

8.3.15

AI25BTECH11035 - SUJAL RAJANI

QUESTION An equilateral triangle is inscribed in the parabola $y^2 = 4ax$, where one vertex is at the vertex of the parabola. Find the length of the side of the triangle.

SOLUTION let the three position vector of the equilateral triangle be **A**, **B** and **C**.
let

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} x \\ y \end{pmatrix}, \mathbf{C} = \begin{pmatrix} x \\ -y \end{pmatrix}$$

both parabola and equilateral triangle are symmetric about x axis that why the y coordinates of **B** and **C** are of different sign.

$$y^2 = 4ax$$

because the position vector of **B** and **C** lie of the parabola.

$$\begin{aligned} \|\mathbf{B} - \mathbf{A}\|^2 &= \|\mathbf{B} - \mathbf{C}\|^2 \\ (\mathbf{B} - \mathbf{A})^\top (\mathbf{B} - \mathbf{A}) &= (\mathbf{B} - \mathbf{C})^\top (\mathbf{B} - \mathbf{C}). \\ y^2 + x^2 &= 4y^2 \\ x &= 12a. \end{aligned}$$

by replacing the value of x in (1) equation :

$$\begin{aligned} y^2 &= 48a^2 \\ y &= \sqrt{48}a \end{aligned}$$

the length of side of equilateral triangle is :

$$2y = 2\sqrt{48}a.$$

