5.13.104

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QUESTION

Let $\mathbf{R}^2$ denote the Euclidean space . Let S=(a, b, c):$a, b, c \epsilon \mathbf{R}$,$ax^2 + 2bxy + cy^2 > 0 \forall (x, y) \epsilon \mathbf{R}^2 - (0, 0)$.Then which of the following statement is (are) TRUE ?

(a) $(2,\frac{7}{2},6) \epsilon$S

(b) IF $(3,b,\frac{1}{12}) \epsilon$S,then $|2b| < 1$.

(c)For any given $(a, b, c) \epsilon$S,the system of linear equations

$$ax + by = 1$$
$$bx + cy = -1$$

has a unique solution .

(d) For any given $(a, b, c) \epsilon$S, the system of linear equations

$$(a + 1)x + by = 0$$
$$bx + (c + 1)y = 0$$

has a unique solution .

Theoretical Solution

solution

as mentioned in the question :

$$ax^2 + 2bxy + cy^2 > 0 \forall (x, y) \epsilon \mathbf{R}^2 - (0, 0) \tag{0.1}$$

let Q be a matric :

$$\mathbf{Q} = \begin{pmatrix} a & b \\ b & c \end{pmatrix} \tag{0.2}$$

then:

$$ax^2 + 2bxy + cy^2 = \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} a & b \\ b & c \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \tag{0.3}$$

for this to happen :

$$a > 0, c > 0 \tag{0.4}$$

$$||\mathbf{Q}|| = ac - b^2 > 0 \tag{0.5}$$

Theoretical solution

option (a):
by using equitation 0.4 and 0.5:
$a = 2 > 0$,c=6,b=$\frac{7}{2}$

$$ac - b^2 = 12 - (\frac{7}{2})^2 = -\frac{1}{4} < 0 \qquad (0.6)$$

option a is not correct .

Theoretical Solution

option b is correct :
by using equitation 0.4 and 0.5:

$$a = 3 > 0, ac - b^2 = \frac{1}{4} - b^2 > 0, |2b| < 1 \tag{0.7}$$

option b is correct .

SOLUTION

option c :

$$ax + by = 1, bx + cy = -1. \tag{0.8}$$

$$\begin{pmatrix} a & b \\ b & c \end{pmatrix} \mathbf{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{0.9}$$

for the solution to be unique:

$$||\mathbf{Q}|| = ac - b^2 \neq 0. \tag{0.10}$$

from the definition of S,$ac - b^2 > 0$.
option c is correct .

option d is not correct

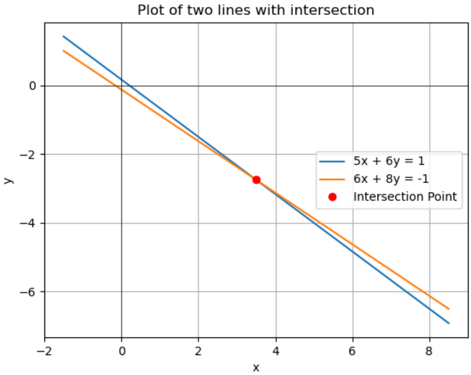
$$(a+1)x + by = 0, bx + (c+1)y = 0. \tag{0.11}$$

this type of homogeneous equation either have one solution or infinite solution .
in case of one solution the solution is (0,0) which we do not get for any

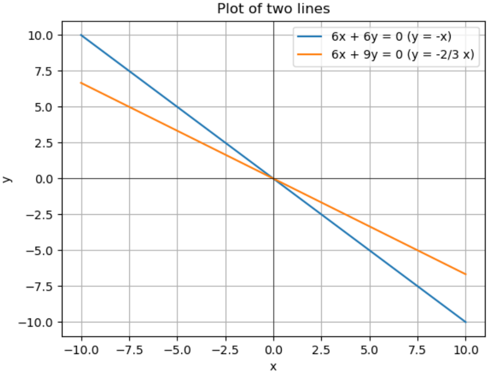
$$(a, b, c) \epsilon S$$

.

Plot


Plot of two lines with intersection
5x + 6y = 1
6x + 8y = -1
Intersection Point
x
y
0
−2
−4
−6
−2
0
2
4
6
8

Plot



Plot of two lines
6x + 6y = 0 (y = -x)
6x + 9y = 0 (y = -2/3 x)
y
x
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0