Problem 8.3.15

Sujal Rajani

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Question

Question: An equilateral triangle is inscribed in the parabola $y^2 = 4ax$, where one vertex is at the vertex of the parabola . Find the length of the side of the triangle .

Solution

SOLUTION let the three position vector of the equilateral triangle be **A**, **B** and **C** .

let

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} x \\ y \end{pmatrix}, \mathbf{C} = \begin{pmatrix} x \\ -y \end{pmatrix} \tag{1.1}$$

both parabola and equilateral triangle are symmetric about x axis that why the y coordinates of **B** and **C** are of different sign .

$$y^2 = 4ax \tag{1.2}$$

because the position vector of **B** and **C** lie of the parabola .

$$||\mathbf{B} - \mathbf{A}||^2 = ||\mathbf{B} - \mathbf{C}||^2 \tag{1.3}$$

$$(\mathbf{B} - \mathbf{A})^\top (\mathbf{B} - \mathbf{A}) = (\mathbf{B} - \mathbf{C})^\top (\mathbf{B} - \mathbf{C}). \tag{1.4}$$

$$y^2 + x^2 = 4y^2 \tag{1.5}$$

$$x = 12a. \tag{1.6}$$

Frame Title

by replacing the value of x in (1) equation :

$$y^2 = 48a^2 \qquad (1.7)$$
$$y = \sqrt{48}a \qquad (1.8)$$

the length of side of equilateral triangle is :

$$2y = 2\sqrt{48}a. \qquad (1.9)$$

Parabola with Equilateral Triangle Inscribed