

Diffie-Hellman Key Exchange

Common Prime q

Primitive root α

Sender - A	Receiver - B
Public key - Y_A	Public key - Y_B
Private key - X_A	Private key - X_B

$$Y_A = \alpha^{X_A} \bmod q$$

$$Y_B = \alpha^{X_B} \bmod q$$

$$K_A = Y_B^{X_A} \bmod q$$

$$K_B = Y_A^{X_B} \bmod q$$

$$q=71, \alpha = 7$$

Sender - A	Receiver - B
Public key - Y_A	Public key - Y_B
Private key - 5	Private key - 12

1) A's Public key?

$$Y_A = \alpha^{x_A} \bmod q$$

$$\begin{aligned} Y_A &= 7^5 \bmod 71 \\ &= 51 \end{aligned}$$

$$5 \Rightarrow 101$$

$$7 \bmod 71 = 7$$

$$7^2 \bmod 71 = 49$$

$$49^2 \bmod 71 = 58$$

$$58 \times 7 \bmod 71 = 51$$

$$Y_B = 7^{12} \bmod 71$$

$$= 4$$

$$12 \Rightarrow 1100$$

$$0: 7 \bmod 71 = 7$$

$$0: 7^2 \bmod 71 = 49$$

$$1: 49^2 \bmod 71 = 58$$

$$1: 58^2 \bmod 71 = 27$$

$$27 \times 58 \bmod 71 = 4$$

$$K_A = Y_B^{x_A} \bmod q$$

$$K_A = 4^5 \bmod 71$$

$$= 30$$

$$K_B = 51^{12} \bmod 71$$

$$= 30$$

$$K_A = K_B$$

Sample mod

$$42^5 \bmod 1073$$

$$5 \Rightarrow 101$$

$$1: 42 \bmod 1073 = 42$$

$$0: 42^2 \bmod 1073 = 691$$

$$1: 691^2 \bmod 1073 = 1069$$

$$1069 \times 42 \bmod 1073 = 905$$

$$691^2 = 477481$$

$$477481 / 1073 = 444.9$$

$$444 \times 1073 = 476412$$

$$477481 - 476412 = 1069$$

$$13^6 \bmod 17$$

$$6 \Rightarrow 110$$

$$13 \bmod 17 \Rightarrow 13$$

$$13^2 \bmod 17 \Rightarrow 16$$

$$16^2 \bmod 17 \Rightarrow 1$$

$$1 \times 16 \bmod 17 \Rightarrow 16$$