5.13.21

AI25BTECH11035 - SUJAL RAJANI

QUESTION

if

$$\begin{pmatrix}1 & 1\\0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 2\\0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 3\\0 & 1\end{pmatrix}\cdots\begin{pmatrix}1 & n-1\\0 & 1\end{pmatrix}=\begin{pmatrix}1 & 78\\0 & 1\end{pmatrix}.$$

then the inverse of $\begin{pmatrix}1 & n\\0 & 1\end{pmatrix}$ is

(a) $\begin{pmatrix}1 & 0\\12 & 1\end{pmatrix}$

(b) $\begin{pmatrix}1 & -13\\0 & 1\end{pmatrix}$

(c) $\begin{pmatrix}1 & -12\\0 & 1\end{pmatrix}$

(d) $\begin{pmatrix}1 & 0\\13 & 1\end{pmatrix}$

solution

as mentioned in the question :

$$\begin{pmatrix}1 & 1\\0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 2\\0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 3\\0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & n-1\\0 & 1\end{pmatrix}=\begin{pmatrix}1 & 78\\0 & 1\end{pmatrix}.$$

let:

$$\mathbf{A}=\begin{pmatrix}1 & n\\0 & 1\end{pmatrix}$$

in these type of question we start the solution by analyzing the pattern.

$$\begin{pmatrix}1 & 1\\0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 2\\0 & 1\end{pmatrix}=\begin{pmatrix}1 & 3\\0 & 1\end{pmatrix}$$

$$\begin{pmatrix}1 & 3\\0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 3\\0 & 1\end{pmatrix}=\begin{pmatrix}1 & 6\\0 & 1\end{pmatrix}$$

$$\begin{pmatrix}1 & 6\\0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 4\\0 & 1\end{pmatrix}=\begin{pmatrix}1 & 10\\0 & 1\end{pmatrix}$$

$$\begin{pmatrix}1 & 10\\0 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 5\\0 & 1\end{pmatrix}=\begin{pmatrix}1 & 15\\0 & 1\end{pmatrix}$$

let the resulting matrix is in the format :

$$\begin{pmatrix}\alpha_{11} & \alpha_{12}\\\alpha_{21} & \alpha_{22}\end{pmatrix}$$

$$\alpha_{11}=1,\alpha_{12}=\frac{n(n+1)}{2},\alpha_{21}=0,\alpha_{22}=1$$

so replacing n with n-1 :

$$\begin{pmatrix} 1 & \frac{n(n-1)}{2} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 78 \\ 0 & 1 \end{pmatrix}$$

$$\frac{n(n-1)}{2} = 78$$

$$n = 13$$

inverse of a matrix

$$\mathbf{Adj}(\mathbf{A})\mathbf{A} = \|\mathbf{A}\|\mathbf{I}$$

$$\mathbf{Adj}(\mathbf{A}) = \|\mathbf{A}\|\mathbf{A}^{-1}$$

$$\mathbf{A}^{-1} = \frac{\mathbf{Adj}(\mathbf{A})}{\|\mathbf{A}\|}$$

adjoint of **A** matrix is :

$$\mathbf{Adj}(\mathbf{A}) = \begin{pmatrix} 1 & -n \\ 0 & 1 \end{pmatrix}$$

$$\|\mathbf{A}\| = 1$$

$$\mathbf{A}^{-1} = \begin{pmatrix} 1 & -n \\ 0 & 1 \end{pmatrix}$$

by equation (1) we are using the value of n :

$$\mathbf{A}^{-1} = \begin{pmatrix} 1 & -13 \\ 0 & 1 \end{pmatrix}$$

option (B) is correct .