CMPE185 Autonomous Mobile Robots

Fall 2022 Homework 1 Solution

Problem 1. (50 pts) Suppose a two-wheel differential drive mobile robot equipped with a 2D range sensor starts at position x = 1.0m, y = 2.0m, with heading θ = π/4. A range sensor is attached to the center of the robot. The range sensor detects an obstacle and returns a reading of α = −π/6 and d = 1.0m.

a. What is the position of the obstacle in the global coordinate frame?

Step1: find the rotation matrix

$$R = \begin{bmatrix} cos\frac{\pi}{4} & -sin\frac{\pi}{4} \\ sin\frac{\pi}{4} & cos\frac{\pi}{4} \end{bmatrix}$$

Step2: find the position

$${}^{R}p = \begin{bmatrix} cos-\frac{\pi}{6} \\ sin-\frac{\pi}{6} \end{bmatrix}, {}^{I}q = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$${}^{I}p = R{}^{R}p + {}^{I}q$$

$${}^{I}p = \begin{bmatrix} cos\frac{\pi}{4} & -sin\frac{\pi}{4} \\ sin\frac{\pi}{4} & cos\frac{\pi}{4} \end{bmatrix} \begin{bmatrix} cos-\frac{\pi}{6} \\ sin-\frac{\pi}{6} \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{6}+\sqrt{2}+4}{4} \\ \frac{\sqrt{6}-\sqrt{2}+8}{4} \end{bmatrix} \approx \begin{bmatrix} 1.97 \\ 2.26 \end{bmatrix}$$

b. For the same robot, suppose the wheel radius is 0.3m and the length of the axles is 1.6m. For the wheel encoder, the total ticks per revolution is 50. After a while, 20 ticks were recorded for the left wheel, and 40 ticks were recorded for the right wheel, will the car collide with the obstacle? Write down all your work.

Step1: find $\triangle S_l$ and $\triangle S_r$

$$\triangle S_l = 2\pi r \frac{\triangle tick_l}{N} = 2\pi * 0.3 * \frac{20}{50} = \frac{6\pi}{25}$$

$$\triangle S_r = 2\pi r \frac{\triangle tick_r}{N} = 2\pi * 0.3 * \frac{40}{50} = \frac{12\pi}{25}$$

Step2: find new position

$$p' = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} + \begin{bmatrix} \frac{\triangle S_r + \triangle S_l}{2} cos(\theta + \frac{\triangle S_r - \triangle S_l}{4L}) \\ \frac{\triangle S_r + \triangle S_l}{2} sin(\theta + \frac{\triangle S_r - \triangle S_l}{4L}) \\ \frac{\triangle S_r - \triangle S_l}{2L} \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ \frac{\pi}{4} \end{bmatrix} + \begin{bmatrix} \frac{12\pi/25+6\pi/25}{2} cos(\pi/4 + \frac{12\pi/25-6\pi/25}{4*0.8}) \\ \frac{12\pi/25+6\pi/25}{2} sin(\pi/4 + \frac{12\pi/25-6\pi/25}{4*0.8}) \\ \frac{12\pi/25-6\pi/25}{2*0.8} \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ \frac{\pi}{4} \end{bmatrix} + \begin{bmatrix} \frac{9\pi}{25} cos(\frac{13\pi}{40}) \\ \frac{9\pi}{25} sin(\frac{13\pi}{40}) \\ \frac{3\pi}{20} \end{bmatrix} \approx \begin{bmatrix} 2.13 \\ 2.02 \\ \frac{2\pi}{5} \end{bmatrix}$$

The car will not collide with the obstacle.

Problem 2. PID Controller (50 pts)

- **a.** Implement a PID go-to-goal controller to control a differential drive mobile robot to move from a starting position to a goal position. Open the "CMPE185_HW_1_p2.m" file and implement the PID controller in the given place. Choose the proper values of the proportional gain K_P, the integral gain K_I, and the derivative gain K_D.
- **b.** Change the values of K_P, K_I, and K_D and observe how the trajectory changes. Plot the corresponding trajectories of the mobile robot and discuss the results.

Submission:

Submit a single Pdf file for Problem 1 and Problem 2.b.
Submit the .m file for Problem 2.a. with the name "first_name + last_name + HW1.m".

Note: the assignment should be completed individually. Do not share results and code with others.