \mathbf{B} and $\mathbf{D} = \mathbf{B} - \mathbf{D} = \begin{pmatrix} 7 \\ 1 \\ 0 \end{pmatrix}$

the position vector joining \mathbf{C} and $\mathbf{A} = \mathbf{C} - \mathbf{A} = \begin{pmatrix} 4 \\ 8 \\ 0 \end{pmatrix}$

the area of quadrilateral ABCD is the vector product of $\frac{1}{2} \|(\mathbf{B}-\mathbf{D}) \times (\mathbf{C}-\mathbf{A})\|$

VECTOR PRODUCT

let \mathbf{N} be a vector :

$$\mathbf{N} = \begin{pmatrix} n_1 \\ n_2 \\ 0 \end{pmatrix} \quad (1)$$

(2)

let \mathbf{M} be a vector :

$$\mathbf{M} = \begin{pmatrix} m_1 \\ m_2 \\ 0 \end{pmatrix} \quad (3)$$

the vector product of two vectors \mathbf{N} and \mathbf{M} is

$$\mathbf{N} \times \mathbf{M} = \begin{pmatrix} n_2 m_3 - n_3 m_2 \\ n_3 m_1 - n_1 m_3 \\ n_1 m_2 - n_2 m_1 \end{pmatrix} = \begin{pmatrix} 1 \times 0 - 0 \times 8 \\ 0 \times 7 - 7 \times 0 \\ 7 \times 8 - 1 \times 4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 52 \end{pmatrix}$$

area of ABCD is :

$$\frac{1}{2} \|(\mathbf{B} - \mathbf{A}) \times (\mathbf{C} - \mathbf{A})\| = \frac{1}{2} \left\| \begin{pmatrix} 7 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 4 \\ 8 \\ 0 \end{pmatrix} \right\| = \frac{1}{2} \sqrt{\begin{pmatrix} 0 \\ 0 \\ 52 \end{pmatrix}^T \begin{pmatrix} 0 \\ 0 \\ 52 \end{pmatrix}} = 26$$

$$n_1=7, n_2=1, m_1=4, m_2=8$$

