

## Problem 8.3.15

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## Question

**Question:** An equilateral triangle is inscribed in the parabola  $y^2 = 4ax$  , where one vertex is at the vertex of the parabola . Find the length of the side of the triangle .

## Solution

**SOLUTION** let the three position vector of the equilateral triangle be **A**, **B** and **C** .

let

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} x \\ y \end{pmatrix}, \mathbf{C} = \begin{pmatrix} x \\ -y \end{pmatrix}$$

both parabola and equilateral triangle are symmetric about x axis that why the y coordinates of **B** and **C** are of different sign .

$$y^2 = 4ax$$

because the position vector of **B** and **C** lie of the parabola .

$$\|\mathbf{B} - \mathbf{A}\|^2 = \|\mathbf{B} - \mathbf{C}\|^2$$

$$(\mathbf{B} - \mathbf{A})^\top (\mathbf{B} - \mathbf{A}) = (\mathbf{B} - \mathbf{C})^\top (\mathbf{B} - \mathbf{C}).$$

$$y^2 + x^2 = 4y^2$$

$$x = 12a.$$

## Frame Title

by replacing the value of  $x$  in (1) equation :

$$y^2 = 48a^2$$

$$y = \sqrt{48}a$$

the length of side of equilateral triangle is :

$$2y = 2\sqrt{48}a.$$

for plotting we are takeing  $a=1$  .

