

Decision - Tree

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31...40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31...40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31...40	medium	no	excellent	yes
31...40	high	yes	fair	yes
>40	medium	no	excellent	no

$$1. \text{Info}(D) = - \sum_{i=1}^m p_i \log_2(p_i)$$

Yes = 9
No = 5

$$\text{Info}(D) = 1 \left(\frac{9}{14} \right) = - \frac{9}{14} \log_2 \left(\frac{9}{14} \right) - \frac{5}{14} \log_2 \left(\frac{5}{14} \right) = 0.940$$

$$2. \text{Info}_A(D) = \sum_{j=1}^v \frac{|D_j|}{|D|} \times \text{Info}(D_j) \quad \text{Feature}$$

$$1. \text{Info}_{\text{age}}(D) = \frac{5}{14} I(1,3) + \frac{4}{14} I(4,0) + \frac{5}{14} I(3,2)$$

$$= \frac{5}{14} \left[- \frac{2}{5} \log_2 \left(\frac{2}{5} \right) - \frac{3}{5} \log_2 \left(\frac{3}{5} \right) \right] + \frac{4}{14} \left[- \frac{4}{4} \log_2 \left(\frac{4}{4} \right) \right] + \frac{5}{14} \left[- \frac{3}{5} \log_2 \left(\frac{3}{5} \right) - \frac{2}{5} \log_2 \left(\frac{2}{5} \right) \right] = 0.692$$

$$2. \text{Info}_{\text{income}}(D) = \frac{4}{14} I(1,2) + \frac{6}{14} I(4,2) + \frac{4}{14} I(3,1)$$

$$= \frac{4}{14} \left[- \frac{2}{4} \log_2 \left(\frac{2}{4} \right) - \frac{2}{4} \log_2 \left(\frac{2}{4} \right) \right] + \frac{6}{14} \left[- \frac{4}{6} \log_2 \left(\frac{4}{6} \right) - \frac{2}{6} \log_2 \left(\frac{2}{6} \right) \right] + \frac{4}{14} \left[- \frac{3}{4} \log_2 \left(\frac{3}{4} \right) - \frac{1}{4} \log_2 \left(\frac{1}{4} \right) \right] = 0.917$$

$$3. \text{Info}_{\text{student}}(D) = \frac{7}{14} I(6,1) + \frac{7}{14} I(3,4)$$

$$= \frac{7}{14} \left[- \frac{3}{7} \log_2 \left(\frac{3}{7} \right) - \frac{4}{7} \log_2 \left(\frac{4}{7} \right) \right] + \frac{7}{14} \left[- \frac{1}{7} \log_2 \left(\frac{1}{7} \right) - \frac{6}{7} \log_2 \left(\frac{6}{7} \right) \right] = 0.799$$

$$4. \text{Info}_{\text{credit_rating}}(D) = \frac{8}{14} I(6,2) + \frac{6}{14} I(3,3)$$

$$= \frac{8}{14} \left[- \frac{6}{8} \log_2 \left(\frac{6}{8} \right) - \frac{2}{8} \log_2 \left(\frac{2}{8} \right) \right] + \frac{6}{14} \left[- \frac{3}{6} \log_2 \left(\frac{3}{6} \right) - \frac{3}{6} \log_2 \left(\frac{3}{6} \right) \right] = 0.892$$

3. $\text{Gain}(A) = \text{Info}(D) - \text{Info}_A(D)$ ค่า Gain สูงสุดจะเป็น root node

3.1 $\text{Gain}(\text{age}) = 0.940 - 0.694 = 0.246 //$

3.2 $\text{Gain}(\text{income}) = 0.940 - 0.911 = 0.029$

3.3 $\text{Gain}(\text{student}) = 0.940 - 0.789 = 0.151$

3.4 $\text{Gain}(\text{credit_rating}) = 0.940 - 0.792 = 0.048$

4. แยกกลุ่ม feature ตามค่าใน root node

4.1 $L = 30$

$$\text{Info}(D) = I(2,3) = 0.991$$

$$\text{Info}_{\text{income}}(D) = \frac{2}{5} I(0,2) + \frac{3}{5} I(1,1) + \frac{1}{5} I(1,0) = 0.4$$

$$\text{Info}_{\text{student}}(D) = \frac{2}{5} I(2,0) + \frac{3}{5} I(0,3) = 0$$

$$\text{Info}_{\text{credit_rating}}(D) = \frac{3}{5} I(1,2) + \frac{2}{5} I(1,1) = 0.951$$

4.2 30...40

age	income	student	credit_rating	buys_computer
31...40	high	no	fair	yes
31...40	low	yes	excellent	yes
31...40	medium	no	excellent	yes
31...40	high	yes	fair	yes

4.3 > 40

$$\text{Info}(D) = I(3,2) = -\frac{3}{5} \log_2\left(\frac{3}{5}\right) - \frac{2}{5} \log_2\left(\frac{2}{5}\right) = 0.991$$

$$\text{Info}_{\text{income}}(D) = \frac{3}{5} I(2,1) + \frac{2}{5} I(1,1) = 0.951$$

$$\text{Info}_{\text{student}}(D) = \frac{3}{5} I(2,1) + \frac{2}{5} I(1,1) = 0.951$$

$$\text{Info}_{\text{credit_rating}}(D) = \frac{3}{5} I(3,0) + \frac{2}{5} I(0,2) = 0$$

คำนวณ Information Gain

$$\text{Gain}(\text{Income}) = 0.991 - 0.4 = 0.591$$

$$\text{Gain}(\text{Student}) = 0.991 - 0 = 0.991 //$$

$$\text{Gain}(\text{Credit_rating}) = 0.991 - 0.951 = 0.02$$

เลือก Gain(Student) เป็น Node

yes = 4 no = 0

เมื่ออายุ 31...40 มั้ย Yes

ใน buys_computer

คำนวณ Information Gain

$$\text{Gain}(\text{Income}) = 0.991 - 0.951 = 0.02$$

$$\text{Gain}(\text{Student}) = 0.991 - 0.951 = 0.02$$

$$\text{Gain}(\text{Credit_rating}) = 0.991 - 0 = 0.991 //$$

เลือก Gain(Credit_rating) เป็น Node

5.

