CS61B Lecture #14: Integers

Integer Types and Literals

Type	Bits	Signed?	Literals
byte	8	Yes	Cast from **int**: (byte) 3
short	16	Yes	None. Cast from **int**: (short) 4096

char	16	No	`'a' // (char) 97`
			`'\n' // newline ((char) 10)`
			`'\t' // tab ((char) 8)`
			`'\\' // backslash`
			`'A', '\101', '\u0041' // == (char) 65`

int	32	Yes	`123`
			`0100 // Octal for 64`
			`0x3f, 0xffffffff // Hexadecimal 63, -1 (!)`

long	64	Yes	`123L, 01000L, 0x3fL` `1234567891011L`

- Negative numerals are just negated (positive) literals.
- "N bits" means that there are 2^N integers in the domain of the type:
  - If signed, range of values is -2^{N-1} .. $2^{N-1}-1$.
  - If unsigned, only non-negative numbers, and range is $0..2^N-1$.

Overflow

- **Problem:** How do we handle overflow, such as occurs in `10000*10000*10000`?
- Some languages throw an exception (Ada), some give undefined results (C, C++)
- Java *defines* the result of any arithmetic operation or conversion on integer types to "wrap around"—*modular arithmetic.*
- That is, the "next number" after the largest in an integer type is the smallest (like "clock arithmetic").
- E.g., `(byte) 128 == (byte) (127+1) == (byte) -128`
- In general,
  - If the result of some arithmetic subexpression is supposed to have type T, an n-bit integer type,
  - then we compute the real (mathematical) value, x,
  - and yield a number, x', that is in the range of T, and that is equivalent to x modulo 2^n.
  - (That means that $x - x'$ is a multiple of 2^n.)

Modular Arithmetic

- Define $a \equiv b \pmod{n}$ to mean that $a - b = kn$ for some integer k.
- Define the binary operation $a \bmod n$ as the value b such that $a \equiv b \pmod{n}$ and $0 \le b < n$ for $n > 0$. (Can be extended to $n \le 0$ as well, but we won't bother with that here.) This is *not* the same as Java's % operation.
- Various facts: (Here, let a' denote $a \bmod n$).

$$\begin{aligned} a'' &= a' \\ a' + b'' &= (a' + b)' = a + b' \\ (a' - b')' &= (a' + (-b)')' = (a - b)' \\ (a' \cdot b')' &= a' \cdot b' = a \cdot b' \\ (a^k)' &= ((a')^k)' = (a \cdot (a^{k-1})')', \text{ for } k > 0. \end{aligned}$$

Modular Arithmetic: Examples

- `(byte) (64*8)` yields 0, since $512 - 0 = 2 \times 2^8$.
- `(byte) (64*2)` and `(byte) (127+1)` yield -128, since $128 - (-128) = 1 \times 2^8$.
- `(byte) (101*99)` yields 15, since $9999 - 15 = 39 \times \cdot 2^8$.
- `(byte) (-30*13)` yields 122, since $-390 - 122 = -2 \times 2^8$.
- `(char) (-1)` yields $2^{16} - 1$, since $-1 - (2^{16} - 1) = -1 \times 2^{16}$.

Modular Arithmetic and Bits

- Why wrap around?
- Java's definition is the natural one for a machine that uses binary arithmetic.
- For example, consider bytes (8 bits):

<table>
<tr><th>Decimal</th><th>Binary</th></tr>
<tr><td>101</td><td>1100101</td></tr>
<tr><td>×99</td><td>1100011</td></tr>
<tr><td>9999</td><td>100111|00001111</td></tr>
<tr><td>− 9984</td><td>100111|00000000</td></tr>
<tr><td>15</td><td>00001111</td></tr>
</table>

- In general, bit n, counting from 0 at the right, corresponds to 2^n.
- The bits to the left of the vertical bars therefore represent multiples of $2^8 = 256$.
- So throwing them away is the same as arithmetic modulo 256.

Negative numbers

- Why this representation for -1?

$$\begin{array}{rl|l} & 1 & 00000001_2 \\ + & -1 & 11111111_2 \\ = & 0 & 1|00000000_2 \end{array}$$

Only 8 bits in a byte, so bit 8 falls off, leaving 0.

- The truncated bit is in the 2^8 place, so throwing it away gives an equal number modulo 2^8. All bits to the left of it are also divisible by 2^8.
- On unsigned types (**char**), arithmetic is the same, but we choose to represent only non-negative numbers modulo 2^{16}:

$$\begin{array}{rl|l} & 1 & 0000000000000001_2 \\ + & 2^{16}-1 & 1111111111111111_2 \\ = & 2^{16}+0 & 1|0000000000000000_2 \end{array}$$

Conversion

- In general Java will silently convert from one type to another if this makes sense and no information is lost from value.
- Otherwise, cast explicitly, as in `(byte) x`.
- Hence, given

```
byte aByte; char aChar; short aShort; int anInt; long aLong;

// OK:
aShort = aByte; anInt = aByte; anInt = aShort;
anInt = aChar; aLong = anInt;

// Not OK, might lose information:
anInt = aLong; aByte = anInt; aChar = anInt; aShort = anInt;
aShort = aChar; aChar = aShort; aChar = aByte;

// OK by special dispensation:
aByte = 13;     // 13 is compile-time constant
aByte = 12+100 // 112 is compile-time constant
```


Promotion

- Arithmetic operations (+, *, ...) *promote* operands as needed.
- Promotion is just implicit conversion.
- For integer operations,
  - if any operand is **long**, promote both to **long**.
  - otherwise promote both to **int**.
- So,

```
aByte + 3 == (int) aByte + 3   // Type int
aLong + 3 == aLong + (long) 3  // Type long
'A' + 2 == (int) 'A' + 2       // Type int
aByte = aByte + 1              // ILLEGAL (why?)
```

- But fortunately,

```
aByte += 1;      // Defined as aByte = (byte) (aByte+1)
```

- Common example:

```
// Assume aChar is an upper-case letter
char lowerCaseChar = (char) ('a' + aChar - 'A'); // why cast?
```


Bit twiddling

- Java (and C, C++) allow for handling integer types as sequences of bits. No "conversion to bits" needed: they already are.
- Operations and their uses:

Mask	Set	Flip	Flip all
00101100	00101100	00101100	
& 10100111	| 10100111	^ 10100111	~ 10100111
00100100	10101111	10001011	01011000

- Shifting:

Left	Arithmetic Right	Logical Right
10101101 << 3	10101101 >> 3	10101100 >>> 3
01101000	11110101	00010101

- What is:
  - (-1) >>> 29?
  - x << n?
  - x >> n?
  - (x >>> 3) & ((1<<5)-1)?

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`(`x` >>> 3) & ((1<<5)-1)`?	5-bit integer, bits 3–7 of x.