Sujay Nayak Anil
ECE Department
NYU Tandon School of Engineering
N10462423 (san440)

INTRODUCTION

The aim of this report is to demonstrate the implementation of the Extended Kalman Filter to estimate the state of the robot. The state of the system is determined based on the information of the previous state of the system and the sensor data obtained from the onboard IMU. Since the sensory data obtained from the onboard IMU is in the body frame of the robot, the dynamics of the system will be non-linear. Hence, the need for the Extended Kalman Filter becomes evident. In the following implementation, the change in the state of the system is determined from the sensor data obtained from the onboard IMU. A process model is designed to predict the state of the robot from this sensory data and the information of the previous state of the system. Once the state of the system is predicted, the measurement data from the Vicon system is used to make the appropriate calibration to the predicted state to determine the estimate of the state of the system.

IMPLEMENTATION

The state of the system to be determined:

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \\ \mathbf{x}_4 \\ \mathbf{x}_5 \end{bmatrix} = \begin{bmatrix} \mathbf{p} \\ \mathbf{q} \\ \dot{\mathbf{p}} \\ \mathbf{b}_g \\ \mathbf{b}_a \end{bmatrix} = \begin{bmatrix} \text{position} \\ \text{orientation} \\ \text{linear velocity} \\ \text{gyroscope bias} \\ \text{accelerometer bias} \end{bmatrix}$$

Since the system is continuous, the state model can be defined as:

x_dot =

$$\begin{pmatrix} V_x \\ V_y \\ V_z \\ \dot{R} \\ \dot{P} \\ \dot{Y} \\ \mathrm{Acc}_x \\ \mathrm{Acc}_y \\ \mathrm{Acc}_z \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Here, the last 6 rows are zero because the gyroscope bias and accelerometer bias are assumed not to vary for short intervals of time.

Email : *san440@nyu.edu*

Uploaded on Mar 25th 2020

The Acc_x, Acc_y, Acc_z and R', P', Y' are the inputs to the system and are not directly available for computation. This information is obtained with the onboard IMU. Since the onboard IMU observes these parameters with the help of its sensors, it introduces some noise into the data during this observation. And the fact that onboard IMU observes acceleration and angular velocity in its body frame, there exists a need for some transformation to convert this data into Acc_x, Acc_y, Acc_z and R', P', Y'.

The pose of the robot is given by :
$\mathbf{q} = [\phi, \theta, \psi]^T = [\text{roll, pitch, yaw}]^T$

Transformation from the sensory data to Acc_x, Acc_y, Acc_z and R', P', Y' :

$[\,R', P', Y'\,]^T =$
$G(\mathbf{x}_2)^{-1}(\omega_m - \mathbf{x}_3 - \mathbf{n}_g)$

Where w_m - is the angular velocity observed by the gyroscope
X_3 - Gyroscope bias
N_g - Noise introduced by the gyroscope

$[\,Acc_x, Acc_y, Acc_z\,]^T =$
$\mathbf{g} + R(\mathbf{x}_2)(\mathbf{a}_m - \mathbf{x}_5 - \mathbf{n}_a)$

Where a_m - is the acceleration observed by the accelerometer
X_3 - Accelerometer bias
N_g - Noise introduced by the accelerometer

Where,
$G(x_2) =$
$$\begin{bmatrix} c\theta & 0 & -c\phi s\theta \\ 0 & 1 & s\phi \\ s\theta & 0 & c\phi c\theta \end{bmatrix}$$
$R(x_2) =$
$$\begin{bmatrix} c\psi c\theta - s\phi s\psi s\theta & -c\phi s\psi & c\psi s\theta + c\theta s\phi s\psi \\ c\theta s\psi + c\psi s\phi s\theta & c\phi c\psi & s\psi s\theta - c\psi c\theta s\phi \\ -c\phi s\theta & s\phi & c\phi c\theta \end{bmatrix}$$

Part A. Process model

At the beginning, the information on the initial state of the robot is provided to the filter. For each stage, the filter has two subroutines that are invoked to predict and estimate the current state based on the measured data. In the prediction routines, the angular velocity and the acceleration data are first extracted from the incoming sensor packet received via wireless network from the robot. Then with the help of the information of the mean and covariance of the previous state variables & the sensory data of the inputs to the state model and the state model of the system, the current state of the system is predicated. Here, the predicted mean of the current state is just the integration of the state model, valued at the previous mean and the input, with the previous mean as its initial condition over dt.

$$\dot{x} = f(x, u, n)$$

$$\bar{\mu}_t = \mu_{t-1} + \delta t\, f(\mu_{t-1}, u_t, 0)$$
$$\bar{\Sigma}_t = F_t\, \Sigma_{t-1}\, F_t^T + V_t\, Q_d\, V_t^T$$

Email : *san440@nyu.edu*

Uploaded on Mar 25th 2020

Where,
dt - time difference between the current and the previous states
$F_t = I + A*dt$

$$A_t = \frac{\partial f}{\partial x}\bigg|_{\mu_{t-1}, u_t, 0}$$

$$U_t = \frac{\partial f}{\partial n}\bigg|_{\mu_{t-1}, u_t, 0}.$$

A and U are the gradient of the function f(x,u,n) w.r.t x and n respectively. These gradients are used due the linearization of the nonlinear system f(x,u,n).

For calculating the predicted covariance, we account for how the current system modifies the previous covariance and covariance of the added noise to the system and then multiply it with the corresponding covariances to obtain the predicted covariance.

B. Part - I

The position and pose of the robot measured by the Vicon system is used to adjust the estimate of the predicted state.

$Z = C*X + V_t$

$$z = \begin{pmatrix} x \\ y \\ z \\ R \\ P \\ Y \end{pmatrix}$$

Here, the measurement is a linear function of the state, x and the noise, v.

We can either use the regular kalman filter equations or extended kalman filter equations to find the current estimate of the system.

In the project submitted, extended kalman filter equations are used to find the current estimate of the system. For that, we need to find C and W :

$$z_t = g(x_t, v_t)$$

$$\mu_t = \bar{\mu}_t + K_t\,(z_t - g(\bar{\mu}_t, 0))$$

$$\Sigma_t = \bar{\Sigma}_t - K_t\, C_t\, \bar{\Sigma}_t$$

$$K_t = \bar{\Sigma}_t\, C_t^T\,(C_t\,\bar{\Sigma}_t\, C_t^T + W_t\, R\, W_t^T)^{-1}$$

Here, the kalman gain, k_t is used to adjust the predicted mean and covariance of the current state to obtain the final estimate of the current state of the system.

C . Part - II

The positional velocity of the robot measured by the Vicon system is used to adjust the estimate of the predicted state.

$Z = C*X + V_t$

$$z = \begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix}$$

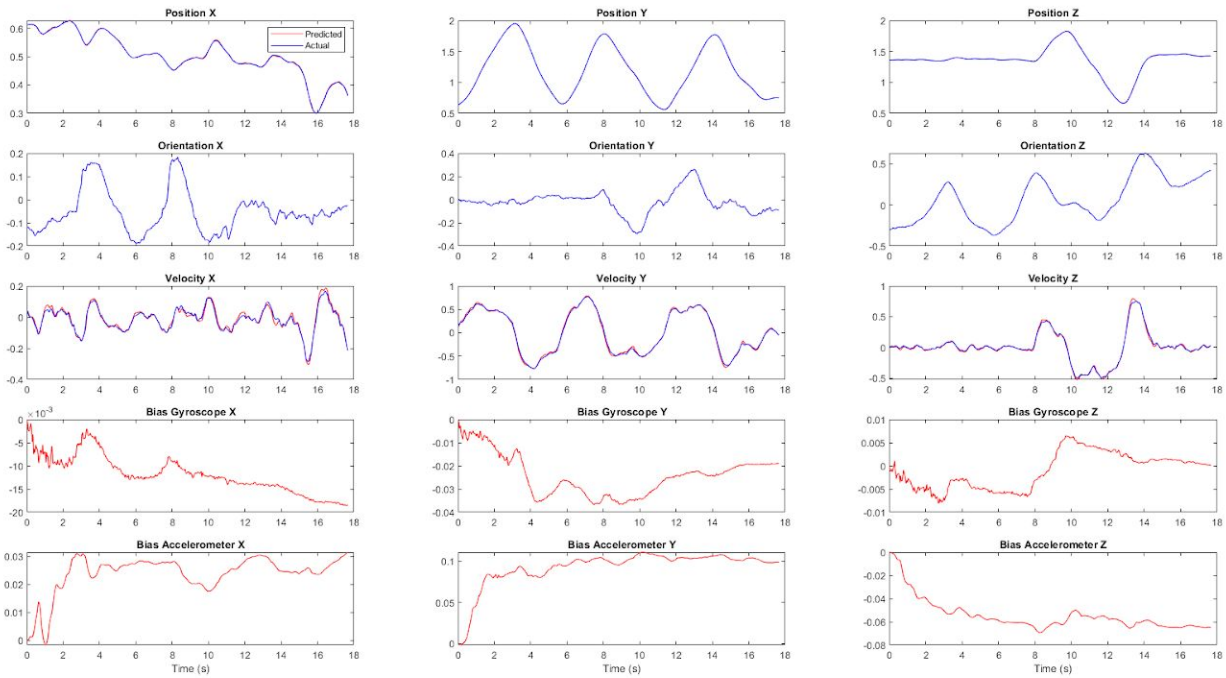
Here, the measurement is a linear function of the state, x and the noise, v.

The extended Kalman filter equation is used to find the current estimate of the system.

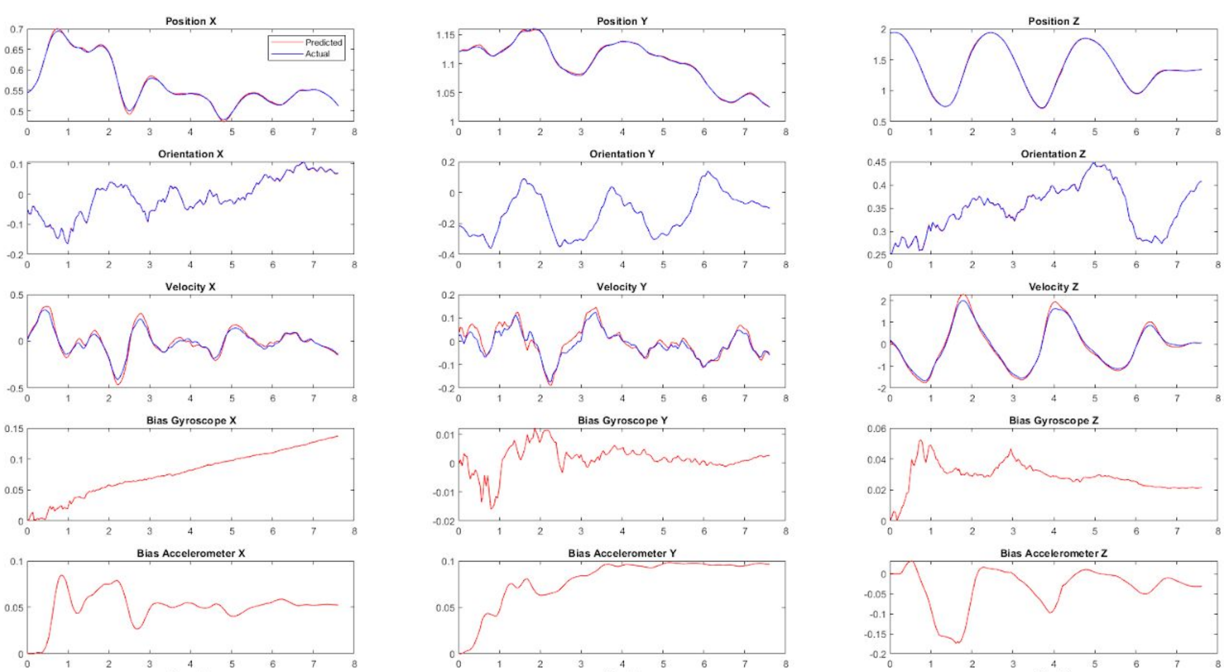
Email : *san440@nyu.edu*

Uploaded on Mar 25th 2020

Plot for Dataset 1 using position and pose measurements :



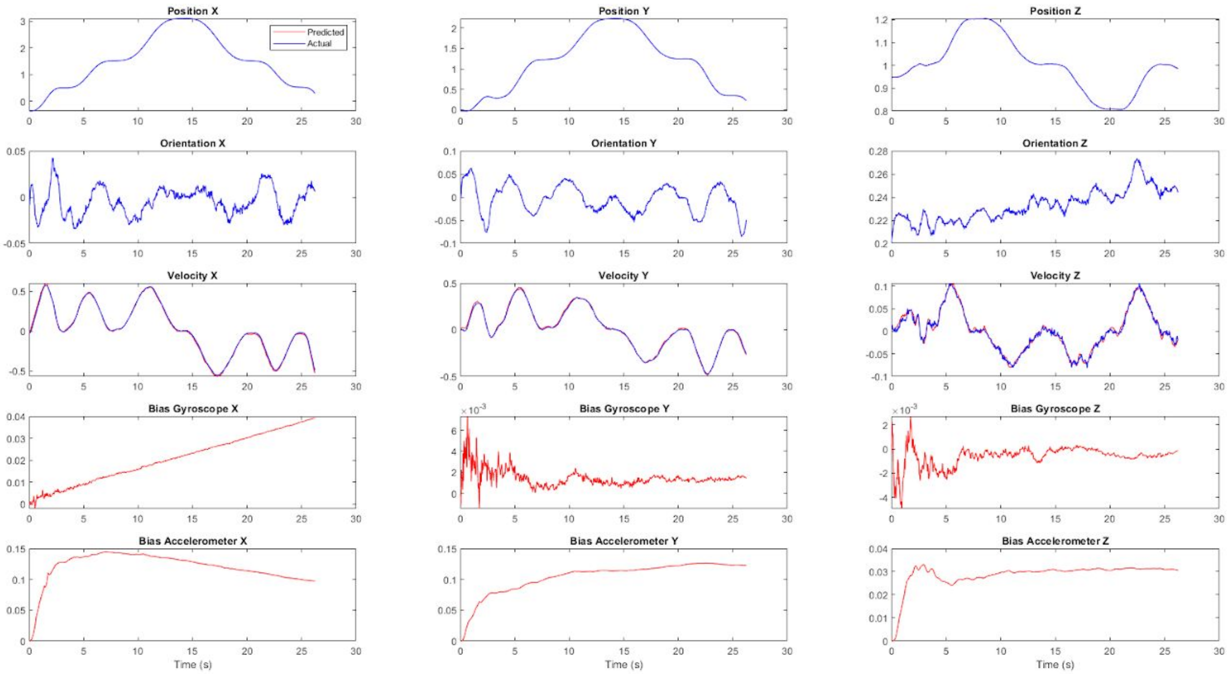
Plot for Dataset 4 using position and pose measurements :



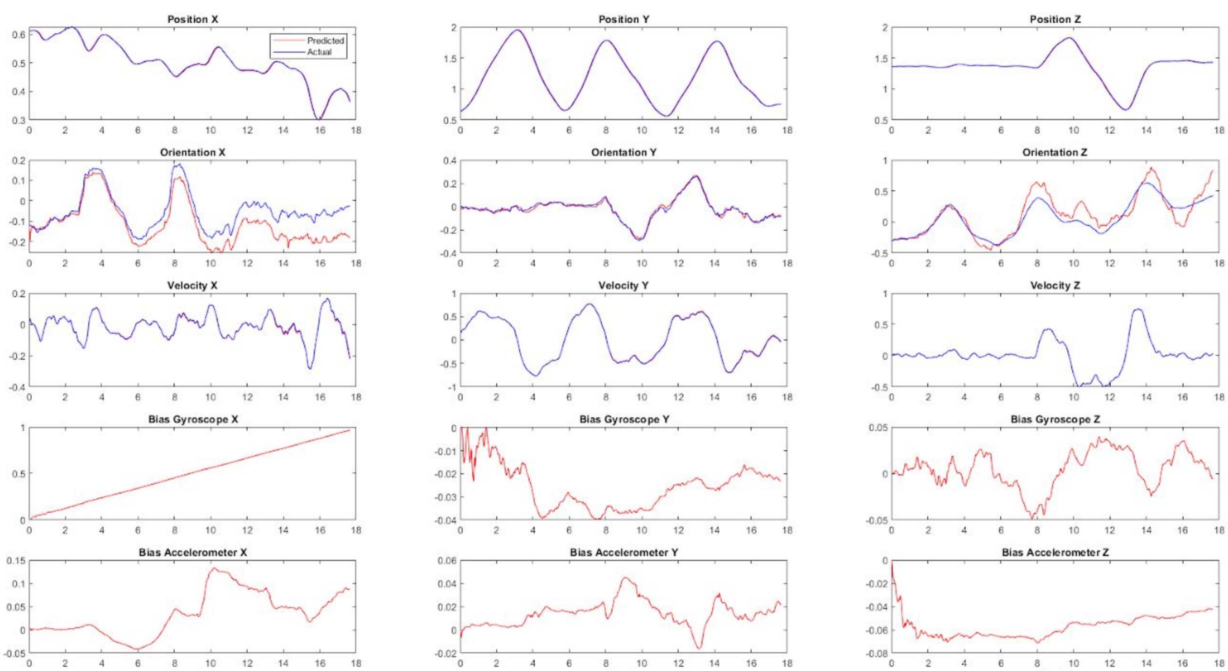
Email : *san440@nyu.edu*

Uploaded on Mar 25th 2020

Plot for Dataset 9 using position and pose measurements :

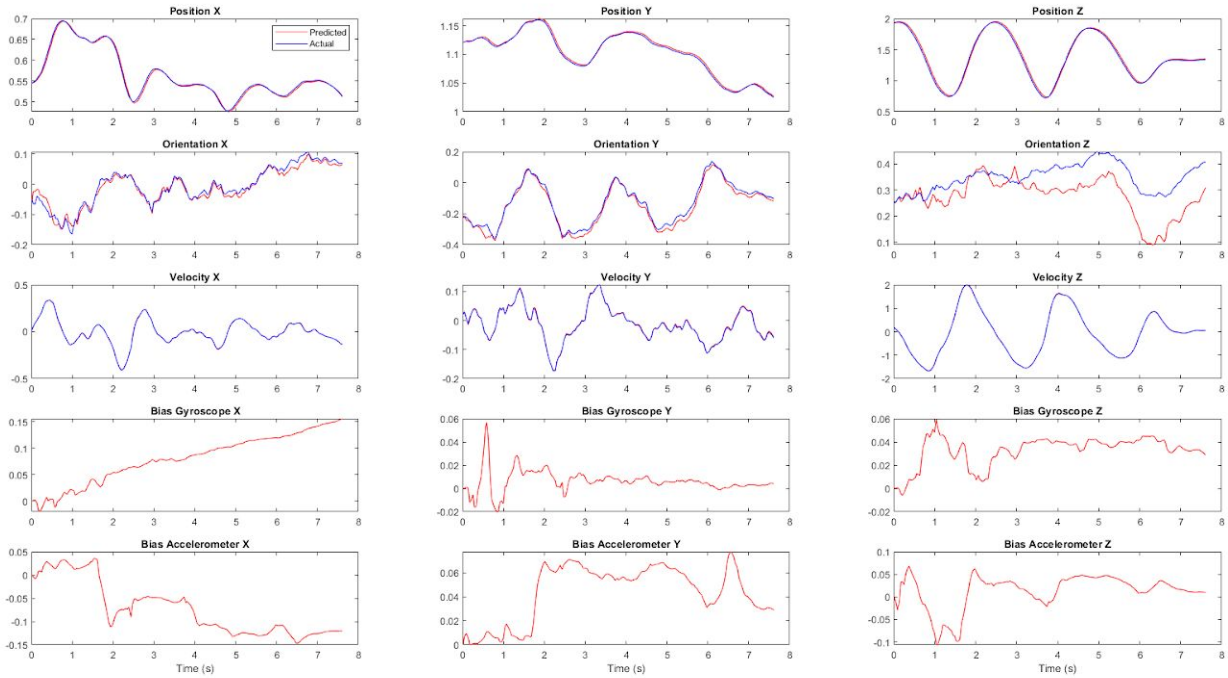


Plot for Dataset 1 using velocity measurements :

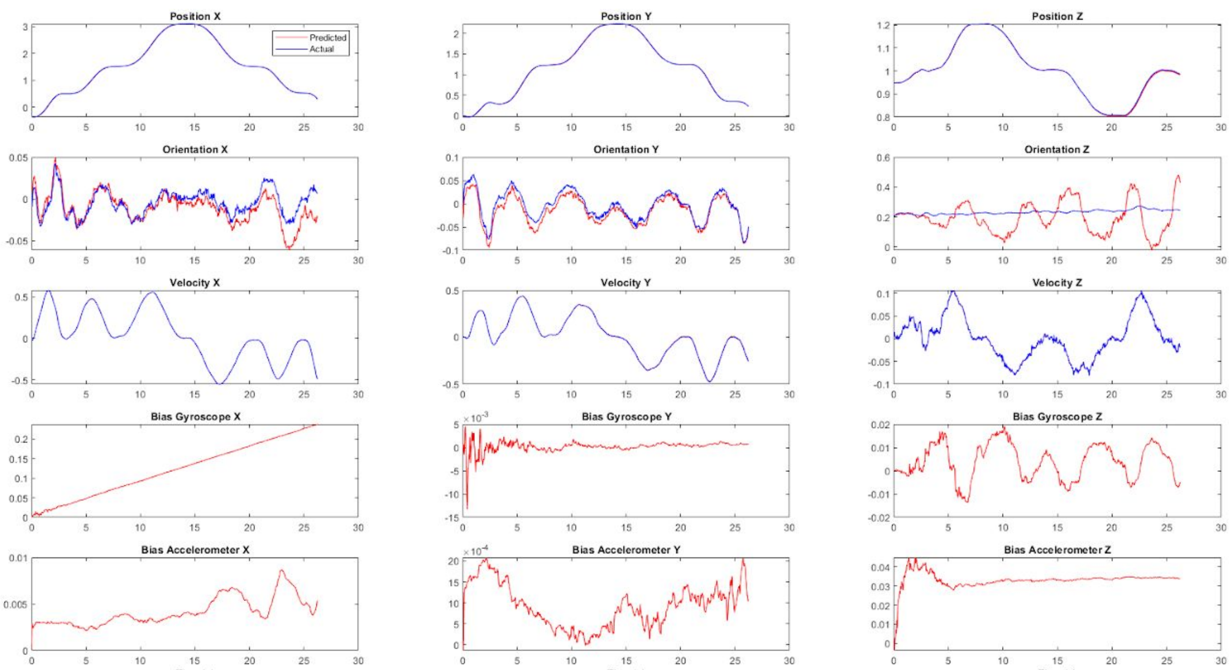


Email : *san440@nyu.edu* Uploaded on Mar 25th 2020

Plot for Dataset 4 using velocity measurements :



Plot for Dataset 9 using velocity measurements :



Email : *san440@nyu.edu*

Conclusion

For part - I , we can observe that the predicted state of the system is almost ideally following the actual value for position and orientation. But, we can also observe that there exists some distortion in the predicted velocity compared to the measured velocity from the Vicon system.

This is a consequence of choosing only the position and pose of the robot for the measurement model. It can be avoided by including velocity measurement into the measurement model.

For part - II , we can observe that the predicted state of the system is almost ideally following the actual value for the velocity. But, we can also observe that there exists some distortion in the predicted position and orientation compared to the measured position and pose from the Vicon system.

This is a consequence of choosing only the velocity of the robot for the measurement model. It can be avoided by including position and pose measurement into the measurement model.

Email : *san440@nyu.edu*
Uploaded on Mar 25th 2020