5.7.8

AI25BTECH11035 - SUJAL RAJANI

QUESTION
Given that $\mathbf{A} = \begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix}$ and $\mathbf{A}^2 = 3\mathbf{I}$,then
(a) $1 + \alpha^2 + \beta\gamma = 0$
(b) $1 - \alpha^2 - \beta\gamma = 0$
(c) $3 - \alpha^2 - \beta\gamma = 0$
(d) $3 + \alpha^2 + \beta\gamma = 0$
solution as mentioned in the question :

$$\mathbf{A} = \begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix}$$

$$\mathbf{A}^2 = 3\mathbf{I}. \tag{1}$$

The characteristic equation of $\mathbf{A}$ is:

$$\left| \begin{pmatrix} \alpha - \lambda & \beta \\ \gamma & -\alpha - \lambda \end{pmatrix} \right| = 0, \tag{2}$$

which expands to

$$\lambda^2 - \alpha^2 - \beta\gamma = 0. \tag{3}$$

By Carlyle-Hamilton theorem

$$\mathbf{A}^2 = (\alpha^2 + \beta\gamma)\mathbf{I}$$

by equation 1 :

$$3\mathbf{I} = (\alpha^2 + \beta\gamma)\mathbf{I} \tag{4}$$

$$3 = \alpha^2 + \beta\gamma$$

. option c is the correct option .