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INTRODUCTION

The aim of this report is to demonstrate the implementation of the Unscented Kalman Filter to estimate the state of the robot. The state of the system is determined based on the information of the previous state of the system and the sensor data obtained from the onboard IMU. Since the sensory data obtained from the onboard IMU is in the body frame of the robot, the dynamic model of the system will be non-linear. Hence, the need for linearization becomes evident. In the following implementation, Unscented transform is used to preserve the accurate mean and covariance of the states across the non-linear transformation in the state space model. The change in the state is determined from the sensor data obtained from the onboard IMU. A process model is designed to predict the state of the robot from this sensory data and the information of the previous state of the system. Once the state of the system is predicted, velocity and pose estimate from the optical data is used to make the appropriate calibration to the predicted state to obtain the estimate of the current

IMPLEMENTATION

The state of the system to be determined:

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \\ \mathbf{x}_4 \\ \mathbf{x}_5 \end{bmatrix} = \begin{bmatrix} \mathbf{p} \\ \mathbf{q} \\ \dot{\mathbf{p}} \\ \mathbf{b}_g \\ \mathbf{b}_a \end{bmatrix} = \begin{bmatrix} \text{position} \\ \text{orientation} \\ \text{linear velocity} \\ \text{gyroscope bias} \\ \text{accelerometer bias} \end{bmatrix}$$

Since the system is continuous, the state model can be defined as:

$$\texttt{x\_dot} = \begin{pmatrix} V_x \\ V_y \\ V_z \\ \dot{R} \\ \dot{P} \\ \dot{Y} \\ \mathrm{Acc}_x \\ \mathrm{Acc}_y \\ \mathrm{Acc}_z \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Here, the last 6 rows are zero because the gyroscope bias and accelerometer bias are assumed not to vary for short intervals of time.

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state of the system.

The Acc_x, Acc_y, Acc_z and R', P', Y' are the inputs to the system and are not directly available for computation. This information is obtained with the onboard IMU. Since the onboard IMU observes these parameters with the help of its sensors, it introduces some noise into the data during this observation. And the fact that onboard IMU observes acceleration and angular velocity in its body frame, there exists a need for some transformation to convert this data into Acc_x, Acc_y, Acc_z and R', P', Y'.

The pose of the robot is given by :
$\mathbf{q} = [\phi, \theta, \psi]^T = [\text{roll, pitch, yaw}]^T$

Transformation from the sensory data to Acc_x, Acc_y, Acc_z and R', P', Y' :

$[R', P', Y']^T =$
$G(\mathbf{x}_2)^{-1}(\omega_m - \mathbf{x}_3 - \mathbf{n}_g)$

Where w_m - is the angular velocity observed by the gyroscope
X_3 - Gyroscope bias
N_g - Noise introduced by the gyroscope

$[Acc_x, Acc_y, Acc_z]^T =$
$\mathbf{g} + R(\mathbf{x}_2)(\mathbf{a}_m - \mathbf{x}_5 - \mathbf{n}_a)$

Where a_m - is the acceleration observed by the accelerometer
X_3 - Accelerometer bias
N_g - Noise introduced by the accelerometer

Where,
$G(x_2) =$

$$\begin{bmatrix} c\theta & 0 & -c\phi s\theta \\ 0 & 1 & s\phi \\ s\theta & 0 & c\phi c\theta \end{bmatrix}$$

$R(x_2) =$

$$\begin{bmatrix} c\psi c\theta - s\phi s\psi s\theta & -c\phi s\psi & c\psi s\theta + c\theta s\phi s\psi \\ c\theta s\psi + c\psi s\phi s\theta & c\phi c\psi & s\psi s\theta - c\psi c\theta s\phi \\ -c\phi s\theta & s\phi & c\phi c\theta \end{bmatrix}$$

Part A. Process model

At the beginning, the information on the initial state of the robot is provided to the filter. For each stage, the filter has two subroutines that are invoked to predict and estimate the current state based on the measured data. In the prediction routines, the angular velocity and the acceleration data are first extracted from the incoming sensor packet received via wireless network from the robot. Then with the help of the information of the mean and covariance of the previous state variables & the sensory data of the inputs to the state model and the state model of the system, the current state of the system is predicated.

Part B. Prediction Step

A set of sigma points are generated from the information of the mean and covariance of the previous state variables.

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$$\mathcal{X}^{(0)} = \boldsymbol{\mu}_{aug}, \qquad \mathcal{X}^{(i)} = \boldsymbol{\mu}_{aug} \pm \sqrt{n' + \lambda'}\left[\sqrt{\boldsymbol{\Sigma}_{aug}}\right]_i$$

Here, the augmented mean and augmented covariance are considered. This is to accommodate the non-linear contribution of the noise parameters in the dynamics of the system. The augmented mean and augmented covariance allows the noise parameters to be considered as one among the states of the system. The augmented mean and augmented covariance is given by :

$$\boldsymbol{\mu}_{aug} = \begin{pmatrix} \boldsymbol{\mu} \\ \mathbf{0} \end{pmatrix}$$

$$\boldsymbol{\Sigma}_{aug} = \begin{pmatrix} \boldsymbol{\Sigma} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{Q} \end{pmatrix}$$

where Q - Covariance of the noise

Once the sigma points are generated, they are passed through the non-linear dynamic model of the system to obtain the new set of sigma points of the predicted state.

$$\mathcal{Y}^{(i)} = g\left(\mathcal{X}^{(i),x}, \mathcal{X}^{(i),q}\right) \qquad i = 0, \dots 2n'$$

From the newly obtained sigma points of the predicted state, the mean and covariance of the predicted state is obtained as follows :

$$\boldsymbol{m}_U = \sum_{i=0}^{2n'} W_i^{(m)\prime} \mathcal{Y}^{(i)}$$

$$W_0^{(m)} = \frac{\lambda'}{n' + \lambda'} \qquad W_i^{(m)\prime} = \frac{1}{2(n' + \lambda')}$$

$$\boldsymbol{S}_U = \sum_{i=0}^{2n'} W_i^{(c)\prime}\left(\mathcal{Y}^{(i)} - \boldsymbol{m}_U\right)\left(\mathcal{Y}^{(i)} - \boldsymbol{m}_U\right)^T$$

$$W_0^{(c)\prime} = \frac{\lambda'}{n' + \lambda'} + (1 - \alpha^2 + \beta) \quad W_i^{(c)\prime} = \frac{1}{2(n' + \lambda')}$$

Part C. Part - I
Estimation using measured pose data

The position and pose of the robot measured with the help of the onboard camera on the quadrotor and identifiable markers placed in known coordinates in the world frame and the measured data is given by, Z :

$Z = C^*X + V_t$

$$z = \begin{pmatrix} x \\ y \\ z \\ R \\ P \\ Y \end{pmatrix}$$

Here, the measurement is a linear function of the state, x and the noise, v.

Here, we can either use the regular kalman filter equations to find the current estimate of the system.

$$\mu_t = \bar{\mu}_t + K_t(z_t - C\,\bar{\mu}_t)$$
$$\Sigma_t = \bar{\Sigma}_t - K_t\, C\, \bar{\Sigma}_t$$
$$K_t = \bar{\Sigma}_t\, C^T\, (C\,\bar{\Sigma}_t\, C^T + R)^{-1}$$

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Part D. Part - II
Estimation using measured velocity data

The velocity of the quadrotor is measured using feature tracking between the consecutive frames captured by the onboard camera on the quadrotor and the measured data is given by, Z :

$$Z = f(x) + n$$

Here, the measurement is a nonlinear function of the state, x and the noise, n.

$$Z = R_{cw} * [\, V_x \;\; V_y \;\; V_z \,]^T - S(\,camAngVel^T\,) * r_{c\,quad}$$

where R_{cw} - Rotation matrix of the world frame w.r.t the camera frame

$r_{c\,quad}$ - Position of the quadrotor w.r.t camera frame represented in camera frame

Since the measurement is a nonlinear function of the state, Unscented transform is used to obtain the equivalent of the measured state from the predicted mean and covariance.

Sigma points of the predicted states :

$$\chi_t^{(0)} = \bar{\boldsymbol{\mu}}_t$$

$$\chi_t^{(i)} = \bar{\boldsymbol{\mu}}_t \pm \sqrt{n+\lambda}\left[\sqrt{\bar{\boldsymbol{\Sigma}}_t}\right]_i$$

Sigma points of the equivalent measured state :

$$Z_t^{(i)} = g\left(\chi_t^{(i)}\right)$$

Mean and covariance of the equivalent measured state :

$$\boldsymbol{z}_{\mu,t} = \sum_{i=0}^{2n} W_i^{(m)} Z_t^{(i)}$$

$$\boldsymbol{S}_t = \sum_{i=0}^{2n} W_i^{(c)} \left(Z_t^{(i)} - \boldsymbol{z}_{\mu k}\right)\left(Z_t^{(i)} - \boldsymbol{z}_{\mu,t}\right)^T + \boldsymbol{R}_t$$

Cross covariance between the equivalent measured state and the predicted state :

$$\boldsymbol{C}_t = \sum_{i=0}^{2n} W_i^{(c)} \left(\chi_t^{(i),} - \bar{\boldsymbol{\mu}}_t\right)\left(Z_t^{(i)} - \boldsymbol{z}_{\mu k}\right)^T$$

Now, we use the unscented kalman filter equations to find the current estimate of the system.

$$\boldsymbol{\mu}_t = \bar{\boldsymbol{\mu}}_t + \boldsymbol{K}_t\,(\boldsymbol{z}_t - \boldsymbol{z}_{\mu,t})$$
$$\boldsymbol{\Sigma}_t = \bar{\boldsymbol{\Sigma}}_t - \boldsymbol{K}_t\,\boldsymbol{S}_t \boldsymbol{K}_t^{\,T}$$
$$\boldsymbol{K}_t = \boldsymbol{C}_t\,\boldsymbol{S}_t^{\,-1}$$

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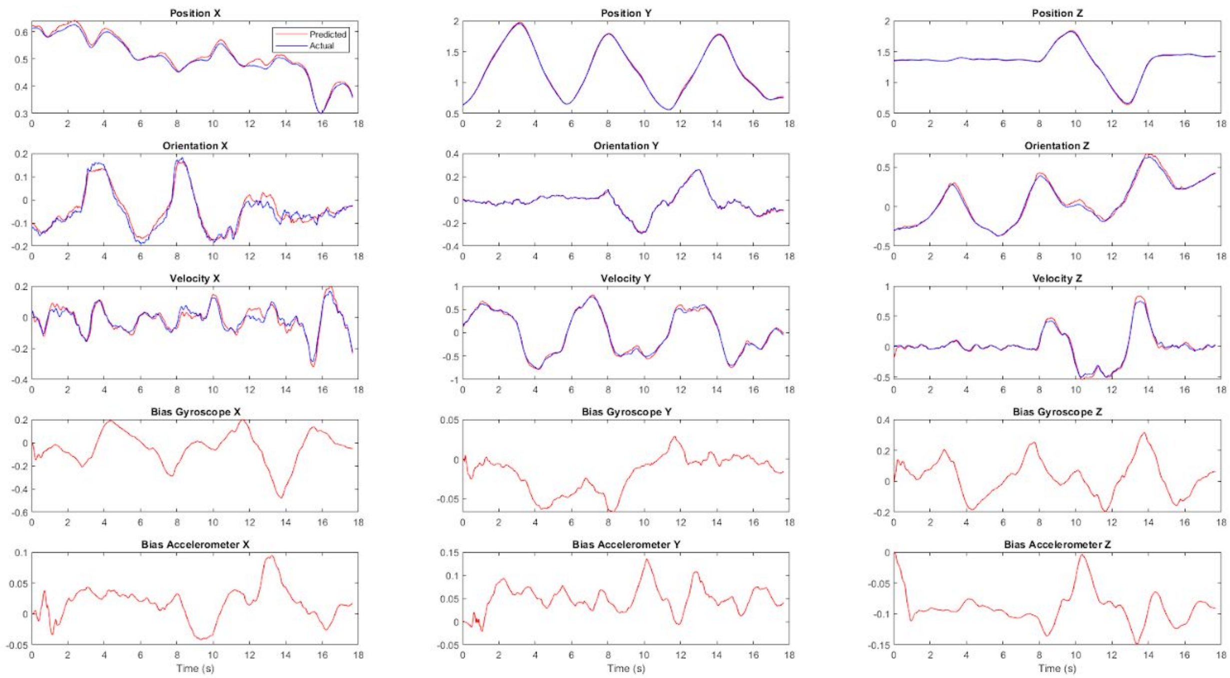
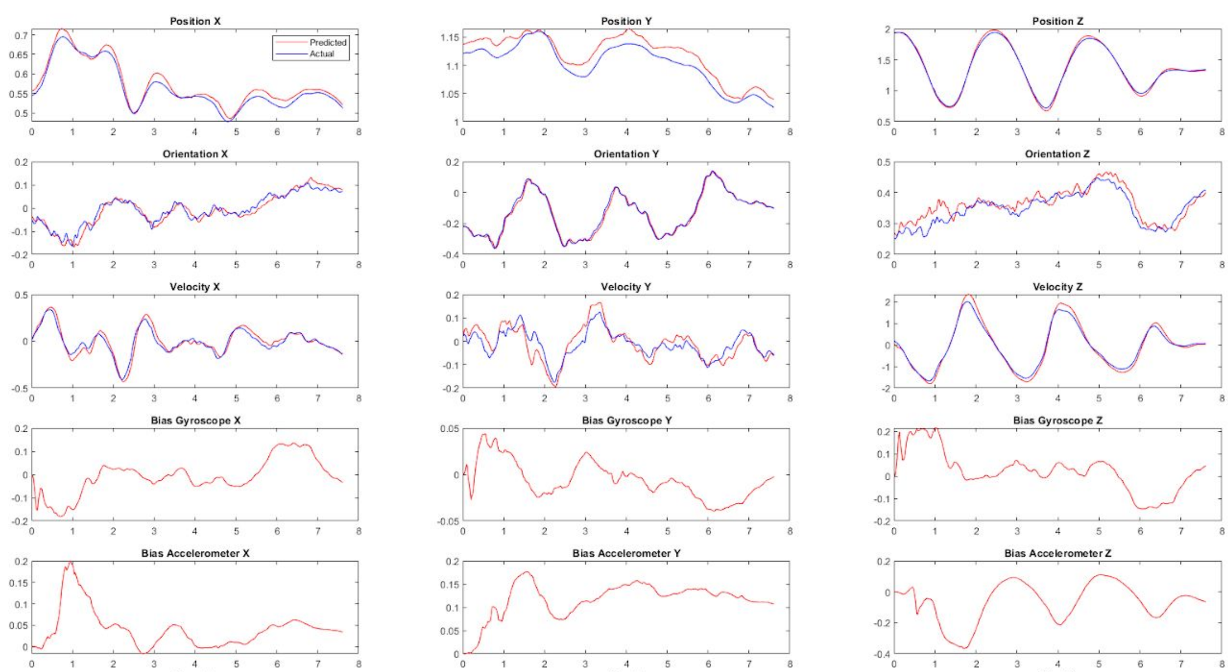
Plot for Dataset 1 using *pose measurement* :



Plot for Dataset 4 using *pose measurement* :



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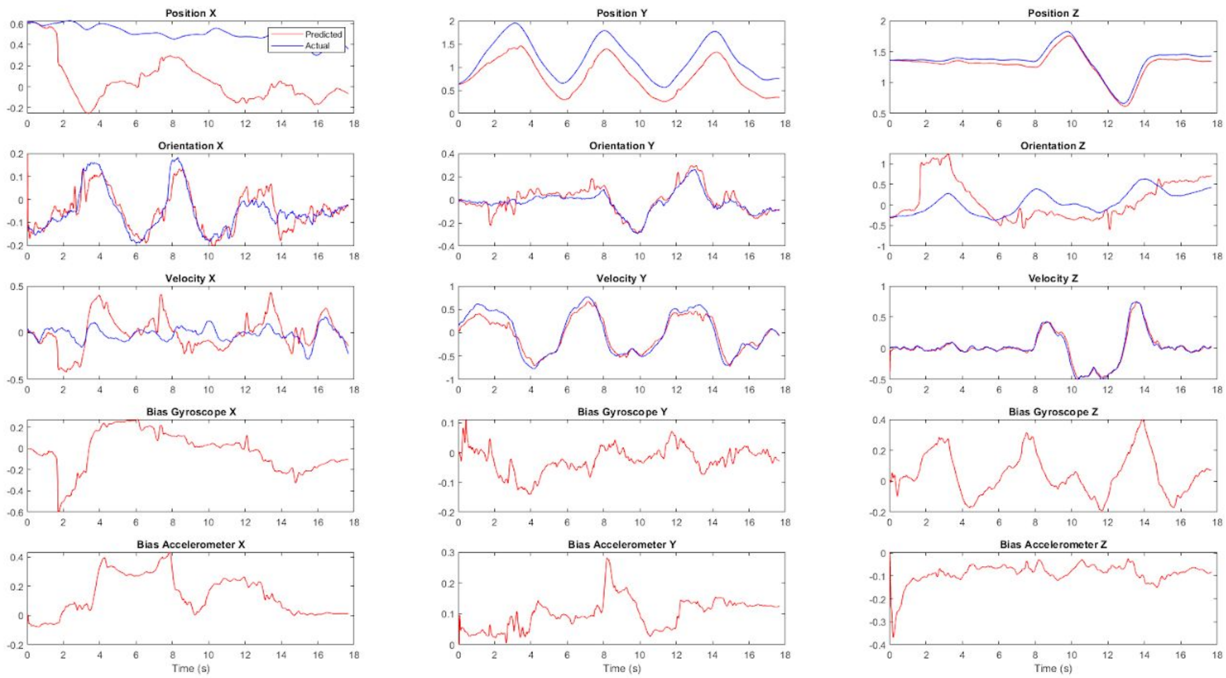
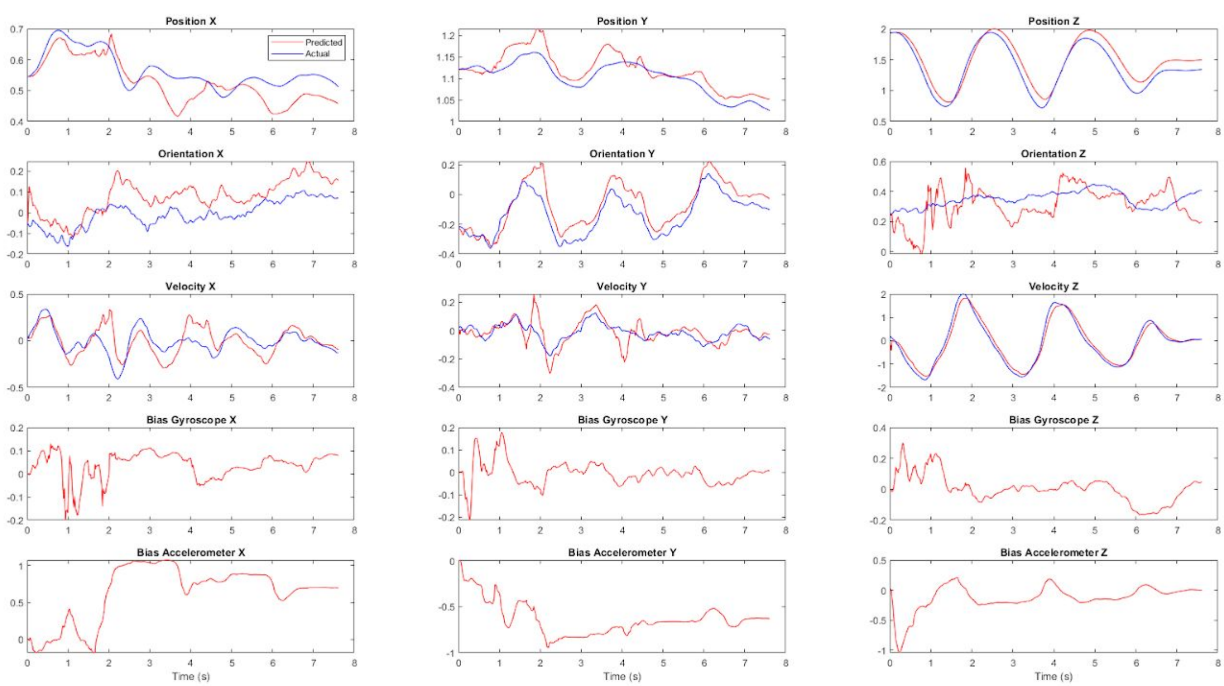
Plot for Dataset 1 using *velocity measurement* :



Plot for Dataset 4 using *velocity measurement* :



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Conclusion

For part - I , we can observe that the predicted state of the system is almost ideally following the actual value of pose and velocity of the quadrotor measured by the Vicon system.

For part - II , we can observe that the predicted state of the system is adequately following the actual value of pose and velocity of the quadrotor measured by the Vicon system. Also, we can observe a lot of distortion present in the predicted pose and velocity compared to the measured Vicon data. This is due to the fact that velocity measured with the help of the optical flow may be subjected to higher distortion than compared to the pose estimated using the identifiable markers.

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