Problem 9.4.7

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Question

Question: Find the roots of the following quadratic equation graphically:
$16x^2 - 8x + 1 = 0$

Solution

First, the equation is already in standard quadratic form:

$$16x^2 - 8x + 1 = 0 \tag{0.1}$$

Input Variables:
The given quadratic can be written in the conic form:

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \tag{0.2}$$

where

$$\mathbf{V} = \begin{pmatrix} 16 & 0 \\ 0 & 0 \end{pmatrix}, \quad \mathbf{u} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}, \quad f = 1 \tag{0.3}$$

Solution

Since the roots correspond to intersections with the x-axis, we represent the line:

$$L : \mathbf{x} = \mathbf{h} + \kappa\mathbf{m} \tag{0.4}$$

with

$$\mathbf{h} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{0.5}$$

Input Parameters

Symbol	Value
V	$\begin{pmatrix} 16 & 0 \\ 0 & 0 \end{pmatrix}$
u	$\begin{pmatrix} -4 \\ 0 \end{pmatrix}$
f	1
h	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
m	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Table: Input Parameters

Solution

The points of intersection of a line with a conic are:

$$\kappa = \frac{1}{\mathbf{m}^T \mathbf{V} \mathbf{m}} \left[-\mathbf{m}^T (\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{[\mathbf{m}^T (\mathbf{V}\mathbf{h} + \mathbf{u})]^2 - g(\mathbf{h})\mathbf{m}^T \mathbf{V}\mathbf{m}} \right] \quad (0.6)$$

where

$$g(\mathbf{h}) = \mathbf{h}^T \mathbf{V} \mathbf{h} + 2\mathbf{u}^T \mathbf{h} + f \quad (0.7)$$

Step-by-Step Evaluation

Step 1: Compute $\mathbf{m}^T\mathbf{V}\mathbf{m}$

$$\mathbf{m}^T\mathbf{V}\mathbf{m} = 16 \tag{0.8}$$

Step 2: Compute $\mathbf{V}\mathbf{h} + \mathbf{u}$

$$\mathbf{V}\mathbf{h} + \mathbf{u} = \begin{pmatrix} -4 \\ 0 \end{pmatrix} \tag{0.9}$$

Step 3: Compute $\mathbf{m}^T(\mathbf{V}\mathbf{h} + \mathbf{u})$

$$\mathbf{m}^T(\mathbf{V}\mathbf{h} + \mathbf{u}) = -4 \tag{0.10}$$

Step 4: Compute $g(\mathbf{h})$

$$g(\mathbf{h}) = 1 \tag{0.11}$$

Final Roots

Step 5: Substitute into κ:

$$\kappa = \frac{4 \pm \sqrt{16 - 16}}{16} = \frac{4}{16} = 0.25 \tag{0.12}$$

Step 6: Intersection point:

$$x = h + \kappa m = (0,0) + (0.25)(1,0) = (0.25, 0) \tag{0.13}$$

Thus, the quadratic $16x^2 - 8x + 1 = 0$ intersects the x-axis at

$$\boxed{x = 0.25} \tag{0.14}$$

Parabola and Root of the Quadratic Equation