MML Assignment - I

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1) Let V be the set of all ordered pairs of real numbers over the field of real numbers defined as $(x_1, x_2, x_3) + (y_1, y_2, y_3) = (x_1+y_1, x_2+y_2, x_3+y_3)$ and $a(x_1, x_2, x_3) = (ax_1, ax_2, ax_3)$ check wether it is a vector space or not.

Sol (i) Closure under Addition

V is the set of all ordered pair of real numbers for any $(x_1, x_2, x_3) \in V$, $(y_1, y_2, y_3) \in V$, their sum $(x_1+y_1, x_2+y_2, x_3+y_3)$ is an ordered triple of real numbers thus belongs to V

$\therefore$ This axiom is satisfied

(ii) Commutative under Addition;

for any $(x_1, x_2, x_3) \in V, (y_1, y_2, y_3) \in V$

$(x_1, x_2, x_3) + (y_1, y_2, y_3) = (x_1+y_1, x_2+y_2, x_3+y_3)$

Since addition of real numbers is commutative

$(x_1+y_1, x_2+y_2, x_3+y_3) = (y_1+x_1, y_2+x_2, y_3+x_3) = (y_1, y_2, y_3) + (x_1, x_2, x_3)$

$\therefore$ This axiom is satisfied

(iii) Associativity under addition;

for any $(x_1, x_2, x_3) \in V$ $(y_1, y_2, y_3) \in V$, $(z_1, z_2, z_3) \in V$

$((x_1, x_2, x_3) + (y_1, y_2, y_3)) + (z_1, z_2, z_3) = (x_1+y_1, x_2+y_2, x_3+y_3) + (z_1, z_2, z_3)$

$= ((x_1+y_1+z_1), (x_2+y_2+z_2), (x_3+y_3+z_3))$

$\Rightarrow (x_1, x_2, x_3) + ((y_1, y_2, y_3) + (z_1, z_2, z_3)) = (x_1, x_2, x_3) + (y_1+z_1, y_2+z_2, y_3+z_3)$

$= (x_1+y_1+z_1, x_2+y_2+z_2, x_3+y_3+z_3)$

$\therefore$ This axiom is satisfied

(iv) Identity :

zero vector is $(0,0,0) \in V$ since for $(x_1, x_2, x_3) \in V$
$(x_1, x_2, x_3) + (0,0,0) = (x_1, x_2, x_3)$
$\therefore$ This axiom is satisfied

(v) Inverse :

for any $(x_1, x_2, x_3) \in V$
it's additive inverse $(-x_1, -x_2, -x_3) \in V$ since $(x_1, x_2, x_3) +$
$(-x_1, -x_2, -x_3) = (0\,0\,0)$
$\therefore$ This axiom is satisfied

(vii) Distributivity of scalar multiplication over vector addition

for any $a \in R$ $(x_1, x_2, x_3) \in V$ and $(y_1, y_2, y_3) \in V$

$$a((x_1\ x_2\ x_3) + (y_1, y_2, y_3)) = a(x_1+y_1, x_2+y_2, x_3+y_3)$$
$$= (a(x_1+y_1), a(x_2+y_2), a(x_3+y_3))$$
$$= (ax_1+ay_1, ax_2+ay_2, ax_3+ay_3)$$
$$= a(x_1, x_2, x_3) + a(y_1, y_2, y_3)$$

$\therefore$ This axiom is satisfied

(viii) Distribution of scalar multiplication over scalar addition

for any $a, b \in R$, $(x_1, x_2, x_3) \in V$

$$(a+b)\cdot(x_1, x_2, x_3) = ((a+b)x_1, (a+b)x_2, (a+b)x_3)$$
$$= (ax_1+bx_1, ax_2+bx_2, ax_3+bx_3)$$
$$= a(x_1, x_2, x_3) + b(x_1, x_2, x_3)$$

$\therefore$ This axiom is satisfied

(ix) compatibility of scalar multiplication

for any $a, b \in R$ and $(x_1, x_2, x_3) \in V$

$$(ab)(x_1, x_2, x_3) = ((ab)x_1, (ab)x_2, (ab)x_3)$$
$$= (a(bx_1), a(bx_2), a(bx_3))$$
$$= a(bx_1, bx_2, bx_3)$$
$$= a(b(x_1, x_2, x_3))$$

This axiom is satisfied

(iv) Identity

let $(x_1, x_2) \in V$

we need (e_1, e_2) such that

$(x_1, x_2) + (e_1, e_2) = (x_1, x_2)$

$x_1 + e_1 = x_1 \Rightarrow e_1 = 0$

$x_2 \cdot e_2 = x_2 \Rightarrow e_2 = 1$

so additive identity is $(0,1)$

$\therefore$ Additive identity satisfied

(v) Additive Inverse

we need (u_1, u_2) such that

$(x_1, x_2) + (u_1, u_2) = (0,1)$

$x_1 = -u_1 \Rightarrow u_1 = -x_1$

$u_2 = \frac{-1}{x_2}$

$\therefore$ The axiom is Not satisfied

(vi) Distributivity of scalar over vector addition

$a((x_1, x_2) + (y_1, y_2)) = (a(x_1 + y_1), a(x_1 y_1)), \forall a \in R$

$(x_1, x_2), (y_1, y_2) \in V$

$a(x_1, x_2) + a(y_1, y_2) = (ax_1 + ay_1, (ax_2)(ay_2))$

$= a(x_1) + ay_1 = ax_2 y_2$

$\therefore$ This axiom is Not satisfied

(vii) Distributivity of scalar addition

let $a \in R$, $(x_1, x_2), (y_1, y_2) \in V$

LHS:

$(a+b)(x_1, x_2) = (a+b)x_1, (a+bx_2)$

RHS:

$(ax_1, ax_2) + (bx_1, bx_2) = (ax_1 + bx_1, (ax_2(by_2))$

$= ((a+b)x_1, (abx_2^2)$

ix. Identity

$1 \in R$ and any $(x_1, x_2, x_3) \in V$,

$1(x_1 x_2, x_3) = (1x_1, 1x_2, 1x_3)$

$= (x_1, x_2, x_3)$

$\therefore$ This axiom is satisfied

2Q Let V be the set of all ordered pairs of real numbers over the field of real numbers defined by $(x_1, x_2) + (y_1, y_2) = (x_1+y_1, x_2+y_2)$ and $a(x_1, x_2) = (ax_1, ax_2)$ check whether it is a vector space or not?

Sol Given $(x_1 x_2) + (y_1 y_2) = (x_1+y_1, x_2+y_2)$ and $a(x_1, x_2) = (ax_1, ax_2)$

(i) closure under addition

Take any two vectors $(x_1, x_2), (y_1 y_2) \in V$

$(x_1 x_2) + (y_1 y_2) = (x_1+y_1, x_2+y_2)$

$\therefore$ This axiom is satisfied

(ii) Commutativity under addition

$(x_1, x_2), (y_1, y_2) \in V$

$(x_1 x_2) + (y_1, y_2) = (x_1+y_1, x_2 \times y_2)$

$(y_1, y_2) + (x_1, x_2) = (y_1+x_1, y_2 \times x_2)$

$((x_1+y_1), (x_2 \times y_2)) = ((y_1+x_1), (y_2 \times x_2))$

$\therefore$ This axiom is satisfied

(iii) Associativity

for $(x_1, x_2), (y_1, y_2), (z_1, z_2) \in V$

$((x_1, x_2) + (y_1 y_2)) + (z_1, z_2) = ((x_1+y_1, x_2 y_2) + (z_1, z_2))$

$= (x_1+y_1+z_1, x_2 \cdot y_2 \cdot z_2)$

$(x_1, x_2) + ((y_1, y_2) + (z_1, z_2)) = (x_1, x_2) + (y_1+z_1, y_2 z_2)$

$= (x_1+y_1+z_1, x_2 \cdot y_2 \cdot z_2)$

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(viii) Compatability of scalar multiplication

Let $a \in R$, $(x_1, x_2) \in V$

$a(b(x_1, x_2)) = (abx_1, abx_2)$

$= ab(x_1, x_2)$

$\therefore$ This axiom is satisfied

(ix) Multiplication identity

Let $(x_1, x_2) \in V$

$a(x_1, x_2) = (x_1, x_2)$

$\therefore$ axiom satisfied

Therefore V with the given operations is not a vectors space over R.

3Q The set $M_{2\times2}$ of all 2×2 matrices, with real numbers over the R. check whether w is a subspace of $V = M_{2\times2}$ where:

(i) $w = \{A \in V : |A| = 0\}$

Clearly $0 \in w$ as determined of zero matrices is 0

Let matrices A & B are in w

Let A and B are in w

$|A| = 0$ and $|B| = 0$

Let $A = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}$, $|A| = 0$ $B = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix}$, $|B| = 0$

$A+B = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$, $|A+B| \neq 0$

$\therefore$ w is Not a subspace of V

(iii) $w = \{A \in V : Tr\,A = 0\}$

Clearly $0 \in w$

Since trace of matrices is 0

Let $A \in w \Rightarrow Tr = 0$

and $B \in w \Rightarrow Tr\,B = 0$

Now $\alpha, \beta \in F$

$Tr(\alpha A + \beta B) = \alpha Tr A + \beta Tr B$
$= \alpha(0) + \beta(0)$
$= 0$
$\alpha A + \beta B \in W = 0$
$\therefore W$ is a subspace of V

(iii) $W = \{A \in V : A^T = A\}$
since Transpose of a zero matrix is 0
let $A \in W = A^T = A$
& $B \in W = B^T = B$
Now $\alpha, \beta \in F$
Now $\alpha, \beta \in F$
$(\alpha A + \beta B)^T = \alpha A^T + \beta B^T$
$= \alpha A + \beta B$
$= \alpha A + \beta B \in W$

(iv) $W = \{A \in V : A^T = -A\}$
clearly $0 \in W$
since Transpose of a zero matrix is 0 matrix
$0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, 0^T = 0$
let $A \in W \Rightarrow A^T = -A$
and $B \in W \Rightarrow B^T = -B$
Now $\alpha, \beta \in F$
$(\alpha A + \beta B)^T = \alpha A^T + \beta B^T$
$= \alpha(-A) + \beta(-B)$
$= -(\alpha A + \beta B) \in W$
$\therefore \alpha A + \beta B \in W$
$\therefore W$ is a subspace of V

4Q Let W_1 & W_2 be two subsets of R^4 such that $W_1 = \{(a,b,c,d) / b-2c+d = 0\}$, $W_2 = \{(a,b,c,d) / a = d, b = 2c\}$
find the basis and dimension of (i) W_1 (ii) W_2 (iii) $W_1 \cap W_2$ (iv) $W_1 + W_2$

$W_2 = \{(x_1,x_2,x_3,x_4) \mid x_1 = 4x_2, x_3 = x_4\}$

$= \{4x_2, x_2, x_4 \mid \forall x_2, x_4 \in R\}$

$\Rightarrow \{x_2(4,1,0,0) + x_4(0,0,1,1)\}$

Now let $S = \{(4,1,0,0),(0,0,1,1)\}$

$= \alpha_1(4,1,0,0) + \alpha_2(0,0,1,1) = (0,0) \forall \alpha_1, \alpha_2 \in R$

$\Rightarrow (4\alpha_1, \alpha_1, 0, 0) + (0,0,\alpha_2,\alpha_2) = (0,0)$

$\Rightarrow 4\alpha_1 = 0, \alpha_1 = 0, \alpha_2 = 0$

$\therefore$ S is bases of W_2

$\boxed{\dim W_2 = 2}$

$W_1 \cap W_2 \Rightarrow \{(x_1,x_2,x_3,x_4 \mid x_1 - 4x_2 + x_3 = 0, x_1 = 4x_2, x_3 = x_4\}$

$\Rightarrow \{4x_2, x_2, 0, 0\} = \{x_2(4,1,0,0)\}$

Now let $S = \{(4,1,0,0)\}$

$= \alpha_1(4,1,0,0) = \bar{0}$

$4\alpha_1 = 0$

$\boxed{\alpha_1 = 0}$

$\therefore$ S is L.I $\therefore$ S is basis of $W_1 \cap W_2$

$L(S) = V(F)$

$\dim(W_1 \cap W_2) = 1$

we have,

$\dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim(W_1 + W_2)$

$= 3 + 2 - 1$

$= 4$

6Q show that the vectors $(1,2,0),(0,3,1),(-1,0,1)$ of $V_3(Q)$ is LI where Q is the field of rational numbers

Sol Given,

$V_1 = (1,2,0), V_2 = (0,3,1)\ V_3 = (-1,0,1)$

let $S = \{(1,2,0),(0,3,1),(-1,0,1)\}$

$\Rightarrow \alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 = \bar{0}$

$\Rightarrow \alpha_1(1,2,0) + \alpha_2(0,3,1) + \alpha_3(-1,0,1) = (0,0,0) \forall \alpha_1, \alpha_2, \alpha_3 \in Q$

Sol: $W_1 = \{(a,b,c,d) / b-2c+d=0\}$

$= \{a, 2c-d, c, d\}$

$= \{a(1,0,0,0)+c(0,2,1,1)+d(0,-1,0,1)\}$

Now let $S = \{(1,0,0,0), (0,2,1,0), (0,-1,0,1)\}$

$\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 = 0$

$\alpha_1(1,0,0,0)+\alpha_2(0,2,1,0), \alpha_3(0,-1,0,1) = (0,0,0)$

$(\alpha_1, 2\alpha_1-\alpha_3, \alpha_2, \alpha_3) = (0,0,0)$

$\alpha_1=0 \quad \alpha_2=0 \quad \alpha_3=0$

$\therefore$ S is L.I

$d(S) = V(F)$

S is basis of W_1

No of Elements in $S = \dim W_1$

$\boxed{\therefore \dim W_1 = 3}$

given $W_2 = \{a,b,c,d\} / a=d, b=2c\}$

$= \{(d, b, b/2, d)\}$

$= \{b(0,1,1/2,0)+d(1,0,0,1)\}$

Let $S = \{(0,1,1/2,0)+(1,0,0,1)\}$

$\alpha_1(0,1,1/2,0)+\alpha_2(1,0,0,1)\} = \bar{0}$

$(\alpha_2, \alpha_1, \frac{\alpha_1}{2}, \alpha_2) = (0,0,0,0)$

$\therefore \alpha_1=0, \alpha_2=0$

$\therefore$ S is L.I

$\alpha(S) = V(F)$

$\therefore$ S is basis of W_2

$\therefore \boxed{\dim W_2 = 2}$

Now $W_1 \cap W_2 = \{(a,b,c,d) \mid b-2c+d=0, a=d, b=2c\}$

$\therefore$ S is basis of W_1

$\therefore$ No of elements in $S = \dim W_1$

$\boxed{\dim W_1 = 3}$

$\Rightarrow (\alpha_1-\alpha_3, 2\alpha_1+3\alpha_2, \alpha_2+\alpha_3) = (0,0,0)$

$\Rightarrow \alpha_1 - \alpha_3 = 0$ — (1)

$\Rightarrow 2\alpha_1 + 3\alpha_2 = 0$ — (2)

$\Rightarrow \alpha_2 + \alpha_3 = 0$ — (3)

By solving (1), (2) & (3) we get

$\therefore \alpha_1 = 0, \alpha_2 = 0, \alpha_3 = 0$

$\therefore$ S is L.I.

7Q Test the set $S = \{(1,2,-1), (1,0,2), (2,1,1)\}$ is bases for R^3 or not.

sol Let $S = \{(1,2,-1), (1,0,2), (2,1,1)\}$ and

$e_1 = (1,2,-1)$ $e_2 = (1,0,2)$, $e_3 = (2,1,1)$, $\forall \alpha_1, \alpha_2, \alpha_3 \in R$

Now $\alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 = 0$

$\alpha_1(1,2,-1) + \alpha_2(1,0,2) + \alpha_3(2,1,1) = \bar{0}$

$(\alpha_1, 2\alpha_1, -\alpha_1) + (\alpha_2, 0, 2\alpha_2) + (2\alpha_3, \alpha_3, \alpha_3) = (0,0,0)$

$\alpha_1 + \alpha_2 + 2\alpha_3 = 0$

$2\alpha_1 + \alpha_3 = 0$

$-\alpha_1 + 2\alpha_3 + \alpha_3 = 0$

$\therefore \alpha_1 = 0$

$\alpha_2 = 0$

$\alpha_3 = 0$

$\therefore$ S is L.I.

Let $t = (x, y, z) \in R^3 \; \forall x, y, z \in R$

$(x, y, z) = \alpha_1(1,2,-1) + \alpha_2(1,0,2) + \alpha_3(2,1,1)$

$= (\alpha_1, 2\alpha_1, -\alpha_1) + (\alpha_2, 0, 2\alpha_2) + (2\alpha_3, \alpha_3, \alpha_3)$

$= (\alpha_1 + \alpha_2 + 2\alpha_3, 2\alpha_1 + \alpha_3, -\alpha_1 + 2\alpha_2 + \alpha_3)$

$x = \alpha_1 + \alpha_2 + 2\alpha_3$ — (1)

$y = 2\alpha_1 + \alpha_3$ — (2)

$z = -\alpha_1 + 2\alpha_2 + \alpha_3$ — (3)

from (2) $\alpha_3 = y - 2\alpha_1$ Sub in (1)

$x = \alpha_1 + \alpha_2 + 2(y - 2\alpha_1)$

$= \alpha_2 - 3\alpha_1 + 2y$

$\boxed{\alpha_2 = x + 3\alpha_1 - 2y}$

$z = -\alpha_1 + 2\alpha_2 + \alpha_3 = -\alpha_1 + 2(x + 3\alpha_1 - 2y) + (y - 2\alpha_1)$

$= 2x + 3\alpha_1 - 3y$

$3\alpha_1 = z - 2x + 3y$

$\boxed{\alpha_1 = \frac{z - 2x + 3y}{3}}$

$\alpha_2 = x + 3\alpha_1 - 2y = x + 3 \cdot \frac{z - 2x + 3y}{3} - 2y$

$= x + z - 2x + 3y - 2y$

$\boxed{\alpha_2 = y + z - x}$

$\alpha_3 = y - 2\alpha_1$

$= y - 2 \cdot \frac{z - 2x + 3y}{3} = \frac{4x - 2z - 3y}{3}$

sub α_1, α_2 and α_3 in above eqn

$(x, y, z) = \left(\frac{z - 2x + 3y}{3}\right)(1, 2, -1) + (y + z - x)(1, 0, 2) + \left(\frac{4x - 2z - 3y}{3}\right)(2, 1, 1)$

$\therefore z$ = linear combination of Element of S

$z \in \alpha(S)$

$\therefore \alpha(S) = R^3$

$\therefore$ S is a bases of R^3

Test the set $S = \{1+2x+x^2, 3+x^2, x+x^2\}$ is basis for R^3 or not

Given set $S = \{1+2x+x^2, 3+x^2, x+x^2\}$

Let $\alpha = 1+2x+x^2$, $\beta = 3+x^2$ $\gamma = x+x^2$

co-ordinate of $\alpha, \beta, \gamma = (1,2,1)(1,0,3)(1,1,0)$

Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 3 \\ 1 & 1 & 0 \end{bmatrix}$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & -2 & 2 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & -2 & 2 \\ 0 & 0 & -4 \end{bmatrix}$

$R_3 \to 2R_3 - R_2$

$(1,2,1), (0,-2,2)$ & $(0,0,-4)$

S is basis of R^3

(90) If W is the subspace of $V_4(R)$ generated by vector $(1,-2,5,-3)$ $(2,3,1,-4)$, $(3,8,-3,-5)$ find basis of W & its dimension

Sol forming row matrix with given vectors

ie $A = \begin{bmatrix} 1 & -2 & 5 & -3 \\ 2 & 3 & 1 & -4 \\ 3 & 8 & -3 & -5 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$

$A = \begin{bmatrix} 1 & -2 & 5 & -3 \\ 0 & 7 & -9 & 2 \\ 0 & 14 & -18 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 & 5 & -3 \\ 0 & 7 & -9 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$R^3 \to R_3 - 2R_2$

$(1,-2,5,-3)$ & $(0,2,-9,2)$ from the least L.I set & hence basis of W

dim $W = 2$

10Q Show that set $W = \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right)$ $(a+b = c+d)$ is subset $M_{2\times 2}(R)$

Sol

$$W = \left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} = a+b = c+d\right\}$$

take $a=b=c=d=0$ Then $0+0 = 0+0$

Let $A = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} \in W$ $B = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \in W$.

so, $a_1+b_1 = c_1+d_1$ & $a_2+b_2 = c_2+d_2$

$$A+B = \begin{bmatrix} a_1+a_2 & b_1+b_2 \\ c_1+c_2 & d_1+d_2 \end{bmatrix}$$

$\Rightarrow (a_1+a_2)+(b_1+b_2) = (a_1+b_1)+(a_2+b_2)$

$\Rightarrow (c_1+d_1)+(c_2+d_2) = (c_1+c_2)+(d_1+d_2)$

Let $\lambda \in R$ & $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in W$

$a+b = c+d$ $\lambda A = \begin{bmatrix} \lambda a & \lambda b \\ \lambda c & \lambda d \end{bmatrix}$

$(\lambda a)+(\lambda b) = \lambda(a+b) = \lambda(c+d) = (\lambda c)+(\lambda d)$

since, W is Non-Empty & closed under addition & scalar multiplication.

$\therefore W$ is subspace of $M_{2\times 2}(R)$.

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