A Brief Introduction To Infinity

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1 Introduction

The concept of infinity has been a topic of fascination and debate throughout the history of mathematics, spanning from ancient philosophy to modern mathematical theories . The mathematical theory of infinity was created, almost single-handedly, by the German mathematician Georg Cantor at the end of the 19th century. His ideas were so radical that many of his contemporaries flatly refused to accept them. The eminent mathematician Henri Poincaré even called Cantor's theory a "pathology" from which mathematicians needed to be cured. Though such harsh criticism caused Cantor much anguish, he stood his ground. And he was vindicated: Today his theory is a cornerstone of all of mathematics.

"Pursuing his ideas, Cantor showed tremendous courage," continued Dr. Frenkel. "Responding to his critics, he wrote: 'The essence of mathematics lies in its freedom.' In math, we have to follow rigorously the chosen axioms and the rules of logic. But within those rules, we can really let our imagination fly. There is no place in mathematics for dogma or prejudice."

Cantor's idea was that infinity is not a number, but rather a property of a *set*. Informally, a set is a collection of objects (called "elements") that have something in common. For instance, it could be the set of all readers of this blog post.

Ancient Indian Understanding of Infinity

1. Philosophical Concepts of Infinity in Hindu Texts:

    - The concept of infinity appears in the *Vedas* (circa 1500 BCE) and *Upanishads* (circa 800–200 BCE), where it is discussed both cosmologically and philosophically. For instance, the *Brihadaranyaka Upanishad* states, "From the Infinite, the Infinite comes, yet the Infinite remains." This reflects a view of infinity as an entity from which more can be drawn without diminishing it—similar to some modern set theory notions.
    - The concept of *ananta* (Sanskrit for "endless" or "infinite") was associated with the divine and the cosmos, portraying infinity as both a spiritual and an existential attribute.

2. Mathematical Advances in Infinity (Jain Mathematics):

    - The Jain mathematicians (circa 500 BCE) were among the first to mathematically formalize infinity. They categorized infinity into several types, including:
        - Infinite in one direction (e.g., the infinite sequence of positive integers),
        - Infinite in two directions (e.g., the entire set of integers, including negatives),
        - Infinite in area or volume, and
        - Infinite in nature (a totality that cannot be counted).

- They also distinguished between *asankhya* (countless or innumerable) and true infinity, using these ideas in practical contexts like geometry and cosmology. Jain scholars defined an infinite number of points on a line segment, a concept later formalized in calculus and real analysis.

3. Bhaskara II and Infinite Series: Bhaskara II (1114–1185 CE) explored infinity in his work *Lilavati*, discussing both positive and negative infinite values. He touched upon the concept of division by zero, a step towards understanding limits, by proposing that dividing by zero results in an "unquantifiable" or infinite result.

1.0.1 Overview of Bhaskara II's Work on Infinite Series

Bhaskara II, also known as Bhaskaracharya (1114–1185 CE), was a prominent Indian mathematician and astronomer who made several notable contributions, particularly in calculus and number theory. His insights on infinite series and infinity are remarkable because they represent an early step toward ideas that would later be formalized in calculus by European mathematicians like Newton and Leibniz. Bhaskara II's most famous work is the "Siddhanta Shiromani," a comprehensive treatise divided into four parts:

1. Lilavati – Arithmetic and algebra
2. Bijaganita – Algebra
3. Grahaganita – Astronomy and planetary motion
4. Ganitadhyaya and Goladhyaya – Spherical trigonometry and related topics

While the "Lilavati" is a more general work that includes arithmetic and mathematical puzzles, the "Bijaganita" and "Grahaganita" contain more sophisticated algebraic ideas, including concepts related to infinity and infinite series.

1.0.2 Bhaskara II's Work on Infinity

1. Concept of Division by Zero:

   - Bhaskara II explored the concept of division by zero in his works, which was a precursor to understanding infinity in mathematical terms.
   - He claimed that a number divided by zero becomes an *unchangeable quantity* or a quantity that is *beyond definite*—essentially infinite. For example, he wrote that if any finite number (e.g., 3 or 5) is divided by zero, it results in something that cannot be measured or quantified.
   - This was not a rigorous mathematical treatment of infinity as understood today, but it showed a sophisticated intuition. He even

attempted to reason about what would happen if positive and negative quantities were divided by zero, anticipating the idea of positive and negative infinities.

2. Understanding of Infinite Processes and Series:

   - Bhaskara's approach to infinite series was grounded in his study of geometric and arithmetic sequences, especially in astronomy and trigonometry, where he examined progressively smaller and larger quantities.
   - For instance, in his astronomical calculations, Bhaskara used iterative methods that implicitly deal with infinitely small steps, foreshadowing calculus-like thinking.

1.0.3 Contributions to Infinite Series

Bhaskara's work with series primarily focused on what we would now call convergent series. He engaged with geometric progressions and had some understanding of how an infinite series could approach a finite value. Some key insights include:

1. Early Use of Infinite Series for Approximations:

   - Bhaskara II used series expansions for trigonometric calculations, which are essential in astronomy.
   - For example, he worked with approximations that involve breaking down a quantity into smaller fractions, similar to how infinite series represent numbers as sums of infinite terms.

2. Development of Techniques Similar to Taylor Series:

   - Though not in the form of modern Taylor series, Bhaskara II's use of polynomial approximations for trigonometric functions demonstrated an understanding that functions could be approximated by a sum of terms with diminishing magnitudes.
   - In particular, his work on sine and cosine functions used iterative calculations, where he approximated values by adding and subtracting progressively smaller terms.

3. Geometric Series and Sum of Fractions: Bhaskara's treatment of fractions as sums of progressively smaller parts reflects an early grasp of geometric series. In "Lilavati," he addresses series that approximate certain values by successive additions, a precursor to the infinite series expansions developed later.

1.0.4 Division by Zero and Concepts of Infinity

Bhaskara II's work includes discussions on the outcomes of division by zero, which was unprecedented in the mathematics of his time. Here are some points related to his ideas:

1. Infinity as an Undefined Quantity:
    - He did not see infinity as a number in the modern sense but recognized it as a kind of *undefined* or *immeasurable* value.
    - For example, he considered what happens when a finite number is divided by zero, saying that it yields something "infinite" or "undetermined." This is similar to the idea of asymptotes in modern mathematics, where a function approaches an undefined value as it moves towards infinity.
2. Positive and Negative Infinity:
    - Bhaskara discussed positive and negative outcomes when dividing positive and negative quantities by zero, intuitively distinguishing between positive and negative infinities.
    - This is an early conceptual precursor to modern limits, where functions can approach positive or negative infinity depending on the direction from which they approach zero.

1.0.5 Bhaskara's Approach Compared to Modern Infinite Series

Although Bhaskara II's work was not as formalized as later developments, his insights were remarkable for his time and had similarities to concepts that would be rigorized in the 17th and 18th centuries. Here's how his ideas align with modern mathematics:

1. Geometric Series: Bhaskara's iterative calculations and handling of fractions foreshadowed the geometric series. For example, in modern terms, the infinite geometric series $12+14+18+\ldots 21+41+81+\ldots$ converges to 1. Bhaskara did not fully formalize convergence but was aware that such series could represent finite values through infinite summation.
2. Limit Concept and Infinitesimals:
    - By discussing division by zero and infinitely small divisions in trigonometric calculations, Bhaskara anticipated some aspects of the limit concept and infinitesimals—fundamental in calculus.
    - Though he lacked the formalism of limits, his work implicitly recognized the idea of values that approach zero but never quite reach it.

1.0.6 Summary of Bhaskara II's Legacy in Infinite Series and Infinity

- Intuitive Understanding of Infinity: Bhaskara II viewed infinity as something beyond measure, rather than a precise number, and he used this concept in his explanations of division by zero and iterative calculations.
- Early Convergence Ideas: His work with iterative summations and trigonometric calculations reveals an early, intuitive grasp of convergence, though he did not formalize this as a concept.
- Foundation for Calculus: Bhaskara's approach to breaking down calculations into smaller steps and handling large series of terms laid the groundwork for the development of calculus, influencing later mathematicians like Madhava and the Kerala school of astronomy.

2 Ancient Greek Philosophy