

# Solved Model Question Paper-II with effect from 2022-23 (CBCS Scheme)

## First/Second Semester B.E. Degree Examination

### Applied Physics for Computer Science Stream

TIME: 03 Hours

Max. Marks: 100

Note:

01. Answer any FIVE full questions, choosing at least ONE question from each MODULE.
02. Draw neat sketches where ever necessary.
03. Constants: Speed of Light ' $c$ ' =  $3 \times 10^8 \text{ ms}^{-1}$ , Boltzmann Constant ' $k$ ' =  $1.38 \times 10^{-23} \text{ JK}^{-1}$ , Planck's Constant ' $h$ ' =  $6.625 \times 10^{-34} \text{ Js}$ , Acceleration due to gravity ' $g$ ' =  $9.8 \text{ ms}^{-2}$ , Permittivity of free space ' $\epsilon_0$ ' =  $8.854 \times 10^{-12} \text{ F m}^{-1}$ .

#### Module -1

**Q.01 a Obtain the expression for Energy Density using Einstein's A and B coefficients and thus conclude on  $B_{12}=B_{21}$ .**

Consider two energy levels  $E_1$  and  $E_2$  of a system of atoms with  $N_1$  and  $N_2$  are population of energy levels respectively.

Let  $U_\nu$  be the energy density of incident beam of radiation of frequency  $\gamma$ . Let us consider the absorption and two emission process

##### 1) Induced absorption:

Induced absorption is the absorption of an incident photon by system as a result of which the system is elevated from a lower energy state to a higher state.

The rate of absorption is proportional to  $N_1 U_\nu$

$$\text{Rate of absorption} = B_{12} N_1 U_\nu \dots\dots\dots (1)$$

Where ' $B_{12}$ ' is the proportionality constant called Einstein Coefficient of induced absorption.

##### 2) Spontaneous emission:

The emission of a photon by the transition of a system from a higher energy state to a lower energy state without the aid of an external energy is called spontaneous emission.

Spontaneous emission depends on  $N_2$  and independent of energy density.

$$\text{The rate of spontaneous emission} = A_{21} N_2 \dots\dots\dots (2)$$

Where ' $A_{21}$ ' is called proportionality constant called Einstein coefficient of spontaneous emission.

### 3) Stimulated emission:

Stimulated emission is the emission of a photon by a system under the influence of a passing photon of just the right energy due to which the system transits from a higher energy state to a lower energy state

The rate of stimulated emission is directly proportional to  $N_2 U_\gamma$ .

The rate of stimulated emission =  $B_{21} N_2 U_\nu$  ..... (3)

Where ' $B_{21}$ ' is the proportionality constant called Einstein's Coefficient of stimulated emission.  
At thermal equilibrium,

Rate of absorption = (Rate of spontaneous emission + Rate of stimulated emission)

$$B_{12} N_1 U_\nu = A_{21} N_2 + B_{21} N_2 U_\nu$$

$$U_\nu (B_{12} N_1 - B_{21} N_2) = A_{21} N_2$$

$$U_\nu = \frac{A_{21} N_2}{(B_{12} N_1 - B_{21} N_2)}$$

$$\text{i.e. } U_\nu = \frac{A_{21}}{B_{21}} \left[ \frac{N_2}{\left( \frac{B_{12} N_1}{B_{21} N_2} - 1 \right)} \right]$$

$$U_\nu = \frac{A_{21}}{B_{21}} \left[ \frac{1}{\left( \frac{B_{12} N_1}{B_{21} N_2} - 1 \right)} \right] \rightarrow (4)$$

By Boltzmann's law,  $N_2 = N_1 e^{-\left(\frac{E_2 - E_1}{KT}\right)} = N_1 e^{-h\nu/KT}$

$$\text{i.e., } N_1/N_2 = e^{h\nu/KT}$$

$$\text{Eqn. (4) becomes } U_\nu = \frac{A_{21}}{B_{21}} \left[ \frac{1}{\left( \frac{B_{12}}{B_{21}} e^{\left(\frac{h\nu}{kT}\right)} - 1 \right)} \right] \rightarrow (5)$$

$$\text{By Planck's law, } U_\nu = \frac{8\pi h \nu^3}{c^3} \left[ \frac{1}{\left( e^{\left(\frac{h\nu}{kT}\right)} - 1 \right)} \right] \rightarrow (6)$$

Comparing equation (5) & (6)

$$\frac{A_{21}}{B_{21}} = 8\pi h \nu^3 / c^3 \quad \& \quad \frac{B_{12}}{B_{21}} = 1 \quad \text{i.e. } B_{12} = B_{21}$$

The probability of induced absorption is equal to the stimulated emission.

Therefore,  $A_{12}$  is written as A and  $B_{12}$ ,  $B_{21}$  written as B.

Equation (5) becomes

$$U_\nu = \frac{A}{B} \left[ \frac{1}{\left( e^{\left( \frac{h\nu}{kT} \right)} - 1 \right)} \right]$$

Above equation is the expression for energy density

**Q.01b Describe attenuation and explain the various fiber losses.**

- Attenuation is the loss of optical power suffered by the optical signal as it propagates through a fiber also called as the fiber loss.
- There are three mechanisms through which attenuation takes place.
- The attenuation of a fiber optic cable is expressed in decibels.

$$\text{i.e., } \alpha = -\frac{10}{L} \log \left[ \frac{P_{out}}{P_{in}} \right] \quad \frac{dB}{km}$$

The main reasons for the loss in light intensity over the length of the cable are due to light absorption, scattering and due to bending losses.

**Attenuation can be caused by three mechanisms.**

**(i) Absorption losses**

- Absorption of photons by impurities like metal ions such as iron, chromium, cobalt and copper in the silica glass of which the fiber is made of.
- During signal propagation photons interact with electrons of impurity atoms and the electrons are excited to higher energy levels.
- Then the electrons give up their absorbed energy either in the form of heat or light energy.
- The re-emission of light energy will usually be in a different wavelength; hence it is referred as loss of energy.
- The other impurity such as hydroxyl (OH) ions which enters into the fiber at the time of fabrication causes significant absorption loss.
- The absorption of photons by fiber itself assuming that there are no impurities and in-homogeneities in it is called as *intrinsic absorption*.

**(ii) Scattering losses**

- Scattering of light waves occurs whenever a light wave travels through a medium having scattering objects whose dimensions are smaller than the wavelength of light.
- Similarly, when a light signal travels in the fiber, the photons may be scattered due to the sharp

changes in refractive index values inside the core over distances and also due to the structural impurities present in the fiber material.

This type of scattering is called as Rayleigh scattering. Scattering of photons also takes place due to trapped gas bubbles which are not dissolved at the time of manufacturing.

A scattered photon moves in random direction and leaves the fiber.

### (iii) Radiation losses

Radiation losses occur due to macroscopic bends and microscopic bends.

- **Macroscopic bending:** All optical fibers are having critical radius of curvature provided by the manufacturer. If the fiber is bent below that specification of radius of curvature, the light ray incident on the core cladding interface will not satisfy the condition of total internal reflection. This causes loss of optical power.
- **Microscopic bending:** Microscopic bends are repetitive small scale fluctuations in the linearity of the fiber axis. They occur due to non-uniformities in the manufacturing and also lateral pressure built up on the fiber. They cause irregular reflections and some of them leak through the fiber. The defect due to non-uniformity (micro-bending) can be overcome by introducing optical fiber inside a good strengthened polyurethane jacket.

**Q.01c Given the Numerical Aperture 0.30 and RI of core 1.49 calculate the critical angle for the core-cladding interface.**

Given:  $NA = 0.30$   
 $n_1 = 1.49$   
 $\theta_c = ?$

$$NA = \frac{\sqrt{n_1^2 - n_2^2}}{n_o}$$

$n_o = 1$

$$\therefore 0.3 = \sqrt{1.49^2 - n_2^2} \rightarrow (1)$$

Squaring (1) on both sides & simplifying for  $n_2$

$$(0.3)^2 = (1.49)^2 - n_2^2$$

$$\Rightarrow n_2^2 = (1.49)^2 - (0.3)^2$$

$$n_2^2 = 2.1301$$

$$\Rightarrow n_2 = 1.459$$



$$\therefore \theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right)$$

$$\theta_c = \sin^{-1} \left( \frac{1.459}{1.49} \right)$$

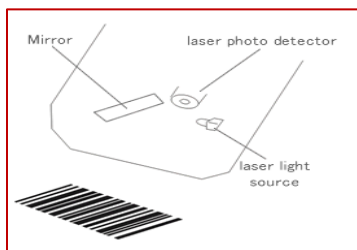
$$\theta_c = \sin^{-1} (0.976)$$

$$\boxed{\theta_c = 76^\circ}$$

OR

**Q.02a Discuss the applications of LASER in bar-code scanner and LASER Cooling.**

**Laser barcode reading**



A bar code consists of a series of strips of dark and white bands. Each strip has a width of about 0.3 mm and the total width of the bar code is about 3 cm.

Laser light reflected off a mirror is shine on the label surface and its reflection is captured by a sensor (laser photo detector) to read a bar code.

Data retrieval is achieved when the photo detector captures the reflected light and replace the black and white bars with binary digital signals.

Reflections are strong in white areas and weak in black areas. A sensor receives reflections to obtain analog waveforms.

The analog signal is converted into a digital signal via an A/D converter.

Data retrieval is achieved when a code system is determined from the digital signal obtained. (Decoding process).

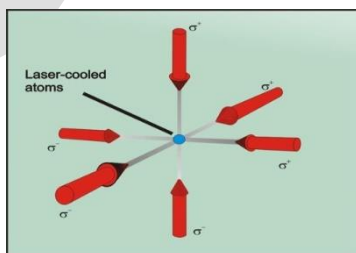
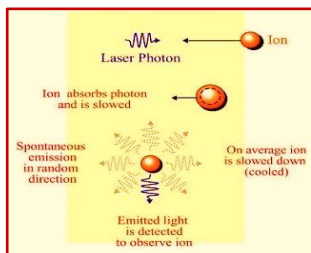
**Laser cooling**

In this technique, heat can be removed optically with the help of laser.

Atoms can be cooled using lasers because light particles from the laser beam are absorbed and re-emitted by the atoms, causing them to lose some of their kinetic energy.

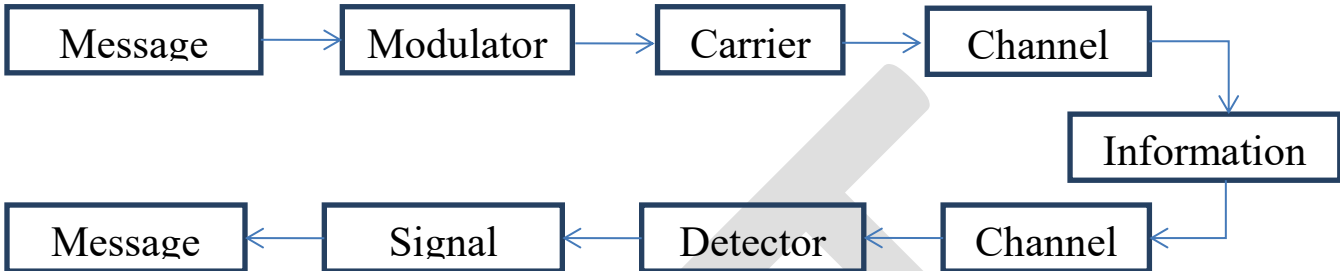
Reduction in the momentum results in the reduction in temperature of atom i.e  $P = \frac{E}{c} = \frac{h}{\lambda}$ .

- After thousands of such impacts, the atoms will be chilled near to zero Kelvin.
- This cooling is also called Doppler cooling.



### Q.02b Discuss Point to Point communication using optical fibers.

Optical fiber communication system consists of transmitter, information channel and receiver. Transmitter converts an electrical signal into optical signal. Information channel carries the signal from transmitter to receiver. The receiver converts optical signal to electrical form. The block diagram of optical fiber communication system is shown in fig.



**Message origin:** It converts a non-electrical message into an electrical signal.

**Modulator:** It converts the electrical message into proper format and it helps to improve the signal onto the wave which is generated by the carrier source.

There are two types of format. They are Analog and digital. Analog signal is continuous and it doesn't make any change in the original format. But digital signal will be either in ON or OFF state.

**Carrier source:** It generates the waves on which the data is transmitted. These carrier waves are produced by the electrical oscillator. Light emitting diodes (LED) and laser diodes (LD) are the different sources.

**Channel Coupler: (Input)** The function of the channel coupler is to provide the information to information channel. It can be an antenna which transfers all the data.

**Information channel:** It is path between transmitter and receiver. There are two types of information channel. They are guided and unguided. Atmosphere is the good example for unguided information channel. Co-axial cable, two-wire line and rectangular wave guide are example for guided channel.

**Channel Coupler: (Output)** The output coupler guides the emerged light from the fiber on to the light detector.

**Detector:** The detector separates the information from the carrier wave. Here a photo-detector converts optical signal to electronic signal.

**Signal processor:** Signal processor amplifies the signals and filters the undesired frequencies.

**Message output:** The output message will be in two forms. Either person can see the information or hear the information. The electrical signal can be converted into sound wave or visual image by using CRO.

**Q.02c Calculate the ratio of population for a given pair of energy levels corresponding to emission of radiation 694.3 nm at a temperature of 300 K.**

Given :  $\lambda = 694.3 \times 10^{-9} \text{ m}$

$T = 300 \text{ K}$

$\frac{N_2}{N_1} = ?$

$\frac{N_2}{N_1} = e^{-hc/\lambda kT}$

$= e^{\frac{-6.625 \times 10^{-34} \times 3 \times 10^8}{694.3 \times 10^{-9} \times 1.38 \times 10^{-23} \times 300}}$

$= e^{\frac{-1.9875 \times 10^{-25}}{2.87 \times 10^{-27}}}$

$= e^{-69.14}$

$\frac{N_2}{N_1} = 9.39 \times 10^{-31}$

## Module-2

**Q.03 a Derive an expression for de Broglie wavelength by analogy and hence discuss the significance of de Broglie waves.**

“Since nature loves symmetry, if the radiation behaves as particles under certain conditions and as waves under certain conditions, then one can expect that, the entities which ordinarily behaves as particles (ex. Like electrons, protons, neutrons) must also exhibit properties attributable to waves under appropriate circumstances”. This is known as **deBroglie hypothesis**

The waves associated with the moving particles are called de Broglie waves or matter waves or pilot waves.

Characteristics of matter waves:

1. Waves associated with moving particles are called matter waves. The wavelength ‘ $\lambda$ ’ of a de-Broglie wave associated with particle of mass ‘ $m$ ’ moving with velocity ‘ $v$ ’ is  
$$\lambda = h/(mv)$$
2. Matter waves are not electromagnetic waves because the de Broglie wavelength is independent of charge of the moving particle.
3. The amplitude of the matter wave depends on the probability of finding the particle in that position.
4. The speed of matter waves depends on the mass and velocity of the particle associated with the wave.

A particle of mass 'm' moving with velocity 'c' possess energy given by

$$E = mc^2 \quad \rightarrow \text{(Einstein's Equation) (1)}$$

According to Planck's quantum theory the energy of quantum of frequency 'v' is

$$E = h\nu \rightarrow (2)$$

From (1) & (2)

$$mc^2 = h\nu = hc / \lambda \quad \text{since } \nu = c/\lambda$$

$$\lambda = hc / mc^2 = h/mc$$

$$\lambda = h/mv \quad \text{since } v \approx c$$

De Broglie wavelength of a free particle in terms of its kinetic energy

Consider a particle, since the particle is free, the total energy is same as

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

Where 'm' is the mass, 'v' is the velocity and 'p' is the momentum of the particle.

$$p = \sqrt{2mE}$$

The expression for de-Broglie wavelength is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

Debroglie Wavelength of an Accelerated Electron:

If an electron accelerated with potential difference 'V' the work done on the 'eV', which is converted to kinetic energy.

$$eV = \frac{1}{2}mv^2 \quad \rightarrow (1)$$

If 'p' is the momentum of the electron, then  $p = mv$

Squaring on both sides, we have

$$p^2 = m^2v^2$$

$$mv^2 = p^2/m$$

Using in equation (1) we have



$$eV = p^2/(2m)$$

$$\text{or } p = \sqrt{2meV}$$

According to de-Broglie  $\lambda = h/p$

$$\text{Therefore } \lambda = \left[ \frac{h}{\sqrt{2meV}} \right]$$

$$\lambda = \frac{1}{\sqrt{V}} \left[ \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 9.11 \times 10^{-31} \times 1.602 \times 10^{-19}}} \right] = \frac{1.226 \times 10^{-9}}{\sqrt{V}} \text{ m}, \quad \lambda = \frac{1.226}{\sqrt{V}} \text{ nm}$$

**Q.03 b Explain the Wave function with mathematical form and Discuss the physical significance of a wave function.**

A physical situation in quantum mechanics is represented by a function called wave function. It is denoted by 'ψ'. It accounts for the wave like properties of particles. Wave function is obtained by solving Schrodinger equation.

Mathematically it is given by

$$\psi = Ae^{i(kx-\omega t)}$$

**Physical significance of wave function:**

The wave function itself has no physical significance, the physical significance is given by a function called probability density or probability function.

$$\text{We know } \psi = Ae^{i(kx-\omega t)}$$

$$\text{Complex conjugate of } \psi \text{ is given by } \Psi^* = Ae^{-i(kx-\omega t)}$$

probability density is  $|\psi|^2 = \psi \psi^* = A^2$ , Where A = Square of amplitude.

According to max born interpretation, as square of the amplitude  $A^2$  for electromagnetic waves represent Intensity of the wave. In quantum mechanics square of the amplitude  $A^2$  represent the probability of finding the particle in certain position.

**Q.03 c Calculate the energy of the first three states for an electron in one dimensional potential well of width 0.1 nm.**

$$E = \frac{n^2 h^2}{8ma^2}$$

given data:  $a = 0.1 \times 10^{-9} \text{ m}$

$$n = 1, 2, 3$$

$$m = 9.1 \times 10^{-31} \text{ kg}$$

$$h = 6.625 \times 10^{-34}$$

$$\begin{aligned} n=1 \\ E_{2,p} &= \frac{1 \times (6.625 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (0.1 \times 10^{-9})^2} \\ &= 6.02 \times 10^{-18} \text{ J} \end{aligned}$$

$$[E_{2,p} = 37.68 \text{ eV}]$$

First excited state,  $n=2$

$$E_I = \frac{E_{2,p}}{n^2} = \frac{E_{2,p}}{4} = \frac{37.68}{4} = 9.42 \text{ eV}$$

For second excited state,  $n=3$

$$E_{II} = \frac{E_{2,p}}{n^2} = \frac{E_{2,p}}{9} = \frac{37.68}{9} = 4.18 \text{ eV}$$

**OR**

**Q.04 a Explain Eigen functions and Eigen Values and hence derive the eigen function of a particle inside infinite potential well of width 'a' using the method of normalization.**

Eigen functions are those wave functions in Quantum mechanics which possesses the following properties:

1. They are single valued.
2. Finite everywhere and
3. The wave functions and their first derivatives with respect to their variables are continuous.

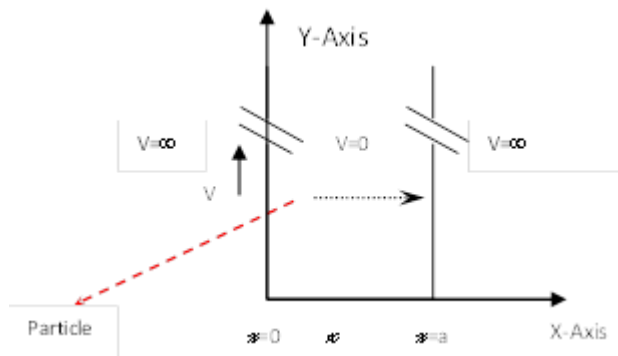
**Eigne values:**

If the wave function is operated by a quantum mechanical operator such that we get back the wavefunction back multiplied by some constant is called as Eigne value.

$$\hat{H}(\psi) = \lambda(\psi)$$

Where  $\lambda = \text{Constant} \rightarrow$  is called as eigen value

If the quantum mechanical operator is Energy operator, then  $\lambda$  is termed as energy eigen value.



Consider a particle of a mass 'm' free to move in one dimension along positive  $x$ -direction between  $x=0$  to  $x=a$ . The potential energy outside this region is infinite and within the region is zero. The particle is in bound state. Such a configuration of potential in space is called infinite potential well. It is also called particle in a box. The Schrödinger equation outside the well is

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2}(E - \infty)\psi = 0 \rightarrow (1) \quad \because V = \infty$$

For outside, the equation holds good if  $\psi = 0$  &  $|\psi|^2 = 0$ . That is particle cannot be found outside the well and also at the walls

The Schrodinger's equation inside the well is:

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2}E\psi = 0 \rightarrow (2) \quad \because V = 0$$

$$\text{Let } \frac{8\pi^2m}{h^2}E = k^2 \rightarrow (3)$$

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0$$

The solution of above equation is:

$$\psi = C \cos kx + D \sin kx \rightarrow (4)$$

$$\text{at } x = 0 \rightarrow \psi = 0$$

$$0 = C \cos 0 + D \sin 0$$

$$\therefore C = 0$$

$$\text{Also } x = a \rightarrow \psi = 0$$

$$0 = C \cos ka + D \sin ka$$

$$\text{But } C = 0$$

$$\therefore D \sin ka = 0 \longrightarrow (5)$$

$$D \neq 0 \quad (\text{because the wave concept vanishes})$$

$$\text{i.e. } ka = n\pi \quad \text{where } n = 0, 1, 2, 3, 4 \dots (\text{Quantum number})$$

$$k = \frac{n\pi}{a} \rightarrow (6)$$

sub eqn (5) and (6) in (4)

$$\psi_n = D \sin \frac{n\pi}{a} x \rightarrow (7)$$

This gives permitted wave functions.

*The Energy Eigen value given by*

Substitute equation (6) in (3)

$$\frac{8\pi^2 m}{h^2} E = k^2 = \frac{n^2 \pi^2}{a^2}$$

$$E = \frac{n^2 h^2}{8ma^2}$$

This is the expression for energy Eigen value.

For  $n = 0$  is not acceptable inside the well because  $\psi_n = 0$ . It means that the electron is not present inside the well which is not true. Thus the lowest energy value for  $n = 1$  is called zero point energy value or ground state energy.

$$\text{i.e. } E_{\text{zero-point}} = \frac{h^2}{8ma^2}$$

The states for which  $n > 1$  are called excited states.

To find out the value of D, normalization of the wave function is to be done.

$$\text{i.e. } \int_0^a |\psi_n|^2 dx = 1 \rightarrow (8)$$

using the values of  $\psi_n$  from eqn (7)

$$\int_0^a D^2 \sin^2 \frac{n\pi}{a} x dx = 1$$

$$D^2 \int_0^a \left[ \frac{1 - \cos(2n\pi/a)x}{2} \right] dx = 1$$

$$\frac{D^2}{2} \left[ \int_0^a dx - \int_0^a \cos \frac{2n\pi}{a} x dx \right] = 1$$

$$\frac{D^2}{2} \left[ x - \frac{a}{2n\pi} \sin \frac{2n\pi}{a} x \right]_0^a = 1$$

$$\frac{D^2}{2} [a - 0] = 1$$

$$\frac{D^2}{2} a = 1$$

$$D = \sqrt{\frac{2}{a}}$$

Substitute D in equation (7)

the normalized wave functions of a particle in one dimensional infinite potential well is:

$$\psi_n = \sqrt{\frac{2}{a}} \sin \frac{n\pi}{a} x \rightarrow (9)$$

**Q.04 b Show that electron does not exist inside the nucleus using Heisenberg's uncertainty principle.**

The energy of a particle is given by

$$E = \frac{1}{2} m v^2 = \frac{p^2}{2m} \quad (1)$$

Heisenberg's uncertainty principle states that

$$\Delta x \cdot \Delta P_x \geq \frac{h}{4\pi} \rightarrow (4)$$

The diameter of the nucleus is of the order  $10^{-14}$  m. If an electron is to exist inside the nucleus, the uncertainty in its position  $\Delta x$  must not exceed  $10^{-14}$  m.

$$\text{i.e. } \Delta x \leq 10^{-14} \text{ m}$$

The minimum uncertainty in the momentum

$$(\Delta P_x)_{\min} \geq \frac{h}{4\pi (\Delta x)_{\max}} \geq \frac{6.63 \times 10^{-34}}{4\pi \times 10^{-14}} \geq 0.527 \times 10^{-20} \text{ kg. m/s}$$

By considering minimum uncertainty in the momentum of the electron

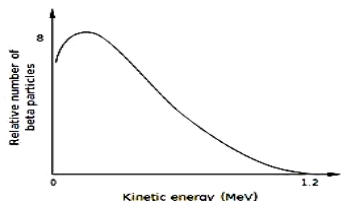
$$\text{i.e., } (\Delta P_x)_{\min} \geq 0.5 \times 10^{-20} \text{ kg.m/s} = p \rightarrow (2)$$

Consider eqn (1)

$$E = \frac{p^2}{2m} = \frac{(0.5 \times 10^{-20})^2}{2 \times 9.1 \times 10^{-31}} = 1.531 \times 10^{-11} = 95.68 \text{ MeV}$$

Where  $m_0 = 9.11 \times 10^{-31} \text{ kg}$

If an electron exists in the nucleus its energy must be greater than or equal to 95.68 MeV. It is experimentally measured that the beta particles ejected from the nucleus during beta decay have energies of about 3 to 4 MeV. This shows that electrons cannot exist in the nucleus. [**Beta decay:** In beta decay process, from the nucleus of an atom, when neutrons are converting into protons in releasing an electron (beta particle) and an antineutrino. When proton is converted into a neutron in releasing a positron (beta particle) and a neutrino. In both the processes energy sharing is statistical in nature. When beta particles carry maximum energy neutrino's carries minimum energy and vice-versa. In all other processes energy sharing is in between maximum and minimum energies. The maximum energy carried by the beta particle is called as the end point energy ( $E_{\max}$ ).



**Q.04 c An electron is associated with a de Broglie wavelength of 1nm. Calculate the energy and the corresponding momentum of the electron.**

Given:  $\lambda = 1 \text{ nm} = 1 \times 10^{-9} \text{ m}$   
 $m = 9.1 \times 10^{-31}$

W.K.T

$$P = \frac{h}{\lambda}$$

$$P = \frac{6.62 \times 10^{-34}}{1 \times 10^{-9}}$$

$$P = 6.62 \times 10^{-25} \text{ kg m/s}$$

Energy

$$E = \left[ \frac{p^2}{2m} \right]$$

$$E = \frac{(6.62 \times 10^{-25})^2}{2 \times 9.1 \times 10^{-31}}$$

$$E = \frac{43.82 \times 10^{-50}}{18.2 \times 10^{-31}}$$

$$E = 2.40 \times 10^{-19}$$

### Module-3

#### Q.05a Discuss the working of phase gate mentioning its matrix representation and truth table.

##### Phase Gate

The phase gate turns a  $|0\rangle$  into  $|0\rangle$  and a  $|1\rangle$  into  $i|1\rangle$ .

The Matrix representation of the S gate is given by

$$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

The effect of S gate on input  $|0\rangle$  is given by  $S|0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$

Similarly the effect of S gate on input  $|1\rangle$  is given by

$$S|1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ i \end{bmatrix} = i|1\rangle$$

The transformation of state  $|\psi\rangle$  is given by

$$S|\psi\rangle = S(\alpha|0\rangle + \beta|1\rangle) = \alpha S|0\rangle + \beta S|1\rangle = \alpha|0\rangle + i\beta|1\rangle$$

The symbol of S gate is given by



The Truth table for S gate is as follows

Input	Output
$ 0\rangle$	$ 0\rangle$
$ 1\rangle$	$i 1\rangle$
$\alpha 0\rangle + \beta 1\rangle$	$\alpha 0\rangle + i\beta 1\rangle$

#### Q.05b Explain Orthogonality and Orthonormality with an example for each.

Two states  $|\psi\rangle$  and  $|\phi\rangle$  are said to be orthogonal if their inner product is Zero. Mathematically  $\langle\psi|\phi\rangle = 0$

The two states are orthogonal means they are mutually exclusive. Like Spin Up and Spin Down of an electron.

Consider the inner product of and  $\langle 0|1\rangle = [1, 0] \begin{bmatrix} 1 \\ 0 \end{bmatrix} = [0 + 0] = 0$

Two states  $|\psi\rangle$  and  $|\phi\rangle$  are said to be orthonormal if their inner product is one.

Mathematically  $\langle\psi|\phi\rangle = 1$

### Q.05c

Given  $|\psi\rangle = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}$  and  $|\phi\rangle = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix}$  Prove that  $\langle\psi|\phi\rangle = \langle\phi|\psi\rangle^*$

Solution :  $\langle\phi|\psi\rangle = \begin{pmatrix} \beta_1^* & \beta_2^* \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \beta_1^* \alpha_1 + \beta_2^* \alpha_2 \dots\dots\dots (1)$

$\langle\psi|\phi\rangle = \begin{pmatrix} \alpha_1^* & \alpha_2^* \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} = \beta_1 \alpha_1^* + \beta_2 \alpha_2^*$

$\langle\psi|\phi\rangle^* = (\beta_1 \alpha_1^* + \beta_2 \alpha_2^*)^* = \beta_1^* \alpha_1 + \beta_2^* \alpha_2 \dots\dots\dots (2)$

Thus from (1) and (2)

$\langle\psi|\phi\rangle = \langle\phi|\psi\rangle^*$

OR

### Q.06a Explain the representation of qubit using Bloch Sphere.

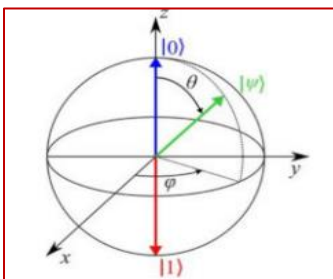
The pure state space qubits (Two Level Quantum Mechanical Systems) can be visualized using an imaginary sphere called Bloch Sphere. It has a unit radius.

The Arrow on the sphere represents the state of the Qubit. The north and south poles are used to represent the basis states  $|0\rangle$  and  $|1\rangle$  respectively. The other locations are the superposition of  $|0\rangle$  and  $|1\rangle$  states and represented by  $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$  with  $|\alpha|^2 + |\beta|^2 = 1$ .

Thus a Qubit can be any point on the Bloch Sphere. The Bloch sphere allows the state of the qubit to be represented unit spherical co-ordinates. They are the polar angle  $\theta$  and the azimuth angle  $\phi$ .

The Bloch sphere is represented by the equation

$$|\psi\rangle = \cos \frac{\theta}{2} |0\rangle + e^{i\phi} \sin \frac{\theta}{2} |1\rangle$$





Case i) For  $\varphi=0$  and  $\theta=0$  then  $|\psi\rangle = |0\rangle$  which is along +z axis.

Case ii) For  $\varphi=0$  and  $\theta = 180$  then  $|\psi\rangle = |1\rangle$  which is along -z axis.

Case iii) For  $\varphi=0$  and  $\theta=\frac{\pi}{2}$  then  $|\psi\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$  which is along +X axis.

Case iv) For  $\varphi=0$  and  $\theta= -\frac{\pi}{2}$  then  $|\psi\rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)$  which is along -X axis.

### Q.06b Explain Single qubit gate and multiple qubit gate with an example for each.

Single Qubit Gates operates on one input Qubits.

The Hadamard Gate is a truly quantum gate and is one of the most important in Quantum Computing. It has similar characteristics of  $\sqrt{NOT}$  Gate. It is a self-inverse gate. It is used to create the superposition of  $|0\rangle$  and  $|1\rangle$  states.

The Matrix representation of Hadamard Gate is as follows  $H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ .

The Hadamard Gate and the output states for the  $|0\rangle$  and  $|1\rangle$  input states are represented as follows.

The Hadamard Gate satisfies Unitary Condition.  $H^\dagger H = I$

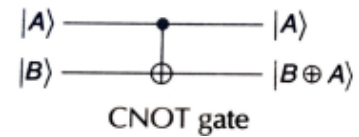
$$\begin{array}{ccc} |0\rangle & \xrightarrow{\quad H \quad} & = \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle \\ |1\rangle & \xrightarrow{\quad H \quad} & = \frac{1}{\sqrt{2}} |0\rangle - \frac{1}{\sqrt{2}} |1\rangle \end{array}$$

The truth-table for the Hadamard Gate is as follows.

Input	Output
$ 0\rangle$	$\frac{ 0\rangle +  1\rangle}{\sqrt{2}}$
$ 1\rangle$	$\frac{ 0\rangle -  1\rangle}{\sqrt{2}}$
$\alpha  0\rangle + \beta  1\rangle$	$\alpha \frac{ 0\rangle +  1\rangle}{\sqrt{2}} + \beta \frac{ 0\rangle -  1\rangle}{\sqrt{2}}$

Multiple Qubit Gates operates on two or more input Qubits. Usually one of them is a control qubit. Controlled Gates 'A' Gate with operation of kind "If 'A' is True then do 'B'" is called Controlled Gate. The  $|A\rangle$  Qubit is called control qubit and  $|B\rangle$  is the Target qubit. The target qubit is altered only when the control qubit is  $|1\rangle$ . The control qubit remains unaltered during the transformations.

### Controlled Not Gate or CNOT Gate



The CNOT gate is a typical multi-qubit logic gate and the circuit is as follows.

The matrix representation of CNOT gate is given by

$$U_{CN} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The Transformation could be expressed as  $|A, B\rangle \rightarrow |A, B \oplus A\rangle$

### Q.06c Explain the Matrix representation of 0 and 1 States and apply identity operator I to $|0\rangle$ and $|1\rangle$ states

Matrix representation of  $|0\rangle$  and  $|1\rangle$

The wave function could be expressed in ket notation as  $|\psi\rangle$  (ket Vector),  $\psi$  is the wave function. The quantum state is given by  $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$  and in matrix form  $|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$ . The matrix form of the states  $|0\rangle$  and  $|1\rangle$  is given by  $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$  and  $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Identity Operator

The operator of type  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  is called identity operator. When an identity operator acts on a state vector it keeps the state intact. By analogy we study identity operator as an identity matrix.

Let us consider the operation of Identity operator on  $|0\rangle$  and  $|1\rangle$  states. As per the principle of identity operation  $I |0\rangle = |0\rangle$  and  $I |1\rangle = |1\rangle$ .

$$I |0\rangle = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = |0\rangle$$

$$I |1\rangle = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$$

Thus the operation of identity matrix (operator) on  $|0\rangle$  and  $|1\rangle$  states leaves the states unchanged.

#### Module-4

#### **Q.07 a Enumerate the failures of classical free electron theory and assumptions of quantum free electron theory of metals.**

Failures of classical free electron theory

Electrical and thermal conductivities can be explained from classical free electron theory. It fails to account the facts such as specific heat, temperature dependence of conductivity and dependence of electrical conductivity on electron concentration.

**i) Specific heat:** The molar specific heat of a gas at constant volume is

$$C_v = \frac{3}{2} R$$

As per the classical free electron theory, free electrons in a metal are expected to behave just as gas molecules. Thus the above equation holds good equally well for the free electrons also.

But experimentally it was found that, the contribution to the specific heat of a metal by its conduction electrons was

$$C_v = 10^{-4} RT$$

which is lower than the expected value. Also according to the theory the specific heat is independent of temperature whereas experimentally specific heat is proportional to temperature.

**ii) Temperature dependence of electrical conductivity:**

Experimentally, electrical conductivity  $\sigma$  is inversely proportional to the temperature  $T$ .

$$\text{i.e. } \sigma_{\text{exp}} \propto \frac{1}{T} \rightarrow (1)$$

According to the assumptions of classical free electron theory

Since  $V_{th} \propto \sqrt{T}$

$$\text{But } \tau \propto \frac{1}{V_{th}}, \quad \tau \propto \frac{1}{\sqrt{T}},$$

substituting in conductivity equation we get

$$\sigma_{CFET} = \frac{ne^2 \tau}{m} = \frac{ne^2}{m \sqrt{T}}$$

$$\text{Or } \sigma_{CFET} \propto \frac{1}{\sqrt{T}} \rightarrow (2)$$

From equations (1) & (2) it is clear that the experimental value is not agreeing with the theory.

### iii) Dependence of electrical conductivity on electron concentration:

According to classical free electron the theory

$$\sigma = \frac{ne^2\tau}{m} \quad \text{i.e., } \sigma \propto n, \quad \text{where } n \text{ is the electron concentration,}$$

Consider copper and aluminum. Their electrical conductivities are  $5.88 \times 10^7 / \Omega\text{m}$  and  $3.65 \times 10^7 / \Omega\text{m}$ . The electron concentrations for copper and aluminum are  $8.45 \times 10^{28} / \text{m}^3$  and  $18.06 \times 10^{28} / \text{m}^3$ . Hence the classical free electron theory fails to explain the dependence of  $\sigma$  on electron concentration

Experimental results:

Metals	Electron concentration(n)	conductivity ( $\sigma$ )
Copper	$8.45 \times 10^{28} / \text{m}^3$	$5.88 \times 10^7 / \Omega\text{m}$
Aluminium	$18.06 \times 10^{28} / \text{m}^3$	$3.65 \times 10^7 / \Omega\text{m}$

### Quantum free electron theory:

#### Assumptions of quantum free electron theory:

- ☐ The energy values of the conduction electrons are quantized. The allowed energy values are realized in terms of a set of energy values.
- ☐ The distribution of electrons in the various allowed energy levels occur as per Pauli's exclusion principle.
- ☐ The electrons travel with a constant potential inside the metal but confined within its boundaries.
- ☐ The attraction between the electrons and the lattice ions and the repulsion between the electrons themselves are ignored.

### **Q.07 b Explain Meissner's Effect and the variation of critical field with temperature.**

A superconducting material kept in a magnetic field expels the magnetic flux out of its body when it is cooled below the critical temperature and thus becomes perfect diamagnet. This effect is called Meissner effect.

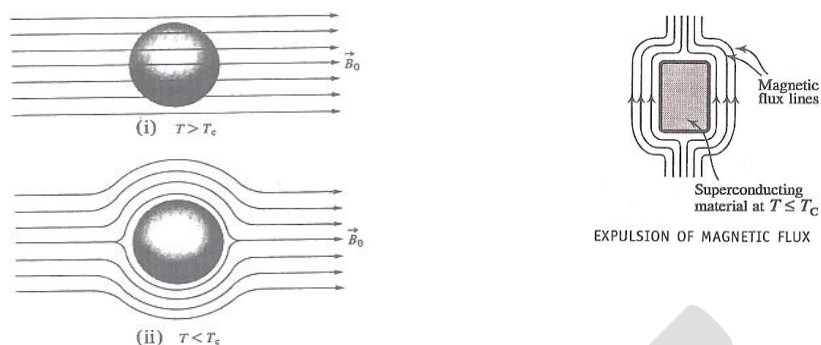


Fig: Superconductor sample subjected to an applied magnetic field with temperature (i) above and (ii) below  $T_c$ . The flux expulsion below  $T_c$  is called Meissner effect

When the temperature is lowered to  $T_c$ , the flux is suddenly and completely expelled, as the specimen becomes superconducting. The Meissner effect is reversible. When the temperature is raised the flux penetrates the material, after it reaches  $T_c$ . Then the substance will be in the normal state.

The magnetic induction inside the specimen

$$B = \mu_0 (H + M)$$

Where 'H' is the intensity of the magnetizing field and 'M' is the magnetization produced within the material.

$$\text{For } T < T_c, \quad B = 0$$

$$\mu_0 (H + M) = 0$$

$$M = -H$$

$$M/H = -1 = \chi$$

Susceptibility is -1 i.e. it is perfect diamagnetism.

Hence superconducting material do not allow the magnetic flux to exist inside the material.

### **Effect of magnetic field:**

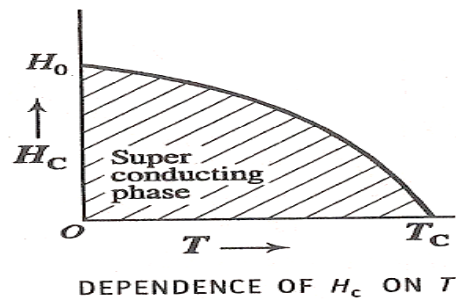
Superconductivity can be destroyed by applying magnetic field. The strength of the magnetic field required to destroy the superconductivity below the  $T_c$  is called critical field. It is denoted by  $H_c(T)$ .

If 'T' is the temperature of the superconducting material, ' $T_c$ ' is the critical temperature, ' $H_c$ ' is the critical field and ' $H_0$ ' is the critical field at  $0^\circ\text{K}$ .

They are related by

$$H_c = H_0[1 - (T/T_c)^2]$$

By applying magnetic field greater than  $H_0$ , the material can never become superconductor whatever may be the low temperature. The critical field need not be external but large current flowing in superconducting ring produce critical field and destroys superconductivity.



**Q.07 c A superconducting tin has a critical temperature of 3.7 K at zero magnetic field and a critical field of 0.0306 Tesla at 0 K. Find the critical field at 2 K.**

Given:  $T_c = 3.7 \text{ K}$ ,  $H_0 = 0.0306 \text{ T}$   
 $H_c = ?$   $T = 2 \text{ K}$

$$H_c = H_0 \left[ 1 - \left( T / T_c \right)^2 \right]$$

$$H_c = 0.0306 \left[ 1 - \left( \frac{2}{3.7} \right)^2 \right]$$

$$H_c = 0.0306 [0.707]$$

$$H_c = 0.021 \text{ T}$$

**OR**

**Q.08 a Explain the phenomenon of superconductivity and Discuss qualitatively the BCS theory of superconductivity for negligible resistance of metal at temperatures close to absolute zero.**

Super conductivity is the phenomenon observed in some metals and materials. Kammerlingh Onnes in 1911 observed that the electrical resistivity of pure mercury drops abruptly to zero at about 4.2K .This state is called super conducting state. The material is called superconductor .The temperature at which they attain superconductivity is called critical temperature  $T_c$ .

### BCS Theory

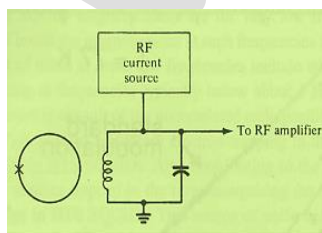
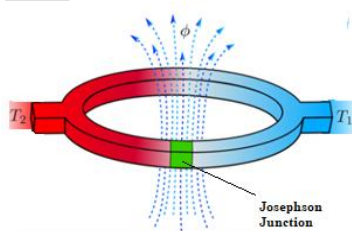
- Bardeen, Cooper and Schrieffer (BCS) in 1957 explained the phenomenon of superconductivity based on the formation of cooper pairs. It is called BCS theory. It is a quantum mechanical concept.

- When a current flow in a superconductor, electrons come near a positive ion core of lattice, due to attractive force. The ion core also gets displaced from its position, which is called lattice distortion. The lattice vibrations are quantized in a term called Phonons.
- Now an electron which comes near that place will interact with the distorted lattice. This tends to reduce the energy of the electron. It is equivalent to interaction between the two electrons through the lattice. This leads to the formation of cooper pairs.
- “Cooper pairs are a bound pair of electrons formed by the interaction between the electrons with opposite spin and momentum in a phonon field”.
- When the electrons flow in the form of cooper pairs in materials, they do not encounter any scattering and the resistance factor vanishes or in other words conductivity becomes infinity which is called as superconductivity.
- In superconducting state electron-phonon interaction is stronger than the coulomb force of attraction of electrons. Cooper pairs are not scattered by the lattice points. They travel freely without slow down as their energy is not transferred. Due to this they do not possess any electrical resistivity.

**Q.08 b Give the qualitative explanation of RF SQUID with the help of a neat sketch.**

SQUID is an acronym for Superconducting Quantum Interference Device.

- The single junction SQUIDS are also known as RF SQUIDS.
- The junction is shorted by superconductor path; therefore the voltage response is obtained by coupling the loop to a RF bias tank circuit.
- The RF (Radio Frequency) SQUID is a **one-junction SQUID loop that can be used as a magnetic field detector.**
- In this configuration, **the RF SQUID is inductively coupled to the inductance  $L_T$  of an  $L_C$  tank circuit.** The tank circuit is driven by an rf current, and the resultant rf voltage is periodic in the flux applied to the SQUID loop with period  $\Phi_0$ .



**Q.08 c Find the temperature at which there is 1% probability that a state with an energy 0.5 eV above Fermi energy is occupied.**

Given:  $E - E_f = 0.5 \text{ eV}$  ,  $T = ?$   
 $= 0.5 \times 10^{-19} \text{ J}$  ,  $f(E) = 1\% = \frac{1}{100}$

$$f(E) = \frac{1}{e^{\frac{E-E_f}{kT}} + 1} \Rightarrow \frac{1}{100} = \frac{1}{e^{\frac{0.5 \times 10^{-19}}{1.38 \times 10^{-23} T}} + 1}$$

$$\frac{1}{100} = \frac{1}{\left[ e^{\frac{5797}{T}} + 1 \right]} \Rightarrow 100 = e^{\frac{5797}{T}} + 1$$

$$99 = e^{\frac{5797}{T}}$$

$$\ln 99 = \frac{5797}{T}$$

$$T = \frac{5797}{4.595}$$

$$T = 1261.5 \text{ K}$$

## Module-5

**Q.09a Elucidate the importance of size & scale and weight and strength in animations.**

The size and scale of characters often play a central role in a story's plot.

We cannot imagine a Superman be without his height and bulging biceps? Some characters, like the Incredible Hulk, are even named after their body types.

We can equate large characters with weight and strength, and smaller characters with agility and speed. As it is noticeable in real life scenarios that, larger people and animals do have a larger capacity for strength, while smaller critters can move and maneuver faster than their large counterparts.

When designing characters, we can run into different situations having to do with size and scale, such as:

1. Human or animal-based characters that are much larger than we see in our everyday experience. Superheroes, Greek gods, monsters,
2. Human or animal-based characters that are much smaller than we are accustomed to, such as fairies and elves.
3. Characters that need to be noticeably larger, smaller, older, heavier, lighter, or more energetic than other characters.



4. Characters that are child versions of older characters. An example would be an animation featuring a mother cat and her kittens. If the kittens are created and animated with the same proportions and timing as the mother cat, they won't look like kittens; they'll just look like very small adult cats.

### Proportion and Scale

Creating a larger or smaller character is not just a matter of scaling everything about the character uniformly.

Example: When we scale a cube, its volume changes much more dramatically than its surface area. Let us say each edge of the cube is 1 unit length. The area of one side of the cube is 1 square unit, and the volume of the cube is 1 cubed unit.

If we double the size of the cube along each dimension, its height increases by 2 times, the surface area increases by 4 times, and its volume increases by 8 times. While the area increases by squares as we scale the object, the volume changes by cubes.

### Weight and strength

Body weight is proportional to volume. The abilities of our muscles and bones, however increase by area because their abilities depend more on cross-sectional area than volume.

To increase a muscle or bone's strength, we need to increase its cross-sectional area.

To double a muscle's strength, for example, you would multiply its width by  $\sqrt{2}$ .

To triple the strength, multiply the width by  $\sqrt{3}$ .

Since strength increases by squares and weight increases by cubes, the proportion of a character's weight that it can lift does not scale proportionally to its size.

Let us take an example of a somewhat average human man. At 6 feet tall, he weighs 180 pounds and can lift 90 pounds. He can lift half his body weight.

If we scale up the body size by a factor of 2, the weight increases by a factor of 8. Such a character could then lift more weight. But since he weighs more than 8 times more than he did before, he cannot lift his arms and legs as easily as a normal man. Such a giant gains strength, but loses agility.

### Q.09b Mention the general pattern of monte Carlo method and hence determine the value of $\pi$ .

#### Monte-Carlo Method:

Monte Carlo Simulation, also known as the Monte Carlo Method or a multiple probability simulation, is a mathematical technique, which is used to estimate the possible outcomes of an uncertain event. The Monte Carlo Method was invented by John von Neumann and Stanislaw Ulam during World War II to improve decision-making under uncertain conditions. It was named after a well-known casino town, called Monaco.

The statistical method of understanding complex physical or mathematical systems by using randomly generated numbers as input into those systems to generate a range of solutions.

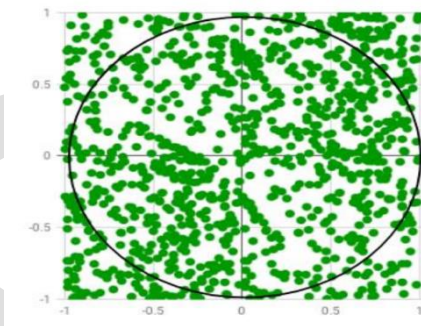
How to use Monte Carlo methods

1. Define a domain of possible inputs
2. Generate inputs randomly from a probability distribution over the domain
3. Perform a deterministic computation on the inputs
4. Aggregate the results

### Estimation of Pi

- The idea is to simulate random (x, y) points in a 2-D plane with the domain as a square of side 2r units centered on (0,0).
- Imagine a circle inside the same domain with the same radius r and inscribed into the square.
- We then calculate the ratio of the number of points that lay inside the circle and the total number of generated points.

Refer to the image below:



We know that the area of the circle  $\pi r^2$ , while that of square  $4r^2$ . The ratio of these two areas is as follows:

$$\frac{\text{area of the circle}}{\text{area of the square}} = \frac{\pi r^2}{4r^2} = \frac{\pi}{4}$$

Now for a very large number of generated points  $\frac{\text{no of points generated inside the circle}}{\text{no of points generated inside the square}} = \frac{\pi}{4}$

$$4 * \frac{\text{no of points generated inside the circle}}{\text{no of points generated inside the square}} = \pi$$

### Q.09c Describe the calculation of Push time and stop time with examples.

**Push time:** The number of frames required to move from 'crouch position' to 'Take off position'.

**Stop time:** The number of frames required to move from 'In air position' to 'Landing position'.

$$JM = \frac{\text{Jump Time}}{\text{Push Time}} = \frac{\text{Jump Height}}{\text{Push Height}} = \frac{\text{Push Acceleration}}{\text{Jump Acceleration}} = \frac{\text{Push Acceleration}}{\text{Gravitational Acceleration}}$$

#### Example:

Push Time: 5 frames

Push Height: 0.4m

Stop Height: 0.5m

Stop Time =  $(5 * 0.5) / 0.4 = 6$  frames

OR

### Q.10a Sketch and explain the motion graphs for linear, easy ease, easy ease in and easy ease out cases of animation.

Timing animation refers to the duration of an action.

In animation, timing of action consists of placing objects or characters in particular locations at specific frames to give the illusion of motion.

**Line of action:** Individual drawings or poses have a **line of action**, which indicates the visual flow of action at that single image.



An object moving with linear motion might speed up, slow down or move with a constant speed and it follows a linear path.

- 1) **Uniform motion:** It is the easiest to animate because the distance the object travels between frames is always the same. The object moves the same distance between consecutive frames. The longer the distance between frames, the higher the speed.



## 2) Ease out / Speed up

The object is speeding up i.e its speed increases gradually, often from a still position. The frames are located such that, initially the frames are closely spaced with gradual increase in the spacings.



## 3) Ease in/ Slowed down.

The object is slowing down, its speed decreases gradually often in preparation for stopping. The frames are located such that, initially the frames are widely spaced with gradual decrease in the spacings of the frames. Timing animation refers to the duration of an action.



## 4) Ease out- Ease in or Ease-Ease.

It is the combination of speed up and slowed down. That is the object initially gets speed up initially and finally comes to still position with slowing down. In the beginning the frames are located such that, initially the frames are closely spaced with gradual increase in the spacings up to middle position.

From the middle position onwards, the frames are widely spaced with gradual decrease in the spacings of the frames towards the still position.



## Q.10b Discuss modeling the probability for proton decay.

Proton decay is a rare type of radioactive decay of nuclei containing excess protons, in which a proton is simply ejected from the nucleus. The mechanism of the decay process is very similar to alpha decay. Proton decay is also a quantum tunneling process.

### Modeling the Probability for Proton Decay

The probability of observing a proton decay can be estimated from the nature of particle decay and the application of Poisson Statistics. The number of protons  $N$  can be modeled by the decay equation

$$N = N_0 e^{-\lambda t}$$

Where:

$N_0$ : is the initial quantity of the element

$\lambda$ : is the radioactive decay constant

$t$ : is time

$N(t)$ : is the quantity of the element remaining after time  $t$ .

Here  $\lambda = 1/t = 10^{-33}/\text{year}$  is the probability that any given proton will decay in a year.

Since the decay constant  $\lambda$  is so small, the exponential can be represented by the first two terms of the Exponential Series.

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots$$

$$e^{-\lambda t} = 1 - \lambda t$$

$$N = N_0(1 - \lambda t)$$

Most recently the experiment on proton decay has been done by Super Kamiokande, Japan which started observation in 1996. It is a large water Cherenkov detector which is the most sensitive detector in the world used to examine proton decay with the huge source with  $7.5 \times 10^{33}$  protons

For one year of observation, the number of expected proton decays is then

$$N_0 - N = N_0 \lambda t$$

$$= (7.5 \times 10^{33} \text{ protons})(10^{-33} / \text{year})(1 \text{ year})$$

$$N_0 - N = 7.5$$

$$N_0 - N = N_0(1 - \lambda t)$$

$$= (7.5 \times 10^{33} \text{ protons})(10^{-33} / \text{year})(1 \text{ year})$$

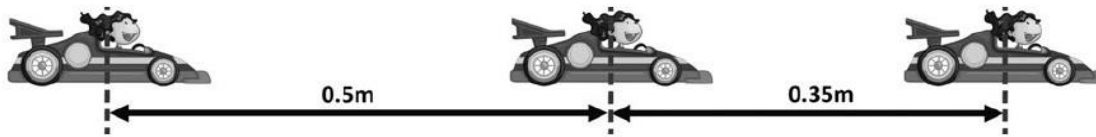
$$N_0 - N = 7.5$$

Proton decay has not been detected experimentally till now probably because of fact that the event is extremely rare. Assuming that  $\lambda = 3$  observed decays per year is mean, then the Poisson distribution function tells us that the probability for zero observations of decay is

$$P(K) = \frac{\lambda^K e^{-\lambda}}{K!} = \frac{3^0 e^{-3}}{0!} = 0.05$$

This low probability for a null result suggests that the proposed lifetime of  $10^{33}$  years is too short.

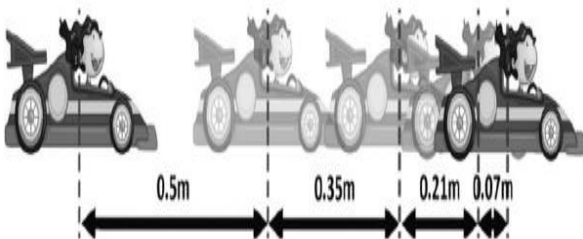
**Q.10c A slowing-in object in an animation has a first frame distance 0.5m and the first slow in frame 0.35m. Calculate the base distance and the number of frames in sequence.**



One of the features of the Odd Rule is that the base distance is always half the difference between any two adjacent distances.

$$\text{Consecutive Frame Multiplier} = \frac{\text{First Distance}}{\text{Base Distance}} = \frac{0.5}{0.07} = 7$$

Thus, Consecutive Frame Multiplier '7' Corresponds to '4' Frames



Frame #	Consecutive frame multiplier	Distance from previous frame
1	7	$7 * 0.07\text{m} = 0.5\text{m}$
2	5	$5 * 0.07\text{m} = 0.35\text{m}$
3	3	$3 * 0.07\text{m} = 0.21\text{m}$
4	1	$1 * 0.07\text{m} = 0.07\text{m}$

\*\*\*\*\* ALL THE BEST \*\*\*\*\*