# CS61B Lecture #17



## Topics

- Overview of standard Java Collections classes.
  - Iterators, ListIterators
  - Containers and maps in the abstract
- Amortized analysis of implementing lists with arrays.

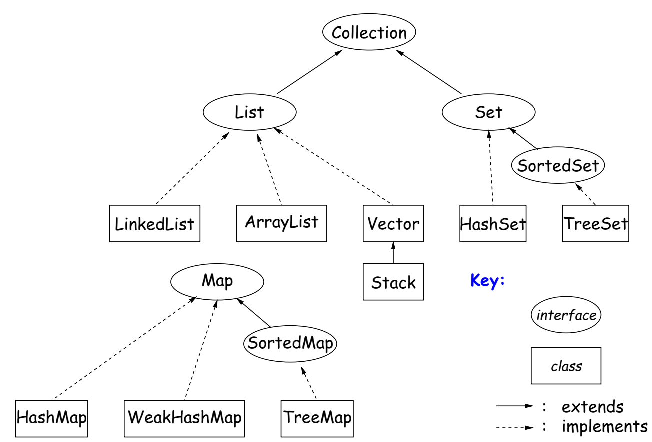


## Data Types in the Abstract

- Most of the time, should *not* worry about implementation of data structures, search, etc.
- What they do for us—their specification—is important.
- Java has several standard types (in `java.util`) to represent collections of objects
  - Six interfaces:
    * `Collection`: General collections of items.
    * `List`: Indexed sequences with duplication
    * `Set`, `SortedSet`: Collections without duplication
    * `Map`, `SortedMap`: Dictionaries (key $\mapsto$ value)
  - Concrete classes that provide actual instances: `LinkedList`, `ArrayList`, `HashSet`, `TreeSet`.
  - To make change easier, purists would use the concrete types only for **new**, interfaces for parameter types, local variables.



## Collection Structures in java.util






## The Collection Interface

- Collection interface. Main functions promised:
  - Membership tests: `contains` ($\in$), `containsAll` ($\subseteq$)
  - Other queries: `size`, `isEmpty`
  - Retrieval: `iterator`, `toArray`
  - *Optional* modifiers: `add`, `addAll`, `clear`, `remove`, `removeAll` (set difference), `retainAll` (intersect)



## Side Trip about Library Design: Optional Operations

- Not all `Collections` need to be modifiable; often makes sense just to get things from them.
- So some operations are optional (`add`, `addAll`, `clear`, `remove`, `removeAll`, `retainAll`)
- The library developers decided to have *all* Collections implement these, but allowed implementations to throw an exception:

  `UnsupportedOperationException`

- An alternative design would have created separate interfaces:

```
interface Collection { contains, containsAll, size, iterator, ... }
interface Expandable extends Collection { add, addAll }
interface Shrinkable extends Collection { remove, removeAll, ... }
interface ModifiableCollection
   extends Collection, Expandable, Shrinkable { }
```

- You'd soon have lots of interfaces. Perhaps that's why they didn't do it that way.

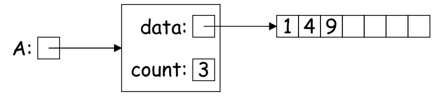
## The List Interface

- Extends `Collection`
- Intended to represent *indexed sequences* (generalized arrays)
- Adds new methods to those of `Collection`:
  - Membership tests: `indexOf`, `lastIndexOf`.
  - Retrieval: `get(`$i$`)`, `listIterator()`, `sublist(`$B, E$`)`.
  - Modifiers: `add` and `addAll` with additional index to say *where* to add. Likewise for removal operations. `set` operation to go with `get`.
- Type `ListIterator<Item>` extends `Iterator<Item>`:
  - Adds `previous` and `hasPrevious`.
  - `add`, `remove`, and `set` allow one to iterate through a list, inserting, removing, or changing as you go.
  - **Important Question:** What advantage is there to saying `List L` rather than `LinkedList L` or `ArrayList L`?



## Implementing Lists (I): ArrayLists

- The main concrete types in Java library for interface `List` are `ArrayList` and `LinkedList`:
- As you might expect, an `ArrayList`, `A`, uses an array to hold data. For example, a list containing the three items 1, 4, and 9 might be represented like this:




- After adding four more items to `A`, its `data` array will be full, and the value of `data` will have to be replaced with a pointer to a new, bigger array that starts with a copy of its previous values.
- Question: For best performance, how big should this new array be?
- If we increase the size by 1 each time it gets full (or by any constant value), the cost of $N$ additions will scale as $\Theta(N^2)$, which makes `ArrayList` look much worse than `LinkedList` (which uses an `IntList`-like implementation.)



## Expanding Vectors Efficiently

- When using array for expanding sequence, best to *double* the size of array to grow it. Here's why.
- If array is size $s$, doubling its size and moving $s$ elements to the new array takes time proportional to $2s$.
- In all cases, there is an additional $\Theta(1)$ cost for each addition to account for actually assigning the new value into the array.
- When you add up these costs for inserting a sequence of $N$ items, the *total* cost turns out to be proportional to $N$, as if each addition took constant time, even though some of the additions actually take time proportional to $N$ all by themselves!



## Amortized Time

- Suppose that the actual costs of a sequence of $N$ operations are $c_0, c_1, \ldots, c_{N-1}$, which may differ from each other by arbitrary amounts and where $c_i \in O(f(i))$.
- Consider another sequence $a_0, a_1, \ldots, a_{N-1}$, where $a_i \in O(g(i))$.
- If
$$\sum_{0 \le i < k} a_i \ge \sum_{0 \le i < k} c_i \textit{ for all } k,$$
we say that the operations all run in $O(g(i))$ *amortized time*.
- That is, the actual cost of a given operation, $c_i$, may be arbitrarily larger than the amortized time, $a_i$, as long as the *total* amortized time is always greater than or equal to the total actual time, no matter where the sequence of operations stops—i.e., no matter what $k$ is.
- In cases of interest, the amortized time bounds are much less than the actual individual time bounds: $g(i) \ll f(i)$.
- E.g., for the case of insertion with array doubling, $f(i) \in O(N)$ and $g(i) \in O(1)$.



## Amortization: Expanding Vectors (II)

| To Insert Item # | Resizing Cost | Cumulative Cost | Resizing Cost per Item | Array Size After Insertions |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 |
| 1 | 2 | 2 | 1 | 2 |
| 2 | 4 | 6 | 2 | 4 |
| 3 | 0 | 6 | 1.5 | 4 |
| 4 | 8 | 14 | 2.8 | 8 |
| 5 | 0 | 14 | 2.33 | 8 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| 7 | 0 | 14 | 1.75 | 8 |
| 8 | 16 | 30 | 3.33 | 16 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| 15 | 0 | 30 | 1.88 | 16 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $2^m + 1$ to $2^{m+1} - 1$ | 0 | $2^{m+2} - 2$ | $\approx 2$ | $2^{m+1}$ |
| $2^{m+1}$ | $2^{m+2}$ | $2^{m+3} - 2$ | $\approx 4$ | $2^{m+2}$ |

- If we spread out (*amortize*) the cost of resizing, we average at most about 4 time units for resizing on each item: "amortized resizing time is 4 units." Time to add $N$ elements is $\Theta(N)$, *not* $\Theta(N^2)$.



## Demonstrating Amortized Time: Potential Method

- To formalize the argument, associate a *potential*, $\Phi_i \ge 0$, to the $i^{\text{th}}$ operation that keeps track of "saved up" time from cheap operations that we can "spend" on later expensive ones. Start with $\Phi_0 = 0$.
- Now we pretend that the cost of the $i^{\text{th}}$ operation is actually $a_i$, the *amortized cost*, defined
$$a_i = c_i + \Phi_{i+1} - \Phi_i,$$
where $c_i$ is the real cost of the operation. Or, looking at potential:
$$\Phi_{i+1} = \Phi_i + (a_i - c_i)$$
- On cheap operations, we artificially set $a_i > c_i$ so that we can increase $\Phi$ ($\Phi_{i+1} > \Phi_i$).
- On expensive ones, we typically have $a_i \ll c_i$ and greatly decrease $\Phi$ (but don't let it go negative—may not be "overdrawn").
- We try to do all this so that $a_i$ remains as we desired (e.g., $O(1)$ for expanding array), without allowing $\Phi_i < 0$.
- Requires that we choose $a_i$ so that $\Phi_i$ always stays ahead of $c_i$.

## Application to Expanding Arrays

- When adding to our array, the cost, $c_i$, of adding element #$i$ when the array already has space for it is 1 unit.
- The array does not initially have space when adding items 1, 2, 4, 8, 16,...—in other words at item $2^n$ for all $n \geq 0$. So,
  - $c_i = 1$ if $i \geq 0$ and is not a power of 2; and
  - $c_i = 2i + 1$ when $i$ is a power of 2 (copy $i$ items, clear another $i$ items, and then add item #$i$).
- So on each operation #$2^n$ we're going to need to have saved up at least $2 \cdot 2^n = 2^{n+1}$ units of potential to cover the expense of expanding the array, and we have this operation and the preceding $2^{n-1} - 1$ operations in which to save up this much potential (everything since the preceding doubling operation).
- So choose $a_0$ = 1 and $a_i = 5$ for $i > 0$. Apply $\Phi_{i+1} = \Phi_i + (a_i - c_i)$, and here is what happens:

| $i$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $c_i$ | 1 | 3 | 5 | 1 | 9 | 1 | 1 | 1 | 17 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 33 | 1 |
| $a_i$ | 1 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 |
| $\Phi_i$ | 0 | 0 | 2 | 2 | 6 | 2 | 6 | 10 | 14 | 2 | 6 | 10 | 14 | 18 | 22 | 26 | 30 | 2 |

*Pretending each cost is 5 never underestimates true cumulative time.*