5.13.21

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Question

if

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \cdots \begin{pmatrix} 1 & n-1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 78 \\ 0 & 1 \end{pmatrix}. \tag{1}$$

then the inverse of $\begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$ is

(a) $\begin{pmatrix} 1 & 0 \\ 12 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & -13 \\ 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & -12 \\ 0 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 0 \\ 13 & 1 \end{pmatrix}$

Theoretical Solution

as mentioned in the question :

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & n-1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 78 \\ 0 & 1 \end{pmatrix}. \tag{2}$$

let:

$$\mathbf{A} = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$$

in these type of question we start the solution by induction :

Theoretical solution

for n=3:

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \tag{3}$$

for n=4 :

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 6 \\ 0 & 1 \end{pmatrix} \tag{4}$$

for n=5 :

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 10 \\ 0 & 1 \end{pmatrix} \tag{5}$$

for n=6 :

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 5 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 15 \\ 0 & 1 \end{pmatrix} \tag{6}$$

Theoretical Solution

let the resulting matrix is in the format :

$$\begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} \tag{7}$$

$$\alpha_{11} = 1, \alpha_{12} = \frac{n(n+1)}{2}, \alpha_{21} = 0, \alpha_{22} = 1 \tag{8}$$

so replacing n with n-1 and by using equation 8 .

$$\begin{pmatrix} 1 & \frac{n(n-1)}{2} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 78 \\ 0 & 1 \end{pmatrix} \tag{9}$$

$$\frac{n(n-1)}{2} = 78 \tag{10}$$

$$n = 13$$

Inverse of a matrix

inverse of a matrix

$$\mathbf{Adj(A)A} = ||\mathbf{A}||\mathbf{I} \tag{11}$$

$$\mathbf{Adj(A)} = ||\mathbf{A}||\mathbf{A}^{-1} \tag{12}$$

$$\mathbf{A}^{-1} = \frac{\mathbf{Adj(A)}}{||\mathbf{A}||} \tag{13}$$

Theoretical Solution

adjoint of **A** matrix is :

$$\mathbf{Adj(A)} = \begin{pmatrix} 1 & -n \\ 0 & 1 \end{pmatrix} \tag{14}$$

$$||\mathbf{A}|| = 1 \tag{15}$$

$$\mathbf{A}^{-1} = \begin{pmatrix} 1 & -n \\ 0 & 1 \end{pmatrix} \tag{16}$$

$$\mathbf{A}^{-1} = \begin{pmatrix} 1 & -13 \\ 0 & 1 \end{pmatrix} \tag{17}$$

option (B) is correct .

C Code -Finding Solution for the system of Equations

```c

#include <stdio.h>

int main() {
    int n;
    int sum = 0;

    // Find n such that (n-1)*n/2 = 78
    for (n = 1; n < 100; n++) {
        sum = (n - 1) * n / 2;
        if (sum == 78) {
            break;
        }
    }
```

C Code -Finding Solution for the system of Equations

```c

printf("Value of n = %d\n", n);

   // Inverse matrix of [[1, n], [0, 1]]
   int inv[2][2] = {{1, -n}, {0, 1}};

   printf("Inverse matrix is:\n");
   printf("[[%d, %d],\n [%d, %d]]\n",
          inv[0][0], inv[0][1], inv[1][0], inv[1][1]);

   return 0;
}
```